STAT4003 Homework Assignment (#3)
(Due Friday, 20 November 2020)

1. Show that the following p.d.f's belong to the exponential family
   (a) Poisson;
   (b) Binomial;
   (c) Negative Binomial;
   (d) Gamma$(\theta; k)$, where $k > 0$ is konwn;
   (e) $N(\theta; 1)$;
   (f) $N(0; \theta)$.

2. Let $X_1, ..., X_n$ denote a random sample of size $n > 2$ from a distribution with p.d.f. $f(x; \theta) = \theta e^{-\theta x}$, $0 < x < \infty$, zero elsewhere, and $\theta > 0$. Then $Y = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ. Prove that $(n-1)/Y$ is the best statistic for θ.

3. Let $X_1, X_2, ..., X_n$ be a random sample from $f(x|\theta) = \theta e^{-\theta x}, \quad x > 0$, where $\theta > 0$.
   (i) Find a $100(1-\alpha)\%$ confidence interval for the mean $1/\theta$;
   (ii) Find a $100(1-\alpha)\%$ confidence interval for the variance $1/\theta^2$;
   (iii) What is the probability that these intervals cover the true mean and true variance, simultaneously?
   (iv) Find a $100(1-\alpha)\%$ confidence interval for $\tau = e^{-\theta}$;

4. Suppose $X_1, ..., X_n$ are independent random variable with distribution $B(1, p)$.
   (a) Find the maximum likelihood estimator of $\theta = (1-p)^2$;
   (b) Show that $\hat{\theta}$ is an unbiased estimator of θ, where $\hat{\theta} = 1$ if $X_1 + X_2 = 0$ and $\hat{\theta} = 0$ otherwise;
   (c) Find the best estimator for θ.

5. In comparing the times until failure (in hours) of two different types of light bulbs, we obtain the sample characteristics $n_1 = 45, \bar{x} = 984$, $s_x^2 = 8742$ and $n_2 = 52, \bar{y} = 1121, s_y^2 = 9411$. Find an approximate 90% confidence interval for the difference of the two population means. Interpret the result and explain why we can use the normal table here despite the fact that the distribution of individual failure times is probably exponential or Weibull.

6. The effectiveness of two methods of teaching statistics is compared. A class of 24 students is randomly divided into two groups and each group is taught according to a different method. Their test scores at the end of the semester show the following characteristics:
   $$n_1 = 13,\ \bar{x} = 74.5,\ s_x^2 = 82.6,\quad n_2 = 11, \bar{y} = 71.8,\ s_y^2 = 112.6$$
   Assuming underlying (approximate) normal distributions with $\sigma_1 = \sigma_2$, find a 95% confidence interval for $\mu_1 - \mu_2$. Compute 90% confidence interval for σ_1/σ_2.