The Chinese University of Hong Kong

Department of Mathematics

MATH1550 Methods of Matrices and Linear Algebra

Assignment 2

Please hand in your assignment to assignment box before 5:30p.m. on Oct 16, 2019 (Wednesday). The assignment box is located at the 2nd floor of LSB and opposites to the Room 223.

2-1: Solve the following systems of linear equations.

(a) $$\begin{cases} x_1 + x_2 + x_3 = 3 \\ x_1 + 2x_2 + 2x_3 = 5 \\ 2x_1 - 5x_2 - 5x_3 = -8 \end{cases}$$

(b) $$\begin{cases} 2x_1 + 5x_2 - 8x_3 + 4x_4 = 3 \\ -3x_1 - 9x_2 + 9x_3 - 7x_4 = -2 \\ x_1 + 2x_2 - 5x_3 = -1 \\ 3x_1 + 10x_2 - 7x_3 + 11x_4 = 7 \end{cases}$$

2-2: Given that the systems of linear equations

$$\begin{cases} 2x_1 + 5x_2 - 6x_3 = 30 \\ x_1 + 3x_2 - 5x_3 = 19 \\ 3x_1 + 5x_2 - ax_3 = b \end{cases}$$

has infinitely many solutions.

(a) Find the value of a and b.

(b) Solve the system.

2-3: A parking lot has 66 vehicles (cars,trucks, motorcycles and bicycles) in it. There are four times as many cars as trucks. The total number of tires (4 per car or truck, 2 per motorcycle or bicycle) is 252. How many cars are there? How many bicycles?

2-4: Compute the null space $\mathcal{N}(A)$ of the matrix A:

$$A = \begin{pmatrix} 2 & 4 & 1 & 3 & 8 \\ -1 & -2 & -1 & -1 & 1 \\ 2 & 4 & 0 & -3 & 4 \\ 2 & 4 & -1 & -7 & 4 \end{pmatrix}.$$

2-5: Let A be a 3×5 matrix and the reduced row echelon form of A is

$$\begin{pmatrix} 1 & -2 & 0 & 0 & a \\ 0 & 0 & 1 & 0 & b \\ 0 & 0 & 0 & 1 & c \end{pmatrix}.$$

Given that $(-3, 1, 6, -2, 1)^T \in \mathcal{N}(A)$. Suppose $\boldsymbol{x}_1 = (2, 0, -3, 1, 4)^T$ is a solution to $A\boldsymbol{x} = \boldsymbol{b}_1$ and $\boldsymbol{x}_2 = (5, -2, 4, 0, 3)^T$ is a solution to $A\boldsymbol{x} = \boldsymbol{b}_2$.

(a) Find the values of a, b and c.

(b) Find $\mathcal{N}(A)$.

(c) Find the solution set of $A\boldsymbol{x} = \boldsymbol{b}_1$.

(d) Find the solution set of $A\boldsymbol{x} = 3\boldsymbol{b}_1 - \boldsymbol{b}_2$.

2-6: Use row operation to find the inverse of the following matrices.

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}.$

(b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{pmatrix}.$