CSCI2100C 2019-20: Solution 4 Part 2

\# This assignment is due at 11:59:59pm, 5th May 2020.

- **Q1. [38 marks]** Consider the directed graph G_1 as shown in Figure 1. Answer the following questions.

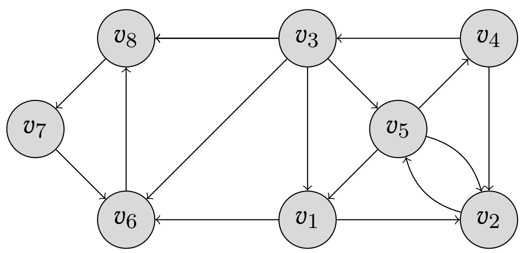


Figure 1. Directed graph for Q1

  - **(i). [4 marks]** Calculate the out-degree of v_3 and the in-degree of v_8. (Refer to CSCI2100C-Lecture22 Page 11)

    The out-degree of v_3 is 4. The in-degree of v_8 is 2.

  - **(ii). [8 marks]** For G_1, show both its adjacency list representation and its adjacency matrix representation. (Refer to CSCI2100C-Lecture22 Pages 17-20)

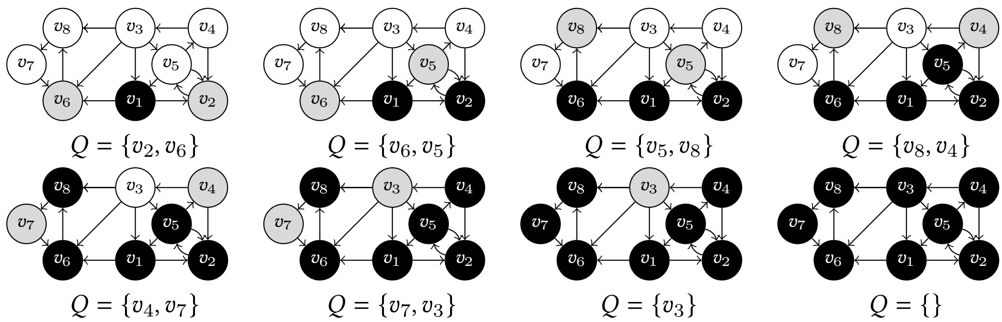
$v_1 \to v_2 \to v_6 \to \varnothing$
$v_2 \to v_5 \to \varnothing$
$v_3 \to v_1 \to v_5 \to v_6 \to v_8 \to \varnothing$
$v_4 \to v_2 \to v_3 \to \varnothing$
$v_5 \to v_1 \to v_2 \to v_4 \to \varnothing$
$v_6 \to v_8 \to \varnothing$
$v_7 \to v_6 \to \varnothing$
$v_8 \to v_7 \to \varnothing$

Adjacency List for Q1(ii)

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1		1				1		
v_2					1			
v_3	1				1	1		1
v_4		1	1					
v_5	1	1		1				
v_6								1
v_7						1		
v_8							1	

Adjacency Matrix for Q1(ii)

  - **(iii). [10 marks]** Traverse G_1 using breadth-first search with v_1 as the source, assuming that the out-neighbors of a node are visited in ascending order of ID. Show the process and the content of the queue Q step by step. You may use 0 to denote the color to be white, 1 to denote the color to be gray, and 2 to denote the color to be black. (Refer to CSCI2100C-Lecture22 Pages 24-28)



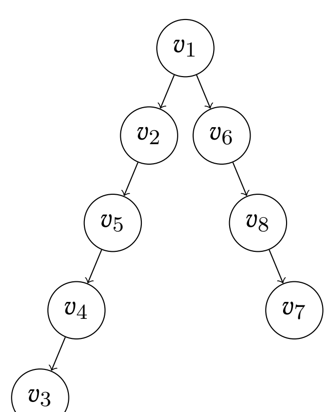
Breadth-First Search for Q1(iii)

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
minlength	0	1	4	3	2	1	3	2
prev	*nil*	v_1	v_4	v_5	v_2	v_1	v_8	v_6

minlength and **prev** Array for Q1(iv)

- **(iv). [8 marks]** According to the results of Part (iii), show the contents of **minlength** array and **prev** array respectively. (Refer to CSCI2100C-Lecture22 Pages 34-35)
- **(v). [4 marks]** Show how to get the minimum length path from the source v_1 to v_4 using the **minlength** array and **prev** array. Justify your answer.

