Homework 1 (Due: March 26th)

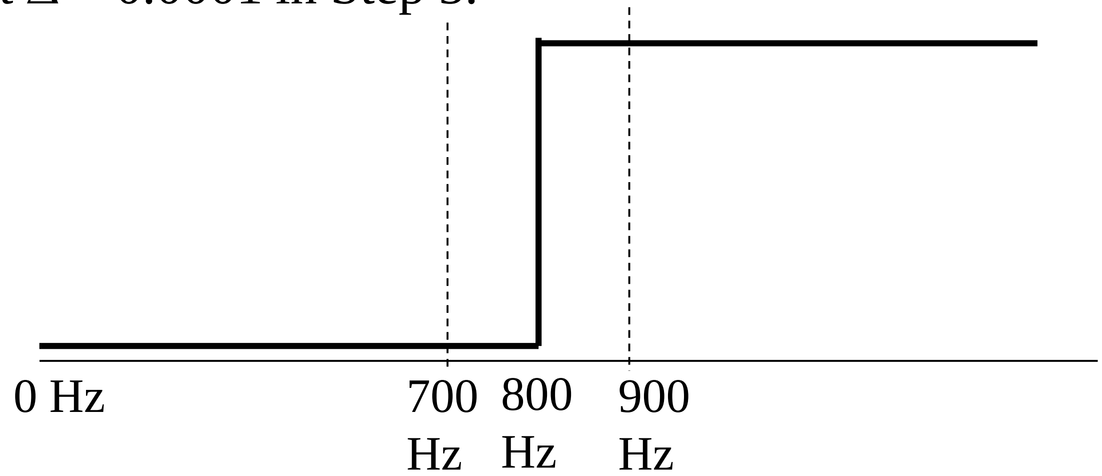
(1) Design a Mini-max **highpass** FIR filter such that (40 scores)

① Filter length = 19, ② Sampling frequency f_s = 4000Hz,

③ Pass Band 800~2000Hz   ④ Transition band: 700~900 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.5 for stop band .

⑥ Set Δ = 0.0001 in Step 5.



※ Matlab or Python code should be handed out by ceiba
E-mail 主旨上註明學號

紙本上要有

(a) the Matlab program, (b) the frequency response,

(c) the impulse response $h[n]$, and (d) the maximal error for each iteration.

(a) the Matlab program

```
clc;clear;

N = 19;
k = (N-1)/2;
delta_F = 0.00025;
delta_Err = 0.0001;
passband = [0.2, 0.5];
stopband = [0, 0.2];
weight_passband = 1;
weight_stopband = 0.5;
transition_band = [0.175, 0.225];
sample_rate_F = 1/delta_F;
Extreme_F = [0, 0.05, 0.10, 0.15, 0.25, 0.30, 0.35, 0.40, 0.45, 0.48, 0.5];
```

```
iteration_Err = []
previous_MaxErr = 0;
F = 0:delta_F:0.5;
Hd = 1.*(passband(1) <= F & F <= passband(2)) + 0.*(stopband(1) <= F
& F <= stopband(2));
WF = weight_passband.*(passband(1) <= F & F <= passband(2)) +
weight_stopband.*(stopband(1) <= F & F <= stopband(2));
WF( transition_band(1)*sample_rate_F+1 :
transition_band(2)*sample_rate_F-1) = 0;
plot(F,WF,'r','lineWidth',1);
figure;
plot(F,Hd,'r','lineWidth',1);
set(gca,'YLim',[-0.5 1.5])
xlabel('F¶b');
ylabel('Hd(F)¶b');
grid on;
```

```
n = 0:1:k;
m = 0:1:k+1;
W = weight_passband.*(passband(1) <= Extreme_F & Extreme_F <=
passband(2)) + weight_stopband.*(stopband(1) <= Extreme_F &
Extreme_F <= stopband(2));
Flip = 1*(mod(m,2)==0) + -1*(mod(m,2)==1);

iteration_count = 0;
while(1)
    iteration_count = iteration_count +1;
    A = [cos(2*pi*Extreme_F'*n) (Flip./W)'];
    b = 1.*(passband(1) <= Extreme_F & Extreme_F <= passband(2)) +
0.*(stopband(1) <= Extreme_F & Extreme_F <= stopband(2));
    s = A\b'

    R = 0.*F;
    for i = (1:length(s)-1)
        R = R + s(i)*cos(2*pi*(i-1)*F);
    end
```

