Pan axis

```
syms W L V_y V_x d x_0 t theta_pan theta_dot_pan H a b c theta_tilt_sym theta_tilt_real

s      = [W  L  V_y V_x d theta_pan theta_dot_pan H];
s_real = [20 40 0   22.6  2 theta_pan theta_dot_pan 7];


x = (V_x * t - 1/2 * L);
y = (d + 1/2 * W);

theta_pan_sym = atan( x / y )
```

theta_pan_sym =

$$-\mathrm{atan}\left(\frac{\frac{L}{2} - V_x\, t}{\frac{W}{2} + d}\right)$$

```
theta_dot_pan_sym = diff(theta_pan_sym)
```

theta_dot_pan_sym =

$$\frac{V_x}{\left(\frac{\left(\frac{L}{2} - V_x\, t\right)^2}{\left(\frac{W}{2} + d\right)^2} + 1\right)\left(\frac{W}{2} + d\right)}$$

```
t_max_sym = L/V_x
```

t_max_sym =

$$\frac{L}{V_x}$$

```



theta_pan_real = subs(theta_pan_sym, s, s_real)
```

theta_pan_real =

$$\mathrm{atan}\left(\frac{113\, t}{60} - \frac{5}{3}\right)$$

```
theta_dot_pan_real = subs(theta_dot_pan_sym, s, s_real)
```

