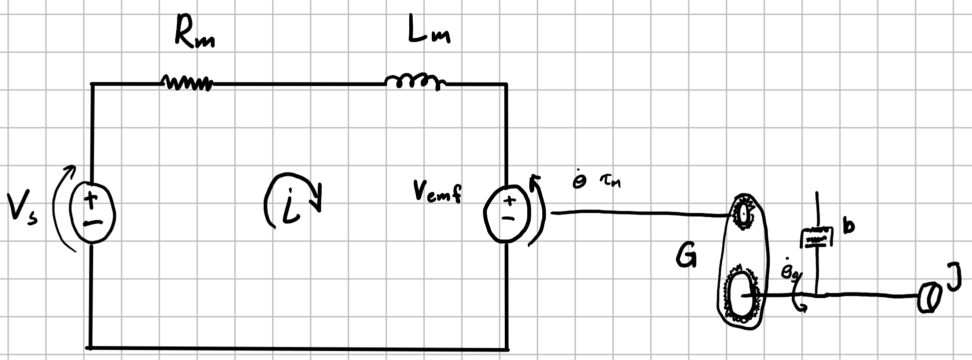
State Space Derivation



Example:
$G \approx 1:3 \Rightarrow G = \frac{1}{3}$

Electrical Circuit

input

$$-V_s + R_m I + L_m \dot{I} + V_{emf} = 0 \qquad V_{emf} = K_e \cdot \dot\theta$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{V_{emf}}{L_m}$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e \cdot \dot\theta_m}{L_m} \rightarrow \dot\theta_m = \frac{1}{G}\dot\theta_g$$

$$\Rightarrow \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e}{L_m G}\dot\theta_g$$

State variables

Physical System

$$\dot\theta_g = \dot\theta_m G$$

$$\ddot\theta_g = \ddot\theta_m G$$

$$\tau_g = \tau_m \cdot \frac{1}{G}$$

$$\tau_m = K_i \cdot I$$

$$J\ddot\theta_g = b\dot\theta_g - \tau_g$$

$$\Rightarrow J\ddot\theta_g = -b\dot\theta_g - \tau_m \cdot \frac{1}{G}$$

$$\Rightarrow J\ddot\theta_g = -b\dot\theta_g - \frac{K_i}{G} \cdot I$$

$$\Rightarrow \ddot\theta_g = \frac{-b}{J}\dot\theta_g - \frac{K_i}{JG} \cdot I$$

State variables

We now have two differential equations

$$\begin{cases} \dot{I} = \frac{V_s}{L_m} - \frac{R_m}{L_m} I - \frac{K_e}{L_m G}\dot\theta_g \\ \ddot\theta_g = \frac{-b}{J}\dot\theta_g - \frac{K_i}{JG} \cdot I \end{cases}$$

Let's create a state space model!

First we define our states and input

$$x = \begin{bmatrix} I \\ \dot\theta_g \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{I} \\ \ddot\theta_g \end{bmatrix}, \quad u = V_s$$

Then we fill in the model

$$\dot{x} = Ax + Bu = \begin{bmatrix} \dot{I} \\ \ddot\theta_g \end{bmatrix} = \begin{bmatrix} \frac{-R_m}{L_m} & \frac{-K_e}{L_m G} \\ \frac{-K_i}{JG} & \frac{-b}{J} \end{bmatrix} \begin{bmatrix} I \\ \dot\theta_g \end{bmatrix} + \begin{bmatrix} \frac{1}{L_m} \\ 0 \end{bmatrix} V_s$$

As the pan/tilt system has two axis, the model is exteded with two extra states.
We can now describe both pan and tilt.

The pan axis inertia is dependent on the tilt angle. We therefore add the positons of the joints as states.

$$J_p = f(\theta_t) = a \cdot \sin(bx) \approx a \cdot bx$$

$$x = \begin{bmatrix} i_p \\ \dot{\theta}_p \\ \theta_p \\ i_t \\ \dot{\theta}_t \\ \theta_t \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{i}_p \\ \ddot{\theta}_p \\ \dot{\theta}_p \\ \dot{i}_t \\ \ddot{\theta}_t \\ \dot{\theta}_t \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{i}_p \\ \ddot{\theta}_p \\ \dot{\theta}_p \\ \dot{i}_t \\ \ddot{\theta}_t \\ \dot{\theta}_t \end{bmatrix} = \begin{bmatrix} -\frac{R_w}{L_m} & -K_e & 0 & 0 \\ \frac{-K_i}{JG^2} & \frac{b}{J_p} & 0 & 0 \\ & & & \\ 0 & 0 & -\frac{R_w}{L_m} & -K_e \\ 0 & 0 & \frac{-K_i}{JG^2} & \frac{b}{J_t} \\ & & & \end{bmatrix} \begin{bmatrix} i_p \\ \dot{\theta}_p \\ \\ i_t \\ \dot{\theta}_t \end{bmatrix} + \begin{bmatrix} \frac{1}{L_n} \\ 0 \\ 0 \\ \frac{1}{L_n} \\ 0 \\ 0 \end{bmatrix} V_s$$

NO WORK