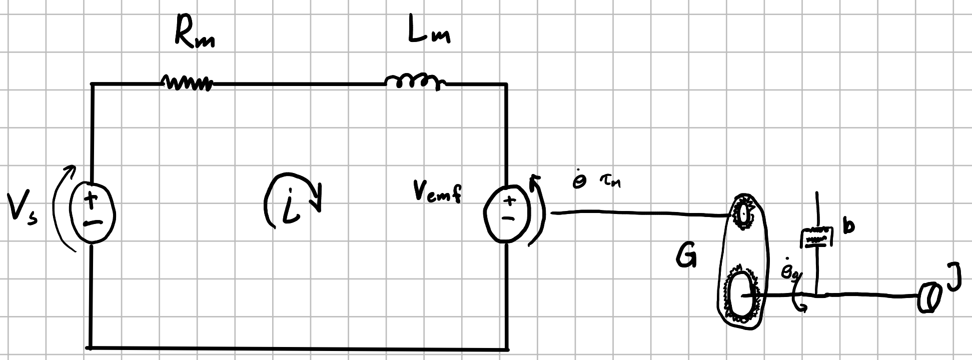
State Space Derivation



Example:
$G \approx 1:3 \Rightarrow G = \frac{1}{3}$

Electrical Circuit

input

$$-V_s + R_m I + L_m \dot{I} + V_{emf} = 0$$

$$V_{emf} = K_e \cdot \dot{\theta}$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{V_{emf}}{L_m}$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e \cdot \dot{\theta}_m}{L_m} \rightarrow \dot{\theta}_m = \frac{1}{G}\dot{\theta}_g$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e}{L_m G}\dot{\theta}_g$$

State variables

Physical System

$$\left.\begin{aligned} \dot{\theta}_g &= \dot{\theta}_m G \\ \ddot{\theta}_g &= \ddot{\theta}_m G \\ \tau_g &= \tau_m \cdot \frac{1}{G} \\ \tau_m &= K_i \cdot I \\ J\ddot{\theta}_g &= b\dot{\theta}_g - \tau_g \end{aligned}\right\}$$

$$\Rightarrow J\ddot{\theta}_g = -b\dot{\theta}_g - \tau_m \cdot \frac{1}{G}$$

$$\Rightarrow J\ddot{\theta}_g = -b\dot{\theta}_g - \frac{K_i}{G} \cdot I$$

$$\Rightarrow \ddot{\theta}_g = \frac{-b}{J}\dot{\theta}_g - \frac{K_i}{JG} \cdot I$$

State variables

We now have two differential equations

$$\begin{cases} \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e}{L_m G}\dot{\theta}_g \\ \ddot{\theta}_g = \frac{-b}{J}\dot{\theta}_g - \frac{K_i}{JG} \cdot I \end{cases}$$

Let's create a state space model!

First we define our states and input

$$x = \begin{bmatrix} I \\ \dot{\theta}_g \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{I} \\ \ddot{\theta}_g \end{bmatrix}, \quad u = V_s$$

Then we fill in the model

$$\dot{x} = Ax + Bu = \begin{bmatrix} \dot{I} \\ \ddot{\theta}_g \end{bmatrix} = \begin{bmatrix} \frac{-R_m}{L_m} & \frac{-K_e}{L_m G} \\ \frac{-K_i}{JG} & \frac{-b}{J} \end{bmatrix} \begin{bmatrix} I \\ \dot{\theta}_g \end{bmatrix} + \begin{bmatrix} \frac{1}{L_m} \\ 0 \end{bmatrix} V_s$$

As the pan/tilt system has two axis, the model is exteded with two extra states.
We can now describe both pan and tilt.