

## MANOVA 2 test-statistics

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### MANOVA 2 with potential systematic interaction

Model 
$$X_{lkr} \sim N_p(\mu_{lk}, \Sigma_{lk}), \quad l = 1, \dots, g, \quad k = 1, \dots, b, \quad r = 1, \dots, n$$
$$\forall \Sigma_{lk} = \Sigma$$
$$\mu_{lk} = \mu + \tau_l + \beta_k + \gamma_{lk}$$

Sums of squares 
$$SS_T = SS_{B1} + SS_{B2} + SS_{int} + SS_W$$

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### Test for systematic interaction and simplification of model (approximate)

Hypothesis 
$$H_{0,\gamma}: \forall \gamma_{lk} = 0 \quad (\text{no systematic interaction})$$

Wilks lambda 
$$\Lambda^* = \frac{|SS_W|}{|SS_{int} + SS_W|} \quad \text{reject } H_{0,\gamma} \text{ for small } \Lambda^* \text{ (close to zero)}$$

Test-statistic 
$$T = - \left[ gb(n-1) - \frac{(p+1)-(g-1)(b-1)}{2} \right] \log \Lambda^* \sim X_{p(g-1)(b-1)}^2$$

Test 
$$\text{reject } H_{0,\gamma} \text{ for } T > X_{p(g-1)(b-1)}^2(\alpha)$$

Conclusion 
$$\text{reject } H_{0,\gamma} \Rightarrow \text{STOP (model cannot be simplified),}$$
$$\text{estimation of } \mu_{lk}, \Sigma$$
$$\text{accept } H_{0,\gamma} \Rightarrow \text{test for factor 1 and 2 effects}$$

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### Test for systematic factor 1 effect (approximate)

Hypothesis 
$$H_{0,\tau}: \forall \tau_l = 0 \quad (\text{no systematic factor 1 effect})$$

Wilks lambda 
$$\Lambda^* = \frac{|SS_W|}{|SS_{B1} + SS_W|} \quad \text{reject } H_{0,\tau} \text{ for small } \Lambda^* \text{ (close to zero)}$$

Test-statistic 
$$T = - \left[ gb(n-1) - \frac{(p+1)-(g-1)}{2} \right] \log \Lambda^* \sim X_{p(g-1)}^2$$

Test 
$$\text{reject } H_{0,\tau} \text{ for } T > X_{p(g-1)}^2(\alpha)$$

Conclusion 
$$\text{reject } H_{0,\tau} \Rightarrow \text{factor 1 is significant}$$

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### Test for systematic factor 2 effect (approximate)

Hypothesis 
$$H_{0,\beta}: \forall \beta_k = 0 \quad (\text{no systematic factor 2 effect})$$

Wilks lambda 
$$\Lambda^* = \frac{|SS_W|}{|SS_{B2} + SS_W|} \quad \text{reject } H_{0,\beta} \text{ for small } \Lambda^* \text{ (close to zero)}$$

Test-statistic 
$$T = - \left[ gb(n-1) - \frac{(p+1)-(b-1)}{2} \right] \log \Lambda^* \sim X_{p(b-1)}^2$$

Test 
$$\text{reject } H_{0,\beta} \text{ for } T > X_{p(b-1)}^2(\alpha)$$

Conclusion 
$$\text{reject } H_{0,\beta} \Rightarrow \text{factor 2 is significant}$$

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