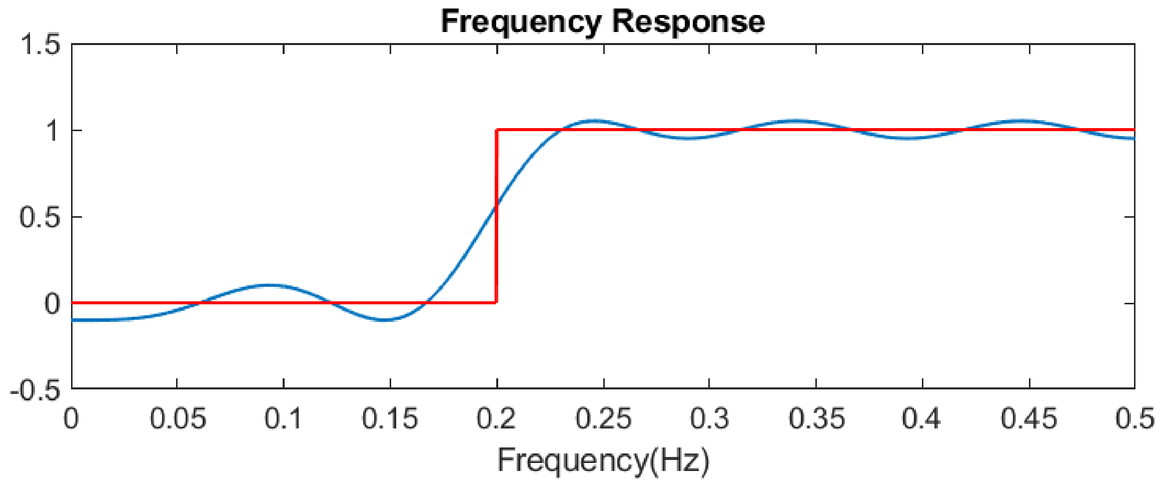
```
Err = (R-Hd).*WF;

tmp_Err = [0 Err 0];
Err_index = [];
Err_value = [];
for i = (2:length(tmp_Err)-1)
    if (tmp_Err(i-1) < tmp_Err(i) && tmp_Err(i) > tmp_Err(i+1)) ||
(tmp_Err(i-1) > tmp_Err(i) && tmp_Err(i) < tmp_Err(i+1))
        Err_value = [Err_value tmp_Err(i)];
        Err_index = [Err_index i-1];
    end
end
```

```
New_Extreme_F = [];
boundaries_F = [];
boundaries_err_Err_value = [];
for i = (1:length(Err_index))
    if Err_index(i) == 1 || Err_index(i) == length(F) || Err_index(i) == int32(transition_band(1)*sample_rate_F) || Err_index(i) == int32(transition_band(2)*sample_rate_F)
        boundaries_F = [boundaries_F (Err_index(i)-1)*delta_F];
        boundaries_err_Err_value = [boundaries_err_Err_value Err_value(i)];
    else
        New_Extreme_F = [New_Extreme_F (Err_index(i)-1)*delta_F];
    end
end

[value, index] = sort(abs(boundaries_err_Err_value), 'descend');
for i = (1:length(Extreme_F)-length(New_Extreme_F))
    New_Extreme_F = [New_Extreme_F boundaries_F(index(i))];
end
```

