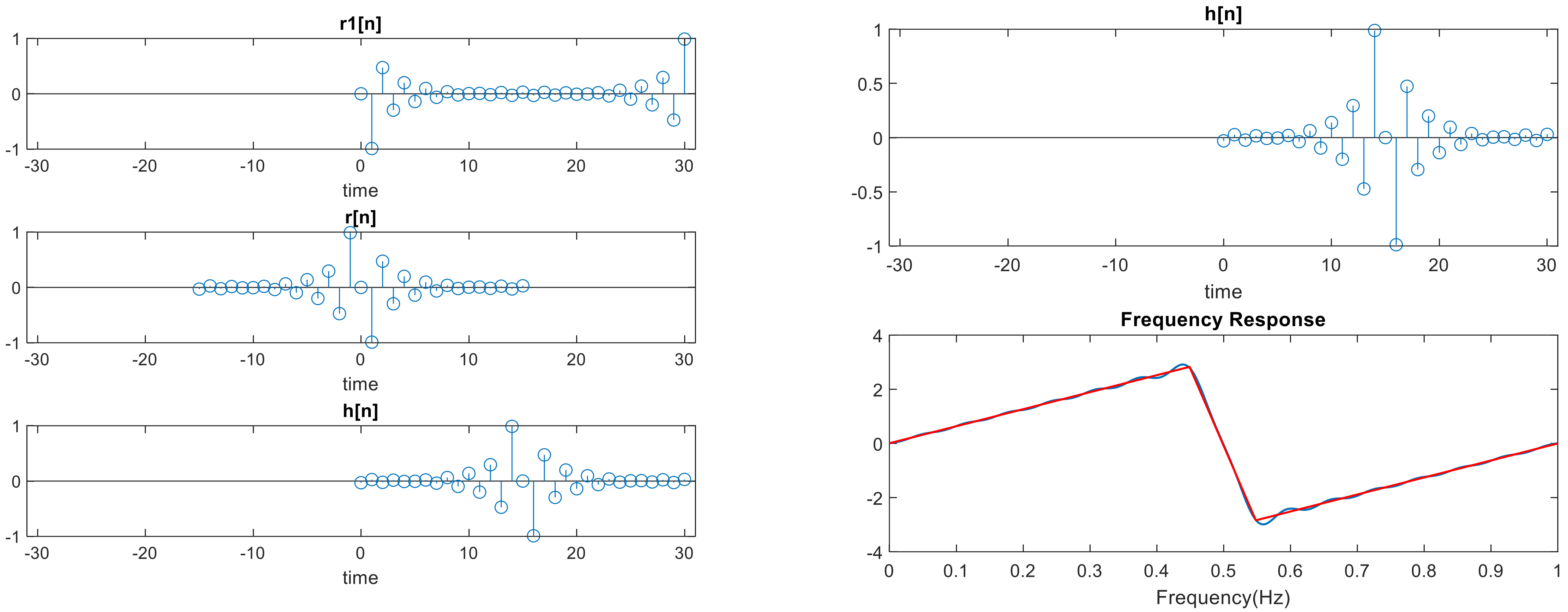
Homework 2 (Due: 4/23)

