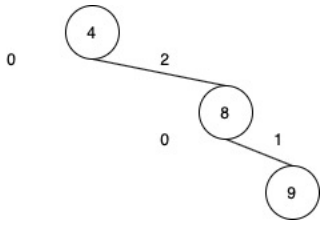
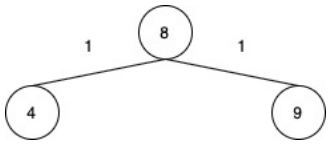
Name: CHAN King Yeung
SID: 1155119394
CSCI2100 Assignment 3

Question 1

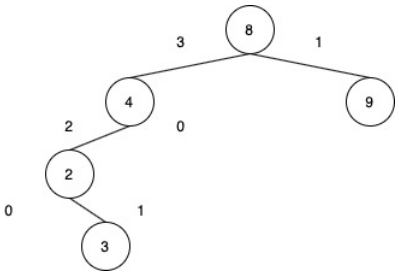
a)



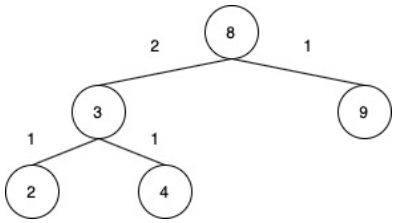
After insert until node 9, the tree is right-right imbalanced, thus, left rotation on node 9 is used



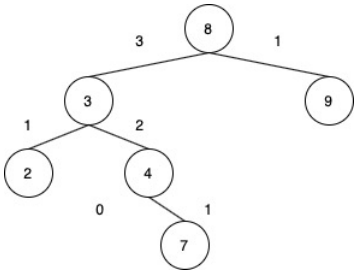
After update the height, the tree is balanced again



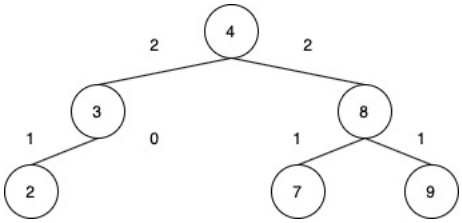
After insert until node 3, the tree is left-right imbalanced, thus, double rotation is used



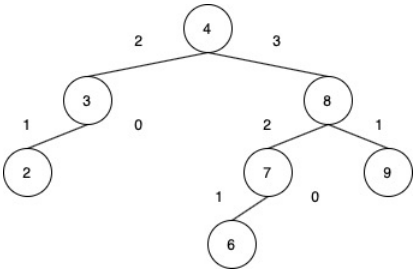
After update the height, the tree is balanced again



After insert until node 7, the tree is left-right imbalanced, thus, double rotation is used

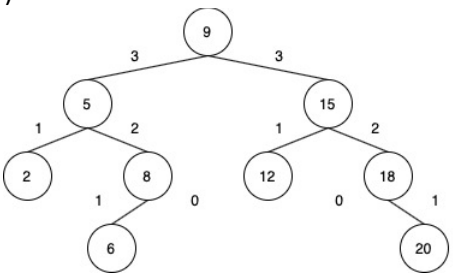


After update the height, the tree is balanced again

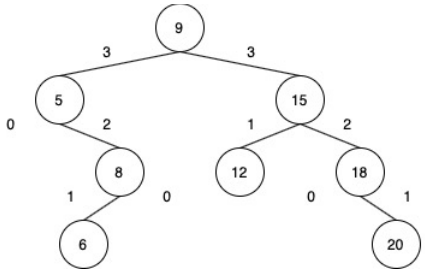


The insertion is finished

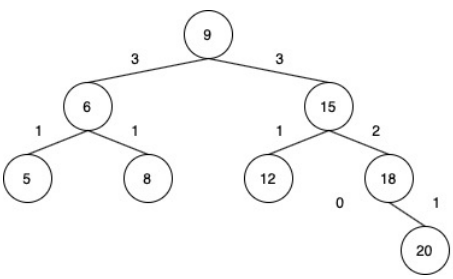
b)



