2018-19 MATH1520AB

Midterm I (2018 Oct 4)

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Session: A / B			

- There are 17 pages (including this page). Please check the number of pages.
- There are 8 questions (7 main questions + 1 bonus question). You have to answer all the main questions (Q1 Q7). The total score is 100 (+15).
- Show your steps unless otherwise stated.
- Good luck!

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
6	8	26	10	9	9	20	0	88+0
,		8		- 1				-
		18			5,60			

- 1. (8pts) Find the natural domain of the following functions:
 - (a) $F(x) = \ln\left(-\frac{x^2}{4} + 4\right)$
- (b) $g(x) = \sqrt{\frac{x}{(2x-4)(x+1)}}$

 $F(\pi)$ is well defined only when $-\frac{\pi}{4} + 4 > 97$



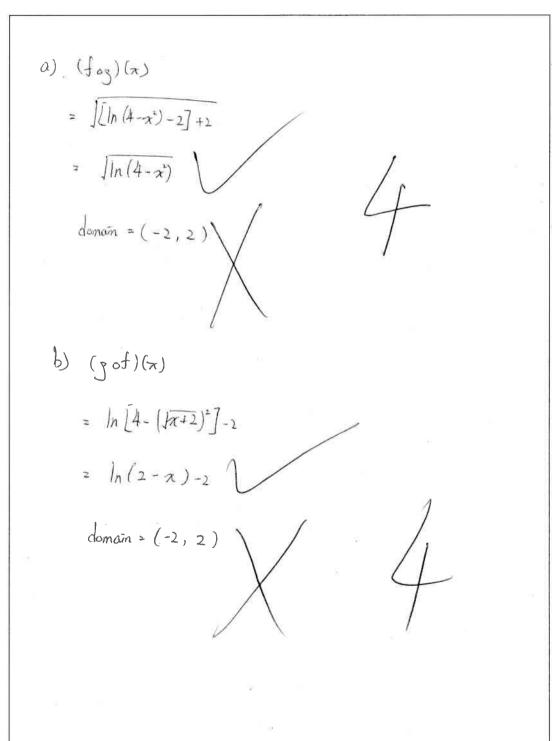
b) g(x) is well defined when

 $\frac{\chi}{(2x-4)(x+1)} = 0 \qquad (2x-4)(x+1) \neq 0$ $\chi \neq -1 \quad U \quad 2$

2. (12pts) Let $f(x) = \sqrt{x+2}$ and $g(x) = \ln(4-x^2)-2$. Expand and simplify the following expression. State the domain of each new function.

(a)
$$(f \circ g)(x)$$

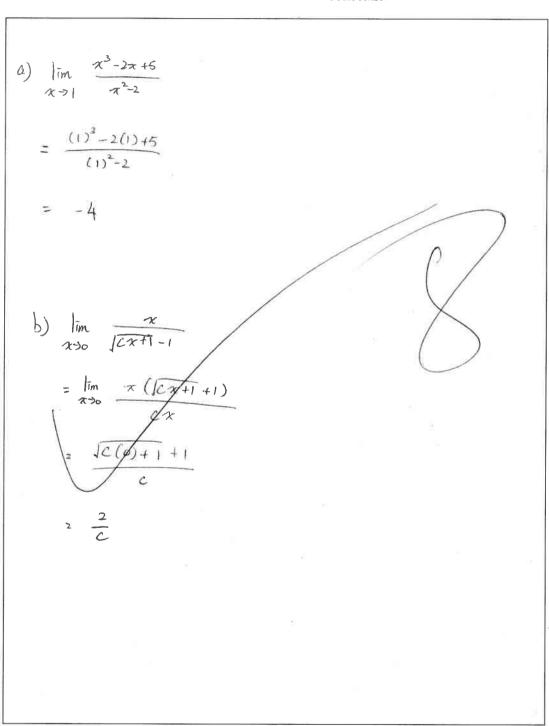
(b)
$$(g \circ f)(x)$$



3. (32pts) Without using L'Hôpital's rule, evaluate the following limits or state that it does not exist. If the limit does not exist but diverges to plus or minus infinity, please indicate so, and determine the correct sign.

(a)
$$\lim_{x \to 1} \frac{x^3 - 2x + 5}{x^2 - 2}$$

(b)
$$\lim_{x\to 0} \frac{x}{\sqrt{cx+1}-1}$$
, where c is a nonzero constant.





