

STAT2001 Assignment 4

Do all 7 questions. Show your steps clearly.

Deadline for this assignment is 22nd Nov. 5:00p.m. You can submit to the assignment locker (next to LSB 125) or submit on Blackboard system.

1. (10 marks) A die is rolled twice. Find the joint probability mass function of X and Y if X denotes the value on the first roll and Y denotes the minimum of the values of the two rolls.
2. (15 marks) Let X and Y be independent random variables both following $b(n, p)$.
 - (a) Compute the covariance of $X + Y$ and $X - Y$.
 - (b) Are $X + Y$ and $X - Y$ independent? Please explain.

3. (15 marks) Let X and Y have the joint p.d.f.

$$f(x, y) = 2(x + y), \quad 0 < x < y < 1.$$

Find the marginal p.d.f. of X and the marginal p.d.f. of Y . Determine whether X and Y are independent.

4. (15 marks) The joint p.d.f. of X and Y is given by

$$f(x, y) = \exp(-(x + y)), \quad 0 \leq x < \infty, 0 \leq y < \infty$$

Find $P(X + Y > 2)$.

5. (15 marks) Suppose that the joint p.d.f. of X and Y is

$$f(x, y) = 6y, \quad 0 < y < x < 1.$$

Find the conditional mean and conditional variance of Y given $X = 0.3$.

6. (15 marks) Using moment generating function, show that if X and Y both follow Geometric distribution with parameter p , $X + Y$ follows a negative binomial distribution. Assume that X and Y are independent.

7. (15 marks) Let X and Y have the joint p.m.f. described in the table:

(x,y)	(1,1)	(2,1)	(3,1)	(1,2)	(1,3)	(2,2)
f(x,y)	1/15	4/15	4/15	1/15	2/15	3/15

Calculate the correlation coefficient of X and Y .

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