

- 3. If a news reporter stated that the number of cases of kidney problems in newborns was nearly four times as many as was usually expected, do you think pregnant mothers living in that area would be overly concerned?
- 4. Is it unlikely that 11 occurred by chance?
- 5. Are there any other statistics that could better inform the public?
- 6. Assume that 3 out of 2500 babies were born with kidney problems in that three-county area the year before the accident. Also assume that 11 out of 2500 babies were born with kidney problems in that three-county area the year after the accident. What is the real percentage increase in that abnormality?
- 7. Do you think that pregnant mothers living in that area should be overly concerned after looking at the results in terms of rates?

See page 309 for the answers.

Exercises 5-2

1. **Defective DVDs** From past experience, a company found that in cartons of DVDs, 90% contain no defective DVDs, 5% contain one defective DVD, 3% contain two defective DVDs, and 2% contain three defective DVDs. Find the mean, variance, and standard deviation for the number of defective DVDs.

2. **Suit Sales** The number of suits sold per day at a retail store is shown in the table, with the corresponding probabilities. Find the mean, variance, and standard deviation of the distribution.

Number of suits sold X	19	20	21	22	23
Probability $P(X)$	0.2	0.2	0.3	0.2	0.1

If the manager of the retail store wants to be sure that he has enough suits for the next 5 days, how many should the manager purchase?

3. **Number of Credit Cards** A bank vice president feels that each savings account customer has, on average, three credit cards. The following distribution represents the number of credit cards people own. Find the mean, variance, and standard deviation. Is the vice president correct?

Number of cards X	0	1	2	3	4
Probability $P(X)$	0.18	0.44	0.27	0.08	0.03

4. **Trivia Quiz** The probabilities that a player will get 5 to 10 questions right on a trivia quiz are shown below. Find the mean, variance, and standard deviation for the distribution.

X	5	6	7	8	9	10
$P(X)$	0.05	0.2	0.4	0.1	0.15	0.1

5. **Cellular Phone Sales** The probability that a cellular phone company kiosk sells X number of new phone

contracts per day is shown below. Find the mean, variance, and standard deviation for this probability distribution.

X	4	5	6	8	10
$P(X)$	0.4	0.3	0.1	0.15	0.05

What is the probability that they will sell 6 or more contracts three days in a row?

6. **Traffic Accidents** The county highway department recorded the following probabilities for the number of accidents per day on a certain freeway for one month. The number of accidents per day and their corresponding probabilities are shown. Find the mean, variance, and standard deviation.

Number of accidents X	0	1	2	3	4
Probability $P(X)$	0.4	0.2	0.2	0.1	0.1

7. **Leading Digits** Suppose that we wanted to check the occurrence of leading digits in real-life data such as stock prices, population numbers, death rates, lengths of rivers to see if they occur randomly. Disregarding zero as a leading digit, we might expect the other nine to occur with equal likelihood. Construct the probability distribution for the leading digits 1–9, assuming equal probability for each, and calculate the mean, variance, and standard deviation for this distribution.

8. **Benford's Law** The leading digits in actual data, such as stock prices, population numbers, death rates, and lengths of rivers, do not occur randomly as one might suppose, but instead follow a distribution according to Benford's law. Below is the probability distribution for the leading digits in real-life lists of data. Calculate the mean for the distribution.

X	1	2	3	4	5	6	7	8	9
$P(X)$	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

9. **Students Using the Math Lab** The number of students using the Math Lab per day is found in the distribution below. Find the mean, variance, and standard deviation for this probability distribution.

<i>X</i>	6	8	10	12	14
<i>P(X)</i>	0.15	0.3	0.35	0.1	0.1

What is the probability that fewer than 8 or more than 12 use the lab in a given day?

10. **Pizza Deliveries** A pizza shop owner determines the number of pizzas that are delivered each day. Find the mean, variance, and standard deviation for the distribution shown. If the manager stated that 45 pizzas were delivered on one day, do you think that this is a believable claim?