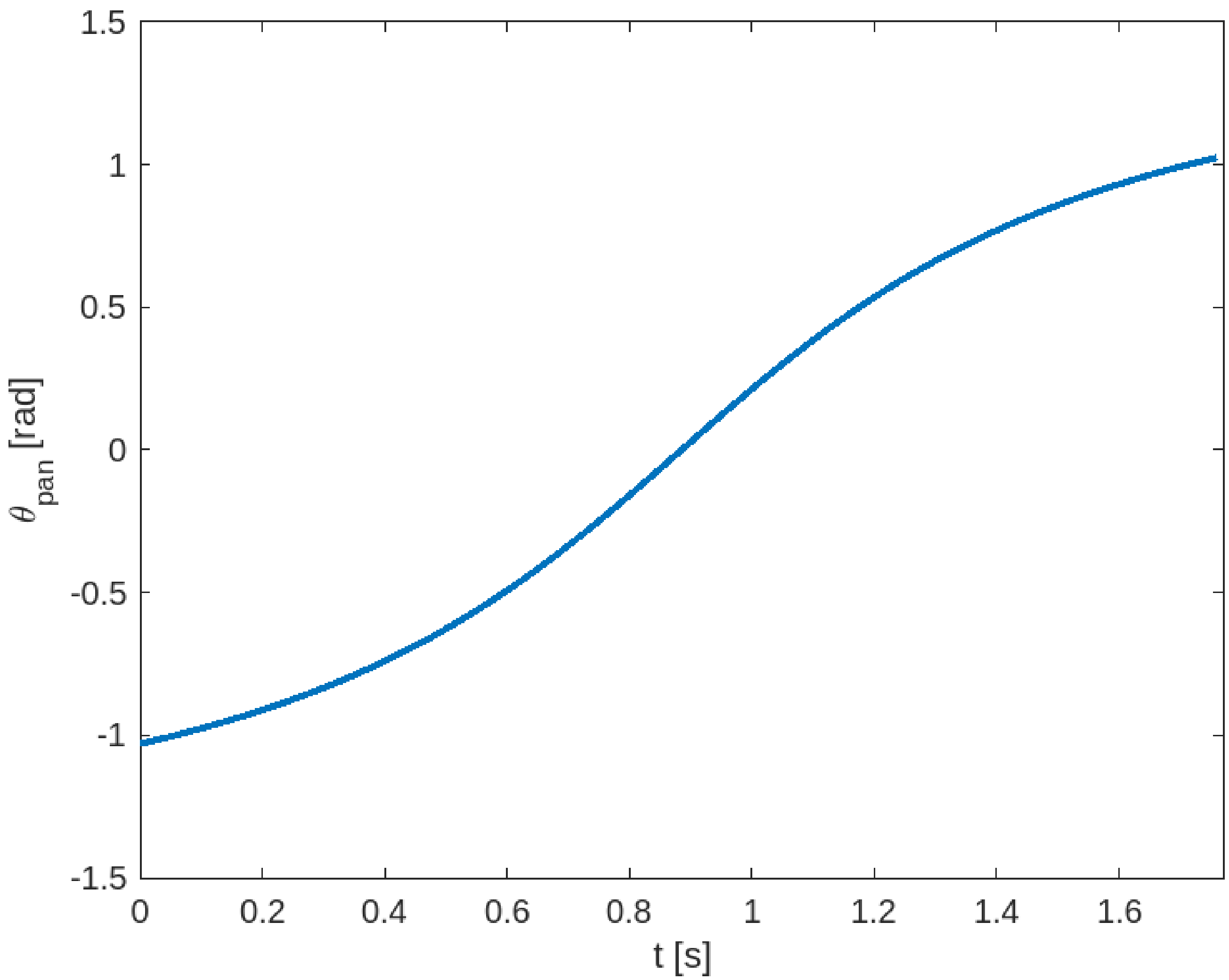
theta_dot_pan_real =

$$\frac{113}{60\left(\frac{\left(\frac{113\, t}{5} - 20\right)^2}{144} + 1\right)}$$

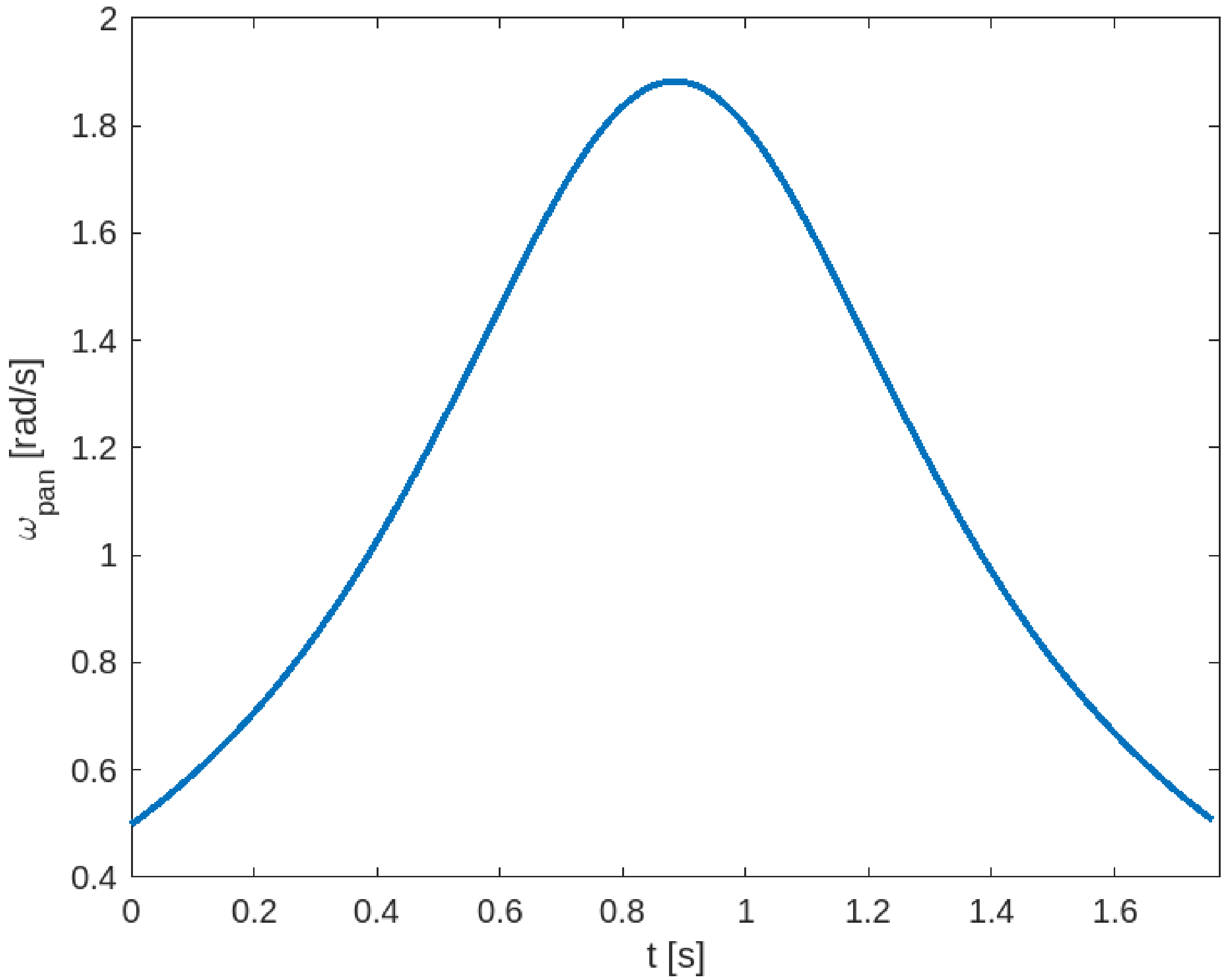
```
t_max = double(subs(t_max_sym, s, s_real))
```

```
t_max = 1.7699
```

```
theta_pan = @(time) subs(theta_pan_real, t, time);
theta_dot_pan = @(time) subs(theta_dot_pan_real, t, time);
t_vals = 0:0.01:t_max
plot(t_vals, theta_pan(t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('\theta_{pan} [rad]')
exportgraphics(gcf, 'figures/reference_theta_pan.png')
```




```
plot(t_vals, theta_dot_pan(t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('\omega_{pan} [rad/s]')
exportgraphics(gcf, 'figures/reference_omega_pan.png')
```



