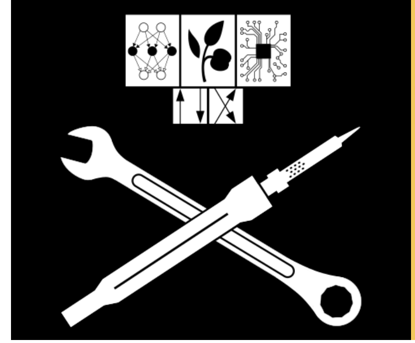

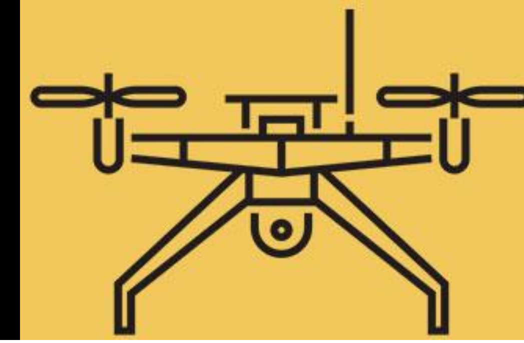

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Practicalities of the course

- Every Monday or Friday
- Course in parallel in Odense and Sønderborg
- About 10 or 11 lectures
- Slides and Lecture notes will be posted on Itslearning for each lecture
- Written exam with theoretical and programming questions

its learning

Examination

- 10 questions
- 2 points each question = 20 max points for the total
- 50% = 10 points to pass (grade 02), 20 points to ace (grade 12)
- theoretical (pen and paper) questions and programming questions (ie implement this or that in Matlab/Simulink)
- Theoretical questions: be precise and justify your answer
- Programming questions: it has to work and give the expected behavior
- Programming questions: I need to be able to open the program on my computer (also: do not program like a “bozo” ☺!)

Syllabus

- Dynamical systems
- State-space representations
- IO representation and PID control
- Stability and state-feedback control
- Tracking and dynamic state-feedback
- MPC and feedback linearization
- State estimation and observers
- Output feedback and ADRC
- Parameter estimation
- Extremum Seeking control
- Controllability and feedforward control

Lectures and teaching philosophy

- Learning by doing is the essence of this course

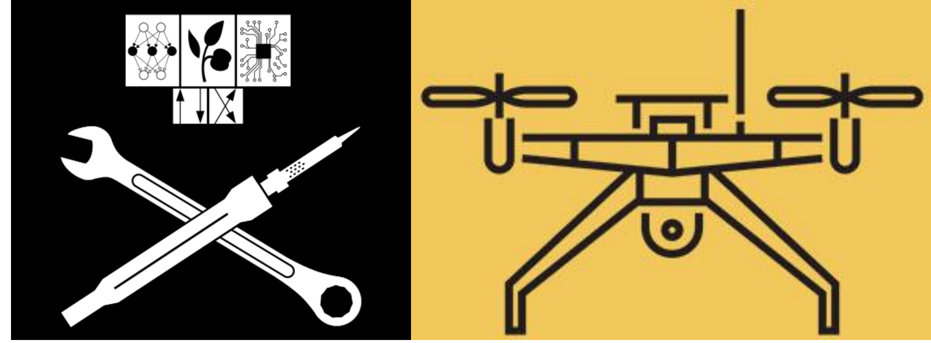
- Each lecture will have a 50 to 60-minute presentation to introduce a concept/technique, followed by a computer session where we practise the technique

- All exercises should be finished for the next lecture

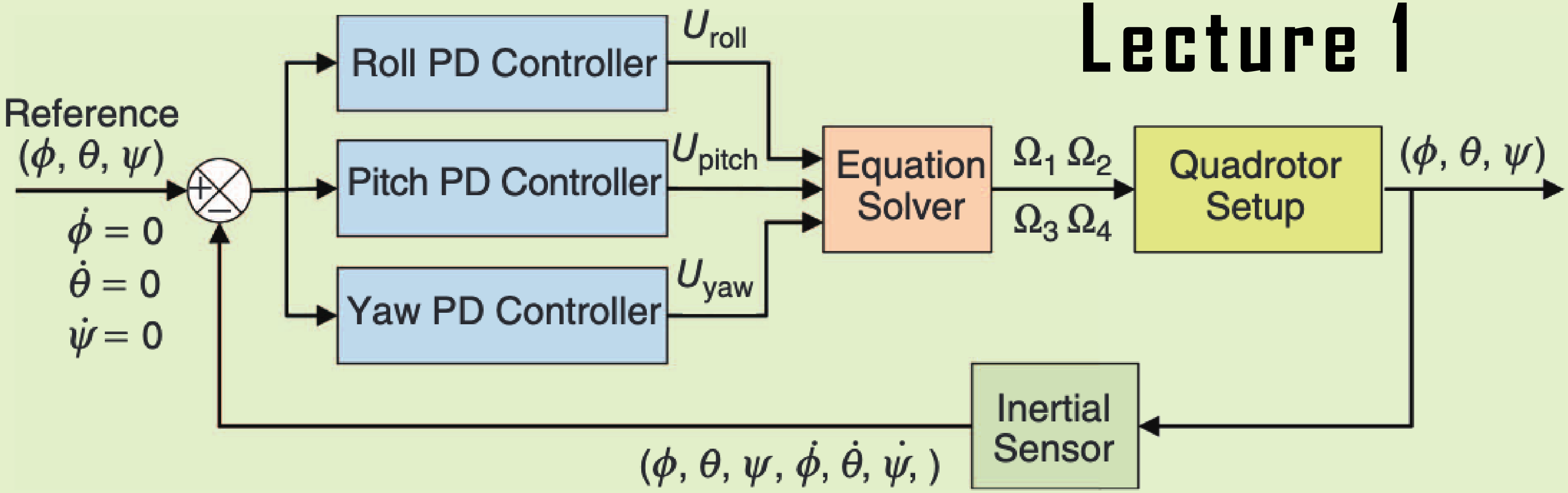
- Attitude vs skills… (be professional, prepared, contribute to the class, etc.)

