STAT2001 Assignment 2

Do all 7 questions. Show your steps clearly.

Deadline for this assignment is *18th Oct. 5:00p.m.* You can submit to the assignment locker (next to LSB 125) or submit on Blackboard system.

1. (15 marks) There are 100 prizes, with one worth \$1, one worth \$2, . . . , and one worth \$100. There are 100 boxes, each of which contains one of the prizes. You get 5 prizes by picking random boxes one at a time, without replacement. Denote X to be the value of the most expensive prize out of the 5 prizes that you get. Find the probability mass function of X (as a simple expression in terms of binomial coefficients).

2. (15 marks) Let X has probability mass function,
$$f(x) = \frac{1}{-\ln(1-p)}\frac{p^x}{x} \quad \text{for } x = 1,2,\ldots$$
where p is a parameter with $0 < p < 1$. Find the mean and variance of X.

3. (10 marks) Let $M(t)$ be the moment generating function of a random variable. The cumulant generating function is defined to be $R(t) = \ln(M(t))$. The jth cumulant of the random variable is defined to be $R^{(j)}(0)$. Now $X \sim Poisson(\lambda)$. Find the jth cumulant of X, for all $j \geq 1$.
(Remark: one can show that the first and second cumulants are the mean and variance of a distribution respectively. See Ex 2.5-5 of textbook for more details.)

4. (15 marks) Given that $E(X+3)=9$, $E[(X+3)^2]=112$, determine (a) $Var(X-4)$, (b) $E(X)$, (c) $Var(X)$.

5. (15 marks) An urn contains 7 white and 5 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white. What is the probability that we shall make at least 8 selections?

6. (15 marks) You are given the moment generating function of X:
$$M(t) = (0.2 + 0.8e^t)^{12}$$
Find the mean of X, second moment of X, and P(X<3).

7. (15 marks) Let X be a discrete random variable with probability mass function:
$$f(x) = c\left(\frac{1}{5}\right)^x, \quad x = 4,5,6,\ldots$$

(a) Find the constant c.
(b) Find the moment generating function of X.

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