Delete node 2 in the AVL tree



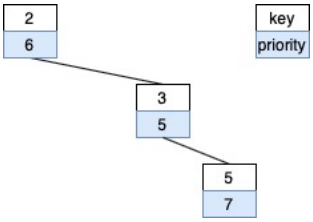
The AVL tree suffers from right-left imbalance, thus, double rotation is used



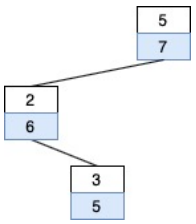
After update the height, the AVL tree is balanced again
The deletion is finished

Question 2

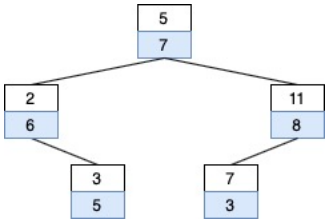
a)



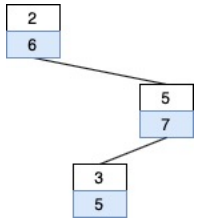
After insert until node 5 with priority 7, it violates the max tree property, thus, move node 5 upward



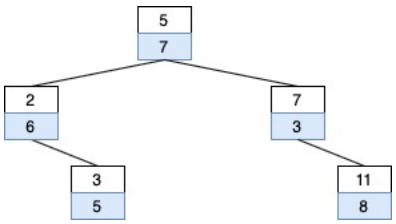
After rotation, there is no more violation



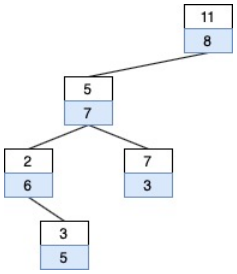
After rotation, it still violates the max tree property, thus, move node 11 upward again



After rotation, it still violates the max tree property, thus, move node 5 upward again

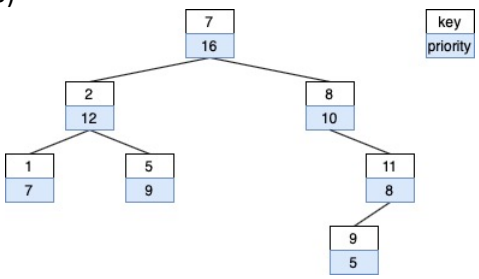


After inset unitl node 11 with priority 8, it violates the max tree property, thus, mode node 11 upward

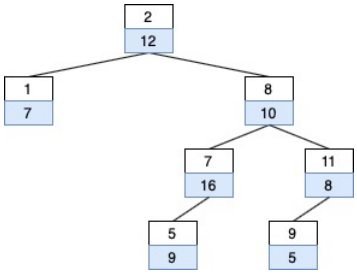


After rotation, there is no more violation
The insertion is finished

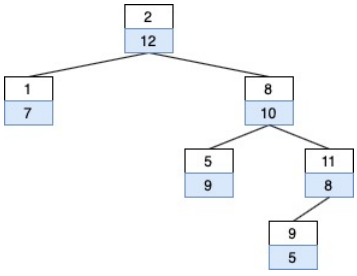
b)



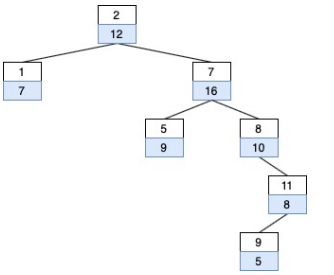
Attempt to delete node 7 with priority 16, we rotate node 2 , which has a larger priority, upward