- Come to class with PAPER AND PEN!

- There will be an assignment midway through the semester



Lecture 1



Dynamical systems and block diagrams

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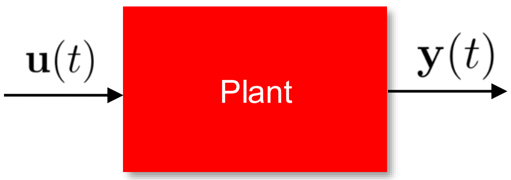
Signal and system representations

In the beginning there was...



the dynamical system Σ

where: - u(t) signal having effect on the behavior of system Σ,

ie u(t) is called (control) input

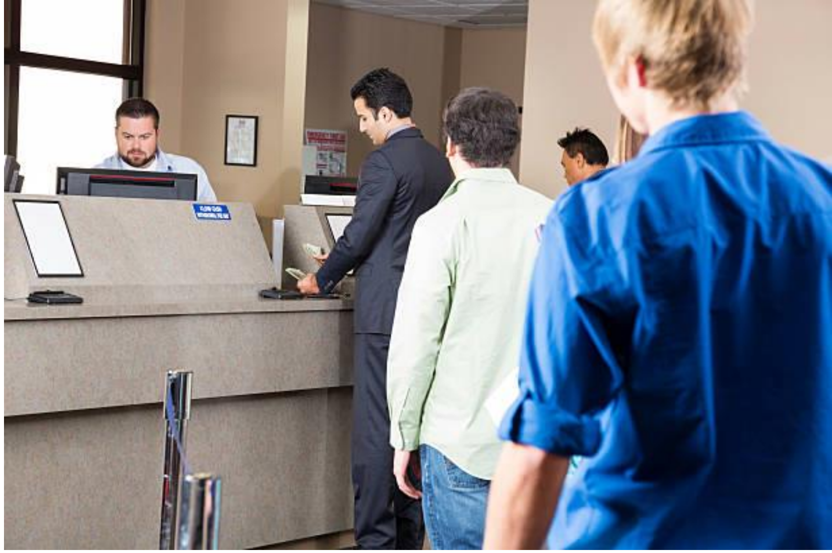
- y(t) signal representing a selected part of the dynamical system/plant dynamics,

ie y(t) is called output

A few systems (1/7)

Bank account with interest rates

Maybe one of the simplest dynamical there exist...



At year t=0, a bank account with 3% interest rate is opened with initial deposit (say 10000DKK).
The evolution of the amount y at year t, ie y(t), is given by

$$y(t+1) = (1+0.03)y(t), \quad \text{where} \quad y(0) = y_0 = 10000\,\text{DKK}$$

recurrence relation

initial condition

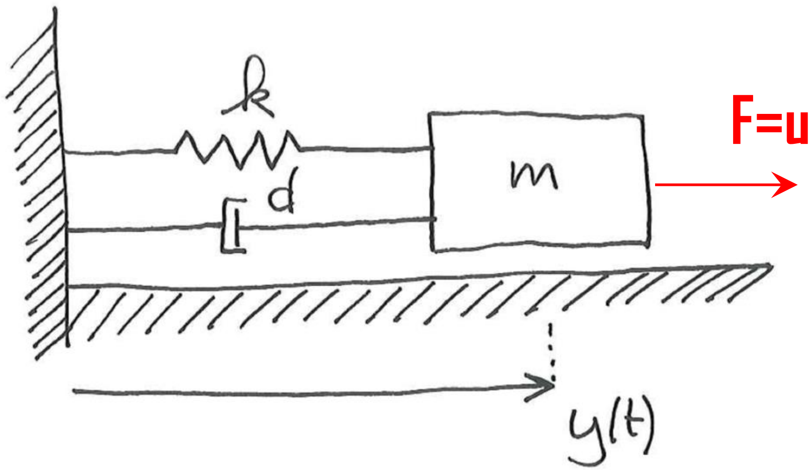
The owner of the account adding/withdrawing some money can be represented by an input variable:

$$y(t+1) = 1.03y(t) + u(t)$$

Remark: 1 input, 1 output, linear system, discrete-time

A few systems (2/7)