(c)
$$\lim_{x \to -2^-} \frac{x-4}{x^2+2x}$$

(d)
$$\lim_{t\to 3^-} \frac{t^2-2t-3}{|t-3|}$$

c)
$$\lim_{\chi \to -2^-} \frac{\chi - 4}{\chi^2 + 2\chi}$$

d)
$$\lim_{t \to 3^{-}} \frac{t^2 - 2t - 3}{1t - 3}$$

$$\frac{1}{2} \lim_{t \to 3^{-}} \frac{(t-3)(t+1)}{-(t-3)}$$

(e)
$$\lim_{x \to +\infty} \frac{\pi + x^{2/3} - x}{x^{1/2} + x - 10}$$

(f)
$$\lim_{x \to +\infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 - 5} \right)$$

e)
$$\lim_{x \to +\infty} \frac{x + x^{\frac{2}{3}} - x}{x^{\frac{1}{2}} + x - 10}$$

$$\lim_{x \to +\infty} \frac{\frac{\pi}{x} + \frac{1}{x^2} - 1}{\frac{1}{x^2} + 1 - \frac{10}{x}}$$

$$f) \lim_{\chi \to +\infty} \left(\overline{\chi^2 + 3\chi} - \overline{\chi^2 - 5} \right)$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}$$

$$\frac{1}{2} \lim_{x \to +\infty} \frac{3 + \frac{5}{2}}{\sqrt{1 + \frac{3}{2}} + \sqrt{1 - \frac{5}{2}}}$$

(g)
$$\lim_{x \to -\infty} \frac{x-2}{\sqrt{x^2-4}}$$

$$\left(\begin{array}{c} \left(\text{h}\right) \lim_{x \to -\infty} \left(1 - \frac{1}{x^2}\right)^x \end{array}\right)$$

g)
$$\lim_{x\to-\infty} \frac{x-2}{\sqrt{x^2-4}}$$

$$\frac{1}{2} \lim_{3 \to -\infty} \frac{1 - \frac{2}{\pi}}{-11 - \frac{4}{\pi}}$$

let
$$\frac{1}{y} = -\frac{1}{x}$$

$$= \lim_{x \to -\infty} \left(1 + \frac{1}{y}\right) \sqrt{\frac{1-y}{1}}$$

$$= \lim_{\chi \to -\infty} \left(1 + \frac{1}{\chi} \right)^{1/2}$$

$$= \lim_{\chi \to -\infty} \left(1 + \frac{1}{\chi} \right)^{1/2}$$

$$= \lim_{\chi \to -\infty} \left(\frac{1}{1} + \frac{1}{\chi} \right)^{1/2}$$

$$= \lim_{\chi \to -\infty} e^{\frac{1}{17}}$$

4. (10pts) Let

$$f(x) = \begin{cases} \ln(x^2 + 1) + 2, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ |3^x - 3|, & \text{if } x < 0. \end{cases}$$

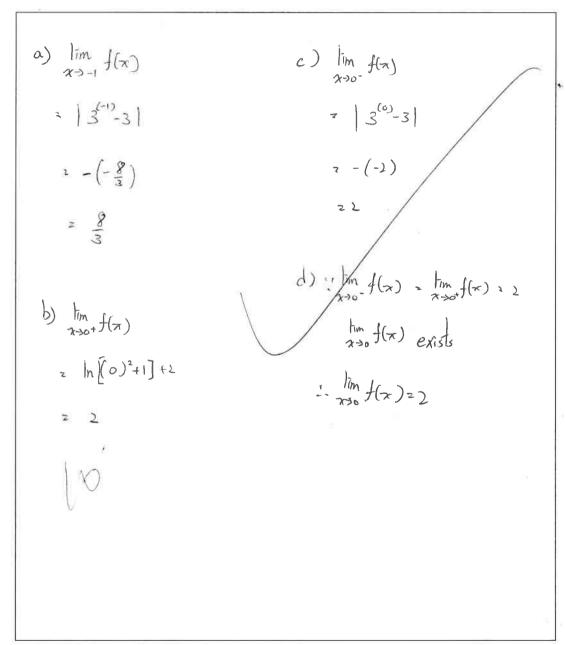
Evaluate the following limits. Furthermore, if the limit doesn't exist but diverges to $\pm \infty$, please indicate so and determine the correct sign.

(a) $\lim_{x \to -1} f(x)$

(c) $\lim_{x\to 0^-} f(x)$

(b) $\lim_{x \to 0^+} f(x)$

(d) $\lim_{x\to 0} f(x)$



5. (9pts) Show that there is a real number a such that $e^a + 2a = 3$.

let f(x) = 3 - e2-2x

Since f(x) is continuous on (R as it is pulynamic), we can apply intermedicle value theorem, $f(-\infty) = 3 - \frac{1-\infty}{2} - 2(-\infty) = \infty$ so; and $f(\infty) = 3 - \frac{1-\infty}{2} - 2(-\infty) = -\infty$ so and thus, there is a number $\alpha \in \mathbb{R}$ such that $f(\alpha) = \alpha f(\alpha)$ mean there is a real number α such that $\alpha = 1$ such t

6. (9pts) Using the first principle, find the derivative of $f(x) = x + \sqrt{x}$.

f'(x) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ = kin x+h + tath - x - Tx $\frac{1}{2}\lim_{h\to 0}\frac{h+\sqrt{x+h}-\sqrt{x}}{h}$