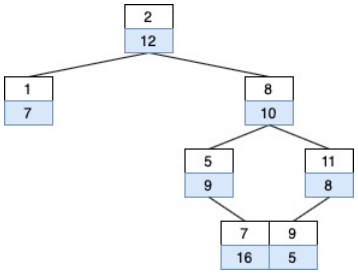
We rotate node 5 with priority 9 upward



The deletion is finished



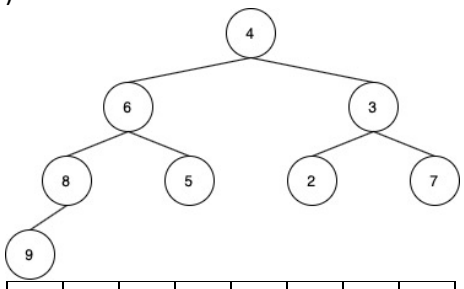
Again, we rotate node 8 upward as it has the larger priority



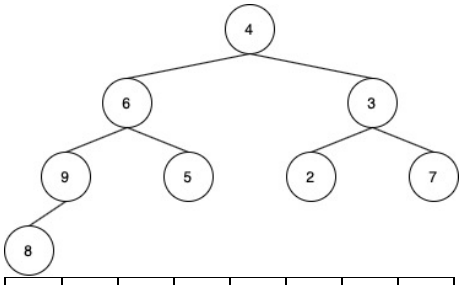
Node 7 became a leaf node, and we remove it

Question 3

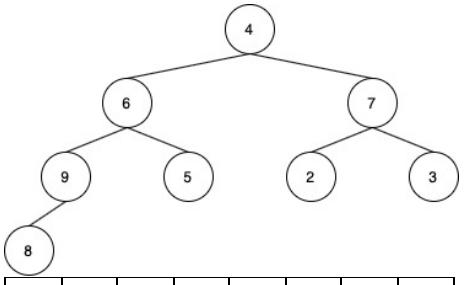
a)



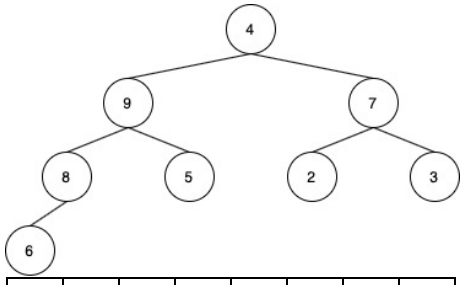
4	6	3	8	5	2	7	9



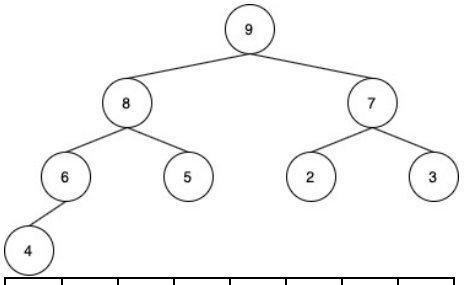
4	6	3	9	5	2	7	8



4	6	7	9	5	2	3	8

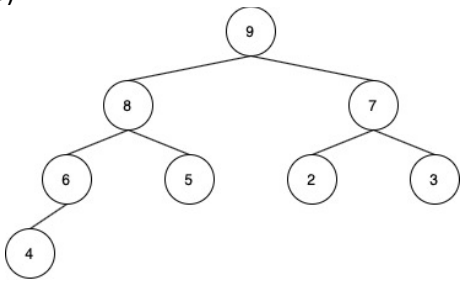


4	9	7	8	5	2	3	6

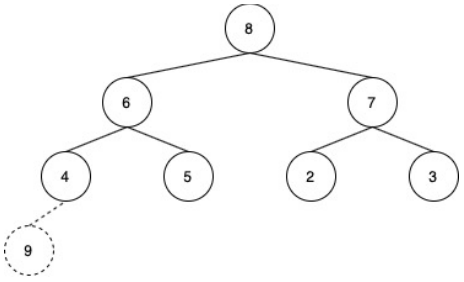


9	8	7	6	5	2	3	4

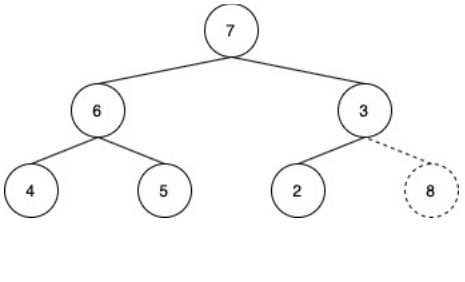
b)