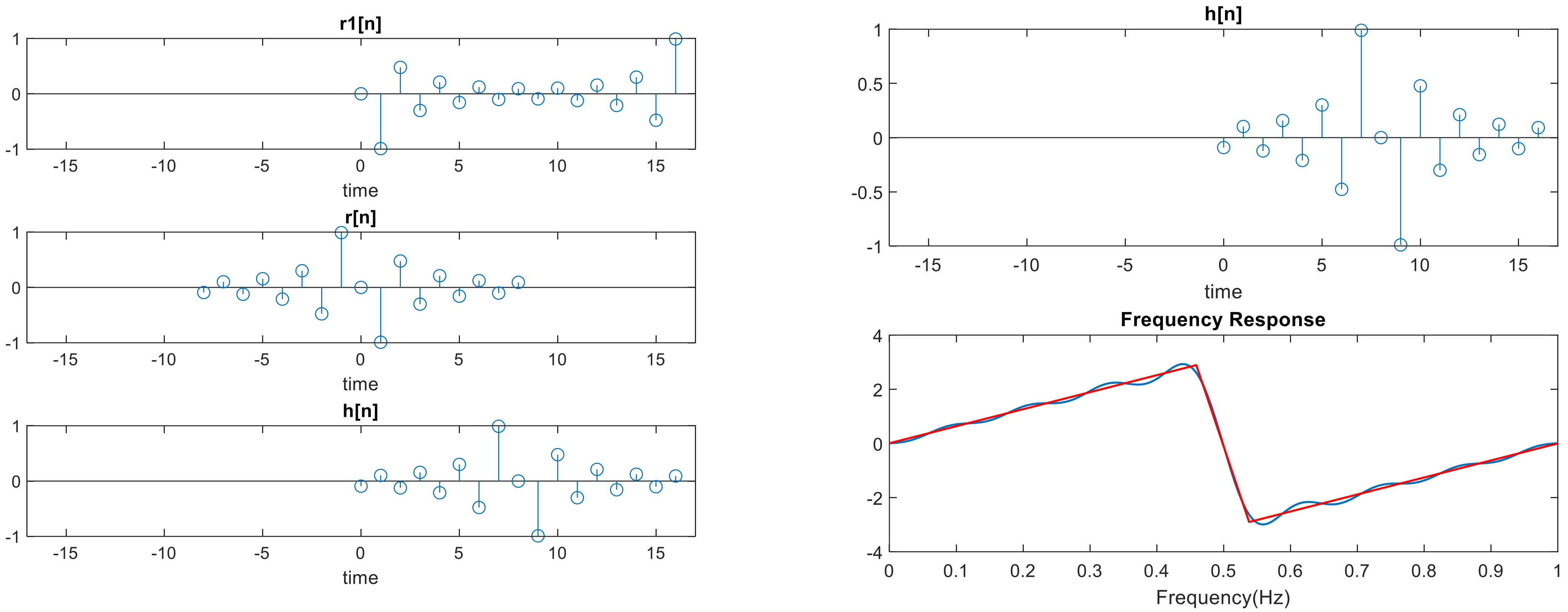
(1) Write a Matlab or Python program that uses the frequency sampling method to design a (2k+1)-point discrete differentiation filter $H(F) = j2\pi F$ (k is an input parameter and can be any integer). (25 scores)

The transition band can be assigned to reduce the error (unnecessary to optimize). The impulse response of the designed filter should be shown. The code should be handed out by ceiba.

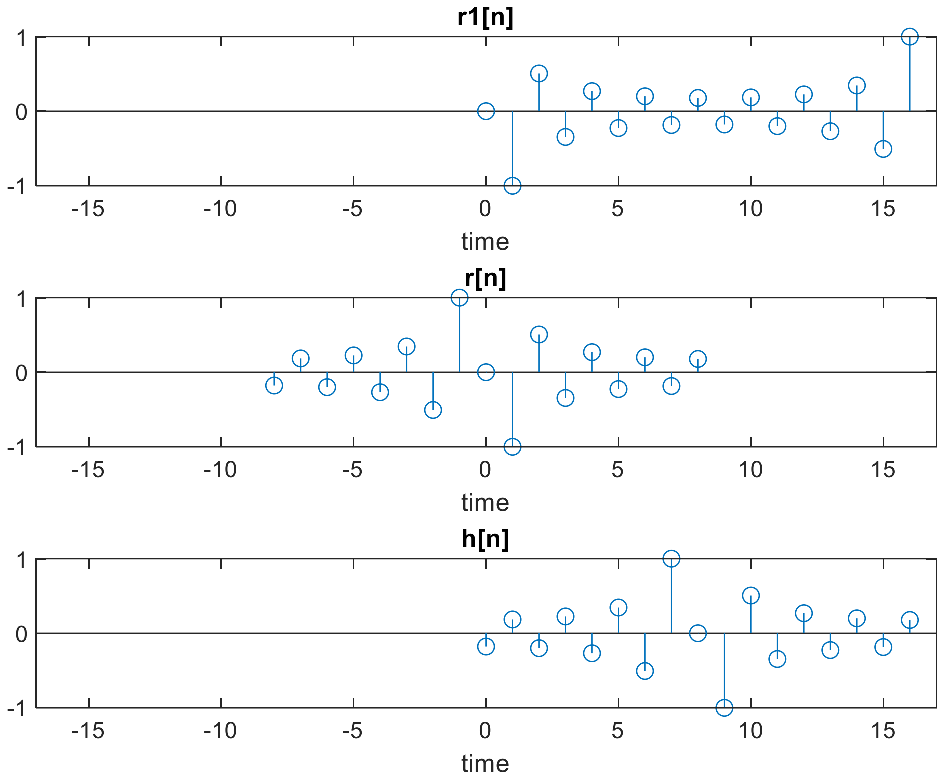
>> m10907314_HW2(15, [0.45 0.55])   k = 15, transition_band = [0.45 0.55]