Mass-Spring-Damper system



Dynamical system represented by an Ordinary Differential Equation (ODE):

$$m\ddot{y}(t) + d\dot{y}(t) + ky(t) = u(t)$$

with the initial conditions $y(0)$ initial position of the mass

$\dot{y}(0)$ initial velocity of the mass

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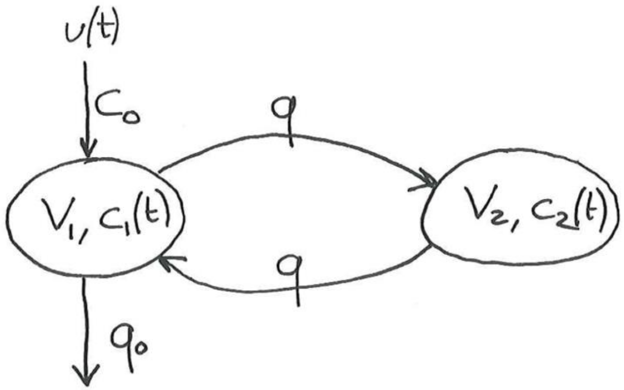
Remark: 1 input, 1 output, linear system, continuous-time

A few systems (3/7)

Pharmacokinetics

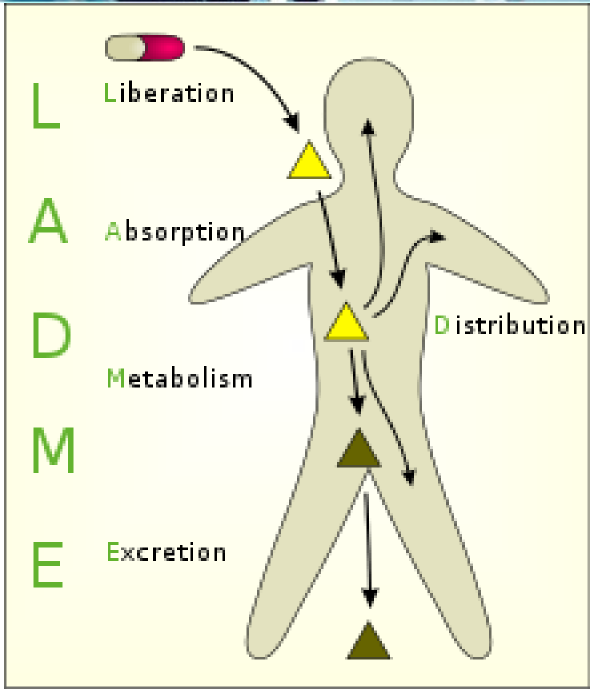


Represented by

$$
\begin{cases}
V_1 \dot{c}_1(t) = q\left(c_2(t) - c_1(t)\right) - q_0 c_1(t) + c_0 u(t) \\
V_2 \dot{c}_2(t) = q\left(c_1(t) - c_2(t)\right))
\end{cases}
$$

with C_1, C_2 concentration of drug in compartment i, u is volume flow rate of drug intake, V_1, V_2 volumes of compartments i.

