CITY UNIVERSITY OF HONG KONG STUDENTS' UNION
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STAT 3008
Assignment 4

Question 1

a) $X'X = \begin{pmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{pmatrix}$ $(X'X)^{-1} = \frac{1}{n\,SXX}\begin{pmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{pmatrix}$ $H = \frac{1}{n\,SXX}\begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}\begin{pmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{pmatrix}\begin{pmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{pmatrix}$

$$h_{ii} = \frac{1}{n\,SXX}\left(\Sigma x_j^2 - 2x_i \Sigma x_j + n x_i^2\right)$$
$$= \frac{\Sigma x_j^2 - n\bar{x}^2}{n\,SXX} + \frac{\bar{x}^2 - 2x_i\bar{x} + x_i^2}{SXX}$$
$$= \frac{1}{n} + \frac{(x_i - \bar{x})^2}{SXX}$$

b) $\sum_{j=1}^{n} x_j = \sum_{i=1}^{n-1} x_i + x_n$

$= (n-1)(a+\delta) + a - (n-1)\delta$

$a = \bar{x}$

$SXX = \sum_{i=1}^{n-1} x_i^2 + x_n^2 - n\bar{x}^2$

$= (n-1)(a^2 + 2a\delta + \delta^2) + a^2 - 2a(n-1)\delta + (n-1)^2\delta^2 - na^2$

$= (n-1)n\delta^2$

$$h_{nn} = \frac{1}{n} + \frac{(a-(n-1)\delta - a)^2}{(n-1)n\delta^2}$$
$$= \frac{1}{n} + \frac{n-1}{n}$$
$$= 1$$

c) $$h_{ii} = \frac{1}{n} + \frac{(a+\delta-a)^2}{(n-1)n\delta^2}$$
$$= \frac{1}{n} + \frac{1}{(n-1)n}$$
$$= \frac{1}{n-1}, \quad i \in [1, n-1]$$

d) $\sum_{j=1}^{2m+1} x_j = \sum_{i=1}^{m} x_i + \sum_{k=m+1}^{2m} x_k + x_{2m+1}$

$= m(a+\delta) + m(a-\delta) + a$

$= (2m+1)a$

$\bar{x} = a$

$SXX = \sum_{i=1}^{m} x_i^2 + \sum_{k=m+1}^{2m} x_k^2 + x_{2m+1}^2 - (2m+1)\bar{x}^2$

$= m(a^2 + 2a\delta + \delta^2) + m(a^2 - 2a\delta + \delta^2) + a^2 - (2m+1)a$

$= 2m\delta^2$

$= (n-1)\delta^2$

$$h_{ii} = \begin{cases} \frac{1}{n} + \frac{(a+\delta-a)^2}{(n-1)\delta^2}, & i \in [1, m] \\ \frac{1}{n} + \frac{(a-\delta-a)^2}{(n-1)\delta^2}, & i \in [m+1, 2m] \\ \frac{1}{n} + \frac{(a-a)^2}{(n-1)\delta^2}, & i = 2m+1 \end{cases}$$

$$= \begin{cases} \frac{2n-1}{(n-1)n}, & i \in [1, 2m] \\ \frac{1}{n}, & i = 2m+1 \end{cases}$$