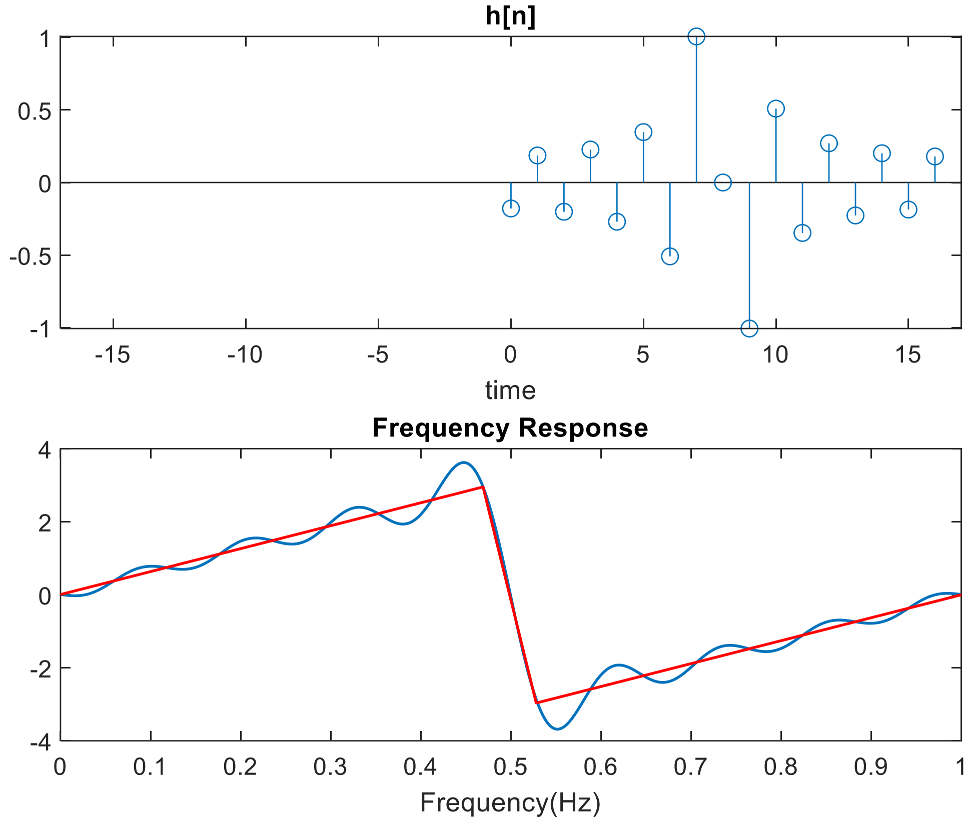


>> m10907314_HW2(8, [0.46 0.54]) k = 8, transition_band = [0.46 0.54]



