

**STAT 4005 Time Series**  
**Assignment 2**  
**Due date: 24 Feb 2021; 5pm**

Let  $a_t \sim WN(0, \sigma^2)$  if not specified otherwise.

1. Let  $X_t$  be a stationary time series with mean  $\alpha$  and autocovariances  $\gamma_k = 0.8^k$  and  $\bar{X} = \sum_{t=6}^{10} X_t/5$ 
  - (a) Find  $E(\bar{X})$ .
  - (b) Find  $\text{Var}(\bar{X})$ .
2. Consider the process

$$Z_t = a_t + a_{t-1} + 0.25a_{t-2}, \quad \sigma_a^2 = 20.$$

- (a) Identify the order of the ARIMA model for the process.
- (b) Is  $\{Z_t\}$  stationary?
- (c) Is  $\{Z_t\}$  invertible?
- (d) Find the ACVF  $\gamma(k)$  and ACF  $\rho(k)$  of  $\{Z_t\}$  for  $k = 0, 1, 2, 3, \dots$
- (e) Find the values of  $\pi_k$ ,  $k = 0, 1, 2, 3, \dots$  if the process is written as

$$a_t = \sum_{i=0}^{\infty} \pi_i Z_{t-i}.$$

3. Consider the AR(2) process

$$Z_t = 0.5Z_{t-1} - 0.06Z_{t-2} + a_t,$$

where  $a_t$ s are independently and identically distributed as  $N(0, 1)$ .

- (a) Find the roots of the AR characteristic equation.
  - (b) Is the process  $Z_t$  stationary and causal? Why?
  - (c) Find the autocovariances  $\gamma(0)$ ,  $\gamma(1)$  and  $\gamma(2)$ .
4. Find ACVF  $\gamma(k)$ ,  $k=0,1,2,3,\dots$  of the process

$$Z_t = 0.7Z_{t-4} + a_t.$$

5. Find the AR and MA representation of the process

$$Z_t = 0.6Z_{t-1} + a_t + 0.2a_{t-1}, \quad a_t \sim WN(0, 4).$$

6. Identify the following as specific ARIMA models:

- a)  $Z_t = 1.5Z_{t-1} - 0.5Z_{t-2} + a_t - 0.3a_{t-1} + 0.6a_{t-2}$ .
- b)  $Z_t = 3Z_{t-1} - 3Z_{t-2} + Z_{t-3} + a_t + 0.1a_{t-1}$ .

7. Consider the ARMA(2,1) model

$$Z_t = 0.6Z_{t-1} - 0.09Z_{t-2} + a_t - 0.2a_{t-1}, \quad a_t \sim WN(0, 1).$$

- a) Find the AR representation of  $\{Z_t\}$ .
- b) Find the ACF  $\rho(k)$  of  $\{Z_t\}$  for  $k \in \mathbb{Z}$ .

8. Show that for  $|\phi| > 1$ ,

$$Z_t = \frac{a_t}{\phi^2} - \left(1 - \frac{1}{\phi^2}\right) \sum_{k=1}^{\infty} \frac{a_{t+k}}{\phi^k}.$$

is a white noise process with  $\text{Var}(Z_t) = \frac{\sigma^2}{\phi^2}$ .