STAT1011 Assignment 2 Solution

1. a. If there are no restrictions, the number of possible arrangements is $8! = 40320$.

   b. If A and B must sit next to each other, consider them as 1 person first, number of ways $= 7!$. Now A could be on the right or on the left. Therefore, the total number of possible ways $= 2(7!) = 10080$.

2. Number of possible committee without restriction $= {}_{15}C_6$.

$$P(4m, 2w) = \frac{{}_7C_4 \cdot {}_8C_2}{{}_{15}C_6} = 0.1958.$$

3.

$$P(\text{blackjack}) = P(\text{1 ace and 1 card from \{K,Q,J,10\}}) = \frac{{}_4C_1 \cdot {}_{16}C_1}{{}_{52}C_2} = 0.04827.$$

4. Let event A = first lands on 6, event B = Dice land on different number, then the required probability is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^5/_{36}}{{}^{30}/_{36}} = \frac{1}{6}$$

5. (a)

$$P(\text{offspring is Aa}) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

$$\begin{aligned} &P(\text{male parant is AA} \mid \text{offspring is Aa}) \\ &= \frac{P(\text{male parant is AA} \cap \text{offspring is Aa})}{P(\text{offspring is Aa})} = \frac{\frac{1}{2} \cdot 1}{\frac{3}{4}} = \frac{2}{3} \end{aligned}$$

   (b)

$$\begin{aligned} &P(\text{male parant is AA} \mid \text{both offsprings are Aa}) \\ &= \frac{P(\text{male parant is AA} \cap \text{both offsprings are Aa})}{P(\text{both offsprings are Aa})} = \frac{\frac{1}{2} \cdot 1}{\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}} = \frac{4}{5} \end{aligned}$$

   (c)

$$\begin{aligned} &P(\text{male parant is Aa} \mid \text{one offspring is aa}) \\ &= \frac{P(\text{male parant is Aa} \cap \text{one offspring is aa})}{P(\text{one offspring is aa})} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{2}} = 1 \end{aligned}$$

Indeed, the answer is trivial since an AA male parent cannot produce an aa offspring.