CITY UNIVERSITY OF HONG KONG STUDENTS' UNION
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STAT 4006

Problem Sheet 1

Question 1

a) RV: attitude toward gun control
EV: gender, mother's education

b) RV: heart disease
EV: blood pressure, cholesterol level

c) RV: vote for president
EV: race, religion, annual income

d) RV: quality of life
EV: marital status

Question 2

$H_0: \pi = \frac{2}{3}$ vs $H_1: \pi \neq \frac{2}{3}$

$$Z_{wald} = \frac{\frac{854}{1103} - \frac{2}{3}}{\sqrt{\frac{854}{1103}\left(\frac{249}{1103}\right)/1103}}$$

$$\approx 8.5465$$

$\text{p-value}_{wald} \approx 0$

$$Z_{score} = \frac{\frac{854}{1103} - \frac{2}{3}}{\sqrt{\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)/1103}}$$

$$\approx 7.5796$$

$\text{p-value}_{score} \approx 3.4639 \times 10^{-14}$

$$G^2_{LR} = 2(854)\ln\left(\frac{\frac{854}{1103}}{\frac{2}{3}}\right) + 2(249)\ln\left(\frac{\frac{249}{1103}}{\frac{1}{3}}\right)$$

$$\approx 61.4468$$

$\text{p-value}_{LR} \approx 4.5519 \times 10^{-15}$

Since all the p-values < 0.05, we reject H_0 at $\alpha = 0.05$

Question 3

a) $$2\sum y_i \ln\left(\frac{\hat{\pi}}{\pi_0}\right) + 2\left(n - \sum y_i\right)\ln\left(\frac{1-\hat{\pi}}{1-\pi_0}\right) \leq \chi^2_{1;\alpha}$$

$$2n\ln\left(\frac{1}{1-\pi_0}\right) \leq z^2_{\alpha/2}$$

$$\ln(1-\pi_0) \geq -\frac{z^2_{\alpha/2}}{2n}$$

$$\pi_0 \leq 1 - e^{-\frac{z^2_{\alpha/2}}{2n}}$$

$$\therefore \pi_0 \in \left[0, 1 - e^{-\frac{z^2_{\alpha/2}}{2n}}\right]$$

$$e^{-\frac{z^2_{\alpha/2}}{2n}} = \sum \frac{\left(-\frac{z^2_{\alpha/2}}{2n}\right)^i}{i!}$$

$$= 1 - \frac{z^2_{\alpha/2}}{2n}$$

$$\therefore \pi_0 \in \left[0, \frac{z^2_{\alpha/2}}{2n}\right]$$

$$\approx \left[0, \frac{(2)^2}{2n}\right]$$

$$\approx \left[0, \frac{1.92}{n}\right]$$

b) $$\left|\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}\right| \leq z_{\alpha/2}$$

$$\pi_0^2 \leq z^2_{\alpha/2}\frac{\pi_0(1-\pi_0)}{n}$$

$$\Rightarrow (z^2_{\alpha/2} + n)\pi_0^2 + (-z^2_{\alpha/2})\pi_0 = 0$$

$$\Rightarrow \pi_0 = \frac{z^2_{\alpha/2} \pm \sqrt{z^4_{\alpha/2}}}{2(z^2_{\alpha/2} + n)}$$

$$= \left[0, \frac{z^2_{\alpha/2}}{z^2_{\alpha/2} + n}\right]$$

Question 4

$$\left|\frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}\right| \leq z_{\alpha/2}$$

$$p^2 - 2p\pi + \pi^2 \leq z^2_{\alpha/2}\frac{\pi(1-\pi)}{n}$$

$$\Rightarrow \left(1 + \frac{z^2_{\alpha/2}}{n}\right)\pi^2 + \left(-2p - \frac{z^2_{\alpha/2}}{n}\right)\pi + p^2 = 0.$$

$$\Rightarrow \pi = \frac{2p + \frac{z^2_{\alpha/2}}{n} \pm \sqrt{\left(-2p - \frac{z^2_{\alpha/2}}{n}\right)^2 - 4\left(1 + \frac{z^2_{\alpha/2}}{n}\right)p^2}}{2\left(1 + \frac{z^2_{\alpha/2}}{n}\right)}$$

$$= \frac{p + \frac{z^2_{\alpha/2}}{2n}}{1 - \frac{z^2_{\alpha/2}}{n}} \pm \frac{z_{\alpha/2}}{2\left(1 - \frac{z^2_{\alpha/2}}{n}\right)}\sqrt{\frac{4p}{n} + \frac{z^2_{\alpha/2}}{n^2} - \frac{4p^2}{n}}$$

$$= \frac{p + \frac{z^2_{\alpha/2}}{2n}}{1 - \frac{z^2_{\alpha/2}}{n}} \pm \frac{z_{\alpha/2}}{1 - \frac{z^2_{\alpha/2}}{n}}\sqrt{\frac{p(1-p)}{n} + \frac{z^2_{\alpha/2}}{4n^2}}$$

Question 5

a) Count: 10 13 21 23 29

EX_i: 9.6 9.6 19.2 33.6 24

$H_0: \pi_1 = \pi_2 = 0.1, \pi_3 = 0.2, \pi_4 = 0.35, \pi_5 = 0.25$ vs H_1: not H_0

$$X^2 = \frac{(10-9.6)^2}{9.6} + \frac{(13-9.6)^2}{9.6} + \frac{(21-19.2)^2}{19.2} + \frac{(23-33.6)^2}{33.6} + \frac{(29-24)^2}{24}$$

