



CITY UNIVERSITY OF HONG KONG

STUDENTS' UNION

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STAT 3008

Assignment 2

Question 1

Variable	Coefficient	Std.Error	t-statistic	p-value	$t_0 \text{ for } \beta_0 = \frac{-23.4325 - (-10)}{12.74} \approx -1.8393$
Constant	-23.4325	12.74	-1.8393	0.0842	$\beta_0 = 8.32 \times 0.1528 \approx 1.2713$
X	1.2713	0.1528	8.32	2	

Source	df	ss	ms	F	p-value	$F_0 = (8.32)^2 = 69.2224$
Regression	1	1848.76	1848.76	69.2224	1.3956×10^{-7}	$MS_{\text{reg}} = \frac{1848.76}{1} = 1848.76$
Residual	18	480.735	26.7075			$MS_{\text{res}} = \frac{1848.76}{69.2224} \approx 26.7075$
Total	19	2329.495				$SS_{\text{total}} = 26.7075(18) = 480.735$

$$SS_{\text{total}} = 1848.76 + 480.735 = 2329.495$$

b) As the slope of the regression is positive, r_{xy} is also positive

$$r_{xy} = \sqrt{\frac{1848.76}{2329.495}} \approx 0.8903$$

c) $H_0: \beta_0 = -10$ vs $H_1: \beta_0 \neq -10$

$$t_0 = \frac{-23.4325 - (-10)}{12.74} \quad p\text{-value} = 0.3056$$

$$\approx -1.0544$$

Since $p\text{-value} > 0.05$, we do not reject H_0 at $\alpha = 0.05$

We do not have sufficient evidence that β_0 is different from -10

Question 2

$$\begin{aligned} a) E(\hat{Y}'\hat{Y}) &= E(\hat{\beta}'X'X\hat{\beta}) \\ &= E(Y'HY) \quad \text{where } H = X(X'X)^{-1}X' \\ &= E(\beta'X'HX\beta + \beta'X'H\epsilon + \epsilon'HX\beta + \epsilon'H\epsilon) \\ &= \beta'X'X\beta + E[\text{tr}(H\epsilon\epsilon')] \\ &= \beta'X'X\beta + \sigma^2 \text{tr}[(X'X)^{-1}X'X] \\ &= \beta'X'X\beta + (p+1)\sigma^2 \end{aligned}$$

$$\begin{aligned} b) \sum_i E(\hat{y}_i^2) &= E(Y'Y) \\ &= E(\beta'X'X\beta + \beta'X'\epsilon + \epsilon'X\beta + \epsilon'\epsilon) \\ &= \beta'X'X\beta + E[\text{tr}(\epsilon\epsilon')] \\ &= \beta'X'X\beta + \text{tr}(\sigma^2 I_n) \\ &= \beta'X'X\beta + n\sigma^2 \\ &= \beta'X'X\beta + (p+1)\sigma^2 + (n-p-1)\sigma^2 \\ &= E(\hat{Y}'\hat{Y}) + E(\hat{\epsilon}'\hat{\epsilon}) \\ &= \sum_i E(\hat{y}_i^2) + \sum_i E(\hat{\epsilon}_i^2) \end{aligned}$$

Question 4

a)

$$X'X = n \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{u}^2 & 0 \\ 0 & 0 & \bar{v}^2 \end{pmatrix} \quad (X'X)^{-1} = \begin{pmatrix} \frac{1}{n} & 0 & 0 \\ 0 & \frac{1}{S_{UU}} & 0 \\ 0 & 0 & \frac{1}{S_{VV}} \end{pmatrix} \quad X'Y = \begin{pmatrix} n\bar{y} \\ SUY \\ SVY \end{pmatrix}$$

$$\hat{\beta} = (X'X)^{-1}X'Y = \begin{pmatrix} \bar{y} \\ \frac{SUY}{S_{UU}} \\ \frac{SVY}{S_{VV}} \end{pmatrix} \Rightarrow \hat{\beta}_0 = \bar{y}, \hat{\beta}_1 = \frac{SUY}{S_{UU}}, \hat{\beta}_2 = \frac{SVY}{S_{VV}}$$

b) As $E(Y|U=u) = \alpha_0 + \alpha_1 u$ is a simple linear regression, we can directly state that

$$\hat{\alpha} = \left(\bar{y} - \frac{SUY}{S_{UU}}(\bar{u}) \right) = \begin{pmatrix} \bar{y} \\ \frac{SUY}{S_{UU}} \end{pmatrix} \Rightarrow \hat{\alpha}_0 = \bar{y} = \hat{\beta}_0, \hat{\alpha}_1 = \frac{SUY}{S_{UU}} = \hat{\beta}_1.$$

Question 5

a) For the true model, $E(Y) = E(X_2\beta + e) = X_2\beta$

$$\begin{aligned} E(\hat{\alpha}) &= (X'X)^{-1}X'E(Y) \\ &= (X'X)^{-1}X'X_2\beta \end{aligned}$$

b) $X'X = \begin{pmatrix} n & \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i^3 \end{pmatrix} \quad (X'X)^{-1} = \frac{1}{n\sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \begin{pmatrix} \sum_{i=1}^n x_i^2 & -\sum_{i=1}^n x_i \\ -\sum_{i=1}^n x_i & n \end{pmatrix} \quad X'X_2 = \begin{pmatrix} n & \sum_{i=1}^n x_i^2 \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^3 \end{pmatrix}$

$$(X'X)^{-1}X'X_2 = \frac{1}{n\sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \begin{pmatrix} n\sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2 & (\sum_{i=1}^n x_i^2)^2 - \sum_{i=1}^n x_i \sum_{i=1}^n x_i^3 \\ 0 & -\sum_{i=1}^n x_i \sum_{i=1}^n x_i^2 - n\sum_{i=1}^n x_i^3 \end{pmatrix}$$

$$E(\hat{\alpha}) = \frac{1}{n^2(\bar{x}^2 - \tilde{x}^2)} \begin{pmatrix} n^2(\bar{x}^2 - \tilde{x}^2)\beta_0 + n^2[(\bar{x}^2)^2 - \tilde{x}(\bar{x}^3)]\beta_1 \\ n^2[-\tilde{x}(\bar{x}^2) + \bar{x}^3]\beta_1 \end{pmatrix} \Rightarrow i) E(\hat{\alpha}_0) = \beta_0 + \frac{[(\bar{x}^2)^2 - \tilde{x}(\bar{x}^3)]\beta_1}{\bar{x}^2 - \tilde{x}^2}, ii) E(\hat{\alpha}_1) = \frac{[-\tilde{x}(\bar{x}^2) + \bar{x}^3]\beta_1}{\bar{x}^2 + \tilde{x}^2}$$

c) i) $E(\lim_{n \rightarrow \infty} \hat{\alpha}_0) = \beta_0 + \frac{\sigma_x^4 - (O)(K\sigma_x^2)}{\sigma_x^2 - (O)^2} \beta_1 = \beta_0 + \sigma_x^2 \beta_1 \neq \beta_0$

ii) $E(\lim_{n \rightarrow \infty} \hat{\alpha}_1) = \frac{-(O)(\sigma_x^2) + K\sigma_x^2}{\sigma_x^2 + (O)^2} \beta_1 = K\sigma_x^2 \beta_1 \neq \beta_1$

Comment Summary