Number of deliveries <i>X</i>	35	36	37	38	39
Probability <i>P(X)</i>	0.1	0.2	0.3	0.3	0.1

11. **Grab Bags** A craft store has 25 assorted grab bags on sale for \$3.00 each. Fifteen of the bags contain \$3.00 worth of merchandise, six contain \$2.00 worth, two contain \$5.00 worth of merchandise, and there are one each containing \$10.00 and \$20.00 worth of merchandise. Suppose that you purchase one bag; what is your expected gain or loss?
12. **Job Bids** A landscape contractor bids on jobs where he can make \$3000 profit. The probabilities of getting 1, 2, 3, or 4 jobs per month are shown.

Number of jobs	1	2	3	4
Probability	0.2	0.3	0.4	0.1

Find the contractor's expected profit per month.

13. **Rolling Dice** If a person rolls doubles when she tosses two dice, she wins \$5. For the game to be fair, how much should she pay to play the game?
14. **Dice Game** A person pays \$2 to play a certain game by rolling a single die once. If a 1 or a 2 comes up, the person wins nothing. If, however, the player rolls a 3, 4, 5,

or 6, he or she wins the difference between the number rolled and \$2. Find the expectation for this game. Is the game fair?

15. **Lottery Prizes** A lottery offers one \$1000 prize, one \$500 prize, and five \$100 prizes. One thousand tickets are sold at \$3 each. Find the expectation if a person buys one ticket.
16. In Exercise 15, find the expectation if a person buys two tickets. Assume that the player's ticket is replaced after each draw and that the same ticket can win more than one prize.
17. **Winning the Lottery** For a daily lottery, a person selects a three-digit number. If the person plays for \$1, she can win \$500. Find the expectation. In the same daily lottery, if a person boxes a number, she will win \$80. Find the expectation if the number 123 is played for \$1 and boxed. (When a number is "boxed," it can win when the digits occur in any order.)

18. **Life Insurance** A 35-year-old woman purchases a \$100,000 term life insurance policy for an annual payment of \$360. Based on a period life table for the U.S. government, the probability that she will survive the year is 0.999057. Find the expected value of the policy for the insurance company.

19. **Roulette** A roulette wheel has 38 numbers, 1 through 36, 0, and 00. One-half of the numbers from 1 through 36 are red, and the other half are black; 0 and 00 are green. A ball is rolled, and it falls into one of the 38 slots, giving a number and a color. The payoffs (winnings) for a \$1 bet are as follows:

Red or black	\$1	0	\$35
Odd or even	\$1	00	\$35
1-18	\$1	Any single number	\$35
9-36	\$1	0 or 00	\$17

If a person bets \$1, find the expected value for each.

- a. Red
b. Even
c. 00
d. Any single number
e. 0 or 00

Extending the Concepts

20. **Rolling Dice** Construct a probability distribution for the sum shown on the faces when two dice are rolled. Find the mean, variance, and standard deviation of the distribution.
21. **Rolling a Die** When one die is rolled, the expected value of the number of dots is 3.5. In Exercise 20, the mean number of dots was found for rolling two dice. What is the mean number of dots if three dice are rolled?

22. The formula for finding the variance for a probability distribution is

$$\sigma^2 = \sum[(X - \mu)^2 \cdot P(X)]$$

Verify algebraically that this formula gives the same result as the shortcut formula shown in this section.

- a. The probability that at least one began online
- b. The probability that two or three began online
- c. What is the probability that exactly one began online?

8. Multiple-Choice Exam A student takes a 20-question, multiple-choice exam with five choices for each question and guesses on each question. Find the probability of guessing at least 15 out of 20 correctly. Would you consider this event likely or unlikely to occur? Explain your answer.

