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STAT 4005

Assignment 1

Question 1

$\hat{T}_t = \sum a_r T_{t+r}$

$= \frac{1}{3}[a+\beta(t-1)^2 + a+\beta(t)^2 + a+\beta(t+1)^2]$

$= a + \beta t^2 + \frac{2\beta}{3}$

$\neq T_t$

the quadratic trend does not pass through the moving average filter

Question 2

a) $E(Z_t) = 8 + 4t$

$\gamma(k) = Cov(8+4t+2X_t, 8+4(t+k)+2X_{t+k})$

$= 4\gamma_k$

b) No, since $E(Z_t)$ depends on time t

c) $\Delta Z_t = 4 + 2(X_t - X_{t-1})$

$E(\Delta Z_t) = 4$

$$\gamma(k) = \begin{cases} 4\gamma_k + 4\gamma_k & \text{for } k=0 \\ -4\gamma_k & \text{for } k=\pm 1 \\ 0 & \text{otherwise} \end{cases}$$

d) Yes, both $E(\Delta Z_t)$ and $\gamma(k)$ are independent of time t

Question 3

a) $E(Z_t) = 0$

$$\gamma(k) = \begin{cases} \frac{1}{3}\sigma^2 & k=0 \\ \frac{1}{9}\sigma^2 & k=1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

con't a) Since both $E(Z_t)$ and $\gamma(k)$ are independent of time t, Z_t is weakly stationary

b)
$$\rho(k) = \begin{cases} 1 & \text{for } k=0 \\ \frac{1}{3} & \text{for } k=1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

c) $Var(\frac{1}{5}\sum Z_t) = \frac{1}{5^2}[5\cdot\frac{1}{3}\sigma^2 + 2(4+3+2)\cdot\frac{1}{9}\sigma^2]$

$= \frac{11}{75}\sigma^2$

Question 4

a) $E(Z_t) = 0.2E(Z_{t-1})$

the equality holds only when $E(Z_t)=0$, given Z_t is stationary, thus, $E(Z_t)=0$

b) $\gamma(0) = Var(0.2Z_{t-1} + a_t) + 2Cov(0.2Z_{t-1}, a_t)$

$= 0.04\gamma(0) + \sigma^2$

$= \frac{\sigma^2}{0.96}$

c) $Cov(Z_t, Z_{t-k}) = Cov(0.2Z_{t-1} + a_{t-1}, Z_{t-k})$

$= 0.2^k Cov(Z_{t-k}, Z_{t-k})$

given base case is $t=0$, such that

$= 0.2^k \gamma(0)$

$= 0.2^k \cdot \frac{\sigma^2}{0.96}$

Question 5

a) $Z_t = 0.2Z_{t-1} + a_t$

$= \sum_{k=0}^{t-2} 0.2^{k+1} a_{t-k-1} + a_t$

$= \sum_{k=1}^{t-1} 0.2^k a_{t-k} + a_t$

$= \sum_{k=0}^{t-1} 0.2^k a_{t-k}$

b) $E(Z_t) = 0$

con't b) $$\gamma(0) = \sum_{k=0}^{t-1} 0.2^{2k} \, Var(a_{t-k})$$

$$= \sigma^2 \sum_{k=0}^{t-1} 0.04^k$$

$$= \frac{1-0.04^k}{0.96} \sigma^2$$

c) $$Cov(Z_t, Z_{t-k}) = Cov\left(\sum_{i=0}^{t-1} 0.2^i a_{t-i}, \sum_{j=0}^{t-k-1} 0.2^j a_{t-k-j}\right)$$

$$= Cov\left(\sum_{i=0}^{t-1} 0.2^i a_{t-i}, \sum_{j=k}^{t-1} 0.2^{j-k} a_{t-j}\right)$$

$$= Cov\left(\sum_{i=k}^{t-1} 0.2^i a_{t-i}, \sum_{j=k}^{t-1} 0.2^{j-k} a_{t-j}\right)$$

$$= \sum_{i=k}^{t-1} 0.2^{2i-k} \sigma^2$$

$$= \sum_{i=0}^{t-k-1} 0.2^{2i+k} \sigma^2$$

$$= \frac{0.2^k - 0.2^{2t-k}}{0.96} \sigma^2$$

Question 6

please kindly refer to the next page.

