STAT2005 Programming Languages for Statistics
Assignment 1
Due: 5pm, 2 October, 2019

1. Using `rep()` and `seq()` as needed to create the following vectors. (The use of `c()` function is prohibited in this question.)

(a) `10 12 14 16 18 20 22 24 26 28 30`

(b) `1 2 3 4 5 2 3 4 5 6 3 4 5 6 7 4 5 6 7 8 5 6 7 8 9`

2. Search for an R function that solves the roots of polynomial equations.

(a) Find all roots of the following equation and save it as `roots`.

$$5x^5 + 4x^4 + 3x^3 + 2x^2 + x = 0.$$

(b) What is the mode of `roots`?

(c) Use a single line command to sort the values in `roots` by ascending order of its imaginary part.

(Note: the imaginary part of a complex number $a+bi$ is b, where $i = \sqrt{-1}$. You can read the help document of the `order()` function by entering `help(order)`.)

3. A standard deck of playing cards can be created in R as a data frame with the following commands.

```
suits <- c("D","C","H","S")
# D = ♦ Diamond, C = ♣ Club, H = ♥ Heart, S = ♠ Spade
ranks <- 2:14
# 11 = Jack, 12 = Queen, 13 = King, 14 = Ace

deck <- matrix(, nrow = 52, ncol = 2)
colnames(deck) = c("suit", "rank")
deck <- as.data.frame(deck)
deck$suit <- rep(suits, 13)
deck$rank <- rep(ranks, 4)
```

(a) Describe the structure of the data frame `deck`, what are the information contained in its row and column?

(b) A poker hand is a set of five playing cards. Sample a poker hand using the data frame `deck` and name it as `hand`.

(c) A flush is a hand that contains five cards all of the same suit. Create a logical value named `is.flush` which is `TURE` if and only if `hand` is a flush.

Hint: You may use `hand <- deck[c(17,9,1,49,41),]` as a test case. The `unique()` function would be useful.

(d) A straight is a hand that contains five cards of sequential rank. Note that both A♦ K♣ Q♣ J♦ 10♠ and 5♥ 4♠ 3♥ 2♣ A♦ are considered to be straight, but Q♠ K♠ A♣ 2♥ 3♦ is not. Create a logical value named `is.straight` which is `TURE` if and only if `hand` is a straight. Use a test case similar to that in (c) to verify your answer.

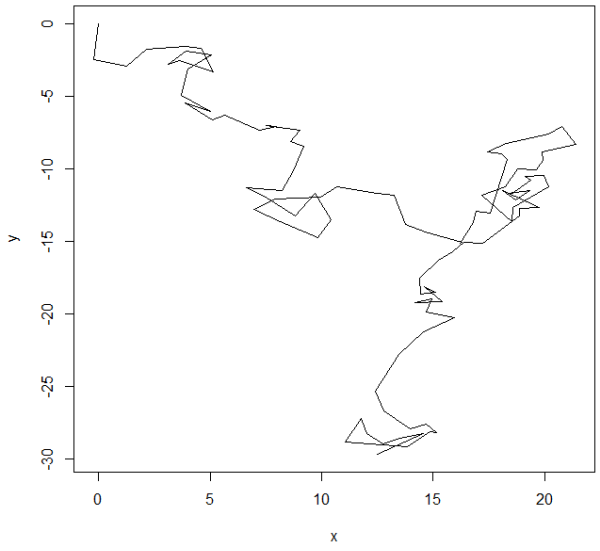
Hint: The `all()` function would be useful.

(e) A straight flush is a hand that is both a straight and a flush. Create a logical value named `is.straightflush` which is `TURE` if and only if `hand` is a straight flush. Modify the logical values `is.flush` and `is.straight` in (c) and (d) such that they becomes `FALSE` if `hand` is a straight flush. Use a test case similar to that in (c) to verify your answer.

4. (a) Consider a two-dimensional random walk

$$X_{t+1} = X_t + Z_{t+1},\ X_0 = 0,$$
$$Y_{t+1} = Y_t + W_{t+1},\ Y_0 = 0,$$

where, Z_t, W_t, $t = 1,2,3,\ldots$ are independent and identically distributed standard normal random variables. Simulate and plot the sample path of (X_t, Y_t) for $t = 0,1,\ldots,100$. A sample is shown below.



(b) Let $0 \le \rho \le 1$, and $U_t = \rho Z_t + \sqrt{1-\rho^2} W_t$, with Z_t, W_t defined as in (a). Redefine the two-dimensional random walk as

$$X_{t+1} = X_t + Z_{t+1},\ X_0 = 0,$$
$$Y_{t+1} = Y_t + U_{t+1},\ Y_0 = 0.$$

i. Simulate 100 sample of Z_t and U_t with $\rho = -0.5$. Check the normality of the sample from U_t with the normal QQ plot. Verify that $E(U_t) = 0$, $Var(U_t) = 1$, and $\mathrm{Corr}(Z_t, U_t) = -0.5$ using simulation.
ii. With $\rho = 0.99$, simulate and plot the sample path of (X_t, Y_t) for $t = 0,1, \ldots, 100$. How is it different from the plot in (a)?

You should submit a file `asg1.r` via Blackboard, which contains all the R codes you use to finish this assignment. The codes should be commented as clearly as possible. Written work (if any) should also be submitted to the assignment drop-box.