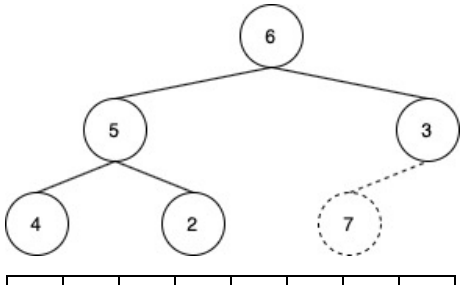
9	8	7	6	5	2	3	4



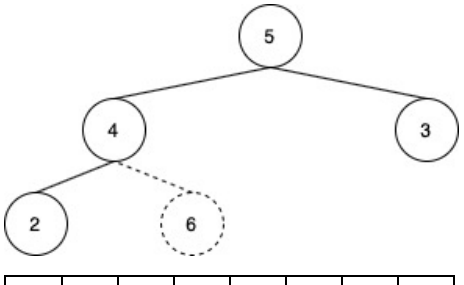
8	6	7	4	5	2	3	9



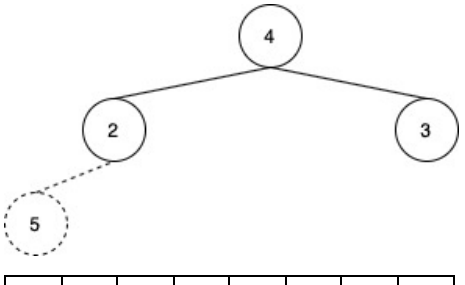
7	6	3	4	5	2	8	9



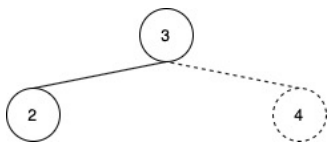
6	5	3	4	2	7	8	9



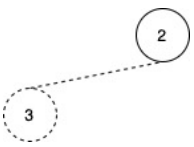
5	4	3	2	6	7	8	9



4	2	3	5	6	7	8	9



3	2	4	5	6	7	8	9

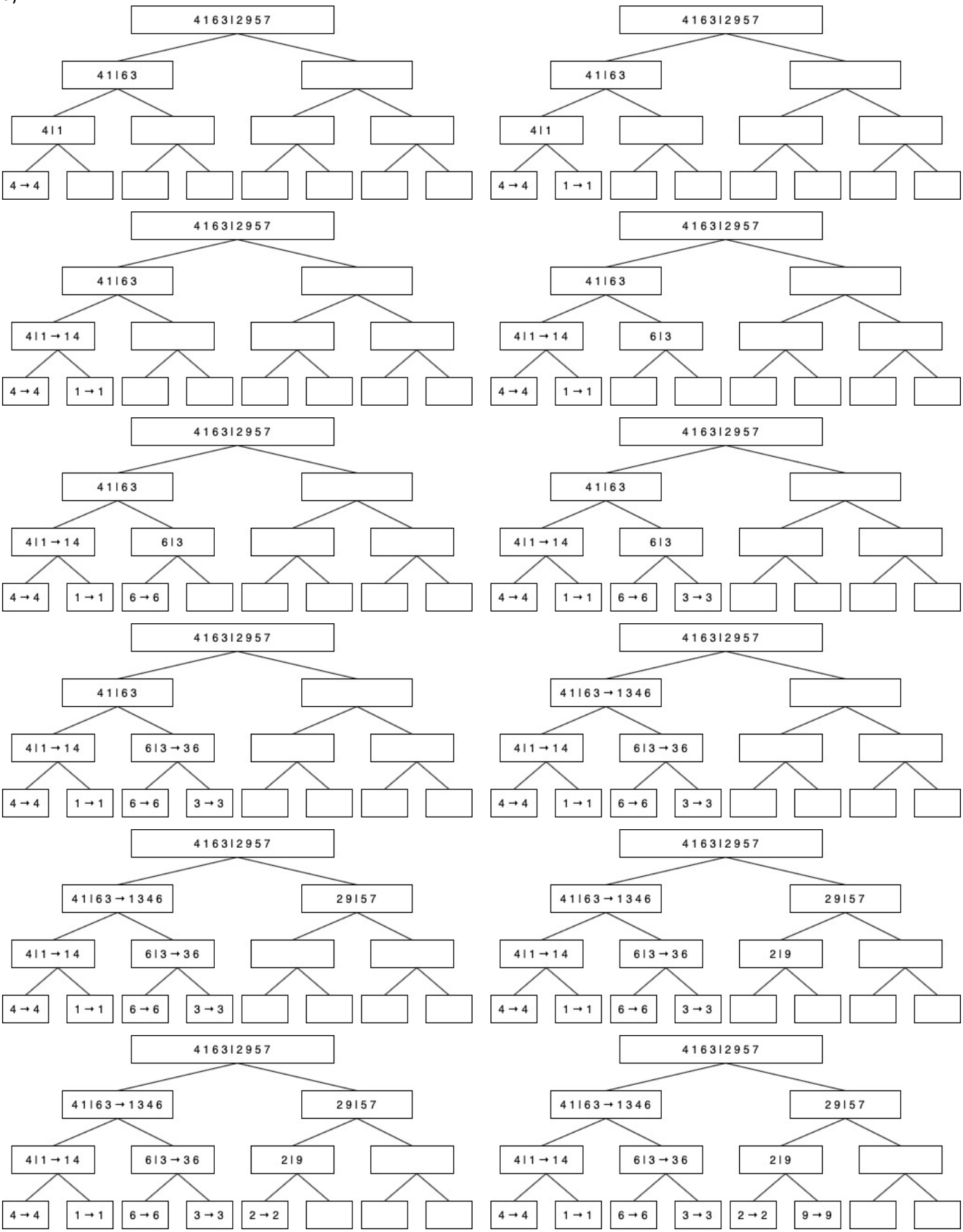


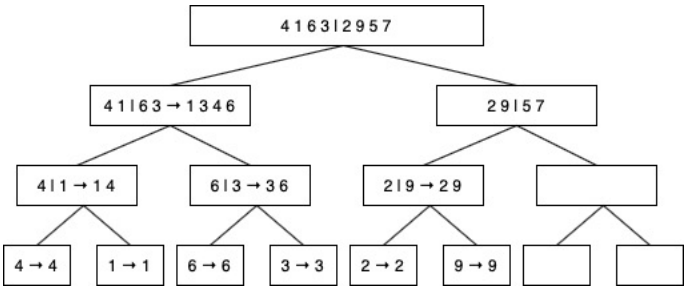