≈ 5.7753

critical value = 9.488

Since $X^2 < 9.488$, we do not reject H_0 at $\alpha = 0.05$

$$G^2 = 2\left[10\ln\left(\frac{10}{9.6}\right) + 13\ln\left(\frac{13}{9.6}\right) + 21\ln\left(\frac{21}{19.2}\right) + 23\ln\left(\frac{23}{33.6}\right) + 29\ln\left(\frac{29}{24}\right)\right]$$

≈ 6.0036

critical value = 9.488

Since $G^2 < 9.488$, we do not reject H_0 at $\alpha = 0.05$

b) $\pi_1 = \pi_2,\ \pi_3 = \pi_4,\ \pi_5 = 1 - 2\pi_1 - 2\pi_3$

$$\ell(\pi) = (n_1+n_2)\ln(\pi_1) + (n_3+n_4)\ln(\pi_3) + n_5\ln(1-2\pi_1-2\pi_3)$$

$\left.\frac{\partial \ell}{\partial \pi_1}\right|_{\hat{\pi}} = 0$

$0 = \frac{n_1+n_2}{\hat{\pi}_1} - \frac{2n_5}{1-2\hat{\pi}_1-2\hat{\pi}_3}$

$\frac{n_1+n_2}{\hat{\pi}_1} = \frac{2n_5}{1-2\hat{\pi}_1-2\hat{\pi}_3}$

$\Rightarrow \frac{n_1+n_2}{\hat{\pi}_1} = \frac{n_3+n_4}{\hat{\pi}_3}$

$\hat{\pi}_1 = \frac{n_1+n_2}{n_3+n_4}\hat{\pi}_3$

$\left.\frac{\partial \ell}{\partial \pi_3}\right|_{\hat{\pi}} = 0$

$0 = \frac{n_3+n_4}{\hat{\pi}_3} - \frac{2n_5}{1-2\hat{\pi}_1-2\hat{\pi}_3}$

$\frac{n_3+n_4}{\hat{\pi}_3} = \frac{2n_5}{1-2\hat{\pi}_1-2\hat{\pi}_3}$

$\Rightarrow \frac{n_1+n_2}{\frac{n_1+n_2}{n_3+n_4}\hat{\pi}_3} = \frac{2n_5}{1-2\hat{\pi}_3\left(\frac{n_1+n_2}{n_3+n_4}\right)-2\hat{\pi}_3}$

$\frac{2n_5\hat{\pi}_3}{n_3+n_4} = 1 - 2\hat{\pi}_3\left(\frac{n_1+n_2}{n_3+n_4}\right) - 2\hat{\pi}_3$

$\hat{\pi}_3 = \frac{n_3+n_4}{2n}$

$$\Rightarrow \hat{\pi}_1 = \hat{\pi}_2 = \frac{n_1+n_2}{2n},\quad \hat{\pi}_3 = \hat{\pi}_4 = \frac{n_3+n_4}{2n},\quad \hat{\pi}_5 = \frac{n_5}{n}$$

c) Count: 10 13 21 23 29

EX_i: 11.5 11.5 22 22 29

$H_0: \pi_1 = \pi_2, \pi_3 = \pi_4$ vs H_1: not H_0

$$X^2 = \frac{(10-11.5)^2}{11.5} + \frac{(13-11.5)^2}{11.5} + \frac{(21-22)^2}{22} + \frac{(23-22)^2}{22} + \frac{(29-29)^2}{29}$$

≈ 0.4822

$$G^2 = 2\left[10\ln\left(\frac{10}{11.5}\right) + 13\ln\left(\frac{13}{11.5}\right) + 21\ln\left(\frac{21}{22}\right) + 23\ln\left(\frac{23}{22}\right) + 29\ln\left(\frac{29}{29}\right)\right]$$

≈ 0.4834

critical value = 5.991

Since both X^2 and $G^2 < 5.991$, we do not reject H_0 at $\alpha = 0.05$

Question 6

$X \sim Poi(4.1)$

X_i	0	1	2	3	4	5	6	7	8	9+
π_i	0.017	0.068	0.139	0.19	0.195	0.16	0.109	0.064	0.033	0.025
Count:	5	11	18	29	26	25	15	10	7	4
EX_i	2.55	10.2	20.85	28.5	29.25	24	16.35	9.6	4.95	3.75

H_0: $X \sim Poi(4.1)$ vs H_1: X is not $\sim Poi(4.1)$

$$X^2 = \frac{(5-2.55)^2}{2.55} + \frac{(11-10.2)^2}{10.2} + \frac{(18-20.85)^2}{20.85} + \frac{(29-28.5)^2}{28.5} + \frac{(26-29.25)^2}{29.25} + \frac{(25-24)^2}{24} + \frac{(15-16.35)^2}{16.35} + \frac{(10-9.6)^2}{9.6} + \frac{(7-4.95)^2}{4.95} + \frac{(4-3.75)^2}{3.75}$$

≈ 4.2116

$$G^2 = 2\left[5\ln\left(\frac{5}{2.55}\right) + 11\ln\left(\frac{11}{10.2}\right) + 18\ln\left(\frac{18}{20.85}\right) + 29\ln\left(\frac{29}{28.5}\right) + 26\ln\left(\frac{26}{29.25}\right) + 25\ln\left(\frac{25}{24}\right) + 15\ln\left(\frac{15}{16.35}\right) + 10\ln\left(\frac{10}{9.6}\right) + 7\ln\left(\frac{7}{4.95}\right) + 4\ln\left(\frac{4}{3.75}\right)\right]$$

≈ 3.6271

critical value = 15.51

Since both X^2 and $G^2 < 15.51$, we do not reject H_0 at $\alpha = 0.05$