STAT 1011: Assignment 3 solutions

Section 5.2

4.

$$\begin{aligned}
\mu &= \sum_x xp(x) \\
&= 5 \cdot 0.05 + 6 \cdot 0.2 + 7 \cdot 0.4 + 8 \cdot 0.1 + 9 \cdot 0.15 + 10 \cdot 0.1 \\
&= 7.4 \\
& \sum_x x^2 p(x) \\
&= 5^2 \cdot 0.05 + 6^2 \cdot 0.2 + 7^2 \cdot 0.4 + 8^2 \cdot 0.1 + 9^2 \cdot 0.15 + 10^2 \cdot 0.1 \\
&= 56.6 \\
\sigma^2 &= \sum_x x^2 p(x) - \mu^2 \\
&= 56.6 - 7.4^2 = 1.84 \\
\sigma &= \sqrt{\sigma^2} \\
&= \sqrt{1.84} \\
&= 1.356
\end{aligned}$$

12.Let X be the number of jobs,

$$\begin{aligned}
Expected\ Profit &= [\sum_x xp(x)] \cdot 3000 \\
&= [1 \cdot 0.2 + 2 \cdot 0.3 + 3 \cdot 0.4 + 4 \cdot 0.1] \cdot 3000 \\
&= 7200.
\end{aligned}$$

Section 5.3

10.Let x= number of students drop out, then $X \sim Binomial(10, 0.103)$.

a.

$$\begin{aligned}
P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\
&= {}_nC_0(0.103)^0(1 - 0.103)^10 + {}_nC_1(0.103)^1(1 - 0.103)^9 + {}_nC_2(0.103)^2(1 - 0.103)^8 \\
&= 0.337 + 0.387 + 0.2 = 0.925
\end{aligned}$$

b.

$$\begin{aligned}
P(X \leq 4) &= P(X \leq 2) + P(X = 3) + P(X = 4) \\
&= 0.925 + 0.06127 + 0.0123 = 0.99857
\end{aligned}$$

c. $P(X=0)=0.337$.

18.$\mu(X)=np, Var(X)=np(1-p)$, if $X \sim Binomial(n,p)$

a. $\mu(X)=1000\cdot 0.1=100,\ Var(X)=1000\cdot 0.1\cdot 0.9=90,\ \sigma(X)=\sqrt{90}=9.49$

b. $\mu(X)=500\cdot 0.25=125,\ Var(X)=500\cdot 0.25\cdot 0.75=93.75,\ \sigma(X)=\sqrt{90}=9.68$

c. $\mu(X)=50\cdot 0.4=20,\ Var(X)=50\cdot 0.4\cdot 0.6=12,\ \sigma(X)=\sqrt{90}=3.464$

d. $\mu(X)=36\cdot\frac{1}{6}=6,\ Var(X)=36\cdot\frac{1}{6}\cdot\frac{5}{6}=5,\ \sigma(X)=\sqrt{90}=2.236$.

Section 5.4

4.Let X_1= # of trailer trucks with no violations, X_2= # of trailer trucks with 1 violation, X_3= # of trailer trucks with 2 or more violations, (X_1,X_2,X_3) satisfies multinomial distribution with $n=5, p_1=0.50, p_2=0.40, p_3=0.10$.

$$P(X_1=3,X_2=1,X_3=1)=\frac{5!}{3!1!1!}(0.5)^3(0.4)^1(0.1)^1=0.1$$

12.Let X= # of orders received with 100 advertisements, then X satisfies possion distribution with λ, where $\lambda=\frac{5}{500}\cdot 100=1$

$$\begin{aligned}\therefore P(X\geq 2) &= 1-P(X\leq 1)\\ &= 1-P(X=0)-P(X=1)\\ &= 1-\frac{1^0\cdot e^{-1}}{0!}-\frac{1^1\cdot e^{-1}}{1!}\\ &= 1-2\cdot 0.36788=0.2642411.\end{aligned}$$

20. Let X= # of defective keyboards in the sample, then X satisfies hypergeometric distribution with n=4, a=6, b=18.

$$\begin{aligned}\therefore P(X\geq 1) &= 1-P(X=0)\\ &= 1-\frac{{}_6C_{0\,18}C_4}{{}_{24}C_4}\\ &= 1-\frac{1\cdot 3060}{10626}=0.712.\end{aligned}$$