>> m10907314_HW2(8, [0.47 0.53])   k = 8, transition_band = [0.47 0.53]





```
function m10907314_HW2(k, transition_band)

if nargin<2
transition_band = [0.46 0.54];
end
if nargin<1  k = 8;
end

N = 2*k + 1;
delta_F = 0.001;  sample_rate_F = 1/delta_F;

F = -0.5:delta_F:0.5;
Hd = fftshift(1i*2*pi*F);  F = 0:delta_F:1;
transition_linear =
linspace(Hd(transition_band(1)*sample_rate_F),Hd(transition_band(2)*sample_rate_F  transition_band(1)*sample_rate_F);
Hd(transition_band(1)*sample_rate_F:transition_band(2)*sample_rate_F-1)
= transition_linear;
%plot(F,imag(Hd),'r','lineWidth',1);

%hold on
```

```
Rn = [];
fn = [];
for m=0:N-1
  Rn = [Rn Hd(floor(m*sample_rate_F/N)+1)];
  fn = [fn F(floor(m*sample_rate_F/N)+1)];
end
%plot(fn,imag(Rn),'ro');

rn1 = ifft(Rn);
rn = fftshift(rn1);

RF = zeros(1,length(F));
for n=1:N
  RF = RF + rn(n)*exp(-1i*2*pi*F*(n-k-1));
end
hn = rn;
```

```
figure;
subplot(3,1,1)
x = 0:1:N-1;
stem(x,real(rn1))
xlim([-N N])
title('r1[n]')
xlabel('time')

subplot(3,1,2)

x = -k:1:k;
stem(x,real(rn))


xlim([-N N])
title('r[n]')
xlabel('time')
```

```
subplot(3,1,3)
x = 0:1:N-1;
stem(x,real(hn))
xlim([-N N])
title('h[n]')
xlabel('time')

figure;
subplot(2,1,1)
x = 0:1:N-1;
stem(x,real(hn))
xlim([-N N])
title('h[n]')
xlabel('time')

subplot(2,1,2)
plot(F,imag(RF),F,imag(Hd),'r','lineWidth',1);
title('Frequency Response')
xlabel('Frequency(Hz)')
```

(2) What are the advantages of using the Wiener filter for equalizer? (10 scores)

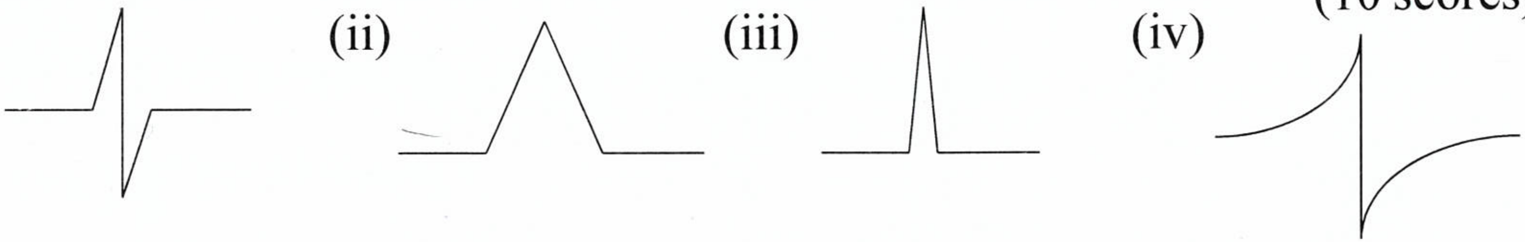
從原本的 $H(F) = \frac{1}{K(F)}$ 改良成 $H(F) = \frac{1}{\frac{C}{K^*(F)} + K(F)}$

當 SNR 小時 c 就會大
當 SNR 大時 c 就會小

優點：① $K(F) \to 0$ 時 $H(F)$ 不會趨近 ∞
② 解決 $\frac{1}{K(F)}$ 放大高頻雜訊問題

(4) The following figures are the impulse responses of some filters. Which one is suitable for ridge detection when the SNR is low? Also illustrate the reasons. (10 scores)

(i) (ii) (iii) (iv)



ridge filter 是 even symmetric 所以將 (i) 和 (iv) 排除，
SNR 小的話表示雜訊較大，當 ridge filter 比較寬時
抗雜訊能力比較高因為白雜訊有正有負平均值為 0
，當夠寬時雜訊較容易相互抵消，故選 (ii) #