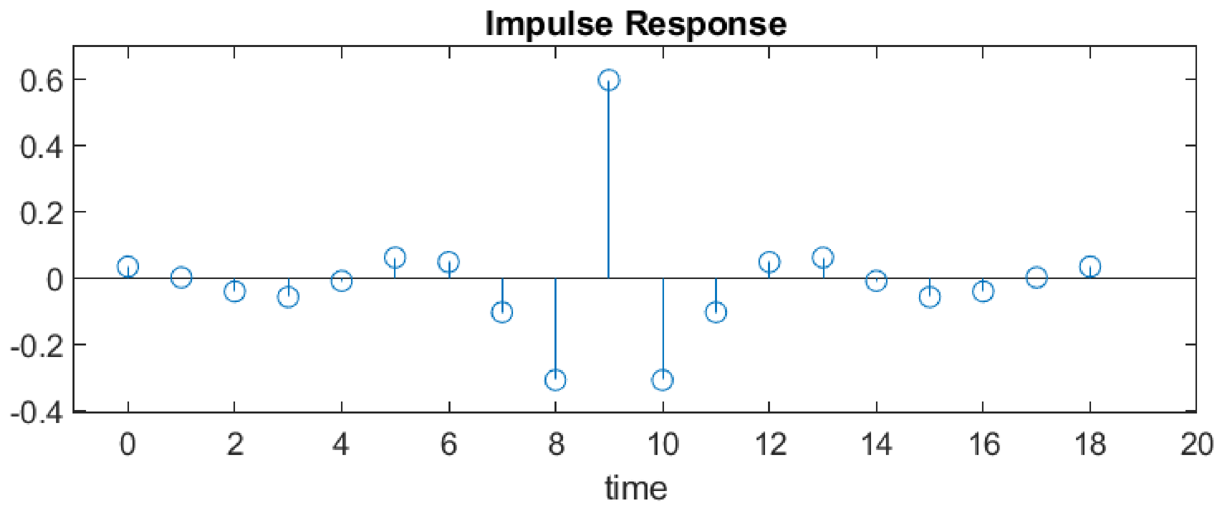
```
 now_MaxErr = max(abs(Err))
    iteration_Err = [iteration_Err now_MaxErr];
    if 0 <= previous_MaxErr - now_MaxErr && previous_MaxErr -
now_MaxErr <= delta_Err
        break;
    else
        Extreme_F = sort(New_Extreme_F)
        previous_MaxErr = now_MaxErr;
    end
end
iteration_count
```

```
subplot(2,1,1)
h_f = [(fliplr(s(2:end-1).'))/2 s(1) s(2:end-1).'/2];
x = 0:1:length(h_f)-1;
stem(x,h_f)
xlim([-1 length(h_f)+1])
ylim([min(h_f)-0.1 max(h_f)+0.1])
title('Impulse Response')
xlabel('time')

subplot(2,1,2)
plot(F,R,F,Hd,'r','lineWidth',1);
axis([0,0.5,-0.5,1.5])
title('Frequency Response')
xlabel('Frequency(Hz)')
```

(b) the frequency response



(c) the impulse response $h[n]$

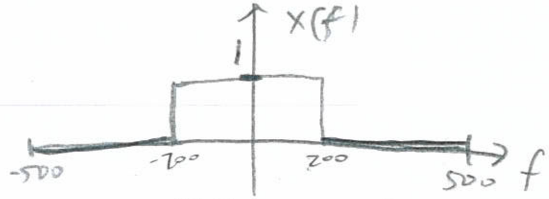


(d) the maximal error for each iteration

	iteration1	iteration2	iteration3	iteration4	iteration5	iteration6
maximal error	0.1831	0.1093	0.0852	0.0506	0.0503	0.0503

(2) Suppose that $X(f)$ is the discrete-time Fourier transform of $x(n\Delta_t)$. Also suppose that we have known that $\Delta_t = 0.001$ sec and

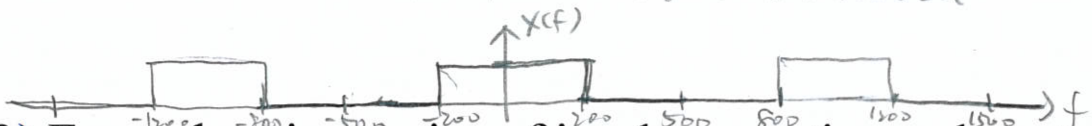
$$X(f) = 1 \text{ for } |f| < 200 \text{ and } X(f) = 0 \text{ for } 200 < |f| < 500.$$



Determine (a) $X(900)$, (b) $X(-1900)$, (c) $X(6100)$. (10 scores)

DTFT 會將離散非週期訊號 轉成 連續週期訊號

$$X(f) = \begin{cases} 1, & \text{if } -200+1000n < f < 200+1000n \\ 0, & \text{if } 200+1000n < f < 500+1000n \\ 0, & \text{if } -500+1000n < f < -200+1000n \end{cases}, \quad n \in \mathbb{Z}$$



(a) $X(900) = X(900-1000) = X(-100) = 1$ #

(b) $X(-1900) = X(-1900+2000) = X(100) = 1$ #

(c) $X(6100) = X(6100-6000) = X(100) = 1$ #

(3) From the view point of implementation, what are the <u>disadvantages</u> of the discrete Fourier transform? (5 scores)