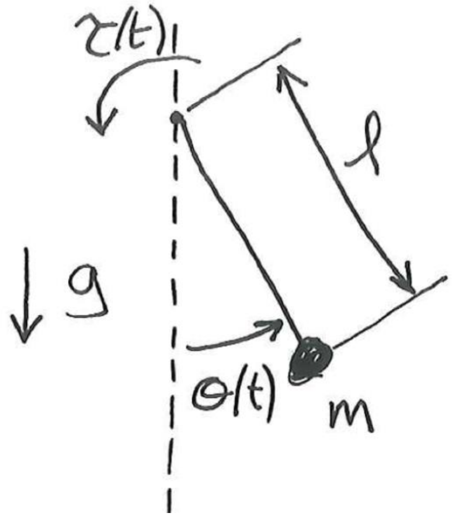
monitored output $y(t) = c_2(t)$

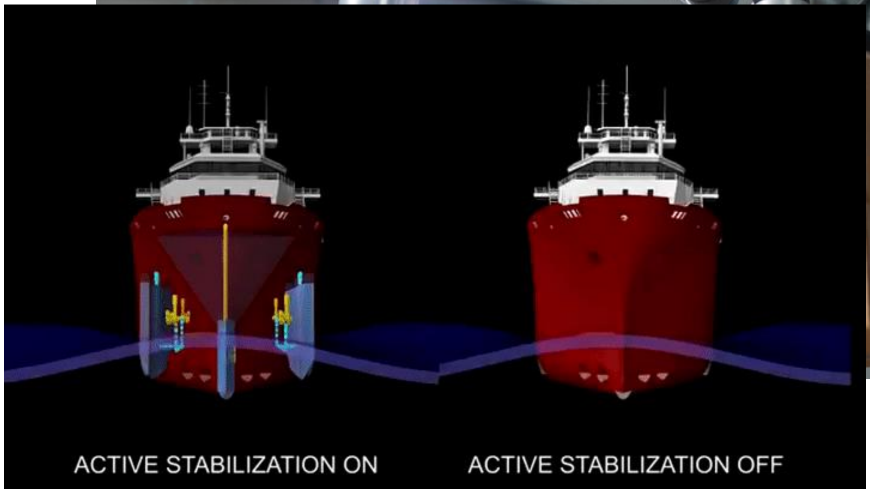


Remark: 2 coupled dyn. syst., 1 input, 1 output, linear

A few systems (4/7)

Pendulum





Represented by

$$ml^2\ddot{\theta}(t) + mgl\sin\theta(t) = 0 \qquad \text{with} \qquad \theta(0) = \theta_0, \quad \dot{\theta}(0) = \omega_0$$

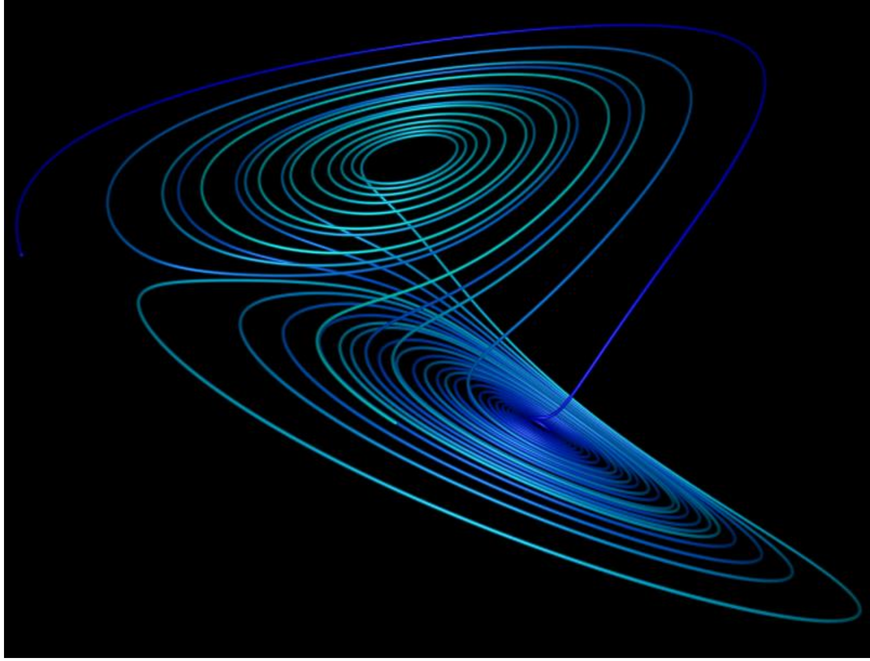
where we measure $y(t) = \theta(t)$ **Remark: nonlinear system, no input!**

add damping: $$ml^2\ddot{\theta}(t) + d_\theta l^2\dot{\theta}(t) + mgl\sin\theta(t) = 0$$

add input/torque:

$$ml^2\ddot{\theta}(t) + d_\theta l^2\dot{\theta}(t) + mgl\sin\theta(t) = \tau(t) \qquad \text{with} \qquad u(t) = \tau(t)$$

A few systems (5/7)



Lorenz system

Ever heard about the butterfly effect? That is the equation:

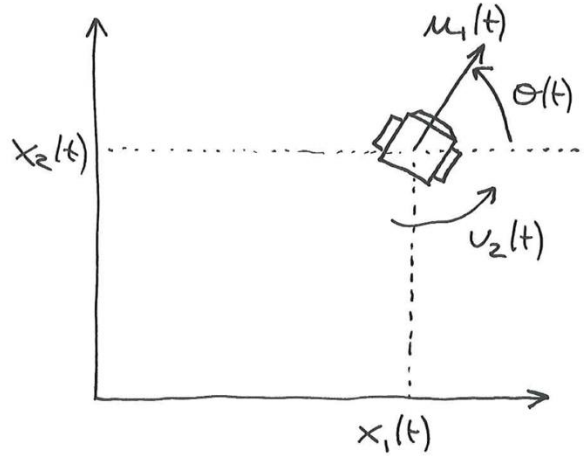
$$\begin{cases} \dot{x}_1(t) = -p x_1(t) + p x_2(t) \\ \dot{x}_2(t) = -x_1(t) x_3(t) - x_2(t) \\ \dot{x}_3(t) = x_1(t) x_2(t) - x_3(t) \end{cases}$$

where p is a constant

Remark: 3 coupled nonlinear dyn. syst., <u>no</u> input

A few systems (6/7)

