Solution of Assignment 1

1. (5 points) ${}_{10}C_2 = 45$

2. (8 points)
   P(some district had more than 1 robbery)
   =1-P(each district had and only had 1 robbery)
   $=1 - \frac{6!}{6^6}$

3. (10 points)
   Total: ${}_7C_1 \times_8 C_1 = 56$
   Same R: ${}_2C_1 \times_1 C_1 = 2$
   Same E: ${}_3C_1 \times_1 C_1 = 3$
   Same V: ${}_1C_1 \times_1 C_1 = 1$
   P(same letter is chosen)$=\frac{2+3+1}{56} = \frac{3}{28}$

4. (a) (2 points) Suggested answer: It is reasonable. For example given A^c, that means A is not guilty. Together with A and B are not relatives of each other, it is natural to assume that the chance that A's blood types matches with the guilty party's, is 10% as in the population. Similar argument works for $P(C|A) = 0.1$. Other justifiable answers can also be accepted.

   (b) (8 points)
   Notations:
   M: A's blood type matches that of the guilty party
   A: A is guilty, B: B is guilty, so $B = A^c$

   $$P(A|M) = \frac{P(M|A)P(A)}{P(M|A)P(A) + P(M|B)P(B)} = \frac{1/2}{1/2 + (1/10)(1/2)} = \frac{10}{11}$$

   (c) (10 points)
   C: B's blood type matches that of the guilty party

   $$P(C|M) = P(C|M, A)P(A|M) + P(C|M, B)P(B|M) = \frac{1}{10} \times \frac{10}{11} + \frac{1}{11} = \frac{2}{11}$$

5. (a) (8 points)
   P($A > B$) = P(A=4) = 2/3
   P($B > C$) = P(C=2) = 2/3
   P($C > D$) = P(C=6) + P(C=2,D=1) = P(C=6) + P(C=2)P(D=1)
   $= \frac{1}{3} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$
   P($D > A$) = P(D=5) + P(D=1,A=0) = P(D=5) + P(D=1)P(A=1)
   $= \frac{1}{2} + \frac{1}{2} \times \frac{1}{3} = \frac{2}{3}$

   (b) (10 points)
   P($A > B, B > C$) = P(A=4,C=2) = P(A=4)P(C=2) = $\frac{4}{9}$ = P($A > B$)P($B > C$)

Therefore, event $A > B$ is independent of the event $B > C$.
P($B > C, C > D$) = P($3 > C > D$) = P(C=2,D=1) = P(C=2)P(D=1)
$= \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} \neq$ P($B > C$)P($C > D$)
Therefore, event $B > C$ is NOT independent of the event $C > D$.

6. (12 points)
$\because A = A \cap (C \cup C^c) = (A \cap C) \cup (A \cap C^c)$, $(A \cap C) \cap (A \cap C^c) = \phi$,
$\therefore$ P(A) = P($(A \cap C) \cup (A \cap C^c)$)
= P($A \cap C$) + P($A \cap C^c$)
= P($A|C$)P(C)+P($A|C^c$)P(C^c)
> P($B|C$)P(C)+P($B|C^c$)P(C^c) = P(B)

7. (15 points)
Notations:
J: get a job, F: fail to get a job
S: strong recommendation, M: moderately good recommendation,
W: weak recommendation

P($J|S$)=0.8, P($J|M$)=0.4, P($J|W$)=0.1,
P(S)=0.6, P(M)=0.3, P(W)=0.1
$\therefore$ P($W|F$) = $\frac{P(F|W)P(W)}{P(F)}$,
P($F|W$) = 1 - P($J|W$) = 0.9,
P(J) = P($J|S$)P(S) + P($J|M$)P(M) + P($J|W$)P(W) = 0.61
P(F) = 1 - P(J) = 0.39
Therefore, P($W|F$) = $\frac{P(F|W)P(W)}{P(F)} = \frac{0.9 \times 0.1}{0.39} = \frac{3}{13}$

8. (12 points)
P($(A \cup B) \cap C$)
=P($(A \cap C) \cup (B \cap C)$)
=P($A \cap C$)+P($B \cap C$)-P($(A \cap C) \cap (B \cap C)$)
=P($A \cap C$)+P($B \cap C$)-P($A \cap B \cap C$)
=P(A)P(C)+P(B)P(C)-P(A)P(B)P(C) (by mutually independence)
=[P(A)+P(B)-P(A)P(B)]P(C)
=P($A \cup B$)P(C)