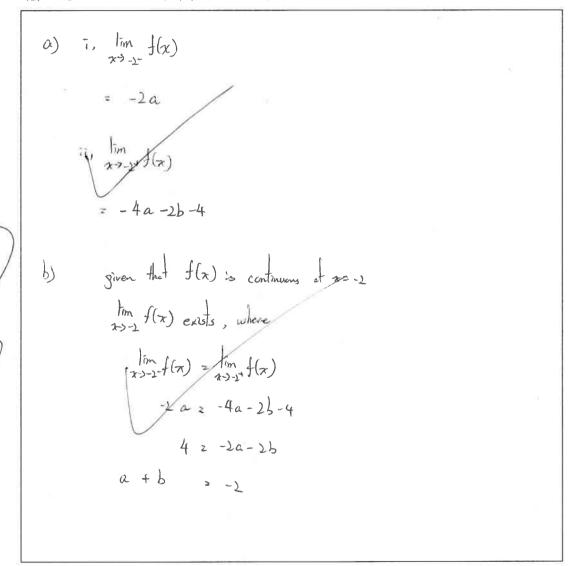
7. (20pts) Let

$$f(x) = \begin{cases} ax, & \text{if } x < -2, \\ -ax^2 + bx - 4, & \text{if } x \ge -2. \end{cases}$$

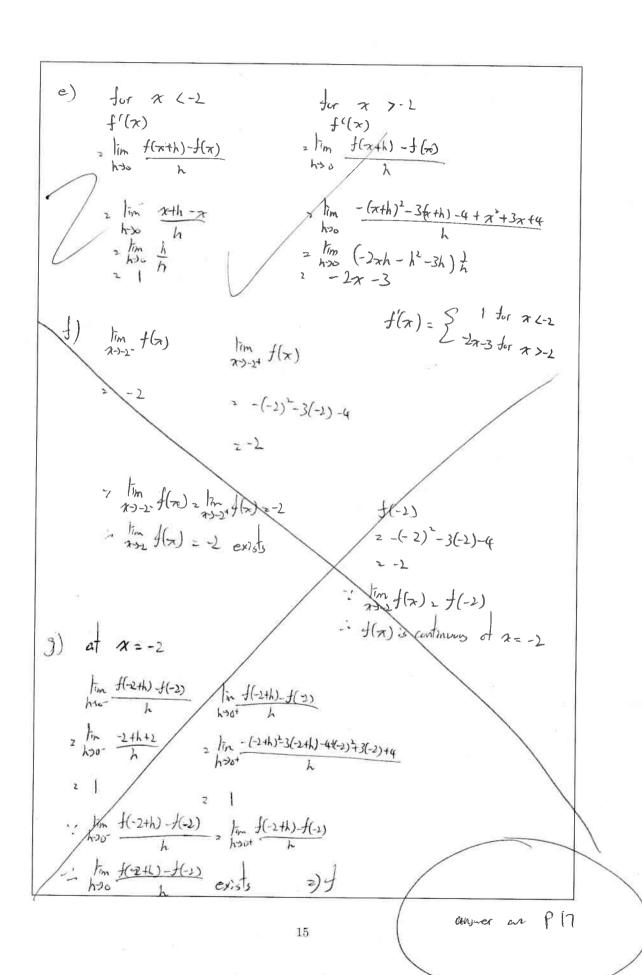
- (a) Find (i) $\lim_{x\to -2^-} f(x)$ and (ii) $\lim_{x\to -2^+} f(x)$.
- (b) Given that f(x) is continuous at x = -2, show that a + b = -2.
- (c) Given that f(x) is differentiable at x = -2, show that 3a + b = 0.
- (d) Hence, find the values of a and b such that f(x) is continuous and differentiable at x = -2, and find f'(-2).

*Use the values of a and b from (d) to answer the following questions. *

- (e) Find f'(x) for $x \neq -2$.
- (f) Explain whether f'(x) is continuous at x = -2.
- (g) Explain whether f'(x) is differentiable at x = -2.



given that f(x) is distensible it x=2 lim f(x+h)-f(x) exists, where tim f(xth)-f(x) lim f(xth)-f(x)
hoo h tim hoo - a (-2+h) + 4a+2b+4 = lim -a (-2+h) = 4b (-2+h) -4 +4c +2b +4 tim -2a+4a+2b+4 ta 2 tim -ch +4a+b az 4ath 3a+b = 0 d) to make f(x) differentiable of a=-1, both tim f(x) and tim f(xth)-f(x) exists such flat z 4 (1) + (-3)



Question 7 Tim f(x) = 1 tim 7(x) = 1 7 /m + (x) = /m + (x) = 1 - /m + (x) exists of (2) is continuen of 2--9) tin # tath et 7=-2 lim f(-2+h)-f(-2) hro- 1 tin f(-2+h)-f(-2) int(x) not alltertal of x:-2

- The End

8. Bonus Question (15pts) Show that the function defined by $f(x) = x^2 - (a+b-1)x + ab$ takes on the value $\frac{a+b}{2}$, where a and b are any two real numbers.