2-wheeled robot



u_1: longitudinal velocity of robot,
u_2: rotational velocity

This gives
$$\begin{cases} \dot{x}_1(t) = u_1(t)\cos\theta(t) \\ \dot{x}_2(t) = u_1(t)\sin\theta(t) \\ \dot{\theta}(t) = u_2(t) \end{cases}$$

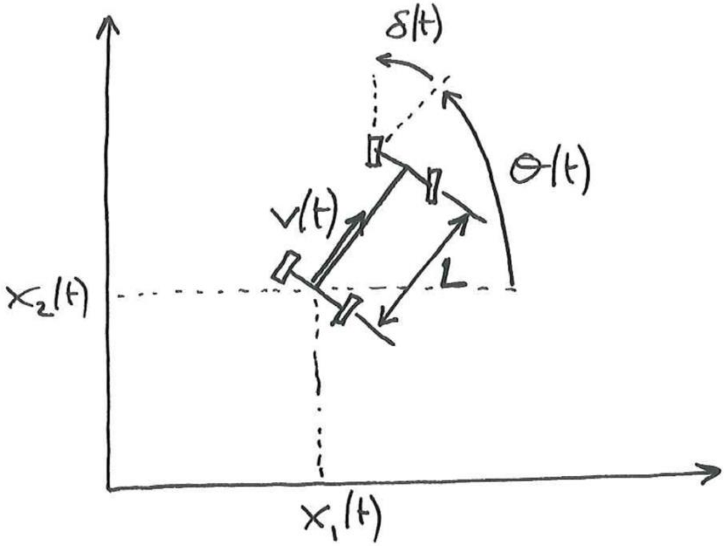
We want to monitor all 3 dynamical variables, ie we have the outputs

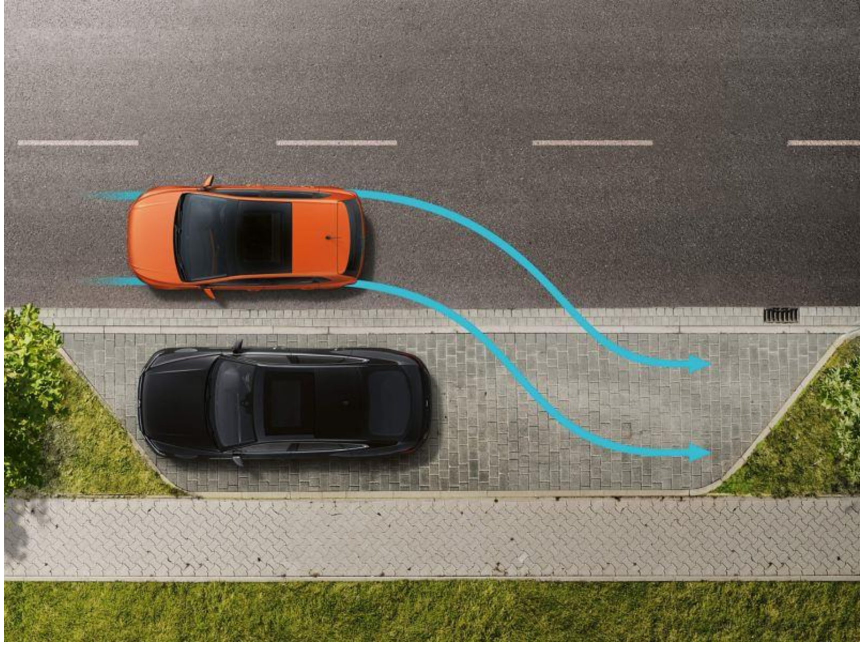
$$y_1(t) = x_1(t), \qquad y_2(t) = x_2(t), \qquad y_3(t) = \theta(t)$$

Remark: nonlinear dyn. syst., 2 inputs, 3 outputs

A few systems (7/7)

Car-like robot





Very similar to the 2-wheeled robot:

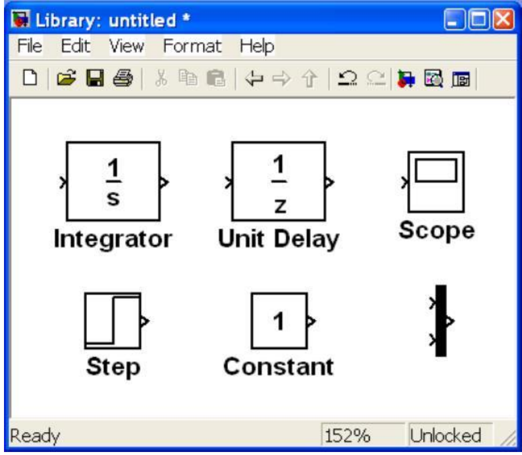
$$\begin{cases} \dot{x}_1(t) = v(t)\cos\theta(t) \\ \dot{x}_2(t) = v(t)\sin\theta(t) \\ \dot{\theta}(t) = \dfrac{v(t)}{L}\tan\delta(t) \end{cases}$$

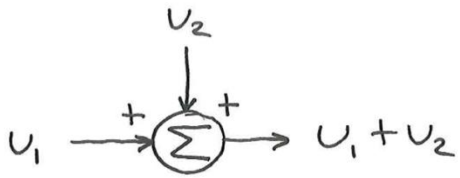
with $u_1(t) = v(t), \qquad u_2(t) = \delta(t)$

Block diagrams: the basic blocks

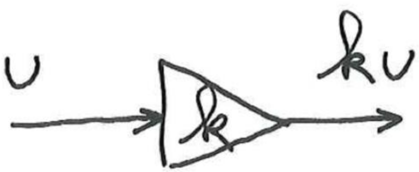
Graphical representation for dynamical systems.

Use a library of blocks…

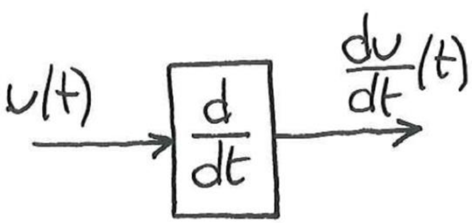




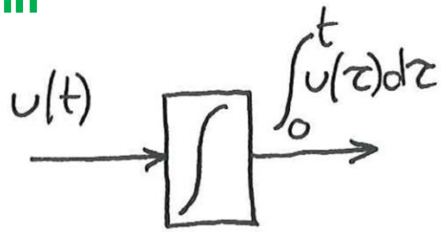
summator (sum)



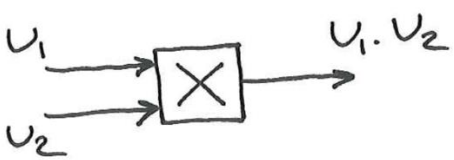
gain



differentiator



integrator



multiplier

…and connect them to each other to obtain a block diagram.

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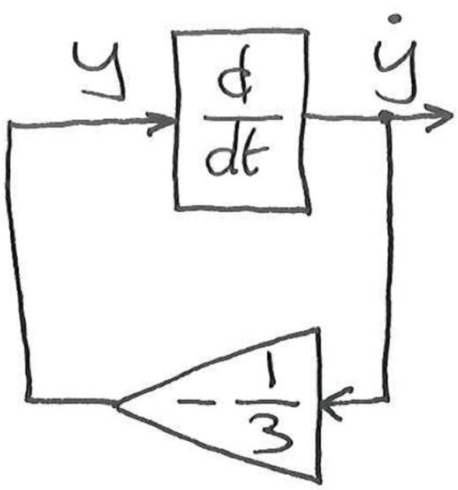
Block diagram on a simple example (1/2)

Objective: make a block diagram of the system

$$\frac{d}{dt}y(t) = -3y(t), \qquad \text{with} \qquad y(0) = 6$$

Let us give it a try and connect blocks like this:



BUT:
- Initial condition not represented
- Differential operator very sensitive to noise!

NOT THE RIGHT WAY

Another way?

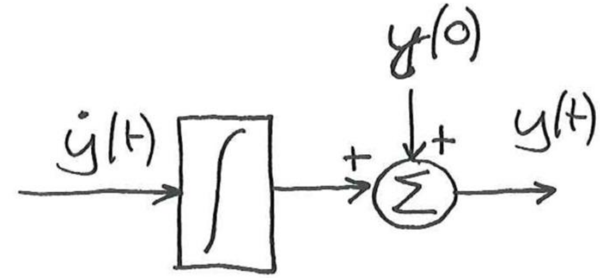
Block diagram on a simple example (2/2)

Let's try something else:

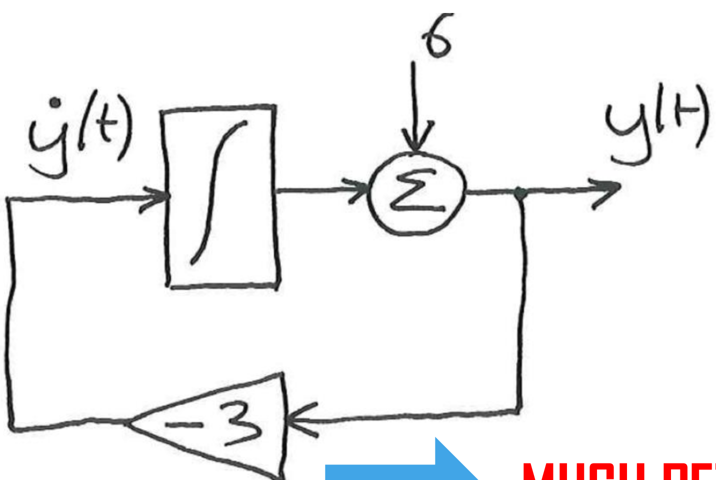
Use a nice trick/property: $\int_0^t \dot{y}(\tau)d\tau = y(t) - y(0)$

which gives

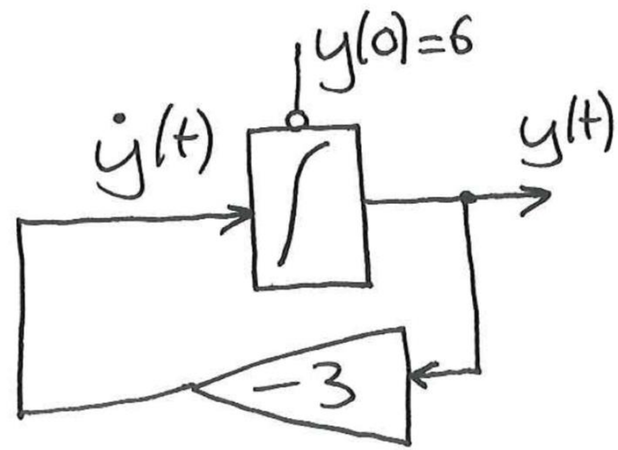
$$y(t) = \int_0^t \dot{y}(\tau)d\tau + y(0)$$



So that $\frac{d}{dt}y(t) = -3y(t)$, gives the block diagram



OR



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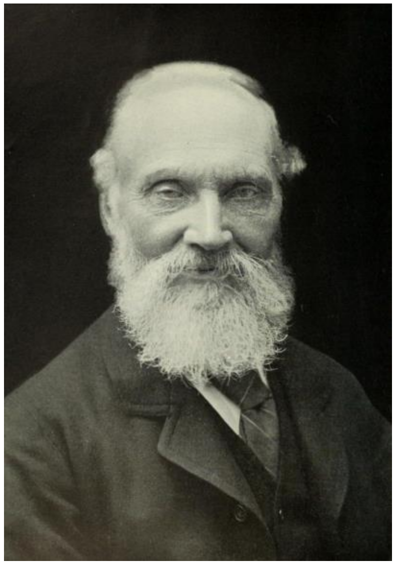
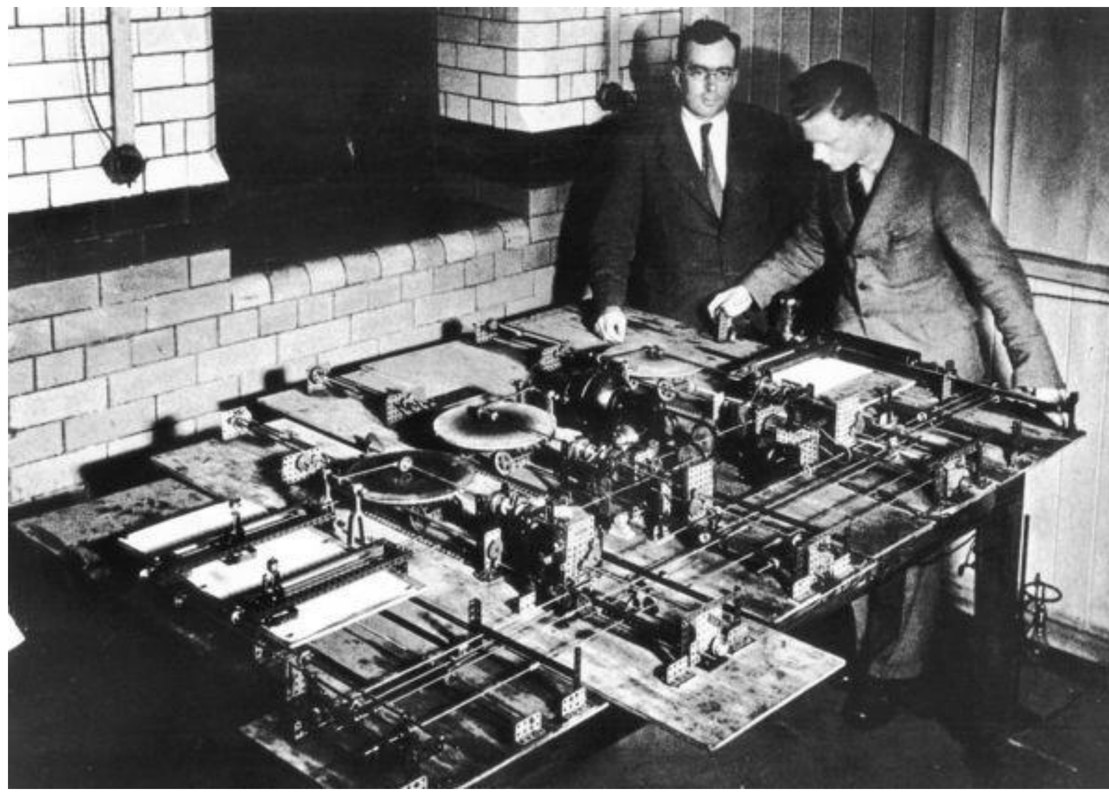
MUCH BETTER ☺ (with noise, initial cond.)