Tilt axis

```
syms H a b c

z = @(t) a*t^2 + b*t + c;

eqs = [
    z(0) == 0;
    z(t_max/2) == H;
    z(t_max) == 0;
]
```

eqs =

$$\begin{pmatrix} c = 0 \\ \frac{10000\,a}{12769} + \frac{100\,b}{113} + c = H \\ \frac{40000\,a}{12769} + \frac{200\,b}{113} + c = 0 \end{pmatrix}$$

```
[A B] = equationsToMatrix(eqs, [a b c])
```

A =

$$\begin{pmatrix} 0 & 0 & 1 \\ \frac{10000}{12769} & \frac{100}{113} & 1 \\ \frac{40000}{12769} & \frac{200}{113} & 1 \end{pmatrix}$$

B =

$$\begin{pmatrix} 0 \\ H \\ 0 \end{pmatrix}$$

```
sol = solve(eqs, [a b c])
```

```
sol = struct with fields:
    a: -(12769*H)/10000
    b: (113*H)/50
    c: 0
```

```
sol.a
```

ans =

$$-\frac{12769\,H}{10000}$$

```
sol = subs(sol, s, s_real)
```

```
sol = struct with fields:
    a: -89383/10000
    b: 791/50
    c: 0
```

```
z = sol.a * t.^2 + sol.b * t + sol.c
```

