Assignment 4 Solution

Section 6.1

8. $P(0 < z < 1.77) = 0.9616 - 0.5 = 0.4616$

14. $P(z < -0.75) = 0.2266$

18. $P(-0.96 < z < -0.36) = 0.3594 - 0.1685 = 0.1909$

48. (a) $z = 0.12$

    (b) $z = 0.52$

    (c) $z = 1.18$

Section 6.2

18. Let the limits be d and c, i.e.

$$P(d \leq X \leq c) = 0.5$$

where X = charitable contributions itemized per income tax return

then

$$P(\frac{d-792}{103} \leq z \leq \frac{c-792}{103}) = 0.5$$

But

$$P(-0.67 \leq z \leq 0.67) = 0.5$$

Therefore,

$$\frac{c-792}{103} = 0.67 \text{ and } \frac{d-792}{103} = -0.67$$

$$c = 861.01 \text{ and } d = 722.99$$

24. Let the standard distribution be σ
Let X = age of the train cars

Given that

$$\begin{aligned} P(X > 22.8) &= 0.2 \\ P(X \leq 22.8) &= 0.8 \\ P(z \leq \frac{22.8 - 19.4}{\sigma}) &= 0.8 \end{aligned}$$

From table,

$$\begin{aligned} P(z \leq 0.84) &= 0.8 \\ \frac{22.8 - 19.4}{\sigma} &= 0.84 \\ \sigma &= 4.048 \end{aligned}$$

28. Let X = amount of water drank by an American in 2008

then

$$X \sim N(23.2, 2.7^2)$$

$$\begin{aligned} P(X > 25) &= P(z > \frac{25 - 23.2}{2.7}) \\ &= P(z > 0.6667) \\ &= 1 - 0.7486 \\ &= 0.2514 \\ P(22 < X < 30) &= P(\frac{22 - 23.2}{2.7} < z < \frac{30 - 23.2}{2.7}) \\ &= P(-0.44 < z < 2.52) \\ &= 0.9941 - 0.33 \\ &= 0.6641 \end{aligned}$$

Section 6.3

8. Let X = amount of glass garbage generated by one family

$$n = 55, X \sim D(17.2, 2.5^2)$$

where D is an unknown distribution

By Central Limit Theorem,

$$\bar{X} \sim N(17.2, \frac{2.5^2}{55})$$

$$\begin{aligned}
P(17 < \bar{X} < 18) &= P(\frac{17 - 17.2}{\sqrt{\frac{2.5^2}{55}}} < z < \frac{18 - 17.2}{\sqrt{\frac{2.5^2}{55}}}) \\
&= P(-0.59 < z < 2.37) \\
&= 0.9911 - 0.2776 \\
&= 0.7135
\end{aligned}$$

16. By Central Limit Theorem,

$$\bar{X} \sim N(24.3, \frac{2.6^2}{33})$$

where X = lifetime of cell phones

$$\begin{aligned}
P(\bar{X} < 23.8) &= P(z < \frac{23.8 - 24.3}{\frac{2.6}{\sqrt{33}}}) \\
&= P(z < -1.1) \\
&= 0.1357
\end{aligned}$$

Section 6.4

10.

$$p = 0.56, n = 500$$

$$\begin{aligned}
np &= (500)(0.56) = 280 \geq 5 \\
n(1-p) &= (500)(1 - 0.56) = 220 \geq 5
\end{aligned}$$

Therefore,

$$\begin{aligned}
X &\sim N(np, np(1-p)) \\
X &\sim N(280, 123.2)
\end{aligned}$$

$$
\begin{aligned}
& P(\text{at least 250 will be enrolled in school}) \\
= \; & P(X \geq 250) \\
= \; & P(z \geq \frac{249.5 - 280}{\sqrt{123.2}}) \\
= \; & P(z \geq -2.75) \\
= \; & 0.997
\end{aligned}
$$

12.

$$p = 0.08, n = 600$$

$$
\begin{aligned}
np &= (600)(0.08) = 48 \geq 5 \\
n(1-p) &= (600)(1-0.08) = 552 \geq 5
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& X \sim N(np, np(1-p)) \\
& X \sim N(48, 44.16)
\end{aligned}
$$

$$
\begin{aligned}
& P(X < 40) \\
= \; & P(z < \frac{39.5 - 48}{\sqrt{44.16}}) \\
= \; & P(z < -1.28) \\
= \; & 0.1003
\end{aligned}
$$