① 有複數運算且計算量龐大

② 取樣的頻率至少要訊號最高頻率兩倍

(4) Suppose that $x[n] = y(0.0002n)$ and the length of $x[n]$ is 15000 and $X[m]$ is the FFT of $x[n]$. Find m_1 and m_2 such that $X[m_1]$ and $X[m_2]$ correspond to the 200Hz and -300Hz components of $y(t)$, respectively. (10 scores)

$\Delta t = 0.0002$ 取樣頻率為 $f_s = \frac{1}{\Delta t} = 5000\,Hz$

$N = 15000$

$m_1 = N\frac{f_1}{f_s} = 15000 \times \frac{200}{5000} = 600$

$m_2 = N\frac{f_2}{f_s} = 15000 \times \frac{-300}{5000} = -900$

$\boxed{m_1 = 600 \quad m_2 = -900}$ #

(5) Which of the following filters are <u>odd</u>? (i) bandpass filter, (ii) edge detector, (iii) differentiation 2 times, (iv) integration 3 times, (v) particle filter, (vi) the Hilbert. (10 scores)

odd filters $\begin{cases} \text{(ii) edge detector} \\ \text{(iv) integration 3 times} \\ \text{(vi) the Hilbert} \end{cases}$ 奇數次積分是 odd，偶數次積分是 even

(ii) - (iv) - (vi) #

(6) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta_t = 0.0002$, and the transition band is from 1600Hz to 1800Hz. (10 scores)

$\delta_1 = \delta_2 = 0.01$

$f_s = \frac{1}{\Delta t} = 5000$

$\Delta F = \frac{f_2}{f_s} - \frac{f_1}{f_s} = \frac{1800}{5000} - \frac{1600}{5000} = \frac{1}{25}$

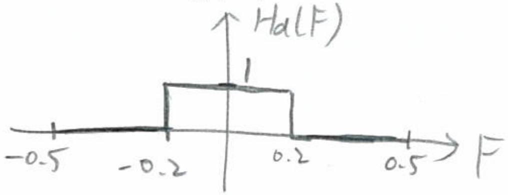
$$N = \frac{2}{3} \frac{1}{\Delta F} \log_{10}\left(\frac{1}{10\delta_1\delta_2}\right)$$

$$= \frac{2}{3} \frac{1}{\frac{1}{25}} \log_{10}\left(\frac{1}{10 \times 0.01 \times 0.01}\right)$$

$$= 50$$

$\underline{N = 50}$ #

(7) Use the MSE method to design the 9-point FIR filter that approximates the lowpass filter of $H_d(F) = 1$ for $|F| < 0.2$ and $H_d(F) = 0$ for $0.2 < |F| < 0.5$. (15 scores)



$$S[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\,dF = \int_{-0.2}^{0.2} 1\,dF$$

$$= F\Big|_{-0.2}^{0.2} = 0.4$$

$$S[n] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi n F) H_d(F)\,dF = 2\int_{-0.2}^{0.2} \cos(2\pi n F)\,dF = 2\,\frac{1}{2\pi n}\sin(2\pi n F)\Big|_{-0.2}^{0.2}$$

$$= \frac{1}{\pi n}\sin\left(\frac{2}{5}\pi n\right) - \frac{1}{\pi n}\sin\left(-\frac{2}{5}\pi n\right) = \frac{2}{\pi n}\sin\left(\frac{2}{5}\pi n\right)$$

$$S[1] = \frac{2}{\pi}\sin\left(\frac{2}{5}\pi\right) = 0.6055$$

$$S[2] = \frac{1}{\pi}\sin\left(\frac{4}{5}\pi\right) = 0.1871$$

$$S[3] = \frac{2}{3\pi}\sin\left(\frac{6}{5}\pi\right) = -0.1247$$

$$S[4] = \frac{1}{2\pi}\sin\left(\frac{8}{5}\pi\right) = -0.1514$$

$h[k] = S[0]$ $\quad k = \frac{N-1}{2} = 4$

$h[k+n] = S[n]/2$, $h[k-n] = S[n]/2$ for $n = 1, 2, 3, \ldots$

$$\boxed{\begin{aligned} h[4] &= S[0] = 0.4 \\ h[3] = h[5] &= S[1]/2 = 0.30275 \\ h[2] = h[6] &= S[2]/2 = 0.09355 \\ h[1] = h[7] &= S[3]/2 = -0.06235 \\ h[0] = h[8] &= S[4]/2 = -0.0757 \end{aligned}}$$

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(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 5, 6, 7)