

Question 2

- a) After executed the R code, it returns

$$\begin{aligned}\hat{\eta}_0 &= 15952.1026 \\ \hat{\eta}_1 &= 244.5008 \\ \hat{\beta} &= 409.8953 \\ \hat{\eta}_{02} &= 4383.1081 \\ \hat{\eta}_{03} &= 8975.9729 \\ \hat{\eta}_{12} &= -1059.1879 \\ \hat{\eta}_{13} &= 1582.9473 \\ \hat{\sigma}^2 &= 5916548\end{aligned}$$

- b)  $\hat{y} = 15952.1026 + 244.5008(1) + 409.8953(1) = 16606.5$   
 The estimated annual salary for Mary is USD 16606.5

- c) After executed the R code, it returns  
 $RSS = 266244659$

- d) After executed the R code, it returns

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Regression	4	642448811	160612203	27.146	1.708e-11
Residual	45	266244659	5916548		
Total	49	908693470			

- e) Since the p-value  $< 0.05$ , we reject  $H_0$  at  $\alpha = 0.05$

We have sufficient evidence that Rank is important to explain the Salary, which  $E(Y|S = s, R = j, X = x) = \eta_0 + \eta_1 s + \beta x + \sum(\eta_{0j} U_j + \eta_{1j} U_j s)$  is more appropriate model comparing to  $E(Y|S = s, R = j, X = x) = \eta_0 + \eta_1 s + \beta x$

- f)  $H_0: E(Y|R = j, X = x) = \eta_0 + \beta x + \sum \eta_{0j} U_j$

- g)  $H_1: E(Y|S = s, R = j, X = x) = \eta_0 + \eta_1 s + \beta x + \sum(\eta_{0j} U_j + \eta_{1j} U_j s)$

- h) After executed the R code, it returns

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Regression	3	10748075	3582692	0.6055	0.6148
Residual	45	266244659	5916548		
Total	48	276992734			

- i) Since the p-value  $> 0.05$ , we do not reject  $H_0$  at  $\alpha = 0.05$

We do not have sufficient evidence that Sex is playing an important role to describe the salary in between male and female, that is,  $\eta_1$ ,  $\eta_{02}$  and  $\eta_{03}$  are not differ from 0

### Question 3

#### a) Forward selection

Start: AIC=253.58

$y \sim 1$

	Df	Sum of Sq	RSS	AIC
+ x4	1	1661.66	884.73	181.57
+ x2	1	1120.45	1425.95	214.99
+ x1	1	504.47	2041.92	240.12
+ x3	1	462.22	2084.18	241.55
+ x6	1	360.22	2186.17	244.90
+ x5	1	281.47	2264.92	247.38
<none>			2546.39	253.58

Step: AIC=181.58

$y \sim x4$

	Df	Sum of Sq	RSS	AIC
+ x3	1	141.523	743.21	171.37
+ x5	1	90.016	794.72	176.06
+ x6	1	49.592	835.14	179.54
+ x2	1	25.046	859.69	181.56
<none>			884.73	181.57
+ x1	1	8.291	876.44	182.92

Step: AIC=171.37

$y \sim x4 + x3$

	Df	Sum of Sq	RSS	AIC
+ x6	1	45.431	697.78	168.96
+ x2	1	21.519	721.69	171.32
<none>			743.21	171.37
+ x1	1	8.535	734.68	172.56
+ x5	1	1.637	741.57	173.22

Step: AIC=168.96

$y \sim x4 + x3 + x6$

	Df	Sum of Sq	RSS	AIC
<none>			697.78	168.96
+ x1	1	15.1423	682.64	169.42
+ x2	1	13.2307	684.55	169.62
+ x5	1	4.2635	693.52	170.53

The parsimonious model under forward selection is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$

#### Backward selection

Start: AIC=172.67

$y \sim x1 + x2 + x3 + x4 + x5 + x6$

	Df	Sum of Sq	RSS	AIC
- x2	1	2.93	678.29	170.98
- x5	1	2.95	678.31	170.98
- x1	1	7.01	682.38	171.40
<none>			675.36	172.67
- x6	1	44.82	720.18	175.17
- x3	1	59.18	734.54	176.55
- x4	1	646.12	1321.48	217.66

