The Chinese University of Hong Kong

Department of Mathematics

MATH1550 Methods of Matrices and Linear Algebra

Assignment 1

Please hand in your assignment to assignment box before 5:30p.m. on Sept. 25, 2019 (Wednesday). The assignment box is located at the 2nd floor of LSB and opposites to the Room 223.

1-1: Find the solution set of the linear system

$$\begin{cases} x & + & 2y & + & 3z & = & 1 \\ 2x & + & 4y & + & 7z & = & 2 \\ 3x & + & 7y & + & 11z & = & 8 \end{cases}$$

1-2: (1.1 no. 31) Find the polynomial $f(t) = a + bt + ct^2$ of degree 2 whose graph passes through the points $(1, -1)$, $(2, 3)$, and $(3, 13)$.

1-3: Consider the linear system (1.1 no. 19)

$$\begin{cases} x & + & y & - & z & = & -2 \\ 3x & - & 5y & + & 13z & = & 18 \\ x & - & 2y & + & 5z & = & k \end{cases}$$

where k is an arbitrary number.

(a) For which value(s) of k does this system have solution(s)?

(b) Find all solutions for each value of k found in part (a).

1-4: A three-digit number has two properties. The tens-digit and the ones-digit add up to 5. If the number is written with the digits in the reverse order, and then subtracted from the original number, the result is 792. Use a system of equations to find all of the three-digit numbers with these properties.

1-5: Let J be an $m \times n$ matrix with all entries equal to 1, i.e., $(J)_{ij} = 1$ for $1 \le i \le m$ and $1 \le j \le n$. Let B be an $m \times n$ matrix with

$$B = \begin{pmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 1 & 2 & 3 & \cdots & n-1 & n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & \cdots & n-1 & n \end{pmatrix}.$$

(a) Find $J^T B$.

(b) Find BJ^T.

1-6: Let A be an $m \times n$ matrix. Show that AA^T is a symmetric matrix.

1-7: Let A be an $n \times n$ matrix. Prove that there exists a symmetric matrix X and a skew-symmetric matrix Y such that $A = X + Y$.