9. Driving to Work Alone It is reported that 77% of workers aged 16 and over drive to work alone. Choose 8 workers at random. Find the probability that

- a. All drive to work alone
- b. More than one-half drive to work alone
- c. Exactly 3 drive to work alone

Source: www.factfinder.census.gov

10. High School Dropouts Approximately 10.3% of American high school students drop out of school before graduation. Choose 10 students entering high school at random. Find the probability that

- a. No more than 2 drop out
- b. At least 6 graduate
- c. All 10 stay in school and graduate

Source: www.infoplease.com

11. Survey on Concern for Criminals In a survey, 3 of 4 students said the courts show “too much concern” for criminals. Find the probability that at most 3 out of 7 randomly selected students will agree with this statement.

Source: *Harper's Index*.

12. Union Workers In 2010 almost 15 million U.S. workers belonged to trade unions, constituting 11.9% of the total labor force. Choose 15 U.S. workers at random. What is the probability that exactly one-third of them belong to a trade union? At least one-third? What is the probability that at least 9 did not belong?

Source: *Time Almanac 2012*.

13. College Education and Business World Success R. H. Bruskin Associates Market Research found that 40% of Americans do not think that having a college education is important to succeed in the business world. If a random sample of 5 Americans is selected, find these probabilities.

- a. Exactly 2 people will agree with that statement.
- b. At most 3 people will agree with that statement.
- c. At least 2 people will agree with that statement.
- d. Fewer than 3 people will agree with that statement.

Source: *100% American* by Daniel Evans Weiss.

14. Destination Weddings Twenty-six percent of couples who plan to marry this year are planning destination weddings. In a random sample of 12 couples who plan to marry, find the probability that

- a. Exactly 6 couples will have a destination wedding
- b. At least 6 couples will have a destination wedding
- c. Fewer than 5 couples will have a destination wedding

Source: *Time* magazine.

15. People Who Have Some College Education Fifty-three percent of all persons in the U.S. population have at least some college education. Choose 10 persons at random. Find the probability that

- a. Exactly one-half have some college education
- b. At least 5 do not have any college education
- c. Fewer than 5 have some college education

Source: *New York Times Almanac*.

16. Guidance Missile System A missile guidance system has 5 fail-safe components. The probability of each failing is 0.05. Find these probabilities.

- a. Exactly 2 will fail.
- b. More than 2 will fail.
- c. All will fail.
- d. Compare the answers for parts a, b, and c, and explain why these results are reasonable.

17. Find the mean, variance, and standard deviation for each of the values of n and p when the conditions for the binomial distribution are met.

- a. $n = 100, p = 0.75$
- b. $n = 300, p = 0.3$
- c. $n = 20, p = 0.5$
- d. $n = 10, p = 0.8$

18. Find the mean, variance, and standard deviation for each of the values of n and p when the conditions for the binomial distributions are met.

- a. $n = 1000, p = 0.1$
- b. $n = 500, p = 0.25$
- c. $n = 50, p = \frac{2}{5}$
- d. $n = 36, p = \frac{1}{6}$

19. Social Security Recipients A study found that 1% of Social Security recipients are too young to vote. If 800 Social Security recipients are randomly selected, find the mean, variance, and standard deviation of the number of recipients who are too young to vote.

Source: *Harper's Index*.

20. Tossing Coins Find the mean, variance, and standard deviation for the number of heads when 10 coins are tossed.

21. American and Foreign-Born Citizens In 2009 the percentage of the U.S. population who was foreign-born was 12.2. Choose 60 U.S. residents at random. How many would you expect to be American-born? Find the mean, variance, and standard deviation for the number who are foreign-born.

Source: *World Almanac 2012*.

1. Using the Poisson distribution, find the theoretical values for each number of hits. In this case, the number of bombs was 535, and the number of regions was 576. So

$$\lambda = \frac{535}{576} \approx 0.929$$

For 3 hits,

$$\begin{aligned} P(X) &= \frac{e^{-\lambda} \cdot \lambda^X}{X!} \\ &= \frac{(2.7183)^{-0.929}(0.929)^3}{3!} \approx 0.0528 \end{aligned}$$

Hence, the number of hits is $(0.0528)(576) = 30.4128$.

