STAT 4005 Time Series
Assignment 1
Due date: 3 Feb 2021; 5pm

Let $a_t \sim WN(0, \sigma^2)$

1. Does the quadratic trend $T_t = \alpha + \beta t^2$ pass through the moving average filter $(a_{-1}, a_0, a_1) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$?

2. Suppose $Z_t = 8 + 4t + 2X_t$, where X_t is a zero-mean stationary series with autocovariance function γ_k.

   (a) Find the mean and the autocovariance function of Z_t.

   (b) Is Z_t stationary? Why?

   (c) Find the mean and the autocovariance function of $\Delta Z_t = (1 - B)Z_t$.

   (d) Is ΔZ_t stationary? Why?

3. Suppose that $Z_t = (a_t + a_{t-1} + a_{t-3})/3$

   (a) Show that Z_t is weakly stationary.

   (b) Find ρ_k, $k = 0, 1, 2, 3, ...$

   (c) Find $\text{Var}\left(\frac{1}{5}\sum_{t=1}^{5} Z_t\right)$.

4. Consider the time series $\{Z_t\}$ satisfying

$$Z_t = 0.2Z_{t-1} + a_t.$$

   (a) Assuming that $\{Z_t\}$ is stationary, find the mean $E(Z_t)$.

   (b) Assuming that $\{Z_t\}$ is stationary and $\text{Cov}(Z_s, a_t) = 0$ for $s < t$, find the variance $\text{Var}(Z_t)$. (Hints: take variance on both sides.)

   (c) Find $\text{Cov}(Z_t, Z_{t-k})$ for $k = 1, 2, 3, ...$ (Hints: multiply Z_{t-k} on both sides and take expectation.)

5. Consider the time series $\{Z_t\}$ satisfying

$$Z_1 = a_1;\ Z_t = 0.2Z_{t-1} + a_t \text{ for } t > 1.$$

   (a) By mathematical induction, show that $Z_t = \sum_{k=0}^{t-1} 0.2^k a_{t-k}$.

   (b) Find the mean $E(Z_t)$ and the variance $\text{Var}(Z_t)$.

   (c) Find $\text{Cov}(Z_t, Z_{t-k})$ for $t > k$ and $k \geq 0$.

6. Consider the data set **monthly_milk.csv** in the class website that contains the monthly milk production from 1962 to 1975. Using `R`, decompose the series into three components.