Multivariate statistics
Lecture 03:
One-sample inference for MVN mean vector

Literature and Problems: See course schedule

Agenda (preliminary):

SLIDE: Review of 1D inference for mean of one-sample normal population

Multivariate one-sample inference for mean vector μ
- model
- hypothesis
- point-estimators for μ and ∑
- Hotelling T^2 test-statistic and its distribution
- hypothesis-test using α or p-value

Confidence region for mean vector μ (p-dimensional ellipsoide)

Confidence intervals for mean vector components $\mu_1, \mu_2, \ldots, \mu_p$ (1D intervals)
- marginal, one-at-a-time CI's, ignoring covariances in ∑
- problem with marginal CI's: overall (simultaneous) confidence is too low
- Bonferroni corrected CI's
- true simultaneous CI's
- DEMO: Confidence region and intervals for μ for bivariate normal distribution

Large sample approximate inference for μ
- approximate distribution of T^2
- confidence region
- marginal CI's
- Bonferroni CI's
- simultaneous CI's
- DEMO: Comparison of exact and large sample approximate test-statistic distributions and quantiles

The Likelihood Ratio (LR) test principle applied to inference for MVN μ
- likelihood function for MVN in general and under H_0
- LR test-statistic Λ definition and interpretation
- expression for Λ in present case: inference for MVN μ
- distribution of Λ not necessary in this case since $\Lambda = f(T^2)$, with f monotone
- DEMO: Equivalence between Hotelling T^2 and LR Λ test-statistics
- large sample approximation for LR tests using $-2 \log \Lambda$