Step: AIC=170.98

$y \sim x1 + x3 + x4 + x5 + x6$

	Df	Sum of Sq	RSS	AIC
- x5	1	4.34	682.64	169.42
- x1	1	15.22	693.52	170.53
<none>			678.29	170.98
- x6	1	54.83	733.12	174.42
- x3	1	74.01	752.30	176.22
- x4	1	1190.09	1868.39	239.90

Step: AIC=169.42

$y \sim x1 + x3 + x4 + x6$

	Df	Sum of Sq	RSS	AIC
- x1	1	15.14	697.78	168.96
<none>			682.64	169.42
- x6	1	52.04	734.68	172.56
- x3	1	148.70	831.34	181.22
- x4	1	1222.35	1904.99	239.26

Step: AIC=168.96

$y \sim x3 + x4 + x6$

	Df	Sum of Sq	RSS	AIC
<none>			697.78	168.96
- x6	1	45.43	743.21	171.37
- x3	1	137.36	835.14	179.54
- x4	1	1278.97	1976.75	239.85

The parsimonious model under backward selection is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$

The parsimonious models under both forward selection and backward selection are the same. Thus, the parsimonious based on AIC is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$

## b) Forward selection

Start: AIC=255.82

 $y \sim 1$ 

	Df	Sum of Sq	RSS	AIC
+ x4	1	1661.66	884.73	186.07
+ x2	1	1120.45	1425.95	219.48
+ x1	1	504.47	2041.92	244.62
+ x3	1	462.22	2084.18	246.05
+ x6	1	360.22	2186.17	249.40
+ x5	1	281.47	2264.92	251.87
<none>		2546.39	255.82	

Step: AIC=186.07

 $y \sim x4$ 

	Df	Sum of Sq	RSS	AIC
+ x3	1	141.523	743.21	178.12
+ x5	1	90.016	794.72	182.81
<none>		884.73	186.07	
+ x6	1	49.592	835.14	186.28
+ x2	1	25.046	859.69	188.31
+ x1	1	8.291	876.44	189.66

Step: AIC=178.12

 $y \sim x4 + x3$ 

	Df	Sum of Sq	RSS	AIC
+ x6	1	45.431	697.78	177.95
<none>		743.21	178.12	
+ x2	1	21.519	721.69	180.31
+ x1	1	8.535	734.68	181.56
+ x5	1	1.637	741.57	182.21

Step: AIC=177.95

 $y \sim x4 + x3 + x6$ 

	Df	Sum of Sq	RSS	AIC
<none>		697.78	177.95	
+ x1	1	15.1423	682.64	180.66
+ x2	1	13.2307	684.55	180.86
+ x5	1	4.2635	693.52	181.77

The parsimonious model under forward selection is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$ 

## Backward selection

Start: AIC=188.41

 $y \sim x1 + x2 + x3 + x4 + x5 + x6$ 

	Df	Sum of Sq	RSS	AIC
- x2	1	2.93	678.29	184.47
- x5	1	2.95	678.31	184.47
- x1	1	7.01	682.38	184.89
<none>		675.36	188.41	
- x6	1	44.82	720.18	188.66
- x3	1	59.18	734.54	190.04
- x4	1	646.12	1321.48	231.15

Step: AIC=184.47

 $y \sim x1 + x3 + x4 + x5 + x6$ 

	Df	Sum of Sq	RSS	AIC
- x5	1	4.34	682.64	180.66
- x1	1	15.22	693.52	181.77
<none>		678.29	184.47	
- x6	1	54.83	733.12	185.66
- x3	1	74.01	752.30	187.47
- x4	1	1190.09	1868.39	251.15

Step: AIC=180.67

 $y \sim x1 + x3 + x4 + x6$ 

	Df	Sum of Sq	RSS	AIC
- x1	1	15.14	697.78	177.95
<none>		682.64	180.66	
- x6	1	52.04	734.68	181.56
- x3	1	148.70	831.34	190.21
- x4	1	1222.35	1904.99	248.26

Step: AIC=177.95

 $y \sim x3 + x4 + x6$ 

	Df	Sum of Sq	RSS	AIC
<none>		697.78	177.95	
- x6	1	45.43	743.21	178.12
- x3	1	137.36	835.14	186.28
- x4	1	1278.97	1976.75	246.59

The parsimonious model under backward selection is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$ 

The parsimonious models under both forward selection and backward selection are the same. Thus, the parsimonious based on BIC is  $y = \beta_0 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6$

c) After executed the R code, it returns 6.034562. That is  $VIF_5 = 6.034562$