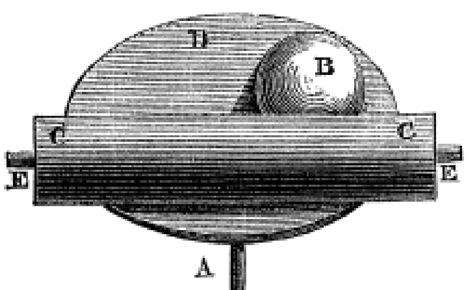
Parenthesis: story behind the integrator trick/method



William Thomson
aka Lord Kelvin

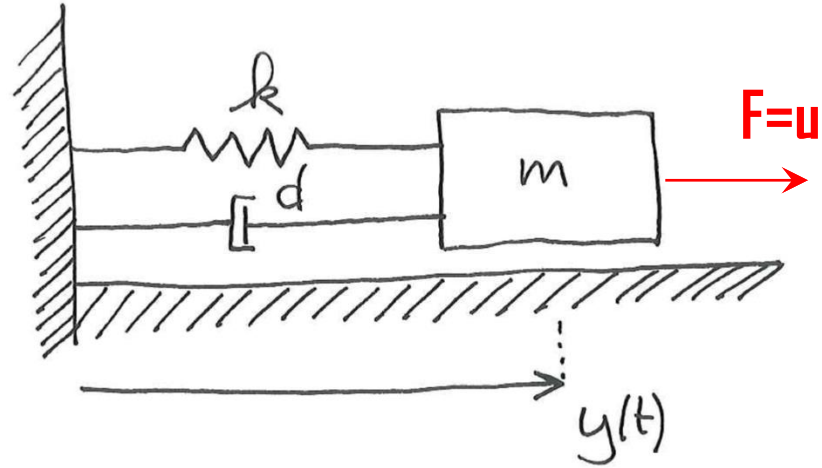
V. "Mechanical Integration of the Linear Differential Equations of the Second Order with Variable Coefficients." By Prof. Sir WILLIAM THOMSON, LL.D., F.R.S. Received January 28, 1876.





"differential analyser"

Exercise (pen and paper/tablet)



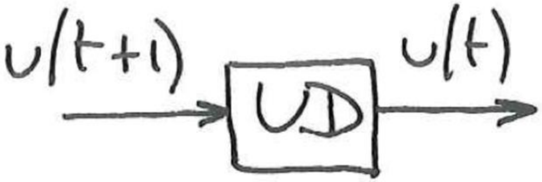
Make a block diagram (with initial conditions) of

$$m\ddot{y}(t) + d\dot{y}(t) + ky(t) = u(t)$$

A word on discrete-time systems

Working with discrete-time systems is very similar to continuous-time ones.

Instead of an integrator, use a unit delay block:

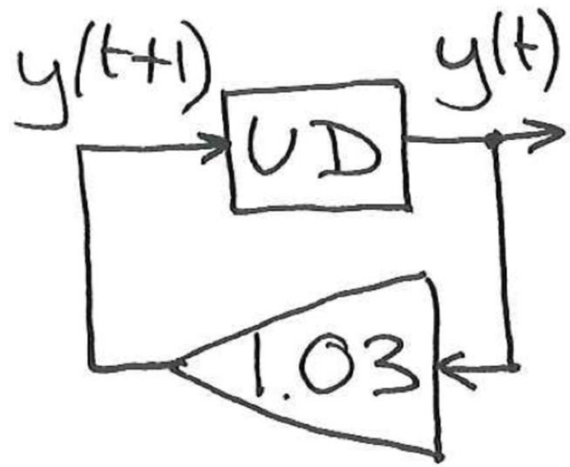


unit delay

As an example, our bank account system represented by

$$y(t+1) = (1+0.03)y(t),$$

gives



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Exercises with MATLAB/Simulink

- Questions for the exercise of this lecture are posted on Itslearning

- Have you downloaded MATLAB/Simulink (R2024b)? ☺