MATH1520 Autumn 2018
Homework 2

You don't need to hand in the homework. Solution will be posted online.

1. Determine the points of discontinuity of the function:
$$f(x) = \frac{x^2 - 7x + 1}{x^2 - 2x}.$$

2. Suppose $f(x)$ and $g(x)$ are continuous at $x = 1$ with $f(1) = 1$, $g(1) = 10$. Compute
$$\lim_{x \to 1} \left| \frac{f(x)^2 - g(x)}{f(x) + 2g(x)} \right|. \tag{1}$$

3. For what values of a and b is
$$f(x) = \begin{cases} -2 & x \leq -1 \\ ax - b & -1 < x < 1 \\ 3 & x \geq 1 \end{cases} \tag{2}$$
continuous at every x?

4. Determine whether $f(x)$ is continuous at $x = 0$:
$$f(x) = \begin{cases} \dfrac{x(x+1)}{|x|}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$$

5. Let $f(x) = x^3 - \dfrac{3}{x}$. Show that there exists $c \in [1, 2]$ such that $f(c) = 3$.

6. Show that there is a root of the equation $x^3 - x - 1 = 0$ between 1 and 2.

7. Use the first principle to find the derivative of $f(x) = x^2 + 2x + x^{-1}$.

8. Use the first principle to find the derivative of $f(x) = \dfrac{x^2}{x+1}$.

9. Use the first principle to find the derivative of $f(x) = \sqrt{x^2 + 1}$.

10. Use the first principle to find the derivative of $f(x) = x^{1/4}$.
Hint: Use $a^4 - b^4 = (a - b)(a^3 + a^2b + ab^2 + b^3)$.

11. Find the value of a that makes the following function differentiable for all x-values.
$$g(x) = \begin{cases} ax, & \text{if } x < 0, \\ x^2 - 5x, & \text{if } x \geq 0. \end{cases}$$

12. Suppose u and v are differentiable functions of x and that

$$u(1) = 2, \quad u'(1) = 0, \quad v(1) = 5, \quad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

(a) $\dfrac{d}{dx}(uv)$

(b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$

(c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$

(d) $\dfrac{d}{dx}(7v - 2u)$

13. Compute the derivatives of the following functions.

(a) $f(x) = 3x^2 + \sqrt{x}$

(b) $g(x) = e^{4x^3}$

(c) $h(x) = \sqrt{x^2 + 1}$

(d) $p(x) = (1 + e^x)(x^2 + 1)$

(e) $q(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(f) (new starting from this one) $y(x) = \dfrac{2x + 5}{3x - 2}$

(g) $w(x) = (2x - 7)^{-1}(x + 5)$

(h) $r(t) = 2\left(\frac{1}{\sqrt{t}} + \sqrt{t}\right)$

(i) $y(x) = \sqrt[3]{x^{8.6}} + 2e^{2.3}$

(j) $w(z) = 3z^2e^{3z}$

(k) $w(x) = \left(\dfrac{1 + 3x}{3x}\right)(3 - x)$

(l) $f(t) = \dfrac{t^2 + 3}{(t - 1)^3 + (t + 1)^3}$

14. By using the logarithmic differentiation, compute $\dfrac{dy}{dx}$:

(a) $y = (2x + 1)^3(x - 1)^4\sqrt{(3x + 2)^5}$.

(b) $y = x^{x^2}$.

(c) $y = (\ln x + 1)^{\ln x}$.