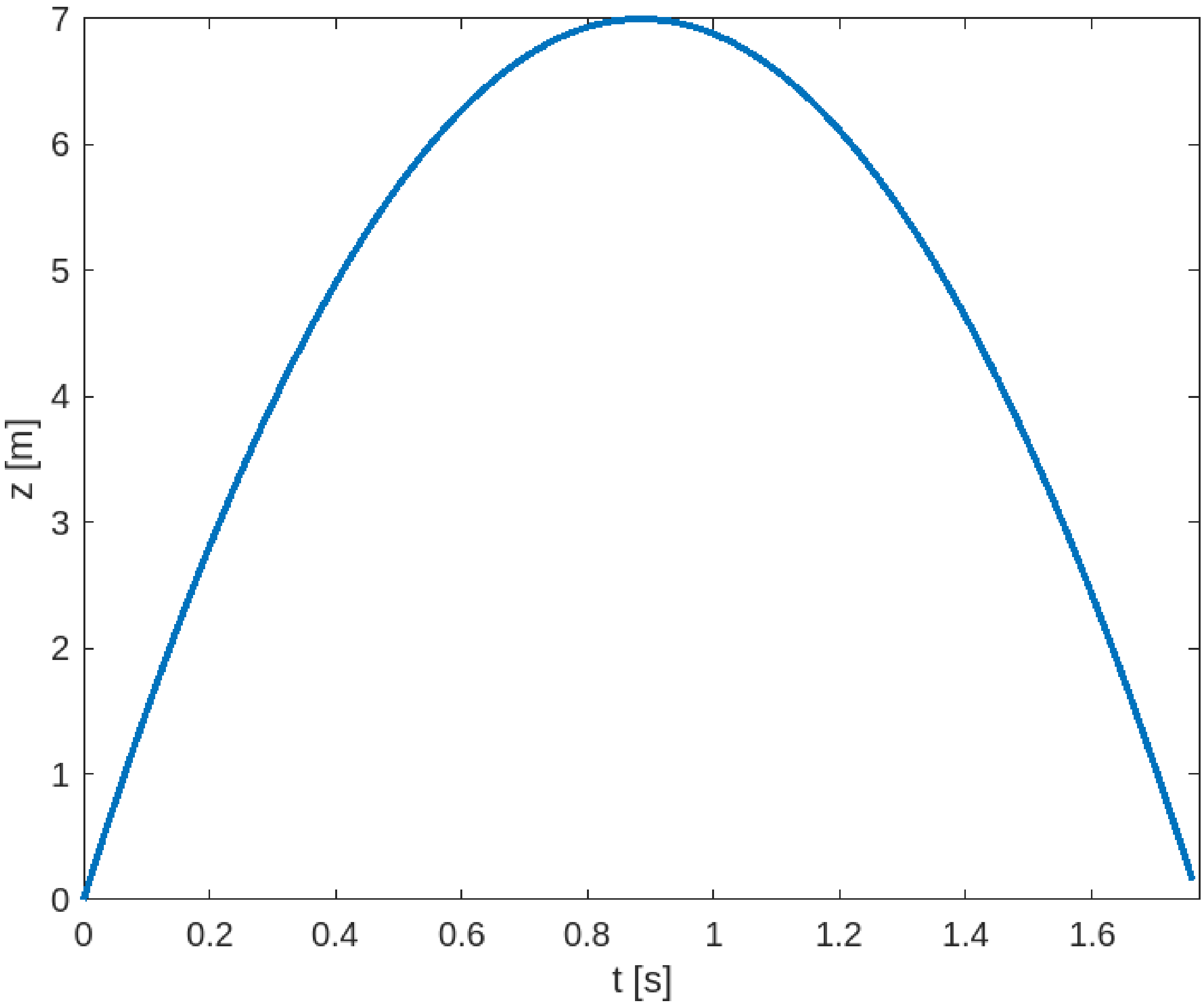
z =

$$\frac{791\,t}{50} - \frac{89383\,t^2}{10000}$$

```
latex(vpa(z,4))
```

```
ans =
'15.82\,t-8.938\,t^2'
```

```
plot(t_vals, subs(z, t, t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('z [m]')
```




```
d = sqrt(x^2 + y^2)
```

d =

$$\sqrt{\left(\frac{L}{2}-V_x\,t\right)^2+\left(\frac{W}{2}+d\right)^2}$$

```
theta_tilt_sym  = atan(z/d)
```

