THE CHINESE UNIVERSITY OF HONG KONG

Department of Statistics

Subject Code:	STAT4003	Course Title:	Statistical Inference
Session:	Semester 1, 2020/2021, Midterm Examination		
Date:	29 October 2020	Time:	15:00pm -16:30pm
Time Allowed:	90 Minutes		

This question paper has __1___ page.

Instructions to Candidates:

1. Attempt **ALL** questions
2. This paper has **4** questions.
3. Give **full details** of your working to the questions in the A4-size answer sheet.
4. Sign your name on **each** page of your answer sheet

Subject Examiner: Professor Yuanyuan LIN

DO NOT TURN OVER THE PAGE UNTIL YOU ARE TOLD TO DO SO

1. (**25 marks**) Let $X_1, X_2, ..., X_n$ be a random sample from the Bernoulli distribution, say, $P(X = 1) = \theta$, $P(X = 0) = 1 - \theta$.

   (a) (**6 marks**) Let $S_n = \sum_{i=1}^{n} X_i$, find the distribution of S_n.

   (b) (**6 marks**) Show $\bar{X}(1 - \bar{X})$ is a biased estimator for $\theta(1 - \theta)$, where $\bar{X} = \sum_{i=1}^{n} X_i/n$.

   (c) (**8 marks**) Find the Cramér Rao Lower Bound for the variance of an unbiased estimator for $\theta(1 - \theta)$.

   (d) (**5 marks**) Does the variance of any unbiased estimator for $\theta(1-\theta)$ achieve this bound? Why? Explain in details.

2. (**25 marks**) Let $X_1, X_2, ..., X_n$ be a random sample from Unif$[0, \theta]$.
   [*Hint* : The probability density function of Unif$[a, b]$ is $f(x) = 1/(b - a)$, for $a \leq x \leq b$; $f(x) = 0$, otherwise.

   (a) (**6 marks**) Find an estimator for θ by the method of moments. Is it unbiased? Hence or otherwise, find an unbiased estimator for θ.

   (b) (**6 marks**) Find an MLE for θ. Is it unbiased? Hence or otherwise, find an unbiased estimator for θ.

   (c) (**8 marks**) Find the variance of unbiased estimators based on (a) & (b). Which unbiased estimator for θ is more efficient?

   (d) (**5 marks**) Given that $\theta > 1$, find an MLE for θ.

3. (**25 marks**) Let $X_1, X_2, ..., X_n$ be a random sample from $N(0, \sigma^2)$ and $\bar{X} = \sum_{i=1}^{n} X_i/n$.

   (a) (**5 marks**) Find an MLE for σ^2. Is it unbiased? Why?

   (b) (**5 marks**) Find the Cramér Rao Lower Bound for the variance of an unbiased estimator for σ^2.

   (c) (**5 marks**) Find c_1, c_2 (which may depend on n) such that $W_1 = c_1\bar{X}^2$ and $W_2 = c_2 \sum_{i=1}^{n}(X_i - \bar{X})^2$ are unbiased for σ^2.

   (d) (**6 marks**) Let $W = aW_1 + (1 - a)W_2$, where a is a constant. Find a such that $Var(W)$ is minimized. Find $Var(W)$.

   (e) (**4 marks**) Is the variance of W in part (d) equal to the Cramér Rao Lower Bound? Explain.

4. (**25 marks**) Let $X_1, X_2, ..., X_n$ be a random sample from Unif$[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$.

   (a) (**5 marks**) Let $\bar{X} = \sum_{i=1}^{n} X_i/n$. Show that $\bar{X}$ is unbiased and consistent.

   (b) (**5 marks**) Let $Y = (X_{(1)}+X_{(n)})/2$, where $X_{(1)} = \min(X_1, ..., X_n)$ and $X_{(n)} = \max(X_1, ..., X_n)$. Is Y an unbiased estimator for θ? Explain in details.

   (c) (**5 marks**) Find the variance of Y.

   (d) (**5 marks**) Consider the two estimators $\bar{X}$ and Y, which is more efficient?

   (e) (**5 marks**) Find a sufficient statistic for θ.

*** End ***