(3) Derive the way to design the FIR filter of Type IV using the method of the FIR filter of Type I. (10 scores)

Type IV: $R(F) = \sum_{n=1}^{k+\frac{1}{2}} s[n]\sin(2\pi(n-\frac{1}{2})F$

因為 $\sin(2\pi(n+\frac{1}{2})F) - \sin(2\pi(n-\frac{1}{2})F) = 2\sin(\pi F)\cos(2\pi nF)$

將 type IV 改寫成

$$R(F) = \sin(\pi F)\sum_{n=0}^{k_1} s_1[n]\cos(2\pi nF)$$

$$= \frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin(2\pi(n+\tfrac{1}{2})F) - \frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin(2\pi(n-\tfrac{1}{2})F)$$

$$= \frac{1}{2}\sum_{n=0}^{k_1} s_1[n-1]\sin(2\pi(n-\tfrac{1}{2})F) - \frac{1}{2}\sum_{n=0}^{k_1} s_1[n]\sin(2\pi(n-\tfrac{1}{2})F)$$

$$= \frac{1}{2}s_1[k_1]\sin(2\pi(k_1+\tfrac{1}{2})F) + \frac{1}{2}\sum_{n=1}^{k_1} s_1[n-1]\sin(2\pi(n-\tfrac{1}{2})F) - \frac{1}{2}s_1[0]\sin(-\pi F) - \frac{1}{2}\sum_{n=1}^{k_1} s_1[n]\sin(2\pi(n-\tfrac{1}{2})F)$$

$$= \frac{1}{2}s_1[0]\sin(\pi F) + \sum_{n=1}^{k_1}\frac{1}{2}(s_1[n-1]-s_1[n])\sin(2\pi(n-\tfrac{1}{2})F) + \frac{1}{2}s_1[k_1]\sin(2\pi(k_1+\tfrac{1}{2})F)$$

(令 $k_1+\frac{1}{2}=k$)

$$= (s_1[0]-\tfrac{1}{2}s_1[1])\sin(\pi F) + \sum_{n=2}^{k-\frac{1}{2}}\frac{1}{2}(s_1[n-1]-s_1[n])\sin(2\pi(n-\tfrac{1}{2})F) + \frac{1}{2}s_1[k-\tfrac{1}{2}]\sin(2\pi kF)$$

$$\boxed{\begin{aligned} &s[1] = s_1[0]-\tfrac{1}{2}s_1[1] \\ &s[n] = \tfrac{1}{2}(s_1[n-1]-s_1[n]) \quad \text{for } n=2,3,\ldots,k-\tfrac{1}{2} \\ &s[k+\tfrac{1}{2}] = \tfrac{1}{2}s_1[k-\tfrac{1}{2}] \\ &k \longrightarrow k-\tfrac{1}{2} = \tfrac{N}{2}-1 \\ &Hd(F) \longrightarrow \csc(\pi F)Hd(F) \\ &W(F) \longrightarrow \sin(\pi F)W(F) \end{aligned}}$$

$$err(F) = [Hd(F) - R(F)]W(F)$$

$$= [Hd(F) - \sin(\pi F)\sum_{n=0}^{k-\frac{1}{2}} s[n]\cos(2\pi nF)]W(F)$$

$$= [\csc(\pi F)Hd(F) - \sum_{n=0}^{k-\frac{1}{2}} s[n]\cos(2\pi nF)]\sin(\pi F)W(F)$$

#

(5) Suppose that the smooth filter is $h[n] = 0.05$ for $|n| \leqq 5$, $h[n] = a$ for $6 \leqq |n| \leqq 15$, and $h[n] = 0$ otherwise. (a) What is the value of a? (b) What is the <u>efficient way</u> to implement the <u>convolution</u> $y[n] = x[n] * h[n]$?

