

Question 1. Assume we have a simplified version of the Animal Classification dataset¹ which includes properties of animals as descriptive features and the animal species as target feature. In our dataset, the animals are classified as being Mammals or Reptiles based on whether they are toothed and have legs, as shown in Table 1. In this question, you are asked to develop a decision tree based on this simplified dataset.

Table 1: Animal Classification dataset

Instance	Toothed	Legs	Species
1	T	T	Mammal
2	T	T	Mammal
3	T	F	Reptile
4	F	T	Mammal
5	T	T	Mammal
6	T	T	Mammal
7	T	F	Reptile
8	T	F	Reptile
9	T	T	Mammal
10	F	T	Reptile

(a) Calculate the resulting Gini index when splitting on the attribute “Toothed” and “Legs”, respectively (i.e. $Gini_{split\text{Toothed}}$ and $Gini_{split\text{Legs}}$). Show your calculation details. Which attribute would be chosen as the first splitting attribute? (10 pts)

(b) Based on the decision in Question 1.a, draw a two-level decision tree if needed using both attributes for splitting. Mark the class label in each leaf node. In case of a tie on the “Mammal” and “Reptile” instances in a leaf node, mark the node as “-”. (4 pts)

(c) **WEKA Tool Practice.** Use the WEKA tool to classify the data with decision tree (J48) under the test option “Use training set”. Copy the result in ‘classifier output’ window to your assignment. (6 pts)

¹ the UCI Zoo Dataset

Question 2. In class, we learn how to solve the sparse recovery problem:

$$\begin{aligned} & \min |x_1| + |x_2| + |x_3| + |x_4| + |x_5| \\ & s. t. \quad \underbrace{\begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 \\ -2 & 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 2 \\ 1 \\ 1 \\ -3 \end{bmatrix}}_b \end{aligned}$$

We find a solution $x = (0,1,0,0,3,0)$ with four entries being zero. Now, instead of finding a solution x to $Ax = b$ with as many zero entries as possible, we want to find a solution to $Ax = b$ that minimizes the first two entries. This motivates the following optimization formulation:

$$\begin{aligned} & \min |x_1| + |x_2| \\ & s. t. \quad \begin{bmatrix} 0 & -1 & 0 & -1 & 1 & 0 \\ -2 & 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ -3 \end{bmatrix} \end{aligned}$$

Please use “cvxpy” to solve the above sparse recovery problem. (10 pts: 8pts for code, 2pts for answer)

Question 3. In lecture we have learned two ideas to tackle the problem of background extraction. These two ideas lead to an optimization formulation as follows:

$$\begin{aligned} & \text{minimize} \quad (|y_1^1| + |y_2^1| + \cdots + |y_n^1|) + \cdots + (|y_1^m| + |y_2^m| + \cdots + |y_n^m|) \\ & \text{subject to} \quad f^i = x + y^i \quad \text{for } i = 1, \dots, m. \end{aligned} \tag{S_1}$$

- (a) We have three figures extracted from one video, i.e., $m = 3$. The three figures are denoted as M1, M2 and M3, with the same size of 130×160 , i.e., $n = 130 \times 160$. Part of the cvxpy code is shown as below. Can you modify the code to obtain an implementation of the above formulation? You can find the figures in attachment. (10 pts)

- (b) Please run your code for (S_1) with your own figure(s) and examine the result. How is the background extraction? Please attach the figures you use and the results you obtain. (bonus: 10 pts)

Cvxdpy Code for Background-Extraction Formulation (S_∞)

```
import numpy as np
import cv2
import cvxdpy as cp
from cvxdpy import *
import matplotlib.pyplot as plt

im1 = cv2.imread('/content/Figure1.png',cv2.IMREAD_GRAYSCALE)
im2 = cv2.imread('/content/Figure2.png',cv2.IMREAD_GRAYSCALE)
im3 = cv2.imread('/content/Figure3.png',cv2.IMREAD_GRAYSCALE)
M_size = im1.shape
size_a = M_size[0]
size_b = M_size[1]
n = size_a*size_b

M1 = im1.reshape(n,-1)
M2 = im2.reshape(n,-1)
M3 = im3.reshape(n,-1)

w = cp.Variable((n,1))

# Please trying to implementing you code here:
#####

plt.figure(figsize=(6,6))
plt.imshow((M1 - w.value).reshape(size_a, size_b), cmap='gray')
plt.figure(figsize=(6,6))
```

```
plt.imshow((M2 - w.value).reshape(size_a, size_b), cmap='gray')
plt.figure(figsize=(6,6))
plt.imshow((M3 - w.value).reshape(size_a, size_b), cmap='gray')
plt.figure(figsize=(6,6))
plt.imshow((w.value).reshape(size_a, size_b), cmap='gray')
```

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