

The Chinese University of Hong Kong
Academic Honesty Declaration Statement

Submission Details

Student Name	CHAN King Yeung (1155119394)		
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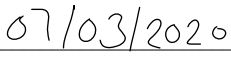
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Signature (CHAN King Yeung, 1155119394)



Date

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Question 1

Total frequency: $3n^2 + 8n + 5$

Time complexity: $O(n^2)$

Question 2

a) By definition, we have $g_i(n) \leq c_i \cdot f_i(n)$ for any $n \geq n_i$

The product of $g_1(n)$ and $g_2(n)$ resulting an inequality as follow

$$g_1(n)g_2(n) \leq c_1c_2 \cdot f_1(n)f_2(n) \text{ for any } n > \max(n_1, n_2)$$

Then we have $g_1(n)g_2(n) = O(f_1(n)f_2(n))$

b) $g(n) = (n^2 + \sqrt{n}) \cdot (n + \log(n)) = O(n^2 \cdot n) = O(n^3)$

c) $g(n) = (n^3 + 3n^2 + 5) \cdot (n^2 + n^4) = \Theta(n^3 \cdot n^4) = \Theta(n^7)$

d) By definition, we have $c_1 \cdot f(n) \leq g(n) \leq c_2 \cdot f(n)$ for any n

The maximum of $f(n)$ and $g(n)$ resulting an inequality as follow

$$c_1(f(n) + g(n)) \leq \max(f(n), g(n)) \leq c_2(f(n) + g(n))$$

Given both $f(n)$ and $g(n)$ are nonnegative function, we have

$$\max(f(n), g(n)) \leq 1 \cdot (f(n) + g(n)) \text{ and } \max(f(n), g(n)) \geq \frac{1}{2} \cdot (f(n) + g(n))$$

Thus, $c_1 = \frac{1}{2}$ and $c_2 = 1$ holds the inequality for any n

Question 3

a)

6	a_h	c	9	d	a_t		b	\emptyset	4	a	d	3	c	b		\emptyset	a
a			b			a_t			c			d			a_h		

b)

7	b	a_t	5	d	a		a	\emptyset	2	a_h	d	3	c	b		\emptyset	c
a			b			a_t			c			d			a_h		

c)

6	c	d	8	d	a_t		d	\emptyset	3	a_h	a	5	a	a_t		\emptyset	c
a			b			a_t			c			d			a_h		

Question 4

a) We have that $a = 4, b = 4, \lambda = 0$

Since $\log_4(4) > 0$, we have that $g(n) = O(n^{\log_b(a)}) = O(n)$

b) We have that $a = 2, b = 4, \lambda = \frac{1}{2}$

Since $\log_4(2) = \frac{1}{2}$, we have that $g(n) = O(n^\lambda \cdot \log(n)) = O(\sqrt{n} \log(n))$

c) We have that $a = 2, b = 4, \lambda = 2$

Since $\log_4(2) < 2$, we have that $g(n) = O(n^\lambda) = O(n^2)$

Question 5

$f_1(n) = 2^{2^{100000}}$ runs in a constant time, thus, $f_1(n) = O(1)$

$f_2(n) = 2^{100000n}$ grows in 2^n , thus, $f_2(n) = O(2^n)$

$f_3(n) = \binom{n}{2}$ rewrite as $\frac{n(n-1)}{2}$, thus, $f_3(n) = O(n^2)$

$f_4(n) = n\sqrt{n}$ known as $n^{\frac{3}{2}}$, thus, $f_4(n) = O(n^{\frac{3}{2}})$

The growth rate, therefore, is $f_2(n) > f_4(n) > f_3(n) > f_1(n)$

Question 6

a) Suppose $T(n) \leq c \cdot n^2$ holds for $n \leq k - 1$

For $n = k$, we have $T(k) \leq T(k - 1) + k \leq c \cdot (k - 1)^2 + k = c \cdot k^2 - 2c \cdot k + c + k$

To make $T(k) \leq c \cdot k^2$, c must satisfy that $-2c \cdot k + c + k \leq 0 \Rightarrow c \geq 1$

By induction, for any $k \geq 1$, we obtain $T(n) \leq 1 \cdot n^2$, therefore, $T(n) = O(n^2)$

b) We have that $a = 1, b = 2, \lambda = 0$

Since $\log_2(1) = 0$, we have that $T(n) = O(n^\lambda \cdot \log(n)) = O(\log(n))$

Question 7

By the definition of Big-Omega, we obtain as follow

$$g(n) = \Omega\left(n^{\log_b(a)} + \sum_{i=0}^y \left(\frac{a}{b^\lambda}\right)^i n^\lambda\right), \text{ where } y = \log_b(n) - 1$$

a) Given $\log_b(a) < \lambda$, we have $a < b^\lambda$ and $g(n) = \Omega(n^{\log_b(a)} + c_0 \cdot n^\lambda)$ for some constant c_0

n^λ grow faster than $n^{\log_b(a)}$, thus by the sum property we obtain $g(n) = \Omega(n^\lambda)$

b) Given $\log_b(a) = \lambda$, we have $a = b^\lambda$ and $g(n) = \Omega(n^{\log_b(a)} + n^\lambda \cdot \log_b(n))$

$n^\lambda \cdot \log_b(n)$ grow faster than $n^{\log_b(a)}$, thus by the sum property we obtain $g(n) = \Omega(n^\lambda \cdot \log(n))$

c) Given $\log_b(a) > \lambda$, we have $a > b^\lambda$ and $g(n) = \Omega(n^{\log_b(a)} + c_0 \cdot n^\lambda)$ for some constant c_0

$n^{\log_b(a)}$ grow faster than n^λ , thus by the sum property we obtain $g(n) = \Omega(n^{\log_b(a)})$
