ASSIGNMENT 1
ANSWER

1. $\hat{T}_t = \sum_{r=-1}^{1} a_r T_{t+r} = \alpha + \beta t^2 + \frac{2}{3}\beta \neq T_t$
   The quadratic trend T_t does not pass through the moving average filter.

2. (a) $E[Z_t] = 8 + 4t$
   $Cov(Z_t, Z_{t+k}) = 4Cov(X_t, X_{t+k}) = 4\gamma_k$
   (b) $\{Z_t\}$ is not stationary because the mean is not constant.
   (c) $\Delta Z_t = 4 + 2(X_t - X_{t-1})$
   $E[\Delta Z_t] = 4$
   $Cov(\Delta Z_t, \Delta Z_{t+k}) = 4(2\gamma_k - \gamma_{k+1} - \gamma_{k-1})$
   (d) $\{\Delta Z_t\}$ is stationary because the mean is constant and the autocovariance only depends on time lag.

3. (a) $E[Z_t] = 0$
   $Cov(Z_t, Z_t) = \frac{1}{3}\sigma^2$
   $Cov(Z_t, Z_{t+k}) = \frac{1}{9}\sigma^2,\ k = 1, 2, 3$
   $Cov(Z_t, Z_{t+k}) = 0,\ k \geq 4$
   $\{Z_t\}$ is stationary because the mean is constant and the autocovariance only depends on time lag.
   (b) $\rho_0 = 1$
   $\rho_k = \frac{1}{3},\ k = 1, 2, 3$
   $\rho_k = 0,\ k \geq 4$

   (c) $Var(\frac{1}{5}\sum_{t=1}^{5} Z_t) = \frac{1}{25}(5\gamma_0 + 8\gamma_1 + 6\gamma_2 + 4\gamma_3 + 2\gamma_4) = \frac{11}{75}\sigma^2$

4. (a) $\mu = 0.2\mu => \mu = 0$
   (b) $\gamma_0 = 0.04\gamma_0 + \sigma^2 => \gamma_0 = \frac{\sigma^2}{0.96}$
   (c) $\gamma_k = 0.2\gamma_{k-1}, k = 1, 2, 3, ...$
   $=> \gamma_k = \frac{\sigma^2}{0.96} * 0.2^k, k = 1, 2, 3, ...$

5. (a) $t = 1,\ Z_1 = 0.2^0 a_1 = a_1$
   $t > 1$, suppose $Z_{t-1} = \sum_{k=0}^{t-2} 0.2^k a_{t-k-1}$
   then $Z_t = 0.2Z_{t-1} + a_t = \sum_{k=0}^{t-2} 0.2^{k+1} a_{t-k-1} + a_t$
   $= \sum_{k=1}^{t-1} 0.2^k a_{t-k} + a_t = \sum_{k=0}^{t-1} 0.2^k a_{t-k}$
   By mathematical induction, $Z_t = \sum_{k=0}^{t-1} 0.2^k a_{t-k}$
   (b) $E[Z_t] = E[\sum_{k=0}^{t-1} 0.2^k a_{t-k}] = 0$
   $Var(Z_t) = Var(\sum_{k=0}^{t-1} 0.2^k a_{t-k}) = \sum_{k=0}^{t-1} 0.04^k \sigma^2 = \frac{\sigma^2}{0.96}(1 - 0.04^t)$

(c) $Cov(Z_t, Z_{t-k}) = Cov(\sum_{i=0}^{t-1} 0.2^i a_{t-i}, \sum_{j=0}^{t-k-1} 0.2^j a_{t-k-j})$
$= Cov(\sum_{i=0}^{t-1} 0.2^i a_{t-i}, \sum_{j=k}^{t-1} 0.2^{j-k} a_{t-j})$
$= Cov(\sum_{i=k}^{t-1} 0.2^i a_{t-i}, \sum_{j=k}^{t-1} 0.2^{j-k} a_{t-j})$
$= \sum_{i=k}^{t-1} 0.2^{2i-k}\sigma^2 = \sum_{i=0}^{t-k-1} 0.2^{2i+k}\sigma^2 = \frac{\sigma^2}{0.96}(0.2^k - 0.2^{2t-k})$