STAT2001 Assignment 1

Do all 8 questions.
Deadline for this assignment is *4th October 5:00p.m.* You can submit to the assignment locker (next to LSB 125) or submit on Blackboard system.

1. (5 marks) A round-robin tournament is being held with 10 tennis players; this means that every player will play against every other player exactly once. How many games are played in total?

2. (8 marks) A city with 6 districts has 6 robberies in a particular week. Assume the robberies are located randomly, with all possibilities for which robbery occurred where equally likely. What is the probability that some district had more than 1 robbery?

3. (10 marks) Suppose that a person chooses a letter at random from R E S E R V E and then choose a letter at random from V E R T I C A L. What is the probability that the same letter is chosen?

4. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them.

   In further investigation at the crime scene, the following are found. The guilty party had a blood type found in 10% of the population. Suspect A does match this blood type. The blood type of Suspect B is not known.

   Define the following 3 events A, M and C,
   A: "A is guilty" (Thus A^c denotes "B is guilty")
   M: "A's blood type matches that of the guilty party"
   C: "B's blood type matches that of the guilty party"

   a. (2 marks) The police reported that suspect A is NOT a relative of suspect B. Is it reasonable to set $P(M|A^c)=P(C|A)=10\%$? Why? (Any justifiable answer can get full mark for this part.)
   b. (8 marks) Assume that $P(M|A^c)=P(C|A)=10\%$. Given the information from the further investigation of the crime scene, what is the probability that A is the guilty party?
   c. (10 marks) Assume that $P(M|A^c)=P(C|A)=10\%$. Given the information from the further investigation of the crime scene, what is the probability that B's blood type matches that of the guilty party?
   (Hint: $P(C|M) = P(C|M,A)P(A|M) + P(C|M,A^c)P(A^c|M)$, as discussed in page 2 of Ch1 suppl2.pdf)

5. Consider four nonstandard dice, whose sides are labeled as follows (the 6 sides on each die are equally likely).

   A: 4, 4, 4, 4, 0, 0
   B: 3, 3, 3, 3, 3, 3
   C: 6, 6, 2, 2, 2, 2
   D: 5, 5, 5, 1, 1, 1

These four dice are each rolled once. Let A be the result for die A, B be the result for die B, etc.
(a) (8 marks) Find P(A > B), P(B > C), P(C >D), and P(D > A).
(b) (10 marks) Is the event A > B independent of the event B > C? Is the event B > C independent of the event C >D? Explain.

6. (12 marks) Let A, B and C be three events of a sample space. Assume $1>P(C)>0$. If $P(A|C)>P(B|C)$ and $P(A|C^c)> P(B|C^c)$. Either prove that $P(A)>P(B)$ or give a counterexample by defining events A, B and C for which that relationship is not true.

7. (15 marks) A worker has asked her supervisor for a confidential letter of recommendation for a new job. She estimates that there is an 80% chance that she will get the job if she receives a strong recommendation, a 40% chance if she receives a moderately good recommendation, and a 10% if she receives a weak recommendation. She further estimates that the probabilities that the recommendation will be strong, moderate and weak are 0.6, 0.3 and 0.1 respectively. Given that she fails to get the job, what is the probability that she received a weak recommendation?

8. (12 marks) Suppose A, B and C are mutually independent events. Show that the events $A \cup B$ and C are independent.

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