theta_tilt_sym =

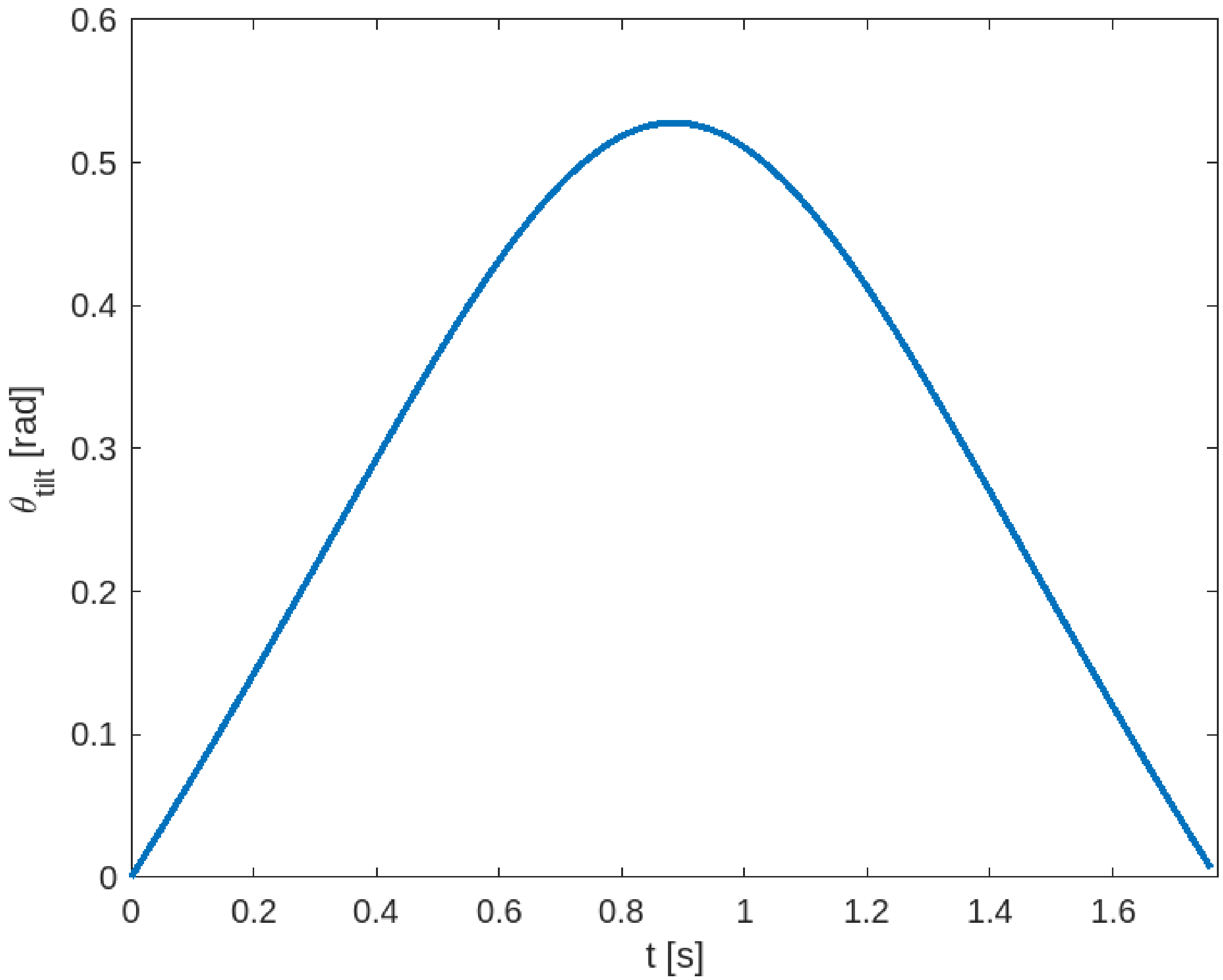
$$\mathrm{atan}\left(\frac{\frac{791\,t}{50}-\frac{89383\,t^2}{10000}}{\sqrt{\left(\frac{L}{2}-V_x\,t\right)^2+\left(\frac{W}{2}+d\right)^2}}\right)$$

```
theta_tilt_real = subs(theta_tilt_sym, [s], [s_real])
```