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STAT4005 Assignment 1

Question 6

After decomposition, we have the following results

Estimated trend

```
  [1]      NA      NA 628.750 663.000 680.000 672.875 645.875 611.000 585.125 572.125 574.000 576.625
 [13] 587.750 611.625 640.750 677.250 696.875 690.750 663.875 627.875 599.875 585.625 588.875 598.375
 [25] 617.625 646.375 677.500 710.000 723.500 714.000 685.000 648.625 622.500 611.375 617.500 625.625
 [37] 641.375 666.750 693.250 725.500 741.375 731.875 702.375 664.625 635.250 620.500 626.000 635.500
 [49] 654.000 685.750 717.500 754.625 774.875 767.500 741.500 706.375 678.750 666.375 671.750 678.250
 [61] 692.875 719.500 747.000 781.250 800.625 793.500 766.250 730.500 700.875 684.500 686.000 691.625
 [73] 707.125 733.750 763.625 797.500 814.750 808.750 782.125 747.500 719.250 702.375 702.875 705.000
 [85] 716.500 741.750 770.625 807.125 828.500 825.375 802.000 768.500 739.375 721.750 721.125 722.250
 [97] 734.875 760.750 789.000 825.000 845.500 841.875 818.500 786.250 758.750 741.250 745.250 754.375
[109] 774.125 808.625 841.750 878.625 899.375 895.000 870.500 837.375 810.000 792.500 792.750 797.125
[121] 812.875 841.250 870.625 904.500 922.000 916.875 892.250 857.625 826.000 803.750 799.000 795.875
[133] 805.875 832.125 860.750 896.625 915.250 907.375 878.125 839.875 808.125 788.625 789.750 793.625
[145] 808.125 836.750 866.875 905.625 929.125 927.125 903.500 867.375 833.625 810.000 805.625 804.250
[157] 815.375 841.500 869.250 905.125 925.000 919.875 895.625 863.250 837.125 822.875      NA      NA
```

Estimated seasonality

```
  [1]      NA      NA 628.750 663.000 680.000 672.875 645.875 611.000 585.125 572.125 574.000 576.625
 [13] 587.750 611.625 640.750 677.250 696.875 690.750 663.875 627.875 599.875 585.625 588.875 598.375
 [25] 617.625 646.375 677.500 710.000 723.500 714.000 685.000 648.625 622.500 611.375 617.500 625.625
 [37] 641.375 666.750 693.250 725.500 741.375 731.875 702.375 664.625 635.250 620.500 626.000 635.500
 [49] 654.000 685.750 717.500 754.625 774.875 767.500 741.500 706.375 678.750 666.375 671.750 678.250
 [61] 692.875 719.500 747.000 781.250 800.625 793.500 766.250 730.500 700.875 684.500 686.000 691.625
 [73] 707.125 733.750 763.625 797.500 814.750 808.750 782.125 747.500 719.250 702.375 702.875 705.000
 [85] 716.500 741.750 770.625 807.125 828.500 825.375 802.000 768.500 739.375 721.750 721.125 722.250
 [97] 734.875 760.750 789.000 825.000 845.500 841.875 818.500 786.250 758.750 741.250 745.250 754.375
[109] 774.125 808.625 841.750 878.625 899.375 895.000 870.500 837.375 810.000 792.500 792.750 797.125
[121] 812.875 841.250 870.625 904.500 922.000 916.875 892.250 857.625 826.000 803.750 799.000 795.875
[133] 805.875 832.125 860.750 896.625 915.250 907.375 878.125 839.875 808.125 788.625 789.750 793.625
[145] 808.125 836.750 866.875 905.625 929.125 927.125 903.500 867.375 833.625 810.000 805.625 804.250
[157] 815.375 841.500 869.250 905.125 925.000 919.875 895.625 863.250 837.125 822.875      NA      NA
```

