STAT 4008 Assignment 1 **Due: Feb. 5, 2021**

1. Suppose a discrete random variable T taking values 1, 3, 5, 7, 9, 12 with probabilities $\frac{1}{6}, \frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{16}$, respectively.

   (a) Find the mean of T

   (b) Find the survival function of T

   (c) Find the area under the curve $S(t)$ in the right upper quadrant, i.e., find

   $$\int_0^\infty S(t)dt$$

   (d) Compare results in (a) and (c).

2. Suppose the lifetime of a electronic component is exponentially distributed with rate θ with density function

   $$f(t) = \theta \exp(-\theta t), t > 0$$

   Find the conditional probability that $T > t + s$ given $T \geq t$, where $s > 0$. Also find the probability that $T > s$.

3. A random variable T is said to be Weibull distributed if its hazard function is of the form

   $$h(t) = \alpha\lambda t^{\alpha-1}, t > 0$$

   where α and λ are positive constants. Find the distribution of $Y = \log T$.

4. Assume the lifetime random variable T is continuous and denote its mean of remaining lifetime given $T \geq t$ by $m(t)$. Then show that the following functions of T can be expressed in term of $m(t)$

   (a) The survival function

   $$S(t) = \frac{m(0)}{m(t)} \exp\left[-\int_0^t \frac{du}{m(u)}\right]$$

   (b) The density function

   $$f(t) = (m'(t) + 1)\left(\frac{m(0)}{m(t)^2}\right) \exp\left[-\int_0^t \frac{du}{m(u)}\right]$$

   (c) The hazard function

   $$h(t) = -\frac{d}{dt} \log[S(t)] = \frac{m'(t) + 1}{m(t)}$$

5. For the geometric random variable with probability mass function

$$\Pr(X = j) = (1 - p)^{j-1}p, j = 1, 2, \ldots,$$

find its hazard function.

6. For the Poisson distribution with probability mass function

$$\Pr(X = j) = e^{-\lambda}\frac{\lambda^j}{j!}, j = 0, 1, 2, \ldots.$$

Show that the hazard function is monotone increasing.

7. Suppose that the mean residual life of a continuous survival time T is given by $m(t) = t + 10$.

   (a) Find the mean of T

   (b) Find $h(t)$

   (c) Find $S(t)$

8. Find the survival function of the Gompertz random variable where its **hazard** function is given by

$$h(t) = \theta e^{\alpha t}, t \geq 0; \theta, \alpha > 0.$$