2	3	4	5	6	7	8	9

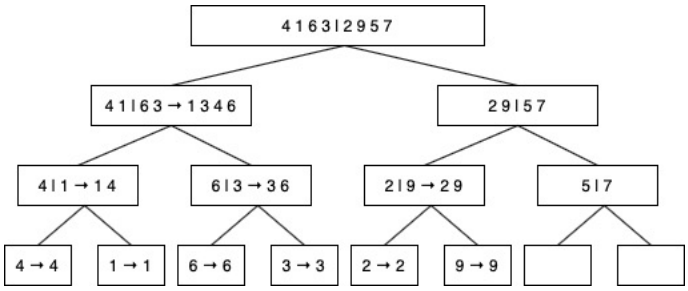
2	3	4	5	6	7	8	9

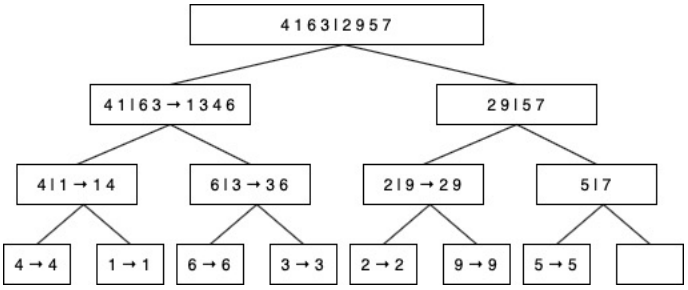
Question 4

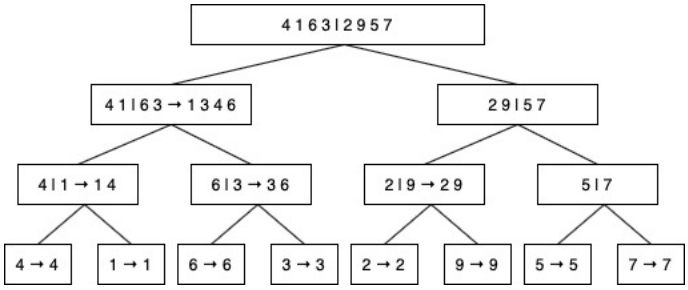
a)

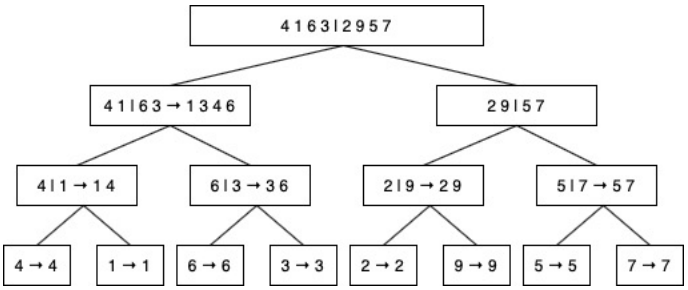

4163|2957
41|63
4|1
4→4
4163|2957
41|63
4|1
4→4
1→1
4163|2957
41|63
4|1→14
4→4
1→1
4163|2957
41|63
4|1→14
6|3
4→4
1→1
4163|2957
41|63
4|1→14
6|3
4→4
1→1
6→6
4163|2957
41|63
4|1→14
6|3
4→4
1→1
6→6
3→3
4163|2957
41|63
4|1→14
