**STAT4008 Mid-Term Test version 4        Date: March 10, 2021**

**Time allowed: $1\frac{1}{2}$ hours**

1. Let $T$ be a random variable with survivor function

$$S(t) = \left(\frac{400}{t+400}\right)^5, t > 0.$$

   Find $E[T]$.

2. The data file "test.csv" contains the following data.

   - time: time to event occurred.
   - failure: 0: alive;1: death.

   Use your seat number as the seed to generate a subsample of size one third of total data. Use this subsample to answer the following questions.

   (a) Construct a 95% confidence interval of the survival probability at $t = 5$ using a transformation which not resulting inadmissible limits.

   (b) Construct a 90% confidence interval of the 80 percentile using a variance stabilizing transformation.

3. The cdf of a random variable $T$ is given by

$$F(t) = 1 - \left(\frac{1}{1+t^6}\right)^6, t > 0$$

   Find its hazard function.

4. Let $t_i$ and $t_{i+1}$ be two consecutive time points where failures occurred in a right-censored sample. Explain why the product limit estimator of the survivor probability of $t \in [t_i, t_{i+1})$ equal to $\hat{S}(t_i)$.

5. You are given the results of a study of time (in weeks) to accomplish a particular skill at work:

| Interview Time | Not yet accomplished | Just accomplished | Already accomplished |
|---|---|---|---|
| 1 | 2 | 3 | 0 |
| 2 | 1 | 2 | 2 |
| 3 | 0 | 1 | 5 |
| 4 | 2 | 2 | 2 |

   What is the estimated probability that the skill can be accomplished within 2 weeks?

6. You study the time between accidents and reports of claims. The study was terminated at time 3.

   You are given:

| Time of Accident | Time between Accident and Claim Report | Number of Reported Claims |
|---|---|---|
| 0 | 1 | 15 |
| 0 | 2 | 7 |
| 0 | 3 | 14 |
| 1 | 1 | 9 |
| 1 | 2 | 9 |
| 2 | 1 | 13 |

   Estimate the conditional probability that the time between accident and claim report is less than 2, given that it does not exceed 3.

7. Suggest a transformation to make the mean and variance of the transformed variable functionally independent if the original random variable $X$ has the following property:
$$\mathrm{Var}(X) \propto [E(X)]^2$$

—END—