# STAT 2006 Assignment 4 (Optional)

1. Data are given for the melting points for 50 metal alloy filaments:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 320 | 326 | 325 | 318 | 322 | 320 | 329 | 317 | 316 | 331 |
| 320 | 320 | 317 | 329 | 316 | 308 | 321 | 319 | 322 | 335 |
| 318 | 313 | 327 | 314 | 329 | 323 | 327 | 323 | 324 | 314 |
| 308 | 305 | 328 | 330 | 322 | 310 | 324 | 314 | 312 | 318 |
| 313 | 320 | 324 | 311 | 317 | 325 | 328 | 319 | 310 | 324 |

   Use 8 classes of equal probability to test the hypothesis, with $\alpha = 0.05$, that these observations come from a normal distribution. Note that you must first estimate two parameters, $\mu$ and $\sigma$.

2. Let $X$ equal the number of male children in a four-child family. Among students who were taking statistics, 79 came from families with four children. For these families, $x = 0, 1, 2, 3$, and 4 for $13, 22, 24, 19$, and 1 families, respectively.

   (a) Test whether the distribution follows Binomial$(4, 0.5)$ for $\alpha = 0.1$.

   (b) Test whether the distribution follows Binomial$(4, \hat{p})$ where $\hat{p}$ is the MLE for $p$ for $\alpha = 0.1$.

3. There is a bag of 224 pieces of candy, each colored brown, orange, green, or yellow. Test the null hypothesis that the machine filling the bag treats the four colors of candy equally likely; that is, test

$$H_0 : p_B = p_O = p_G = p_Y = \frac{1}{4}.$$

   The observed values were 42 brown, 64 orange, 53 green, and 65 yellow candies. Use the significance level $\alpha = 0.05$.

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from Binomial$(1, p)$.

   (a) Show that $\bar{X}$ is an unbiased estimator of $p$ and find $\text{Var}(\bar{X})$.

   (b) Find the Rao-Cramér lower bound for the variance of every unbiased estimator of $p$.

   (c) What is the efficieny of $\bar{X}$ as an estimator of $p$ and what can you conclude?

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$.

   (a) Find the MLE $\widehat{\sigma^2}$ for $\sigma^2$ and show that it is unbaised.

   (b) Find $\text{Var}(\widehat{\sigma^2})$.

   (c) Find the Rao-Cramér lower bound for the variance of every unbiased estimator of $\sigma^2$.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

   (a) Show that the sample variance, $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$, is an unbiased estimator of $\sigma^2$.

   (b) Show that $S^2$ does not attain the Rao-Cramér lower bound.