STAT2001 Assignment 3

Do all 7 questions. Show your steps clearly.
Deadline for this assignment is *8 Nov. 5:00p.m.* You can submit to the assignment locker (next to LSB 125) or submit on Blackboard system.

1. The Rayleigh distribution has p.d.f.

$$f(x) = x\exp\left(-\frac{x^2}{2}\right), \qquad x > 0.$$

Let X has the Rayleigh distribution.
(a) Find $P(1 < X < 3)$.
(b) Find the first quartile, median, and third quartile of X.

2. Alice is trying to transmit to Bob the answer to a yes-no question, using a noisy channel. She encodes "yes" as 1 and "no" as 0, and sends the appropriate value. However, the channel adds noise; specifically, Bob receives what Alice sends plus a noise term that follows standard Normal distribution. If Bob receives a value greater than 1/2 he interprets it as "yes"; otherwise, he interprets it as "no". Find the probability that Bob understands Alice correctly.

3. Fred wants to sell his car, after moving back to Blissville (where he is happy with the bus system). He decides to sell it to the first person to offer at least $15,000 for it. Assume that the offers are independent Exponential random variables with mean $10,000. Find the expected number of offers Fred will have.

4. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10:00 and 10:30.
   (a) What is the probability that you will have to wait longer than 10 minutes?
   (b) If at 10:10 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 2 minutes?

5. A loss (in $100,000) due to fire in a building has a p.d.f. of

$$f(x) = \frac{1}{5}\exp\left(-\frac{x}{5}\right), \qquad 0 \le x < \infty$$

Given that the loss is greater than 5, find the probability that it is greater than 8.

6. Let X follows $N(\mu,\sigma^2)$ so that $P(X<20)=0.9$ and $P(X<30)=0.95$. Find μ and σ^2.

7. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer follow Chi-square distribution with mean μ=12 years. What is the probability that a batch of 20 chips will contain at least 1 whose lifetimes are longer than 21 years?

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