Homework 5 (Due: 7/2)

(1) Write a Matlab or Python program to compute the FFT of two N-point real signals x and y using only one N-point FFT.

$$[Fx, Fy] = \text{fftreal}(x, y)$$

The code should be handed out by ceiba. (20 scores)

```
function [Fx, Fy] = fftreal(x, y)
    if nargin<1
        Fs = 1000;          % Sampling frequency
        T = 1/Fs;           % Sampling period
        L = 1500;           % Length of signal
        t = (0:L-1)*T;      % Time vector
        f = Fs*(0:L-1)/L;

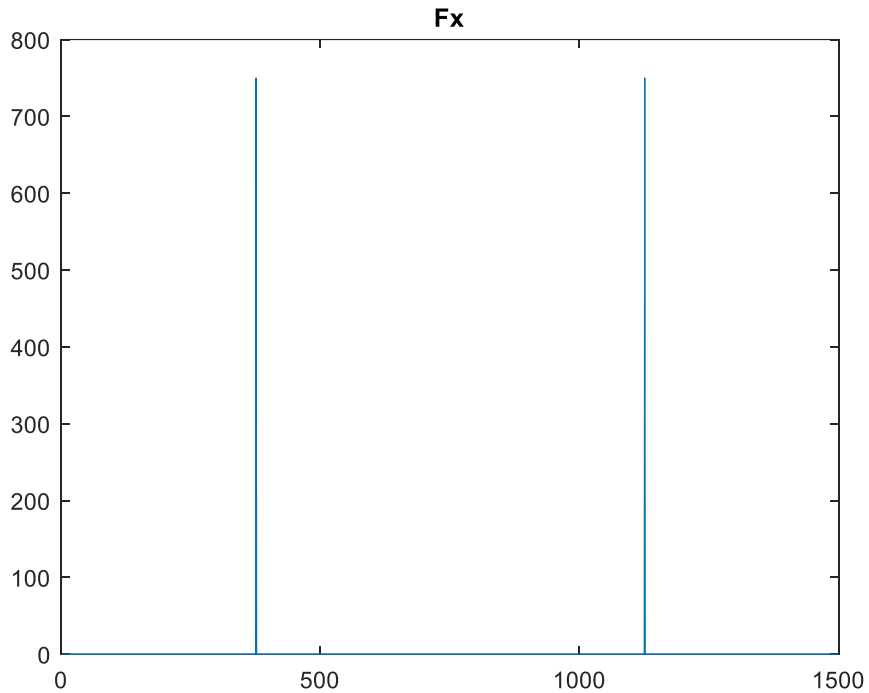
        x = cos(2*pi*150*t);
        y = sin(2*pi*100*t);
    end

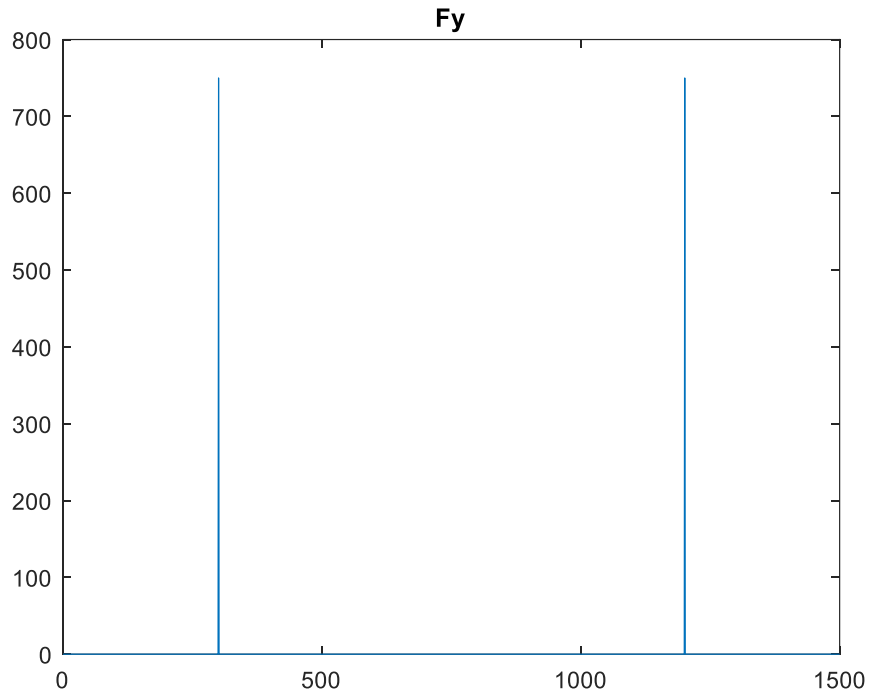
    f3 = x + i*y;
    F3 = fft(f3);
    L = length(F3);
    %第1項為平均值所以跳過
    for m = 2:L
        Fx(m) = (F3(m) + conj( F3(L-m+2) )) / 2;
        Fy(m) = (F3(m) - conj( F3(L-m+2) )) / (i*2);
    end

    figure;
    plot(abs(Fx));
    title("Fx");
    figure;
    plot(abs(Fy));
    title("Fy");
end
```

```
>> Fs = 1000;            % Sampling frequency
   T = 1/Fs;             % Sampling period
   L = 1500;             % Length of signal
   t = (0:L-1)*T;        % Time vector
   f = Fs*(0:L-1)/L;

   x = cos(2*pi*250*t);
   y = sin(2*pi*200*t);
>> fftreal(x,y);
```





