

# Inertia

```
syms theta_tilt L_tilt_vert L_tilt_horz L_pan_vert  
L_pan_horz m_tilt_vert m_tilt_horz m_pan_vert m_pan_horz
```

## Tilt Axis

```
I_tilt = 2 * ( (1/12 * m_tilt_vert * L_tilt_vert^2) + 2 * m_tilt_horz * (L_tilt_vert/2)^2 )
```

$I_{\text{tilt}} =$

$$\frac{L_{\text{tilt,vert}}^2 m_{\text{tilt,vert}}}{6} + L_{\text{tilt,vert}}^2 m_{\text{tilt,horz}}$$

## Pan Axis

Inertia of the base pan frame

```
I_pan_base = (1/12 * m_pan_horz * L_pan_horz^2) + 2 * m_pan_vert * (L_pan_horz/2)^2
```

$I_{\text{pan\_base}} =$

$$\frac{L_{\text{pan,horz}}^2 m_{\text{pan,vert}}}{2} + \frac{L_{\text{pan,horz}}^2 m_{\text{pan,horz}}}{12}$$

Inertia of the tilt system as a function of its angle

```
I_pan_tilt_asm_horz = 2 * ( 1/12 * m_tilt_horz * L_tilt_horz^2 ...  
    + m_tilt_horz * (L_tilt_vert * sin(theta_tilt))^2 );  
I_pan_tilt_asm_vert = 2 * ( 1/12 * m_tilt_vert * (m_tilt_vert*sin(theta_tilt))^2 )  
+ m_tilt_vert * (m_tilt_horz/2)^2;  
I_pan_tilt_asm = I_pan_tilt_asm_horz + I_pan_tilt_asm_vert
```

$I_{\text{pan\_tilt\_asm}} =$

$$2 L_{\text{tilt,vert}}^2 m_{\text{tilt,horz}} \sin(\theta_{\text{tilt}})^2 + \frac{L_{\text{tilt,horz}}^2 m_{\text{tilt,horz}}}{6} + \frac{m_{\text{tilt,vert}}^3 \sin(\theta_{\text{tilt}})^2}{6} + \frac{m_{\text{tilt,vert}} m_{\text{tilt,horz}}^2}{4}$$

Adding them up!

```
I_pan = simplify(I_pan_tilt_asm + I_pan_base)
```

$I_{\text{pan}} =$

$$\frac{m_{\text{tilt,vert}}^3 \sin(\theta_{\text{tilt}})^2}{6} + \frac{L_{\text{pan,horz}}^2 m_{\text{pan,vert}}}{2} + \frac{L_{\text{pan,horz}}^2 m_{\text{pan,horz}}}{12} + \frac{L_{\text{tilt,horz}}^2 m_{\text{tilt,horz}}}{6} + \frac{m_{\text{tilt,vert}} m_{\text{tilt,horz}}^2}{4} + 2 L_{\text{tilt,vert}}^2 m$$

## Actual Lengths and Masses

```
m_pr_meter = 0.4284/0.28
```

$m_{\text{pr\_meter}} = 1.5300$

First, make all the masses functions of rod length

```

s      = [theta_tilt L_tilt_vert L_tilt_horz L_pan_vert L_pan_horz m_tilt_vert
          m_tilt_horz          m_pan_vert          m_pan_horz          ];
s_new = [theta_tilt L_tilt_vert L_tilt_horz L_pan_vert L_pan_horz L_tilt_vert
          *m_pr_meter L_tilt_horz*m_pr_meter L_pan_vert*m_pr_meter L_pan_horz*m_pr_meter];

I_tilt = simplify(subs(I_tilt, s, s_new));
I_pan  = simplify(subs(I_pan, s, s_new));

vpa(I_tilt, 3)

```

$$\text{ans} = 0.255 L_{\text{tilt,vert}}^2 (L_{\text{tilt,vert}} + 6.0 L_{\text{tilt,horz}})$$

```
vpa(I_pan, 3)
```

$$\text{ans} = 0.127 L_{\text{pan,horz}}^3 + 0.765 L_{\text{pan,vert}} L_{\text{pan,horz}}^2 + 0.597 L_{\text{tilt,vert}}^3 \sin(\theta_{\text{tilt}})^2 + 3.06 L_{\text{tilt,vert}}^2 L_{\text{tilt,horz}} \sin(\theta_{\text{tilt}})^2 + 0.895 L$$

Now, for the lengths

```

s      = [ theta_tilt L_tilt_vert L_tilt_horz L_pan_vert L_pan_horz ];
s_real = [ theta_tilt 0.21      0.28      0.207      0.43      ];

I_tilt = subs(I_tilt, s, s_real);
vpa(I_tilt, 3)

```

$$\text{ans} = 0.0213$$

```

I_pan = subs(I_pan, s, s_real);
vpa(I_pan, 3)

```

$$\text{ans} = 0.0433 \sin(\theta_{\text{tilt}})^2 + 0.0598$$

For use in other scripts

```
J_tilt_out = I_tilt
```

$$J_{\text{tilt\_out}} = \frac{4250799}{2000000000}$$

```
J_pan_out = I_pan
```

$$J_{\text{pan\_out}} = \frac{86626088199 \sin(\theta_{\text{tilt}})^2}{2000000000000} + \frac{14939128233}{250000000000}$$