Complete the table for the other number of hits.

Hits	0	1	2	3	4	5
Regions				30.4		

2. Write a brief statement comparing the two distributions.
3. Based on your answer to question 2, can you conclude that the rockets were fired at random?

See page 309 for the answer.

Exercises 5–4

1. Use the multinomial formula and find the probabilities for each.
- a. $n = 6, X_1 = 3, X_2 = 2, X_3 = 1, p_1 = 0.5, p_2 = 0.3, p_3 = 0.2$
 - b. $n = 5, X_1 = 1, X_2 = 2, X_3 = 2, p_1 = 0.3, p_2 = 0.6, p_3 = 0.1$
 - c. $n = 4, X_1 = 1, X_2 = 1, X_3 = 2, p_1 = 0.8, p_2 = 0.1, p_3 = 0.1$
2. Use the multinomial formula and find the probabilities for each.
- a. $n = 3, X_1 = 1, X_2 = 1, X_3 = 1, p_1 = 0.5, p_2 = 0.3, p_3 = 0.2$
 - b. $n = 5, X_1 = 1, X_2 = 3, X_3 = 1, p_1 = 0.7, p_2 = 0.2, p_3 = 0.1$
 - c. $n = 7, X_1 = 2, X_2 = 3, X_3 = 2, p_1 = 0.4, p_2 = 0.5, p_3 = 0.1$
3. **M&M's Color Distribution** According to the manufacturer, M&M's are produced and distributed in the following proportions: 13% brown, 13% red, 14% yellow, 16% green, 20% orange, and 24% blue. In a random sample of 12 M&M's, what is the probability of having 2 of each color?
4. **Truck Inspection Violations** The probabilities are 0.50, 0.40, and 0.10 that a trailer truck will have no violations, 1 violation, or 2 or more violations when it is given a safety inspection by state police. If 5 trailer trucks are inspected, find the probability that 3 will have no violations, 1 will have 1 violation, and 1 will have 2 or more violations.
5. **Reusable Grocery Bags** In a magazine survey, 60% of respondents said that they use reusable grocery bags; 32%, plastic; and 8%, paper. In a random sample of 10 grocery shoppers, what is the probability that 6 will use reusable bags and that 2 each will request paper or plastic?
- Source: *Everyday with Rachel Ray*, April 2012.
6. **Mendel's Theory** According to Mendel's theory, if tall and colorful plants are crossed with short and colorless plants, the corresponding probabilities are $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}$, and $\frac{1}{16}$ for tall and colorful, tall and colorless, short and colorful, and short and colorless, respectively. If 8 plants are selected, find the probability that 1 will be tall and colorful, 3 will be tall and colorless, 3 will be short and colorful, and 1 will be short and colorless.
7. Find each probability $P(X; \lambda)$, using Table C in Appendix A.
- a. $P(5; 4)$
 - b. $P(2; 4)$
 - c. $P(6; 3)$
8. Find each probability $P(X; \lambda)$ using Table C in Appendix A.
- a. $P(10; 7)$
 - b. $P(9; 8)$
 - c. $P(3; 4)$
9. **Study of Robberies** A recent study of robberies for a certain geographic region showed an average of 1 robbery per 20,000 people. In a city of 80,000 people, find the probability of the following.
- a. 0 robberies
 - b. 1 robbery
 - c. 2 robberies
 - d. 3 or more robberies

