MANOVA 1 test-statistics

MANOVA 1 decomposition of variation

Sum of squares	Between groups	$B = \sum_{l=1}^{g} \sum_{j=1}^{n_l} (x_l - \bar{x})(x_l - \bar{x})^T$
	Within groups	$W = \sum_{l=1}^{g} \sum_{j=1}^{n_l} \left(x_{lj} - \bar{x}_l\right)\left(x_{lj} - \bar{x}_l\right)^T$
$\hat{\Sigma}$ estimates	Between groups	$S_B = \hat{\Sigma} = \frac{B}{g-1}$
	Within groups	$S_W = \hat{\Sigma} = \frac{W}{n-g}$
Factor effect ?	S_B "big" compared to S_W is indication of significant factor effect	
Problem	**How do you compare the "big-ness" of two (pxp) matrices ?**	

MANOVA 1 test-statistics

Hypothesis	$H_0\!: \ \forall\tau_l = 0 \Leftrightarrow \forall\mu_l = \mu$	(no significant factor effect)
Wilks lambda	$\Lambda^* = \frac{\lvert W\rvert}{\lvert B+W\rvert}$	reject H_0 for small Λ^* (close to zero)
Hotelling-Lawley	$T_0^{\ 2} = \mathrm{trace}(BW^{-1})$	reject H_0 for large $T_0^{\ 2}$
Pillai	$V = \mathrm{trace}(B(B+W)^{-1})$	reject H_0 for large V
Roy	$R =$ largest eigenvalue of BW^{-1}	reject H_0 for large R

Informal argument for the form of Wilks lambda

Wilks lambda $\quad \Lambda^* = \frac{\lvert W\rvert}{\lvert B+W\rvert} \Rightarrow \frac{1-\Lambda^*}{\Lambda^*} = \frac{\lvert B+W\rvert - \lvert W\rvert}{\lvert W\rvert}$

Treating positive matrix determinants as scalars (illegal in general)

$$\frac{1-\Lambda^*}{\Lambda^*} \ \text{"}=\text{"} \ \frac{\lvert B\rvert}{\lvert W\rvert} = \frac{\lvert S_B\rvert (g-1)^p}{\lvert S_W\rvert (n-g)^p}$$

$$T \overset{\text{def}}{=} \frac{(n-g)^p}{(g-1)^p} \cdot \frac{1-\Lambda^*}{\Lambda^*} = \frac{\lvert S_B\rvert}{\lvert S_W\rvert}$$

reject H_0 for small $\Lambda^* \Leftrightarrow$ large T

Exact distribution cases of Wilks lambda (table also in textbook)

$p = 1,\ g \geq 2$	$T \overset{\text{def}}{=} \frac{n-g}{g-1} \cdot \frac{1-\Lambda^*}{\Lambda^*} \sim F_{g-1,n-g}$
$p = 2,\ g \geq 2$	$T \overset{\text{def}}{=} \frac{n-g-1}{g-1} \cdot \frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}} \sim F_{2(g-1),2(n-g-1)}$
$p \geq 1,\ g = 2$	$T \overset{\text{def}}{=} \frac{n-p-1}{p} \cdot \frac{1-\Lambda^*}{\Lambda^*} \sim F_{p,n-p-1}$
$p \geq 1,\ g = 3$	$T \overset{\text{def}}{=} \frac{n-p-2}{p} \cdot \frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}} \sim F_{2p,2(n-p-2)}$

Large-sample approximation of Wilks lambda

$n \gg 1 \quad T \overset{\text{def}}{=} -\left(n - 1 - \frac{p+g}{2}\right) \cdot \log \frac{\lvert W\rvert}{\lvert B+W\rvert} \sim X^2_{p(g-1)}$ (approximately)