6|3→36
4→4
1→1
6→6
3→3
4163|2957
41|63→1346
4|1→14
6|3→36
4→4
1→1
6→6
3→3
4163|2957
41|63→1346
29|57
4|1→14
6|3→36
4→4
1→1
6→6
3→3
4163|2957
41|63→1346
29|57
4|1→14
6|3→36
2|9
4→4
1→1
6→6
3→3
4163|2957
41|63→1346
29|57
4|1→14
6|3→36
2|9
4→4
1→1
6→6
3→3
2→2
4163|2957
41|63→1346
29|57
4|1→14
6|3→36
2|9
4→4
1→1
6→6
3→3
2→2
9→9

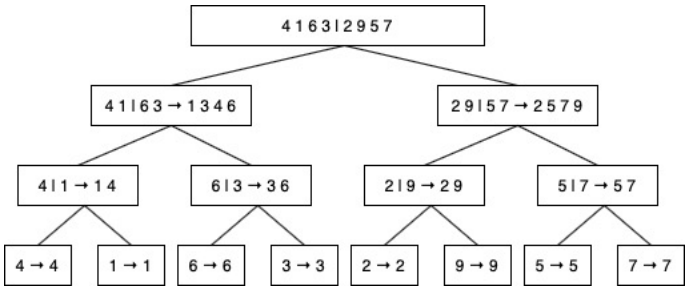


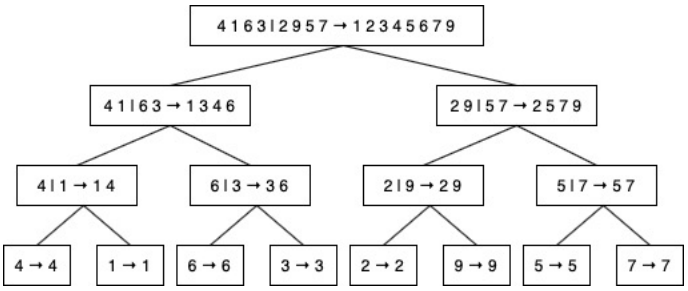












b)