- 10. Misprints on Manuscript Pages** In a 400-page manuscript, there are 200 randomly distributed misprints. If a page is selected, find the probability that it has 1 misprint.
- 11. Colors of Flowers** A nursery provides red impatiens for commercial landscaping. If 5% are variegated instead of pure red, find the probability that in an order for 200 plants, exactly 14 are variegated.
- 12. Mail Ordering** A mail-order company receives an average of 5 orders per 500 solicitations. If it sends out 100 advertisements, find the probability of receiving at least 2 orders.
- 13. Company Mailing** Of a company's mailings 1.5% are returned because of incorrect or incomplete addresses. In a mailing of 200 pieces, find the probability that none will be returned.
- 14. Emission Inspection Failures** If 3% of all cars fail the emissions inspection, find the probability that in a sample of 90 cars, 3 will fail. Use the Poisson approximation.
- 15. Phone Inquiries** The average number of phone inquiries per day at the poison control center is 4. Find the probability it will receive 5 calls on a given day. Use the Poisson approximation.
- 16. Defective Calculators** In a batch of 2000 calculators, there are, on average, 8 defective ones. If a random sample of 150 is selected, find the probability of 5 defective ones.
- 17. School Newspaper Staff** A school newspaper staff is comprised of 5 seniors, 4 juniors, 5 sophomores, and 7 freshmen. If 4 staff members are chosen at random for a publicity photo, what is the probability that there will be 1 student from each class?
- 18. Missing Pages from Books** A bookstore owner examines 5 books from each lot of 25 to check for missing pages. If he finds at least 2 books with missing pages, the entire lot is returned. If, indeed, there are 5 books with missing pages, find the probability that the lot will be returned.
- 19. Hors d'Oeuvres Selection** A plate of hors d'oeuvres contains two types of filled puff pastry—chicken and shrimp. The entire platter contains 15 pastries—8 chicken and 7 shrimp. From the outside the pastries appear identical, and they are randomly distributed on the tray. Choose 3 at random; what is the probability that all 3 have the same filling?
- 20. Defective Computer Keyboards** A shipment of 24 computer keyboards is rejected if 4 are checked for defects and at least 1 is found to be defective. Find the probability that the shipment will be returned if there are actually 6 defective keyboards.
- 21. Defective Electronics** A shipment of 24 electric typewriters is rejected if 3 are checked for defects and at least 1 is found to be defective. Find the probability that the shipment will be returned if there are actually 6 typewriters that are defective.
- 22. Job Applications** Ten people apply for a job at Computer Warehouse. Five are college graduates and five are not. If the manager selects 3 applicants at random, find the probability that all 3 are college graduates.
- 23. Selling Carpet** A person works in a large home improvement store and approaches customers to tell them about the store's carpet sale. He then asks them if they would like to talk to a sales representative. From past experience, the person has found that the probability of getting a "yes" is about 0.32. Find the probability that the person's first "yes" will occur with the fifth customer.
- 24. Winning a Prize** A soda pop manufacturer runs a contest and places a winning bottle cap on every sixth bottle. If a person buys the soda pop, find the probability that the person will (a) win on his first purchase, (b) win on his third purchase, or (c) not win on any of his first five purchases.
- 25. Shooting an Arrow** Mark shoots arrows at a target and hits the bull's-eye about 40% of the time. Find the probability that he hits the bull's-eye on the third shot.
- 26. Amusement Park Game** At an amusement park basketball game, the player gets 3 throws for \$1. If the player makes a basket, the player wins a prize. Mary makes about 80% of her shots. Find the probability that Mary wins a prize on her third shot.

Extending the Concepts

Another type of problem that can be solved uses what is called the *negative binomial distribution*, which is a generalization of the binomial distribution. In this case, it tells the average number of trials needed to get k successes of a binomial experiment. The formula is

$$\mu = \frac{k}{p}$$

where k = the number of successes
 p = the probability of a success

Use this formula for Exercises 27–30.

- 27. Drawing Cards** A card is randomly drawn from a deck of cards and then replaced. The process continues until 3 clubs are obtained. Find the average number of trials needed to get 3 clubs.
- 28. Rolling an 8-Sided Die** An 8-sided die is rolled. The sides are numbered 1 through 8. Find the average number of rolls it takes to get two 5s.
- 29. Drawing Cards** Cards are drawn at random from a deck and replaced after each draw. Find the average number of cards that would be drawn to get 4 face cards.