

MATH1520 Autumn 2018
Homework 4

1. Use the second derivative test to find the relative minimum and the relative maximum of the function

(a) $f(x) = x^3 + 3x^2 + 1$

(b) $f(x) = (x^2 - 9)^2$

(c) $f(x) = x + \frac{1}{x}$

(d) $f(x) = \frac{x^2}{x - 2}$

(e) $h(t) = \frac{1}{1 + t^2}$

2. The second derivative f'' of a function is given. In each case, use this information to determine where the graph of $f(x)$ is concave upward and concave downward and find all values of x for which an inflection point occurs. [You are not required to find $f(x)$ or the y coordinates of the inflection points.]

(a) $f''(x) = x^2(x - 3)(x - 1)$

(b) $f''(x) = \frac{x^2 + x - 2}{x^4 + 2}$

3. Sketch the graph of a function f that has all the following properties:

(a) The graph has discontinuities at $x = -1$ and $x = 3$

(b) $f'(x) > 0$ for $x < 1$, $x \neq -1$

(c) $f'(x) < 0$ for $x > 1$, $x \neq 3$

(d) $f''(x) > 0$ for $x < -1$ and $x > 3$ and $f''(x) < 0$ for $-1 < x < 3$

(e) $f(0) = 0 = f(2)$, $f(1) = 3$

4. Do the global extrema for the function

$$f(x) = x^2 - \frac{2}{x}, \quad x \in [-2, -\frac{1}{2}]$$

exist? If yes, find them.

5. Let $f(x) = \frac{x^2}{(x - 2)^2}$.

(a) Find the domain of f .

(b) Find the intercepts, if any.

(c) Find the location of any vertical asymptotes of f .

(d) Find the horizontal asymptotes.

(e) Find the critical points of f .

(f) Find the intervals of increasing, decreasing.

- (g) Find the possible points of inflection of f .
 - (h) Find the intervals of concave up and down.
 - (i) Sketch the graph of the function.
6. Suppose the graph $y = f(x)$ is concave upward. Show that the graph $y = f(x)$ lies above the tangent to it at $x = a$.

Hint.

- (a) Find the equation of the tangent in terms of a , $f(a)$ and $f'(a)$.
 - (b) Let $g(x) = f(x) - f(a) - f'(a)(x - a)$. Show that $g(x)$ is minimum at $x = a$.
7. Find a point on the curve $y = x^2$ that is closest to the point $(18, 0)$.
8. When a resistor of R ohms is connected across a battery with electromotive force U volts and internal resistance r ohms, a current of I amperes will flow, generating P watts of power, where

$$I = \frac{U}{r + R} \quad \text{and} \quad P = I^2 R$$

Assuming r , U are constants, what choice of R results in maximum power?

9. When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x hundred units, where

$$3p^2 - x^2 = 12.$$

How fast is the supply changing when the price is \$4 per unit and is increasing at the rate of 87 cents per month?

10. A storm at sea has damaged an oil rig. Oil spills from the rupture at the constant rate of $60 \text{ ft}^3/\text{min}$, forming a slick that is roughly circular in shape and 3 inches thick.
- (a) How fast is the radius of the slick increasing when the radius is 70 feet?
 - (b) Suppose the rupture is repaired in such a way that the flow is shut off instantaneously. If the radius of the slick is increasing at the rate of 0.2 ft/min when the flow stops, what is the total volume of oil that spilled onto the sea?