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STAT4005

Assignment 3

Question 1

please kindly refer to page 3

Question 2

$\gamma(0) = \hat{\gamma}(0) \Rightarrow (1+\theta^2)\sigma^2 = 0.6423 \quad \dots (1)$

$\gamma(1) = \hat{\gamma}(1) \Rightarrow \theta\sigma^2 = 0.0233 \quad \dots (2)$

solving the above equations, we have

$\hat{\theta} \approx 0.0363$, 27.53 (rejected)

$\hat{\sigma}^2 \approx 0.6419$

Question 3

$$X = \begin{pmatrix} Y_2 & Y_1 \\ \vdots & \vdots \\ Y_{19} & Y_{18} \end{pmatrix} \quad Y = \begin{pmatrix} Y_3 \\ \vdots \\ Y_{20} \end{pmatrix}$$

from .R attached, we have

$\hat{\phi}_1 = 0.2354$, $\hat{\phi}_2 = -0.223$, $\hat{\sigma}^2 \approx 0.7237$

$\hat{\Gamma}_2^{-1}(1,1) = \hat{\Gamma}_2^{-1}(2,2) = \frac{0.6423}{(0.6423)^2 - (0.0233)^2}$

≈ 1.559

CI for ϕ_1 : $0.2354 \pm 1.96\sqrt{\frac{0.7237}{20}(1.559)}$

$\approx [-0.2301, 0.7009]$

CI for ϕ_2 : $-0.223 \pm 1.96\sqrt{\frac{0.7237}{20}(1.559)}$

$\approx [-0.6885, 0.2425]$

Question 4

$$\begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{pmatrix} = \begin{pmatrix} 0.6423 & 0.0233 \\ 0.0233 & 0.6423 \end{pmatrix}^{-1} \begin{pmatrix} 0.0233 \\ -0.1929 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0.0472 \\ -0.302 \end{pmatrix}$$

Question 5

with $Z_t = Y_t - \phi Y_{t-1} - \theta Z_{t-1}$ condition on $Z_0 = 0$, $Y_0 = 0$

from .R attached, we have

$\hat{\phi} \approx -0.5109$, $\hat{\theta} \approx 0.844$, $\hat{\sigma}^2 \approx 0.681$

Question 6

from .R attached, we have

$\hat{\phi} = -0.3883$, $\hat{\theta} = 1$, $\hat{\sigma}^2 = 0.5254$

the maximised $\ell(\phi, \theta) = -23.08$

Question 7

from .R attached, we have

$FPE_{(p=1)} \approx 0.8042$ $\quad FPE_{(p=4)} \approx 0.9319$

$FPE_{(p=2)} \approx 0.8422$ $\quad FPE_{(p=5)} \approx 0.8477$

$FPE_{(p=3)} \approx 0.8882$

$\Rightarrow$ AR(1) is the best in terms of FPE

Question 8

from .R attached, we have

$AICC_{(q=1)} \approx 53.3839$ $\quad AICC_{(q=4)} \approx 59.4228$

$AICC_{(q=2)} \approx 53.2022$ $\quad AICC_{(q=5)} \approx 63.5976$

$AICC_{(q=3)} \approx 56.2802$

$\Rightarrow$ MA(2) is the best in terms of AICC

Question 9

from .R attached, we have the residuals

0.9405	-1.0002	-0.4272	0	0.0456
0.3873	1.5354	-1.5332	1.1342	-0.0057
-0.1575	1.2402	-0.5411	-0.2208	0.3518
0.3583	0.0985	-0.2304	0.7741	0.6592

$H_0 : \rho_Z(k) = 0 \ \forall k$ vs $H_1 : \rho_Z(k) \neq 0 \ \exists k$

Since $Q(10) \approx 11.7303 < 16.919$, critical value,

we do not reject H_0 at 0.05 level of significant

Question 10

from. R attached, we have

FPE $\approx$ 0.3871 for ARMA(2,4)

AIC $\approx$ 51.7022 for ARMA(0,2)

AICc $\approx$ 52.8748 for ARMA(1,1)

BIC $\approx$ 26.8167 for ARMA(1,4)

which each of them achieved the minimal value

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