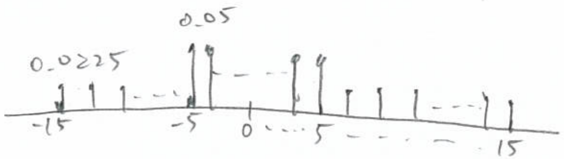
(10 scores)

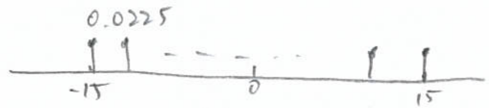
(a) $\sum_{n=-\infty}^{\infty} h[n] = 1$

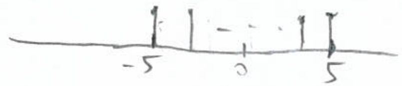
$h[n] = 0.05$ when $n = -5, -4 \cdots 5$ 11個點

$h[n] = a$ when $n = -15, -14 \cdots -6, 6, 7, \cdots 15$ 20個點

$\sum_{n=-\infty}^{\infty} h[n] = 0.05 \times 11 + a \times 20 = 1$ $\qquad \underline{a = 0.0225}$ #

(b) $h[n] =$ 

$=$ $0.0225(u[n+15] - u[n-16])$ 

$+$ $0.0225(u[n+5] - u[n-6])$ $(0.05 - 0.0225) = 0.022$ 

$$Y(z) = H(z)X(z) = X(z) \times 0.0225 \left[\frac{(z^{-1})^{-15}}{1-z^{-1}} - \frac{(z^{-1})^{16}}{1-z^{-1}} + \frac{(z^{-1})^{-5}}{1-z^{-1}} - \frac{(z^{-1})^{6}}{1-z^{-1}} \right]$$

$$Y(z) - z^{-1}Y(z) = 0.0225 \times X(z) \times \left[z^{15} - z^{-16} + z^{5} - z^{-6} \right]$$

$$Y(z) = 0.0225 \times \left[z^{15}X(z) - z^{-16}X(z) + z^{5}X(z) - z^{-6}X(z) \right] + z^{-1}Y(z)$$

$$\Rightarrow \underline{y[n] = 0.0225\left[x[n+15] - x[n-16] + x[n+5] - x[n-6] \right] + y[n-1]}$$ #

設 $h(n)$ 等比級數 $= a^n u[n+k]$

$$H(z) = \sum_n h[n] z^{-n} = \sum_{n=-k}^{\infty} a^n z^{-n}$$

$$= \sum_{n=-k}^{\infty} (a z^{-1})^n$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^{n-k}$$

$$= (a z^{-1})^{-k} \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= (a z^{-1})^{-k} \frac{1}{1 - a z^{-1}}$$

$$H(z)=\frac{(z-2)(3z^2+2z+1)}{2(z+\sqrt{0.5})(z-\sqrt{0.5})}$$

$\Downarrow$ 公式解 $z=\frac{-1\pm\sqrt{2}i}{3}$

$$H(z)=\frac{3(z-2)(z+\frac{1+\sqrt{2}i}{3})(z+\frac{1-\sqrt{2}i}{3})}{2(z+\sqrt{0.5})(z-\sqrt{0.5})}$$

(6) Suppose that an IIR filter is $H(z)=\dfrac{3z^3-4z^2-3z-2}{2z^2-1}$

(a) Find its cepstrum.

(b) Convert it into the minimum phase filter.

(c) Compared to the original IIR filter, what are two advantages of the minimum phase filter? (20 scores)

(a) $H(z)=\dfrac{3(z-2)(z+\frac{1+\sqrt{2}i}{3})(z+\frac{1-\sqrt{2}i}{3})}{2(z+\sqrt{0.5})(z-\sqrt{0.5})}$

$$=\frac{3\times-2\times(1-\frac{1}{2}z)\times z(1+\frac{1+\sqrt{2}i}{3}z^{-1})\times z(1+\frac{1-\sqrt{2}i}{3}z^{-1})}{2\times z\times(1+\sqrt{0.5}z^{-1})\times z\times(1-\sqrt{0.5}z^{-1})}$$

$$=-3\times\frac{(1-\frac{1}{2}z)(1+\frac{1+\sqrt{2}i}{3}z^{-1})(1+\frac{1-\sqrt{2}i}{3}z^{-1})}{(1+\sqrt{0.5}z^{-1})(1-\sqrt{0.5}z^{-1})}$$

$$\hat{x}[n]=\begin{cases}\log(-3)=0.477+1.364i, & n=0\\ \dfrac{-(\frac{-\sqrt{2}i-1}{3})^n}{n}+\dfrac{-(\frac{\sqrt{2}i-1}{3})^n}{n}+\dfrac{(-\sqrt{0.5})^n}{n}+\dfrac{(\sqrt{0.5})^n}{n}, & n>0\\ \dfrac{(\frac{1}{2})^{-n}}{n}, & n<0\end{cases}$$

