Homework 4 (Due: 6/11)

(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two gray images A and B. The sizes of A and B are equivalent.

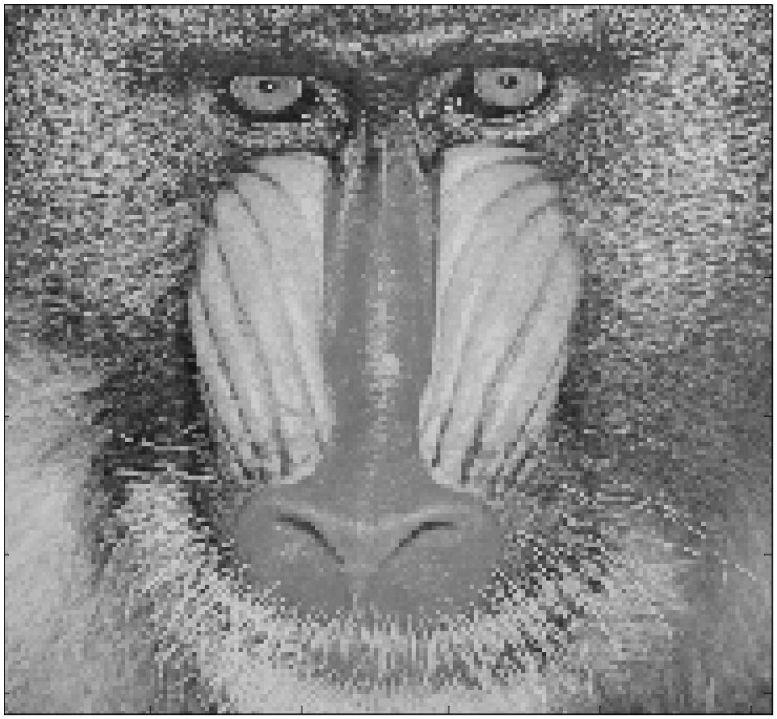
SSIM(A, B, c1, c2)

where c1 and c2 are some adjust constants.

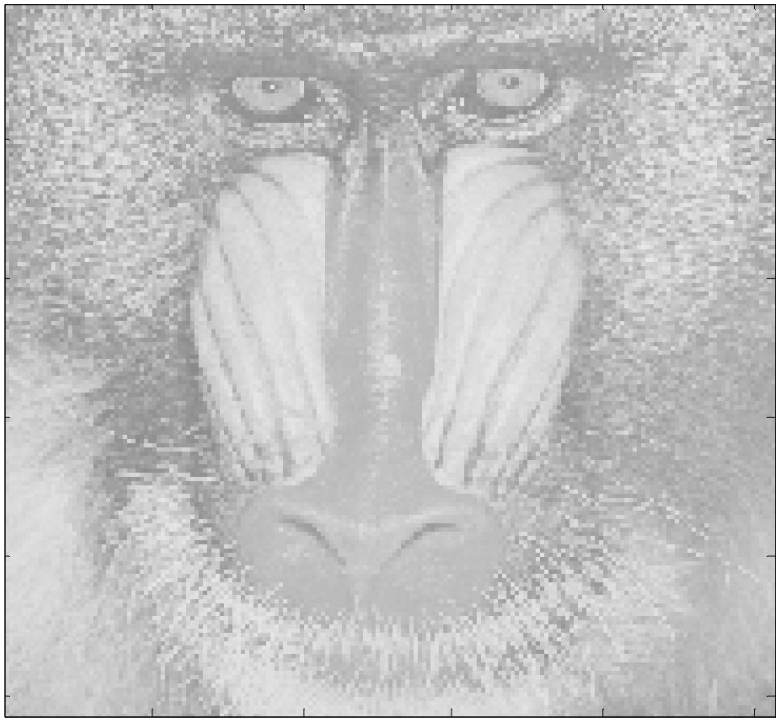
The code should be handed out by ceiba. (20 scores)

```
function ssim = SSIM(A, B, c1, c2)
    if nargin<1
        A = imread('picture1.PNG');
        A = rgb2gray(A);
        B = imread('picture2.PNG');
        B = rgb2gray(B);
        c1 = 1 / sqrt(255);
        c2 = 1 / sqrt(255);
    end

    A = double(A);
    B = double(B);
    L = 255;

    [M, N] = size(A);
    mean_A = mean(mean(A));
    mean_B = mean(mean(B));
    variance_A = sum(sum((A-mean_A).^2)) / (M*N);
    variance_B = sum(sum((B-mean_B).^2)) / (M*N);
    covariance_AB = sum(sum((A-mean_A).*(B-mean_B))) / (M*N);

    ssim = (2*mean_A*mean_B + (c1*L)^2) / (mean_A^2 + mean_B^2 +
 (c1*L)^2) * (2*covariance_AB + (c2*L)^2) / (variance_A + variance_B +
 (c2*L)^2);
end
```

```
>> A = rgb2gray(imread('picture1.PNG'));
>> B = rgb2gray(imread('picture2.PNG'));
>> c1 = 1 / sqrt(255);
>> c2 = 1 / sqrt(255);
>> ssim = SSIM(A, B, c1, c2)

ssim =

    0.6300
```