  Get the previous node of v_4 in prev array, which is v_5; get the previous node of v_5, which is v_2; get the previous node of v_1, i.e., the source. We get the path: $v_1 \longrightarrow v_2 \longrightarrow v_5 \longrightarrow v_4$, which length is 3.
- **(vi). [4 marks]** Draw the BFS tree. (Refer to CSCI2100C-Lecture22 Page 36)



BFS Tree for Q1(vi)

- **Q2. [26 marks]** A directed graph G_2 is shown in Figure 4. Assume that we use depth-first search (DFS) to check if G_2 is a DAG and the permutation of nodes to do DFS on G_2 is $(v_2, v_3, v_4, v_5, v_6, v_1, v_7)$. During a DFS traversal, assume that the out-neighbors of a node are visited in ascending order of ID. Answer the following questions.

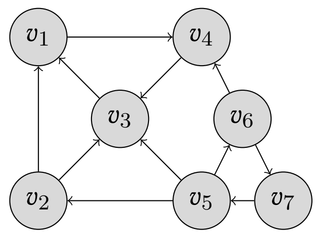
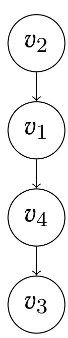


Figure 4. Directed Graph G_2 for Q2

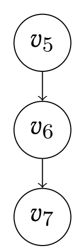
  - **(i). [7 marks]** Show the first discovery time and finish time of each node. (Refer to CSCI2100C-Lecture24 Pages 5-6)

    v_2: 1/8, v_1: 2/7, v_4: 3/6, v_3: 4/5, v_5: 9/14, v_6: 10/13, v_7: 11/12.

  - **(ii). [4 marks]** Draw the DFS trees. (Refer to CSCI2100C-Lecture24 Page 7)



DFS-Tree Rooted at v_2 for Q2(ii)



DFS-Tree Rooted at v_5 for Q2(ii)

  - **(iii). [11 marks]** Classify edges according to the interval of each node derived from Part (i). You should explicitly output the type of each edge. Justify your answer. (Refer to CSCI2100C-Lecture24 Page 8)

    $\langle v_1, v_4\rangle$, $I(v_4) \subset I(v_1)$, forward edge. $\langle v_2, v_1\rangle$, $I(v_1) \subset I(v_2)$, forward edge.
    $\langle v_2, v_3\rangle$, $I(v_3) \subset I(v_2)$, forward edge. $\langle v_3, v_1\rangle$, $I(v_3) \subset I(v_1)$, backward edge.
    $\langle v_4, v_3\rangle$, $I(v_3) \subset I(v_4)$, forward edge. $\langle v_5, v_2\rangle$, $I(v_5) \cap I(v_2) = \emptyset$, cross edge.
    $\langle v_5, v_3\rangle$, $I(v_5) \cap I(v_3) = \emptyset$, cross edge. $\langle v_5, v_6\rangle$, $I(v_6) \subset I(v_5)$, forward edge.
    $\langle v_6, v_4\rangle$, $I(v_6) \cap I(v_4) = \emptyset$, cross edge. $\langle v_6, v_7\rangle$, $I(v_7) \subset I(v_6)$, forward edge.
    $\langle v_7, v_5\rangle$, $I(v_7) \subset I(v_5)$, backward edge.

  - **(iv). [4 marks]** Show why G_2 is (or is not) a DAG using the results in Part (iii). Justify your answer. (Refer to CSCI2100C-Lecture24 Page 11)

    Since there exist backward edges $\langle v_3, v_1\rangle$ and $\langle v_7, v_5\rangle$, G_2 contains a cycle. So G_2 is not a DAG.