



## Question 1

(Please refer to page 3)

can't c)  $H_0: \Delta = 0$  vs  $H_1: \Delta \neq 0$ 

$$|T_0| = \frac{1220 - 1681 - \frac{1}{2}}{\sqrt{420}}$$

$$\approx 2.5129$$

$$p\text{-value} \approx 0.012$$

Since  $p\text{-value} < 0.05$ , we reject  $H_0$  at  $\alpha = 0.05$ .

## Question 2

a)  $H_0: \Delta = 0$  vs  $H_1: \Delta \neq 0$ 

$$\chi_0^2 = \frac{(115 - 81 - 1)^2}{23}$$

$$\approx 1.5652$$

$$p\text{-value} \approx 0.2109$$

Since  $p\text{-value} > 0.05$ , we do not reject  $H_0$  at  $\alpha = 0.05$ .

b) Wilcoxon signed-rank test

-ve count +ve count  $t_i$  avg(rank)

-3 2 3 4 6 20.5

-2 2 2 5 7 14

-1 4 1 6 10 5.5

$$R^+ = 6(5.5) + 5(14) + 4(20.5)$$

$$= 185$$

$$E(R^+) = \frac{23(24)}{4}$$

$$= 138$$

$$\text{Var}(R^+) = \frac{23(24)(47)}{24} - \frac{10^3 + 7^3 + 6^3 - 10 \cdot 7 - 6}{48}$$

$$= 1049$$

 $H_0: \Delta = 0$  vs  $H_1: \Delta \neq 0$ 

$$|T_0| = \frac{1185 - 1381 - \frac{1}{2}}{\sqrt{1049}}$$

$$\approx 1.4357$$

$$p\text{-value} \approx 0.1511$$

Since  $p\text{-value} > 0.05$ , we do not reject  $H_0$  at  $\alpha = 0.05$ c)  $R_i = 220$ 

$$E(R_i) = \frac{12(12+15+1)}{2}$$

$$= 168$$

$$\text{Var}(R_i) = \frac{12(15)}{12} (12+15+1)$$

$$= 420$$

## Question 3

a) Yates-corrected  $\chi^2$  test $H_0$ : the prevalence of otorrhea for the ear drop group is the same as the observation group. vs $H_1$ : the prevalence of otorrhea for the ear drop group is not the same as the observation group.

	with otorrhea	without otorrhea		with otorrhea	without otorrhea
b)					
Eardrops	4	72	Eardrops	22.65	53.35
Observation	41	34	Observation	22.35	52.65
	45	106			
	151				

$$\chi^2 = \frac{(4 - 22.65 - 0.5)^2}{22.65} + \frac{(72 - 53.35 - 0.5)^2}{53.35} + \frac{(41 - 22.35 - 0.5)^2}{22.35} + \frac{(34 - 52.65 - 0.5)^2}{52.65}$$

$$\approx 41.7103$$

$$p\text{-value} \approx 1.0585 \times 10^{-10}$$

Since  $p\text{-value} < 0.05$ , we reject  $H_0$  at  $\alpha = 0.05$ 

## Question 4

a) Fisher's exact test

b)  $H_0$ : the feeding performance for titmice is the same as that of goldfinches. $H_1$ : the feeding performance for titmice is not the same as that of goldfinches.

	striped seeds	black oil seed		striped seeds	black oil seed
titmice	4	1	titmice	$x$	$5-x$
goldfinches	5	19	goldfinches	$9-x$	$15+x$
	9	20		9	20
	29			29	

$$\left| \frac{x}{9} - \frac{5-x}{20} \right| \geq \left| \frac{4}{9} - \frac{1}{20} \right|$$

$$|28x - 45| \geq 71$$

$$\Rightarrow x \geq 4 \text{ or } x \leq -1$$

extreme value  $x = \{4, 5\}$

con't b)  $p\text{-value} = \frac{\binom{8}{4}\binom{20}{5} + \binom{8}{5}}{\binom{29}{9}}$   
 $\approx 0.0223$

Since  $p\text{-value} < 0.05$ , we reject  $H_0$  at  $\alpha=0.05$

c) Yates-corrected  $\chi^2$  test

d)  $H_0$ : the feeding performance of goldfinches is the same on different days vs

$H_1$ : the feeding performance of goldfinches is not the same on different days

	Day 1	Day 2	Day 3	Day 4
Black oil	14.2	14.2	8.88	49.71
Striped	9.8	9.8	6.12	34.29

$$\chi^2 = \sum_{\text{cell}} \frac{(\text{O}_{ij} - \text{E}_{ij} - 0.5)^2}{\text{E}_{ij}}$$

$$\approx 5.0739$$

$$p\text{-value} \approx 0.1665$$

Since  $p\text{-value} > 0.05$ , we do not reject  $H_0$  at  $\alpha=0.05$

### Question 1

#### **PARAMETRIC METHOD**

We examine the equal variances assumption by using F test for equal variances of 2 independent samples

$$H_0: \sigma_{\text{lighter smoking}}^2 = \sigma_{\text{heavier smoking}}^2 \quad \text{vs} \quad H_1: \sigma_{\text{lighter smoking}}^2 \neq \sigma_{\text{heavier smoking}}^2$$

```
> var.test(data$fn1, data$fn2)
      F test to compare two variances
data:  data$fn1 and data$fn2
F = 0.80995, num df = 40, denom df = 40, p-value = 0.5081
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.4319271 1.5188134
sample estimates:
ratio of variances
 0.8099486
```

Since p-value > 0.05, we do not reject  $H_0$  at  $\alpha = 0.05$

We, then, use t test for equal means of 2 independent samples, given their variances are equal

$$H_0: \mu_{\text{lighter smoking}} = \mu_{\text{heavier smoking}} \quad \text{vs} \quad H_1: \mu_{\text{lighter smoking}} \neq \mu_{\text{heavier smoking}}$$

```
> t.test(data$fn1, data$fn2, var.equal = T)
      Two Sample t-test
data:  data$fn1 and data$fn2
t = 0.03004, df = 80, p-value = 0.9761
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.04774195  0.04920536
sample estimates:
mean of x mean of y
0.6648780 0.6641463
```

Since p-value > 0.05, we do not reject  $H_0$  at  $\alpha = 0.05$

#### **NONPARAMETRIC METHOD**

We use Wilcoxon rank-sum test to test whether 2 medians are equal for 2 independent samples

$$H_0: \Delta = 0 \quad \text{vs} \quad H_1: \Delta \neq 0$$

```
> fn = c(data$fn1, data$fn2)
> label = c(rep("LST", length(data$fn1)), rep("HST", length(data$fn2)))
> wilcox.test(fn~label,
+             alternative = "two.sided",
+             mu = 0,
+             paired = F,
+             exact = F,
+             correct = T,
+             conf.int = F)
      Two Sample t-test
data:  data$fn1 and data$fn2
t = 0.03004, df = 80, p-value = 0.9761
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.04774195  0.04920536
sample estimates:
mean of x mean of y
0.6648780 0.6641463
```

Since p-value > 0.05, we do not reject  $H_0$  at  $\alpha = 0.05$

There are no difference in between parametric and nonparametric methods for the given dataset. Those methods suggest that there is no significant difference in mean or median BMD between lighter smoking and heavier smoking.

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