#

(b)

$$H(z)=\frac{3(z-2)(z+\frac{1+\sqrt{2}i}{3})(z+\frac{1-\sqrt{2}i}{3})}{2(z+\sqrt{0.5})(z-\sqrt{0.5})}$$

$$H_{mp}(z)=\frac{3(z-2)(z+\frac{1+\sqrt{2}i}{3})(z+\frac{1-\sqrt{2}i}{3})}{2(z+\sqrt{0.5})(z-\sqrt{0.5})}\times 2\,\frac{z-\frac{1}{2}}{z-2}$$

$$=\frac{3(z-\frac{1}{2})(z+\frac{1+\sqrt{2}i}{3})(z+\frac{1-\sqrt{2}i}{3})}{(z+\sqrt{0.5})(z-\sqrt{0.5})}$$

#

(c)

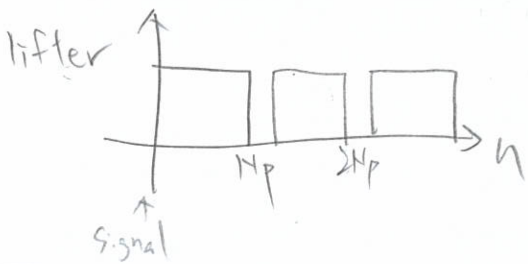
① 可以把能量集中在n=0附近

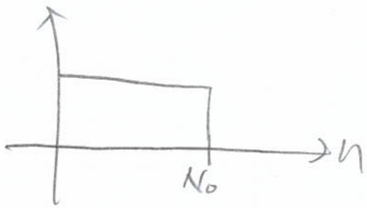
② 可以使正轉換和逆轉換都能穩定

(7) (a) Why the <u>cepstrum</u> is more suitable for dealing with the <u>multipath problem</u> than the equalizer $1/H(z)$ where $H(z)$ is the z transform of the impulse response? (Write at least 2 reasons) (b) Why the <u>Mel-cepstrum</u> is more suitable for dealing with the acoustic signal than the original cepstrum? (Write at least 3 reasons)

(15 scores)

(a)





$N_0 < N_p$ 回音將被濾除

① 不必知道 decade 參數 a 也能設計

② 取一個 N_0 更容易設計，對原本訊號破壞不大，且 N_p 也不一定要事先知道

(b)

① $B_m[k]$ 等比級數，符合人體聽覺

② $\sum |X(k)|^2 B_m[k] = 0$ 的機率較低

③ 不會有相位問題因為 $\sum |X(F)|^2 B(F)$ 實數

④ 使用DCT減少運算量

(Extra): Answer the questions according to your student ID number. (ended with 0, 3, 4, 5, 8, 9)

問： FIR filter of Type IV (a) 點數是 odd 還是 even？

(b) 是奇對稱還是偶對稱？

Type 1: $R(F) = \sum_{n=0}^{k} s[n]\cos(2\pi nF)$ N是奇數，偶對稱

Type 2: $R(F) = \sum_{n=0}^{k+\frac{1}{2}} s[n]\cos(2\pi(n-\frac{1}{2})F)$ N是偶數，偶對稱

Type 3: $R(F) = \sum_{n=1}^{k} s[n]\sin(2\pi nF)$ N是奇數，奇對稱

Type 4: $R(F) = \sum_{n=1}^{k+\frac{1}{2}} s[n]\sin(2\pi(n-\frac{1}{2})F)$ N是偶數，奇對稱 ✻