1	3	6	8	2	5	7	9

$left = 0$ $secondleft = 4$
$left = 1$ $secondleft = 4$
$left = 1$ $secondleft = 5$
$left = 2$ $secondleft = 5$
$left = 2$ $secondleft = 6$
$left = 3$ $secondleft = 6$
$left = 3$ $secondleft = 7$
$left = 4$ $secondleft = 7$

Temporary

1							
1	2						
1	2	3					
1	2	3	5				
1	2	3	5	6			
1	2	3	5	6	7		
1	2	3	5	6	7	8	
1	2	3	5	6	7	8	9

c)

6	3	5	2	7	4	1	8
6	3	8	2	7	4	1	5
3	6	8	2	7	4	1	5
3	2	8	6	7	4	1	5
3	2	4	6	7	8	1	5
3	2	4	1	7	8	6	5
3	2	4	1	5	8	6	7

Swap the pivot data and the last data
$nextsmallpos = 0$
$nextsmallpos = 1$
$nextsmallpos = 2$
$nextsmallpos = 3$
$nextsmallpos = 4$
Swap the last data and the data in nextsmallpos

Question 5

Array A

6	1	2	1	3	4	6	1	3	2

counter

3	2	2	1	0	2
3	5	7	8	8	10

Array B

				2					
				2		3			
		1		2		3			
		1		2		3			6
		1		2		3	4		6
		1		2	3	3	4		6
	1	1		2	3	3	4		6
	1	1	2	2	3	3	4		6
1	1	1	2	2	3	3	4		6
1	1	1	2	2	3	3	4	6	6

counter

3	4	7	8	8	10
3	4	6	8	8	10
2	4	6	8	8	10
2	4	6	8	8	9
2	4	6	7	8	9
2	4	5	7	8	9
1	4	5	7	8	9
1	3	5	7	8	9
0	3	5	7	8	9
0	3	5	7	8	8

Question 6

By counting sort, it builds up an array A for counter. The sort cost (pre-processing time) is $O(n+k)$ for integers are ranged in $[1,k]$. The number of integers fall within a range of $[a,b]$ is $A[b]-A[a]$, such algorithm takes $O(1)$ time

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Question 7

a)

4650
7391
5479
3152
2465
3918
9830

Sort on digit 1

4650
9830
7391
3152
2465
3918
5479

Sort on digit 2

3918
9830
4650
3152
2465
5479
7391

Sort on digit 3

3152
7391
2465
5479
4650
9830
3918

Sort on digit 4

2465
3152
3918
4650
5479
7391
9830

b) We can represent numbers in range of $[1, n^3-1]$ with digit of base n, such that $a \cdot n^2 + b \cdot n + c$, where $a, b, c \in [1, n^3-1]$. By radix sort, $k = n$ and $d = c$, such that $O\big(d \cdot (n+k)\big) = O\big(c \cdot (n+n)\big) = O(n)$

Question 8

a) The expected number of the basic operation `random(left,right)` executed from Lines 1 to 7
$E(X) = 2 \cdot \sum_{i=1}^{\infty} \frac{i}{3^i} = 6$, given that $Pr(X = 3i) = \frac{1}{3^i}$

For `partition` takes $O(n)$ and others take constant time
Thus, the expected time complexity for Lines 1 to 7 is $O(n)$

b) $T(n) \le cn + \frac{1}{n}\sum_{i=1}^{n} max\big(T(i-1), T(n-i)\big)$
$\le cn + \frac{2}{n}\sum_{i=n/2}^{n-1} T(i)$
$\le cn + \frac{2}{n}\left[T(n-1) + T(n-2) + \cdots + T\left(\frac{n}{2}\right)\right]$
$\le 4nc$
$\therefore$ the time complexity is $O(n)$