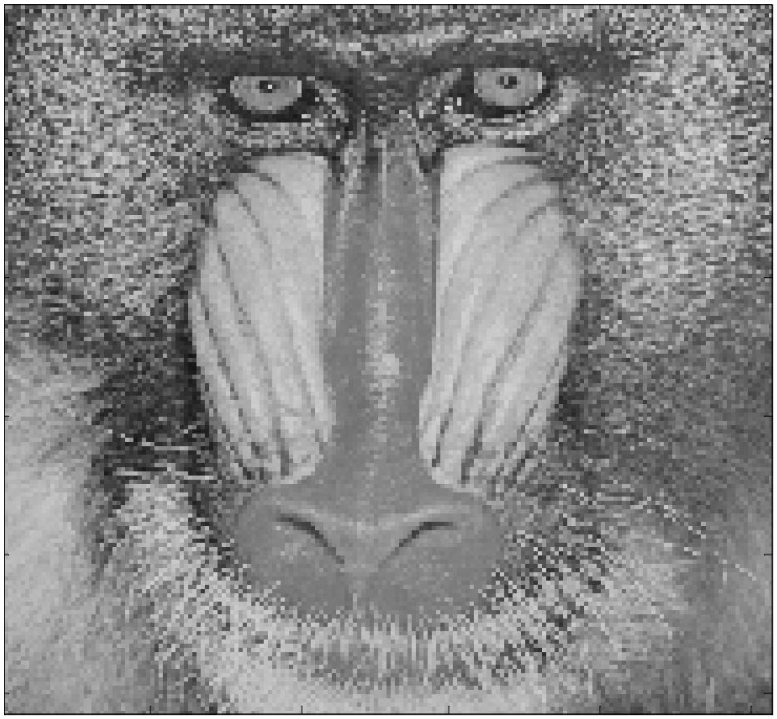


picture1.PNG



picture2.PNG

```
>> A = rgb2gray(imread('picture1.PNG'));
>> B = rgb2gray(imread('picture3.PNG'));
>> c1 = 1 / sqrt(255);
>> c2 = 1 / sqrt(255);
>> ssim = SSIM(A, B, c1, c2)

ssim =

    0.1271
```



picture1.PNG

picture3.PNG

(2) State at least three examples that using the PSNR cannot reflect the similarity of two vocal signals. (10 scores)

1.頻率的變化
2.相位的變化
3.震幅的變化

(3) How do we implement the following matrix operation with the least number of multiplications? (15 scores)

a=0.7010
b=0.9239
c=0.3827

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.7010 & 0.7010 & 0.7010 & 0.7010 \\ 0.9239 & 0.3827 & -0.3827 & -0.9239 \\ 0.7010 & -0.7010 & -0.7010 & 0.7010 \\ 0.3827 & -0.9239 & 0.9239 & -0.3827 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(1) 轉成a、b、c

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(2) 將y_0、y_2跟y_1、y_3分開算

$$\begin{bmatrix} y_0 \\ y_2 \end{bmatrix} = \begin{bmatrix} a & a \\ a & a \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \end{bmatrix} + \begin{bmatrix} a & a \\ -a & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} b & c \\ c & -b \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_1 - x_2 \end{bmatrix}$$

(3) 將4個小矩陣分別求出所需乘法

$$\begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} a & a \\ a & a \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \end{bmatrix}$$

$$z_0 = a(x_0 + x_3)$$
$$z_1 = z_0$$

$\begin{bmatrix} a & a \\ a & a \end{bmatrix} \begin{bmatrix} x_0 \\ x_3 \end{bmatrix}$需要1個乘法

$$\begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} a & a \\ -a & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$z_0 = a(x_1 + x_1)$$
$$z_1 = -z_0$$

$\begin{bmatrix} a & a \\ -a & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$需要1個乘法

$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} b & c \\ c & -b \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_1 - x_2 \end{bmatrix}$$

$$= \begin{bmatrix} c & c \\ c & c \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_1 - x_2 \end{bmatrix} + \begin{bmatrix} b-c & 0 \\ 0 & -b-c \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_1 - x_2 \end{bmatrix}$$

$$z_0 = \mathrm{c}(x_0 - x_3 + x_1 - x_2)$$
$$z_1 = z_0$$
$$z_2 = (b-c)(x_0 - x_3)$$
$$z_3 = (b+c)(x_2 - x_1)$$

$\begin{bmatrix} b & c \\ c & -b \end{bmatrix} \begin{bmatrix} x_0 - x_3 \\ x_1 - x_2 \end{bmatrix}$需要3個乘法

總共需要1+1+3=5個乘法

(4) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and exp($j\ \theta$) required only 2 real multiplications?

