

MANOVA 2 test-statistics

MANOVA 2 with potential systematic interaction

Model $X_{lkr} \sim N_p(\mu_{lk}, \Sigma_{lk}), l = 1, \dots, g, k = 1, \dots, b, r = 1, \dots, n$
 $\forall \Sigma_{lk} = \Sigma$
 $\mu_{lk} = \mu + \tau_l + \beta_k + \gamma_{lk}$

Sums of squares $SS_T = SS_{B1} + SS_{B2} + SS_{int} + SS_W$

Test for systematic interaction and simplification of model (approximate)

Hypothesis $H_{0,\gamma}: \forall \gamma_{lk} = 0$ (no systematic interaction)
Wilks lambda $\Lambda^* = \frac{|SS_W|}{|SS_{int} + SS_W|}$ reject $H_{0,\gamma}$ for small Λ^* (close to zero)

Test-statistic $T = - \left[gb(n - 1) - \frac{(p+1)-(g-1)(b-1)}{2} \right] \log \Lambda^* \sim X^2_{p(g-1)(b-1)}$

Test reject $H_{0,\gamma}$ for $T > X^2_{p(g-1)(b-1)}(\alpha)$

Conclusion reject $H_{0,\gamma} \Rightarrow$ STOP (model cannot be simplified),
estimation of μ_{lk}, Σ
accept $H_{0,\gamma} \Rightarrow$ test for factor 1 and 2 effects

Test for systematic factor 1 effect (approximate)

Hypothesis $H_{0,\tau}: \forall \tau_l = 0$ (no systematic factor 1 effect)
Wilks lambda $\Lambda^* = \frac{|SS_W|}{|SS_{B1} + SS_W|}$ reject $H_{0,\tau}$ for small Λ^* (close to zero)

Test-statistic $T = - \left[gb(n - 1) - \frac{(p+1)-(g-1)}{2} \right] \log \Lambda^* \sim X^2_{p(g-1)}$

Test reject $H_{0,\tau}$ for $T > X^2_{p(g-1)}(\alpha)$

Conclusion reject $H_{0,\tau} \Rightarrow$ factor 1 is significant

Test for systematic factor 2 effect (approximate)

Hypothesis $H_{0,\beta}: \forall \beta_k = 0$ (no systematic factor 2 effect)
Wilks lambda $\Lambda^* = \frac{|SS_W|}{|SS_{B2} + SS_W|}$ reject $H_{0,\beta}$ for small Λ^* (close to zero)

Test-statistic $T = - \left[gb(n - 1) - \frac{(p+1)-(b-1)}{2} \right] \log \Lambda^* \sim X^2_{p(b-1)}$

Test reject $H_{0,\beta}$ for $T > X^2_{p(b-1)}(\alpha)$

Conclusion reject $H_{0,\beta} \Rightarrow$ factor 2 is significant
