Date: Oct. 28, 2019.
Time allowed: 50 minutes.
Full mark: 30.

The Chinese University of Hong Kong
Department of Mathematics
MATH1550 Methods of Matrices and Linear Algebra
Test

1. Let

$$A = \begin{pmatrix} 2 & -4 & 3 & -3 & 5 \\ 1 & 2 & 1 & 5 & 3 \\ 1 & -2 & 4 & -34 & 0 \end{pmatrix}$$

be a 3×5 matrix over $\mathbb{R}$.

(a) Find rref(A). [You may not need to follow the Gauss-Jordan Method.] (5 marks)

(b) Consider the following system of linear equations over $\mathbb{R}$:

$$\begin{cases} 2x_1 - 4x_2 + 3x_3 - 3x_4 = 5 \\ x_1 + 2x_2 + x_3 + 5x_4 = 3 \\ x_1 - 2x_2 + 4x_3 - 34x_4 = 0 \end{cases}$$

Find all the lead variable(s) in terms of free variable(s). Find the solution set of the above system. (5 marks)

2. Let $A = \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & -1 & -1 \\ -2 & 5 & -2 & 0 \end{pmatrix}$. A student wants to find A^{-1}, if any. The student did the following steps:

$$\left(\begin{array}{cccc|cccc} 1 & -2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 0 & 1 & 0 \\ -2 & 5 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & -2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & -2 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & -2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 2 & -2 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \end{array}\right).$$

Now, you are required to help the student to finish the work. (6 marks)

3. Find the condition(s) of a, b, c such that the following system is consistent (i.e., has solution(s)).

$$\begin{cases} x_1 - 2x_2 + 5x_3 = a \\ 4x_1 - 5x_2 + 8x_3 = b \\ -3x_1 + 3x_2 - 3x_3 = c \end{cases}$$

(6 marks)

4. Let

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \text{ and } A = \begin{pmatrix} 3 & 3 & -1 & -1 \\ 3 & 3 & -1 & -1 \\ -1 & -1 & 3 & 3 \\ -1 & -1 & 3 & 3 \end{pmatrix}.$$

(a) Evaluate BB^T. And hence find B^{-1}. (3 marks)

(b) Evaluate $B^{-1}AB$. (2 marks)

(c) Using (b) or otherwise, compute A^n for any positive integer n. [Hint: Consider $(B^{-1}AB)^n$.] (3 marks)