theta_tilt_real =

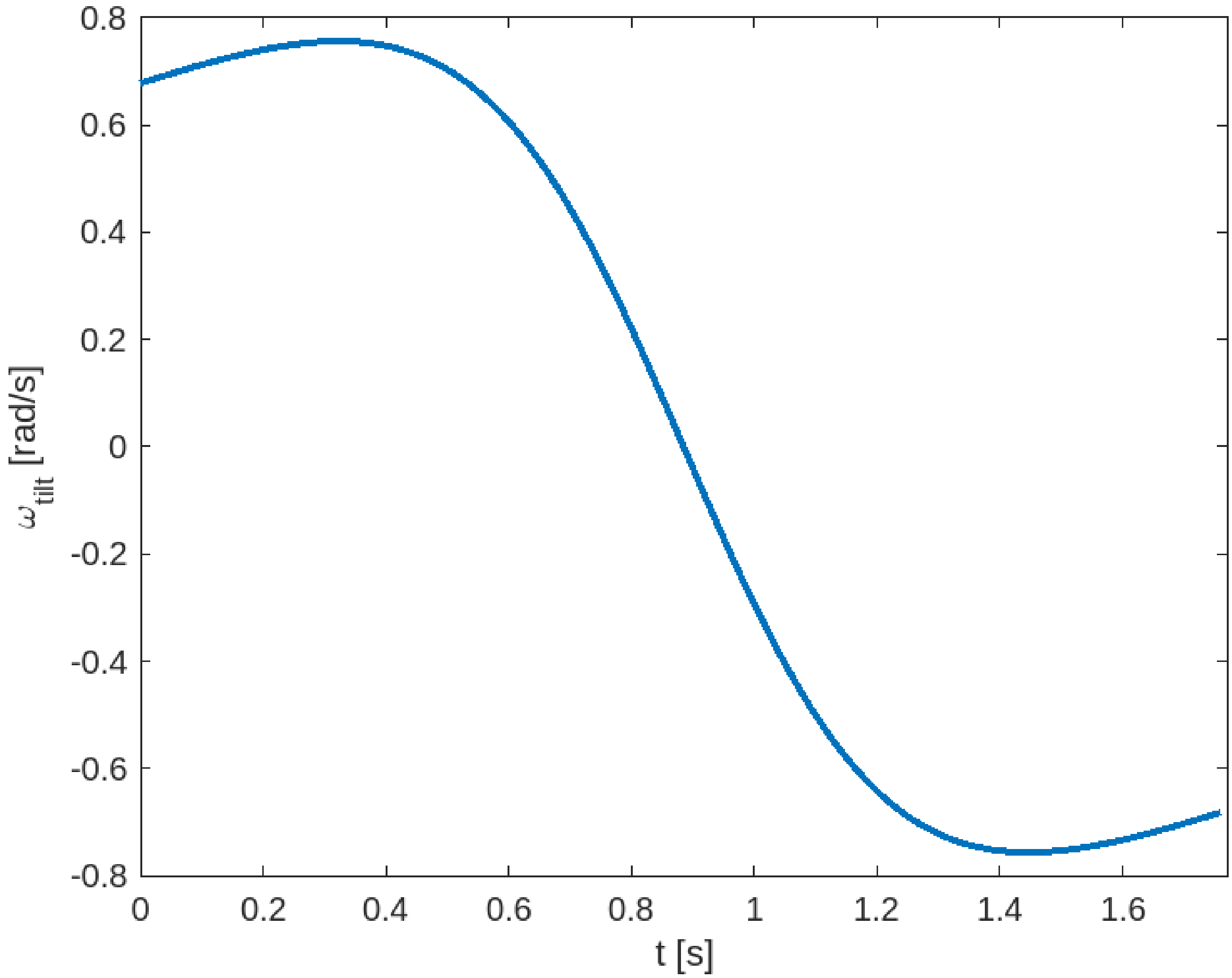
$$\operatorname{atan}\left(\frac{\frac{791\,t}{50}-\frac{89383\,t^2}{10000}}{\sqrt{\left(\frac{113\,t}{5}-20\right)^2+144}}\right)$$

```
theta_dot_tilt_real = diff(theta_tilt_real);
plot(t_vals, subs(theta_tilt_real, t, t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('\theta_{tilt} [rad]')
exportgraphics(gcf, 'figures/reference_theta_tilt.png')
```




```
plot(t_vals, subs(theta_dot_tilt_real, t, t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('\omega_{tilt} [rad/s]')
```