Estimated noise

```
          Qtr1         Qtr2         Qtr3         Qtr4
1           NA           NA  15.04878049  -5.50000000
2   34.05792683  31.76829268  -2.07621951 -10.50000000
3  -30.06707317  12.51829268 -17.20121951   6.87500000
4   -0.69207317 -37.98170732  16.04878049  -2.75000000
5   32.18292683  32.89329268  -0.07621951  -9.37500000
6  -29.81707317   9.01829268 -20.07621951   1.12500000
7   -2.56707317 -20.73170732  14.29878049  -3.50000000
8   33.55792683  29.64329268  -3.20121951  -8.12500000
9  -31.44207317   7.26829268 -19.70121951   9.87500000
10   3.68292683 -37.10670732  19.54878049  -2.00000000
11  27.68292683  31.76829268   3.42378049 -10.12500000
12 -33.19207317   8.14329268 -20.20121951   1.00000000
13  10.05792683 -43.10670732  22.29878049   1.87500000
14  23.18292683  38.14329268  -2.70121951  -7.87500000
15 -30.69207317   8.26829268 -22.95121951  11.25000000
16   7.18292683 -44.85670732  18.79878049   4.25000000
17  23.43292683  31.14329268   4.54878049  -7.00000000
18 -32.81707317  10.14329268 -22.20121951   7.87500000
19  -3.06707317 -30.10670732  15.17378049   0.00000000
20  30.30792683  24.89329268   4.67378049  -6.00000000
21 -31.19207317  11.26829268 -22.07621951   7.50000000
22   4.55792683 -44.10670732  18.17378049  -0.62500000
23  29.55792683  27.26829268   2.79878049  -3.00000000
24 -27.31707317   8.89329268 -27.32621951  13.25000000
25   2.18292683 -46.10670732  21.79878049   0.50000000
26  27.55792683  24.76829268   4.29878049  -1.75000000
27 -31.69207317  13.39329268 -30.45121951  -1.87500000
28  16.93292683 -44.98170732  22.04878049   0.87500000
29  29.68292683  25.64329268   2.29878049  -1.87500000
30 -32.94207317  15.14329268 -25.95121951   4.37500000
31   0.18292683 -34.60670732  23.17378049  -3.00000000
32  26.05792683  25.76829268   5.54878049  -1.12500000
33 -29.94207317  13.89329268 -29.20121951  10.62500000
34   2.18292683 -51.48170732  26.04878049   2.87500000
35  28.80792683  24.26829268   6.67378049  -1.37500000
36 -37.06707317  10.01829268 -25.95121951   9.87500000
37   6.93292683 -51.10670732  25.92378049  -2.12500000
38  26.93292683  27.51829268   8.29878049   1.12500000
39 -31.56707317   9.64329268 -28.82621951  10.25000000
40   5.68292683 -51.85670732  26.54878049  -0.62500000
41  28.05792683  24.76829268   4.17378049  -3.75000000
42 -33.06707317  11.76829268           NA           NA
```

Appendix

```
data = read.csv('monthly_milk.csv')
data = data[1:168, 2]
data = ts(data, frequency = 4)

n = length(data)
T1 = rep(NA,n)
filter = c(0.125, 0.25, 0.25, 0.25, 0.125)
radius = 2
start = 3
end = n - 2
for(k in start:end)
  T1[k] = filter %*% x[(k - radius):(k + radius)]
T1

d = 4
D = data - T1
D.bar = mean(D, na.rm = T)
S.mat = matrix(D - D.bar, ncol = d, byrow = T)
S = apply(S.mat, 2, mean, na.rm = T)
(S = rep(S, n / d))

(N = data - T1 - S)
```