STAT4003 Homework Assignment (#4)
(Due Monday, 7 December 2020)

1. The number of successes in n independent trails is to be used to test the null hypothesis that the parameter θ of a binomial population equals 1/2 against the alternative that it does not equal 1/2.

   (i) Find an expression for the likelihood ratio statistic;

   (ii) Use the result of part (i) to show that the critical region of the likelihood ratio test can be writeen as
   $$x \ln x + (n-x)\ln(n-x) \geq K$$
   where x is the observed number of successes;

   (iii) Study the graph of $f(x) = x\ln x + (n-x)\ln(n-x)$, in particular its minimum and its symmetry, to show that the critical region of this likelihood ratio test can also be written as
   $$|x - n/2| \geq K.$$

2. Given a random sample of size n from a normal population with unknown mean and variance, find an expression for the likelihood ratio statistic for testing $H_0: \ \sigma = \sigma_0$ against $H_1: \ \sigma \neq \sigma_0$.

3. Consider a random sample of size 4 from the uniform distribution $U(0, \theta)$. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics. Let the observed value of Y_4 be y_4. We reject $H_0 : \theta = 1$ and accept $H_1 : \theta \neq 1$ if either $y_4 \leq 1/2$ or $y_4 > 1$. Find the power function $Q(\theta), \theta > 0$, of the test.

4. Given a random sample of size n from a normal population with $\mu = 0$, use the Neyman-Pearson theorem to construct the most powerful critical region of size α to test the null hypothesis $\sigma = \sigma_0$ against the alternative $\sigma = \sigma_1$, where $\sigma_1 > \sigma_0$.

5. Let $X_1, \cdots, X_{20}$ be a random sample of size 20 from a Poisson distribution with mean θ. Show that the critical region defined by $\sum_{i=1}^{20} X_i \geq 5$ is a uniformly most powerful critical region for testing $H_0 : \theta = 0.1$ against $H_1 : \ \theta > 0.1$. What is α, the significance level of the test? Find the power function.