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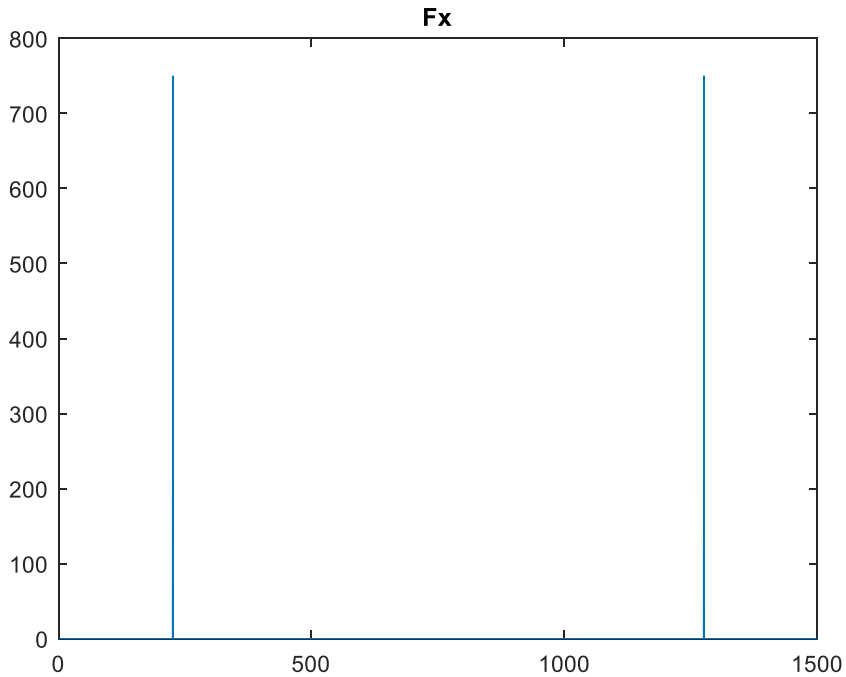
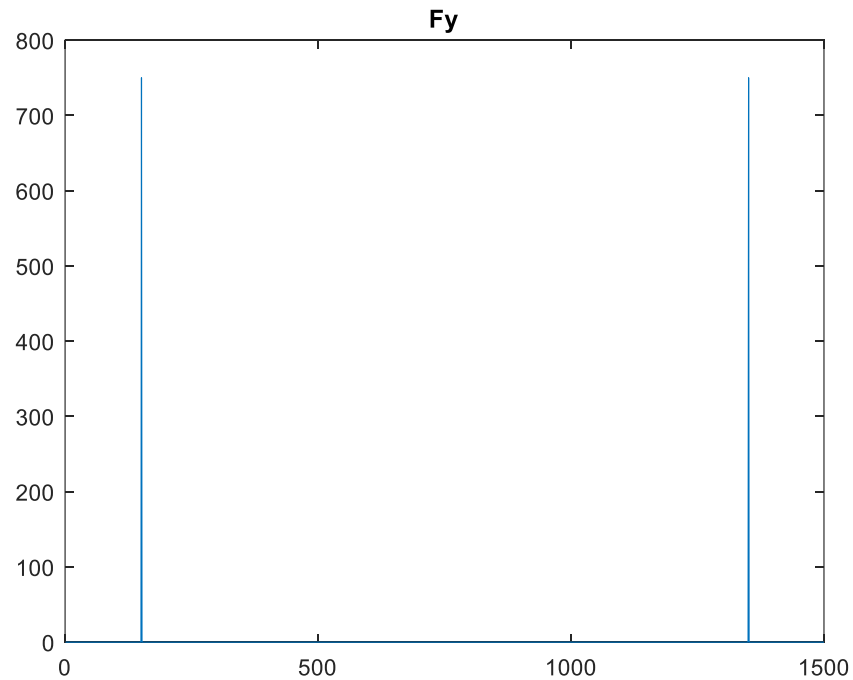
   f = Fs*(0:L-1)/L;

   x = cos(2*pi*150*t);
   y = sin(2*pi*100*t);
>> fftreal(x,y);
```





(2) What are the two main advantages of the sectioned DFT convolution?

(10 scores)

1.運算時間複雜度O(n)

2.P與N獨立不相干，可以不用為不同訊號長度設計不同晶片

(3) How many entries of (a) the N-point Walsh transform and (b) the N-point Haar transform that are equal to 0, 1, and -1? (10 scores)

(a)

$$\mathbf{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

除了第一行全是1之外，

後面的行都是1和-1各一半，所以

Number of 0 = 0

Number of 1 = $N + \frac{N}{2}(N-1)$

Number of -1 = $\frac{N}{2}(N-1)$

(b)

N = 32

$$
\mathbf{H} = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}
$$

第1行都1

1行內有一行數量的1或-1

2行內有一行數量的1或-1

4行內有一行數量的1或-1

8行內有一行數量的1或-1

排除第一行不算，剩下的行中將會有$\log_2 N$行數量的1或-1，
這些數量中1和-1各佔一半，所以

Number of 1 = $\frac{N}{2}\log_2 N + N$

Number of -1 = $\frac{N}{2}\log_2 N$

Number of 0 = $N^2 - N\log_2 N - N$

(4) What are the most important applications of (a) the Walsh transform and (b) the Haar transform nowadays? (10 scores)

(a) CDMA(code division multiple access)
(b) Localized spectrum analysis、edge detection

(Extra): Answer the questions according to your student ID number.

(ended with 4) ID : M10907314

32-point Haar transform 10th row 應該是什麼?

$$
\mathbf{H} = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}
$$

16-poing Haar transform 的第10行

因為觀察Haar transform 可以發現當N-point第m行就會是$\frac{N}{2}$-point第m行重複兩次的結果，32-point Haar transform第10行的就會是

[0 0 0 0 1 1 -1 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]