MATH1520 Autumn 2018
Homework 5

1. Compute

   (a) $\int_{-1}^{1} \frac{5x}{(4+x^2)^2} dx$

   (b) $\int_{0}^{1} x\sqrt{x+1} dx$

   (c) $\int_{2}^{4} \frac{1}{x(\ln x)^2} dx$

   (d) $\int (2x+6)^1 4\, dx$

   (e) $\int \sqrt{4x-1}\, dx$

   (f) $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}\, dx$

   (g) $\int \frac{x}{\sqrt[3]{4-3x}}\, dx$

   (h) $\int_{10}^{30} ve^{-v/5}\, dv$

   (i) $\int_{2}^{1} t \ln 2t\, dt$

   (j) $\int_{-1}^{3} (t-1)e^{1-t}\, dt$

2. Compute the following integral by partial fraction decomposition.

   (a) $\int \frac{x^3 - x + 1}{x^2 - 1} dx$

   (b) $\int \frac{x^4}{(x-1)(x-2)} dx.$

   (c) $\int \frac{(x+2)}{x^3 - x} dx.$

   (d) $\int_{3}^{9} \frac{4-3x}{(x-1)^2}\, dx$

3. Compute the following integrals.

   (a) $\int e^x \sqrt{e^x - 1} dx;$

   (b) $\int \frac{x^3 + 2x + 1}{x+1} dx$

(c) $\int (x^3 - x)e^x dx.$

(d) $\int_2^4 \frac{e^{2x}}{1+e^x}\, dx$

(e) $\int_1^{10} (\ln x)^3\, dx$

4. Find the area between the region enclosed by $y = 2 - x^2$ and $y = 2x - 1$.

5. Evaluate the given limit using appropriate definite integral.

$$\lim_{n\to\infty} n\left[\frac{1}{(2n+1)^2} + \frac{1}{(2n+2)^2} + \cdots + \frac{1}{(2n+n)^2}\right]$$

6. Evaluate the following improper integrals.

(a) $\int_0^{+\infty} xe^{-x^2}\, dx$

(b) $\int_0^{+\infty} 2xe^{-3x}\, dx$

(c) $\int_{-\infty}^0 \frac{1}{(2x-1)^2}\, dx$

(d) $\int_0^{+\infty} xe^{1-x}\, dx$

(e) $\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}\, dx$

7. Suppose that if f is continuous, find the value of the integral $I = \int_0^a \frac{f(x)}{f(x)+f(a-x)} dx$ by making the substitution $u = a - x$ and adding the resulting integral to I.

8. Compute

$$\int \frac{1}{x^2 - a^2}\, dx,\ a \in \mathbb{R}.$$