MATH1550 Methods of Matrices and Linear Algebra

Suggested Answer for Assignment 2

2-1: (a) The matrix corresponding to the system of equations:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 5 \\ 2 & -5 & -5 & -8 \end{array}\right)$$

The RREF of the matrix is

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Thus the solution is $\{(1, -t+2, t) | t \in \mathbb{R}\}$

(b) The The RREF of the corresponding matrix is

$$\left(\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

So the solution set is $\{(5+9t, -3-2t, t, 2) | t \in \mathbb{R}\}$

2-2: (a) The RREF of the corresponding matrix is

$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & -5 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & -a-1 & b-25 \end{array}\right)$$

So we have $a = -1$ and $b = 25$.

(b) From (a), the solution set is $\{(-5-7t, 8+4t, t) | t \in \mathbb{R}\}$

2-3: Let x_1, x_2, x_3, x_4 be the number of cars, trucks, motorcycles and bicycles respectively. The follow systems of equations holds:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 66 \\ x_1 - x_4 &= 0 \\ 4x_1 + 4x_2 + 2x_3 + 2x_4 &= 252 \end{aligned}$$

The RREF of the corresponding matrix is

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 48 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 1 & 6 \end{array}\right)$$

So there are 48 cars and the number of bicycles can be 0, 1, 2, 3, 4, 5 or 6.

2-4: The RREF of A is

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So $\mathcal{N}(A) = \{(-2s-5t, s, 8t, -2t, t) | s, t \in \mathbb{R}\}$

2-5: (a) By setting $Ax_0 = 0$, where $x_0 = (-3, 1, 6, -2, 1)^T$, we have $a = 5, b = -6, c = 2$.

(b) $\mathcal{N}(A) = \{(2s - 5t, s, 6t, -2t, t) | s, t \in \mathbb{R}\}$

(c) The solution set is $\{(2s - 5t + 2, s, 6t - 3, -2t + 1, t + 4) | s, t \in \mathbb{R}\}$

(d) Note that $A(3\mathbf{x}_1 - \mathbf{x}_2) = 3\mathbf{b}_1 - \mathbf{b}_2$. So $3x_1 - x_2 = (1, 2, -13, 3, 9)$ is a solution of $A\mathbf{x} = 3\mathbf{b}_1 - \mathbf{b}_2$. Then the solution set is $\{(2s - 5t + 1, s + 2, 6t - 13, -2t + 3, t + 9) | s, t \in \mathbb{R}\}$

2-6: (a) The inverse is

$$\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

(b) The inverse is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{1}{7} \end{pmatrix}$$