(10 scores)

$$x = a + jb$$

$$e^{j\theta} = cos(\theta) + \mathrm{jsin}(\theta) = \mathrm{c} + \mathrm{jd}$$

$\begin{bmatrix} z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ 原本需要3個乘法，

當θ為$\frac{\pi}{2}$的整數倍時因為值為 0 or 1 or -1 or j or –j 就不需要乘法，

當θ為$\frac{\pi}{6}$的整數倍時sin(θ)或cos(θ)會有一個值為$\frac{1}{2}$所以就只需要2個乘法，

當θ為$\frac{\pi}{4}$的整數倍時sin(θ)=cos(θ)或sin(θ)=-cos(θ) 所以就只需要2個乘法，

當θ為 $\frac{\pi}{4}$ 或 $\frac{\pi}{6}$ 的整數倍且不為$\frac{\pi}{2}$ 的整數倍時只需要2個乘法。

(5) Determining the numbers of real multiplications for the (a) 165-point DFT, (b) 242-point DFT. (10 scores)

(a)

N = 165=3*5*11 ，為互質
the numbers of real multiplications =
55個 3-point DFT + 33個 5-point DFT + 15個 11-point DFT =
55*2 + 33*10 + 15*40 = **1040**

(b)

N = 242 = 2*121=2 *(11*11)
先算11*11
P1 = 11
P2 = 11
D1 = 100
D2 = 0 ，沒有N/12和N/8的倍數，
the numbers of real multiplications =
11個 11-point DFT + 11個 11-point DFT + 3*D1 + 2*D2 =
11*40 + 11*40 + 3*100 = 1180

$$2 * MUL_{121} + 121 * MUL_2 = 2 * 1180 + 121 * 0 = \mathbf{2360}$$

(6) Suppose that we want to implement the convolution of two complex sequences $x[n]$ and $h[n]$ where length($x[n]$) = 300 and length($h[n]$) = 200 .

(a) What is the number of the points of the DFT that should be used for implementation?

(b) How many real multiplications are required? (10 scores)

(a)

M=300

N=200

$P \geq M + N - 1 = 499$

從表中選擇 504-point DFT可以得到最小乘法數

(b)

$P = 504$

$MUL_{504} = 2300$

$MUL = 2300 \times 2 + 3 \times 504 = 6112$

(7) Suppose that a 1-D edge detector is:

$x_s[n] = x[n] * h[n]$ $h[1] = -h[-1] = 0.8$ $h[2] = -h[-2] = 0.15$

$h[3] = -h[-3] = 0.075$ $h[0] = 0$ $h[n] = 0$ otherwise

Design an efficient way with least number of real multiplications to implement the above filter operation. (10 scores)

$$x_s[n] = x[n] * h[n] = \sum_m x[n-m]h[m]$$

$$= 0.075x[n+3] + 0.15x[n+2] + 0.8x[n+1] + 0x[n] + 0.8x[n-1] + 0.15x[n-2] + 0.075x[n-3]$$

$$= 0.075(x[n+3] + x[n-3]) + 0.15(x[n+2] + x[n-2]) + 0.8(x[n+1] + x[n-1])$$

$$= 0.075(x[n+3] + x[n-3] + 2x[n+2] + 2x[n-2]) + 0.8((x[n+1] + x[n-1])$$

使用directly computing因為M長度小且有對稱性，計算每個$x_s[n]$都只需要2個乘法

(8) Suppose that length($x[n]$) = 1600. What is the best way to implement the convolution of two complex sequences $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 450, (b) length($y[n]$) = 30, and

(c) length($y[n]$) = 2, (15 scores)

(a)

Directly computing = $3 \times M \times N$ = 2160000

IFFT(FFT(x)FFT(h)) 方式

$P \geq M + N - 1 = 2049$

If P=2304

Number of real multiplications = $2 \times MUL_P + 3 \times P$ = 2*15868+3*2304=38648

Sectioned convolution

If L = 100

$P \geq L + M - 1 = 549$

If P = 560

S = 16

Number of real multiplications = $2S \times MUL_P +$
$3S \times P$ = 2*16*3100+3*16*560=126080

使用IFFT(FFT(x)FFT(h))最好

(b)

Directly computing = $3 \times M \times N$ = 144000

IFFT(FFT(x)FFT(h)) 方式
$P \geq M + N - 1 = 1629$
If P=1680
Number of real multiplications = $2 \times MUL_P + 3 \times P$ = 2*10420+3*1680=25880

Sectioned convolution
If L = 100
$P \geq L + M - 1 = 129$
If P = 144
S = 16
Number of real multiplications = $2S \times MUL_P +$
$3S \times P =$ 2*16*436+3*16*144=20864

使用Sectioned convolution最好

(b)

Directly computing = $3 \times M \times N$ = 9600

IFFT(FFT(x)FFT(h)) 方式
$P \geq M + N - 1 = 1601$
If P=1680
Number of real multiplications = $2 \times MUL_P + 3 \times P$ = 2*10420+3*1680=25880

Sectioned convolution
If L = 3
$P \geq L + M - 1 = 4$
If P = 4
S = 534
Number of real multiplications = $2S \times MUL_P +$
$3S \times P =$ 2*534*0+3*534*4=6408

使用Sectioned convolution最好

(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 4, 5, 6, 9) ID:M10907314

加分題 8點DCT主要用在什麼地方?

Process of JPEG Image Compression