NAME: CHAN King Yeung ID: 1155119394 DATE: MATH1550 Assignment 1

~~8/10~~ 9/10

Question 1

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 2 \\ 3 & 7 & 11 & 8 \end{array}\right)$$

$$\overset{\substack{-2R_1+R_2 \\ -3R_1+R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 5 \end{array}\right)$$

$$\overset{\substack{R_2 \leftrightarrow R_3 \\ -2R_3+R_2 \\ -3R_3+R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\overset{-2R_2+R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

The solution set is
$\{(-9, 5, 0)\}$

Question 2

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 13 \end{array}\right)$$

$$\overset{\substack{-R_1+R_2 \\ -R_1+R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 8 & 14 \end{array}\right)$$

$$\overset{-2R_2+R_3}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 6 \end{array}\right)$$

$$\overset{\substack{\frac{R_3}{2} \\ -3R_3+R_2 \\ -R_3+R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & 0 & -4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\overset{-R_2+R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Therefore, $f(t) = 1 - 5t + 3t^2$

Question 3

a)
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 3 & -5 & 13 & 18 \\ 1 & -2 & 5 & k \end{array}\right)$$

$$\overset{\substack{-3R_1+R_2 \\ -R_1+R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -8 & 16 & 24 \\ 0 & -3 & 6 & k+2 \end{array}\right)$$

$$\overset{\substack{\frac{R_2}{-8} \\ 3R_2+R_3}}{\sim} \left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & k-7 \end{array}\right)$$

The linear system have solutions if and only if $k = 7$

b)
$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\overset{-R_2+R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The solution set is
$\{(1-a, -3+2a, a) \mid a \in \mathbb{R}\}$

Question 4

Let x_1 be hundreds-digit;
x_2 be tens-digit;
x_3 be ones-digit

property ①:
$x_2 + x_3 = 5$

property ②:
$100x_1 + 10x_2 + x_3 - 100x_3 - 10x_2 - x_1 = 792$
$99x_1 - 99x_3 = 792$

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 5 \\ 99 & 0 & -99 & 792 \end{array}\right)$$

$$\overset{\substack{\frac{R_2}{99} \\ R_1 \leftrightarrow R_2}}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 8 \\ 0 & 1 & 1 & 5 \end{array}\right)$$

$x_1 = 8 + a$
$x_2 = 5 - a$
$x_3 = a$

(continue on the next page)

each digit should be restricted in a range of $[0, 9]$, thus

$$x_1 = 8 + a \quad , \quad a \in [-8, 1]$$
$$x_2 = 5 - a \quad , \quad a \in [-4, 5]$$
$$x_3 = a \quad , \quad a \in [0, 9]$$

where a should be restricted in a range of $[0, 1]$

Therefore, the possible digits are 850 and 941

Question 5

a) $(J^T B)_{ij} = m_j$ for $0 \leq i \leq m$, $0 \leq j \leq n$

b) $(B J^T)_{ij} = [(J^T B)^T]_{ij}$
$= (J^T B)_{ji}$
$= m_i$ for $0 \leq i \leq m$, $0 \leq j \leq n$

-1

Question 6

$(AA^T)^T = (A^T)^T A^T$
$= AA^T$

$\therefore$ AA^T is symmetric

Question 7

Let X be $\frac{1}{2}(A+A^T)$;
Y be $\frac{1}{2}(A-A^T)$

$X + Y = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$
$= A$

Given that

X is a symmetric matrix

$X^T = [\frac{1}{2}(A+A^T)]^T$
$= \frac{1}{2}[A^T + (A^T)^T]$
$= \frac{1}{2}(A+A^T)$
$= X$

Y is a skew-symmetric matrix

$Y^T = [\frac{1}{2}(A-A^T)]^T$
$= \frac{1}{2}[A^T - (A^T)^T]$
$= -\frac{1}{2}(A-A^T)$
$= -Y$