Uni-variate (1D) inference about mean of one-sample normal population

Model $X_1, X_2, \dots, X_n$ iid $X_i \sim N(\mu, \sigma^2)$, μ, σ^2 unknown

Hypothesis $H_0\colon\ \mu = \mu_0$ against $H_1\colon\ \mu \neq \mu_0$

Point-estimators $\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \sim N(\mu, \frac{\sigma^2}{n})$

(ML, unbiased, consistent estimator)

$\hat{\sigma}^2 = S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \sim \frac{\sigma^2}{n-1} X_{n-1}^2$

(not ML, but unbiased, consistent)

Test-statistic $H_0 \Rightarrow\ T = \frac{\bar{x}-\mu_0}{S/\sqrt{n}} \sim t_{n-1}$

Test of hypothesis calculate value of test-statistic $T = t$

approach I:

Select level of significance, α (level of confidence $1-\alpha$)

Reject H_0 if $|t| > t_{n-1,\alpha/2}$

approach II:

calculate p-value: $p = 2P(t_{n-1} > |t|)$

Reject H_0 for all $\alpha > p$

Alternative test-statistic $H_0 \Rightarrow\ T^2 = \left(\frac{\bar{x}-\mu_0}{S/\sqrt{n}}\right)^2 = n(\bar{x}-\mu_0)(S^2)^{-1}(\bar{x}-\mu_0)$

$T^2 \sim "(t_{n-1})^2" = F_{1,n-1}$

T^2 could be used for testing H_0, using the $F_{1,n-1}$ distribution

Confidence interval for μ $P\left(-t_{n-1,\alpha/2} < \frac{\bar{x}-\mu}{S/\sqrt{n}} < t_{n-1,\alpha/2}\right) = 1-\alpha \ \Rightarrow$

$100(1-\alpha)\%$ confidence interval for μ:

$\left[\bar{x} - \frac{S}{\sqrt{n}} t_{n-1,\alpha/2},\ \ \bar{x} + \frac{S}{\sqrt{n}} t_{n-1,\alpha/2}\right]$