Assignment 5 Solution

Section 8.2

8. Let $\mu =$ average salary of all state employees

$$H_0 : \mu = 59593$$
$$H_1 : \mu < 59593 \text{ (one-tailed test)}$$

$$\sigma = 1500, n = 30, \alpha = 0.01$$

$$\begin{aligned} z &= \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \\ &= \frac{58800 - 59593}{1500/\sqrt{30}} \\ &= -2.9 < -2.33 \end{aligned}$$

Therefore, H_0 is rejected.

With $\alpha = 0.01$, we have enough evidence to indicate that the average salary of all state employees is less than \$59593.

24. Let $\mu =$ average number of speeding tickets issued by the police per day

$$H_0 : \mu = 60$$
$$H_1 : \mu \neq 60$$

$$\alpha = 0.05, \bar{X} = 59.93, \sigma = 13.42$$

$$\begin{aligned} z &= \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \\ &= \frac{59.93 - 60}{13.42/\sqrt{30}} \\ &= -0.03 \end{aligned}$$

$$\text{p-value} = 2P(z < -0.03) = 0.976 > 0.05$$

Therefore, H_0 is not rejected.

With $\alpha = 0.05$, we do not have enough evidence to indicate that the average speeding tickets issued by the police is not 60.

Section 8.3

10. Let $\mu =$ average number of words in a novel

$$H_0 : \mu = 50000$$
$$H_1 : \mu > 50000 \text{ (one-sided test)}$$

$$\alpha = 0.1, n = 7, df = n - 1 = 6$$
$$\bar{X} = 50363.57, s = 1113.159$$

$$\begin{aligned} T &= \frac{\bar{X} - 50000}{s/\sqrt{n}} \\ &= \frac{50363.57 - 50000}{1113.159/\sqrt{7}} \\ &= 0.864 < 1.44 = t_{0.1,6} \end{aligned}$$

Therefore, H_0 is not rejected.

With $\alpha = 0.1$, we do not have enogh evidence to show that the mean number of words exceeds 50000.

16. Let $\mu =$ average stipend of teaching assistants in economics in the population.

$$H_0 : \mu = 15000$$
$$H_1 : \mu \neq 15000 \text{ (two-sided test)}$$

$$n = 12, df = n - 1 = 11, \alpha = 0.05$$
$$\bar{X} = 14347.17, s = 2048.54$$

$$\begin{aligned} T &= \frac{\bar{X} - 15000}{s/\sqrt{n}} \\ &= \frac{14347.17 - 15000}{2048.54/\sqrt{12}} \\ &= -1.1 \end{aligned}$$

$$|T| < t_{0.025,11} = 2.201$$

Therefore, H_0 is not rejected.

With $\alpha = 0.05$, there is not enough evidence to conclude that the average stipend differs from \$15000.

Section 8.4

10. Let $p =$ proportion of students who are undergraduates in universities in the particular state.

$$H_0 : p = 0.856$$
$$H_1 : p \neq 0.856 \text{ (two-tailed test)}$$

$$n = 500, \alpha = 0.05$$

$$np_0 = (500)(0.856) = 428 \leq 5$$
$$n(1 - p_0) = (500)(1 - 0.856) = 72 \leq 5$$

Therefore, normal approximation can be used.

$$\begin{aligned} z &= \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \\ &= \frac{\frac{420}{500} - 0.856}{\sqrt{\frac{(0.856)(1-0.856)}{500}}} \\ &= -1.02 \end{aligned}$$

$$|z| < z_{0.025} = 1.96$$

Therefore, H_0 is not rejected.

With $\alpha = 0.05$, we do not have enough evidence to claim that the proportion of students who are undergraduates is different from 0.856 in that state.

12. Let $p =$ proportion of all U.S. households that own two or more TV sets.

$$H_0 : p = 0.83$$
$$H_1 : p < 0.83 \text{ (one-tailed test)}$$

$$n = 300, \alpha = 0.05$$

$$np_0 = (300)(0.83) = 249 \geq 5$$
$$n(1 - p_0) = (300)(1 - 0.83) = 51 \geq 5$$

Therefore, normal approximation can be used.

$$\begin{aligned}
\hat{p} &= \frac{240}{300} = 0.8 \\
z &= \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \\
&= \frac{0.8 - 0.83}{\sqrt{\frac{(0.83)(1-0.83)}{300}}} \\
&= -1.383 > -1.65 = -z_{0.05}
\end{aligned}$$

Therefore, H_0 is not rejected.

With $\alpha = 0.05$, there is not enough evidence to support the claim that the percentage is less than 83%.

Section 8.5

14. Let σ = population standard deviation of the amounts of vitamin C for 100g of selected fruits and vegetables

$$\begin{aligned}
&H_0 : \sigma = 12 \\
&H_1 : \sigma \neq 12 \text{ (two-tailed test)}
\end{aligned}$$

$$n = 13, df = n - 1 = 12, \alpha = 0.01$$

$$s = 14.608$$

$$\begin{aligned}
\chi^2 &= \frac{(n-1)s^2}{\sigma_0^2} \\
&= \frac{(12)(14.608^2)}{12^2} \\
&= 17.783
\end{aligned}$$

$$\chi^2_{12,0.95} = 5.226 < 17.783 < 21.026 = \chi^2_{12,0.05}$$

Therefore, H_0 is not rejected.

With $\alpha = 0.1$, there is not enough evidence to support the claim that the standard deviation differs from 12mg.