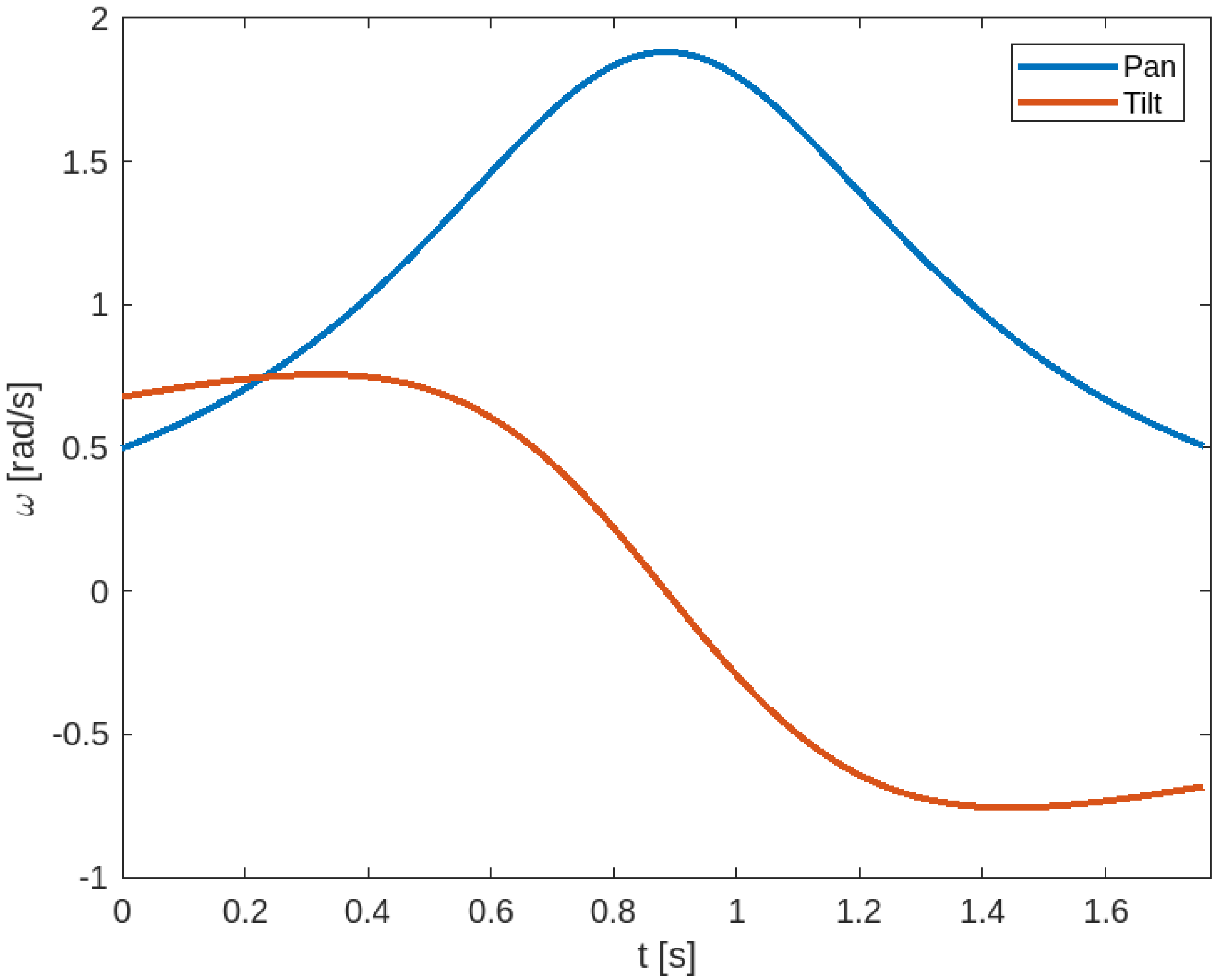
```
exportgraphics(gcf, 'figures/reference_omega_tilt.png')
```



References for Simulink

```
ref = @(time) subs([
    theta_dot_pan_real;
    theta_dot_tilt_real;
], t, time);

plot(t_vals, ref(t_vals), LineWidth=2)
xlim([0 t_max])
xlabel('t [s]')
ylabel('\omega [rad/s]')
legend(["Pan" "Tilt"])
exportgraphics(gcf, 'figures/simulink_references.png')
```



Pan
Tilt
ω [rad/s]
t [s]
2
1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6