Derivatives

CFA一级培训项目

讲师：周琪

学位证书
- 中央财经大学经济学学士、复旦大学工商管理硕士、CFA（特许金融分析师）持证人、FRM（金融风险管理师）持证人

工作职称
- 金程教育财经项目部副总监
- 金程教育资深培训师
- CFRM（注册金融风险管理师）特聘讲师

服务客户
- 中国工商银行、中国银行、中国建设银行、中国农业银行、交通银行、招商银行、民生银行、杭州银行、中国进出口银行
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Mikey Chow
8年授课，5000+授课课时

Topic Weightings in CFA Level I

Topics	Weights (%)
Quantitative Methods	8-12
Economics	8-12
Financial Statement Analysis	13-17
Corporate Issuers	8-12
Equity	10-12
Fixed Income	10-12
Derivatives	5-8
Alternative Investments	5-8
Portfolio Management	5-8
Ethical and Professional Standards	15-20

- **强化班知识点说明和使用指南**

序号	课件元名称（知识点）	必考	高频	低频
9	Principle of forward pricing & valuation	1	0	0
10	Pricing & valuation of different forward contracts	0	1	0
11	Pricing and valuation of futures	0	1	0
12	Pricing and valuation of swaps	0	1	0
13	Option payoff and profit	0	1	0
14	Option valuation	1	0	0

- 必考知识点指的是近10年考试中考试频率大于等于75%的考点，在强化班中重点讲解，必须掌握；

- 高频知识点指的是近10年考试中考试频率介于25%到75%的考点，在强化班中重点讲解，必须掌握；

- 低频知识点指的是近10年考试中考试频率小于25%的考点，在基础班中重点讲解，学员可以根据自己的掌握情况在基础班中巩固学习；

- 本学科知识点合计16个，其中必考知识点4个，高频知识点10个，低频知识点2个，掌握必考和高频考点覆盖了近10年97.35%的题目。

Derivatives

1. Derivative Instrument and Derivative Market Features

2. Forward Commitment and Contingent Claim Features and Instruments

3. Derivative Benefits, Risks, and Issuer and Investor Uses

4. Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

5. Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

6. Pricing and Valuation of Futures Contracts

7. Pricing and Valuation of Interest Rates and Other Swaps

8. Pricing and Valuation of Options

9. Option Replication Using Put–Call Parity

10. Valuing a Derivative Using a One-Period Binomial Model

Framework

Module

Derivative Instrument and Derivative Market Features

1. Derivative Instrument

2. Derivative Market

Derivative Instrument

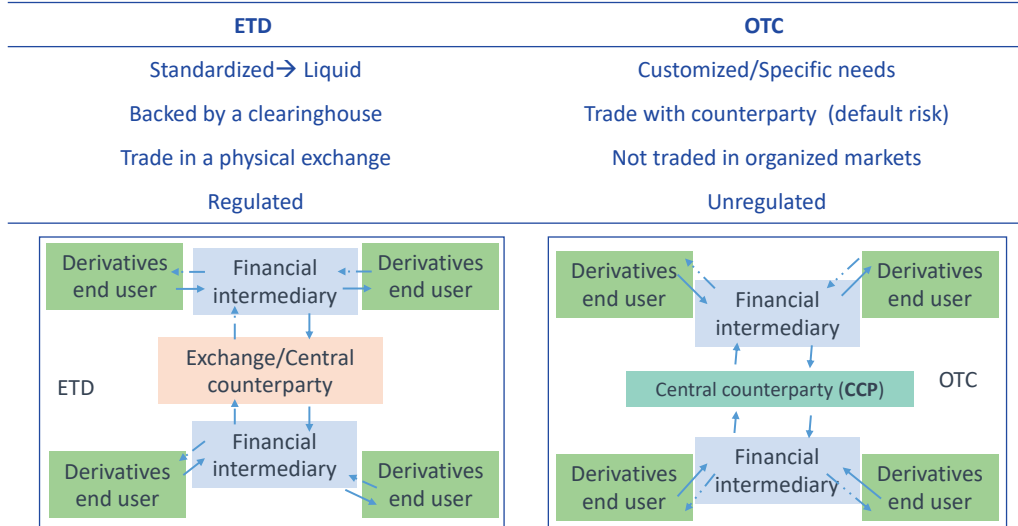| | **Forward** | |
|---|---|
| 概念 | ➢ A <u>forward contract</u> is a <u>private agreement</u> that obligates one party to buy and the other party to sell a specific quantity of an underlying asset, at a <u>set price</u>, at a <u>future date.</u> |
| 分类 | ➢ Commodity forward contract
➢ Financial forward contract |
| 目的 | ➢ Hedge risk
➢ Speculation |
| 交割
★ | ➢ At expiration
  ● Physical settlement
  ● Cash settlement
➢ Prior to expiration |

Derivative Underlyings

Asset Class	Examples Sample	Uses
Equities	• Individual stocks • Equity indexes • **Equity price volatility**	• Change exposure profile (Investors) • Employee compensation (Issuers)
Interest rates	• Sovereign bonds (domestic) • Market reference rates (MRR)	• Change duration exposure (Investors) • Alter debt exposure profile (Issuers)
Foreign exchange	• Sovereign bonds (foreign) • Market exchange rates	• Manage global portfolio risks (Investors) • Manage global trade risks (Issuers)
Commodities	• **Soft and hard commodities** • Commodity indexes	• Manage operating risks (Consumers/Producers) • Portfolio diversification (Investors)
Credit	• Individual refence entities • Credit indexes	• Portfolio diversification (Investors) • Manage credit risk (Financial Intermediaries)
Other	• Weather • Cryptocurrencies • Longevity	• Manage operating risks (Issuers) • Manage portfolio risks (Investors)

Classification

● **According to contract features: Firm commitment & Contingent claim**

  ○ **Firm commitment** (linear derivatives, symmetric payoff profile): a pre-determined amount is agreed to be exchanged at settlement.

    ✓ forward contracts;

    ✓ futures contracts;

    ✓ swaps involving a periodic exchange of cash flows.

  ○ **Contingent claim** (non-linear derivatives, asymmetric payoff profile): one of the counterparties determines whether and when the trade will settle.

    ✓ An **option** is the primary contingent claim.

    ✓ An **embedded derivative** is a derivative within an underlying, such as a callable, puttable, or convertible bond.
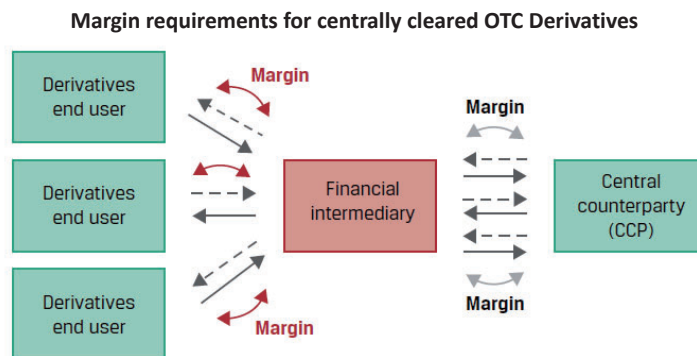
Classification Based On Trading Place

- Differences

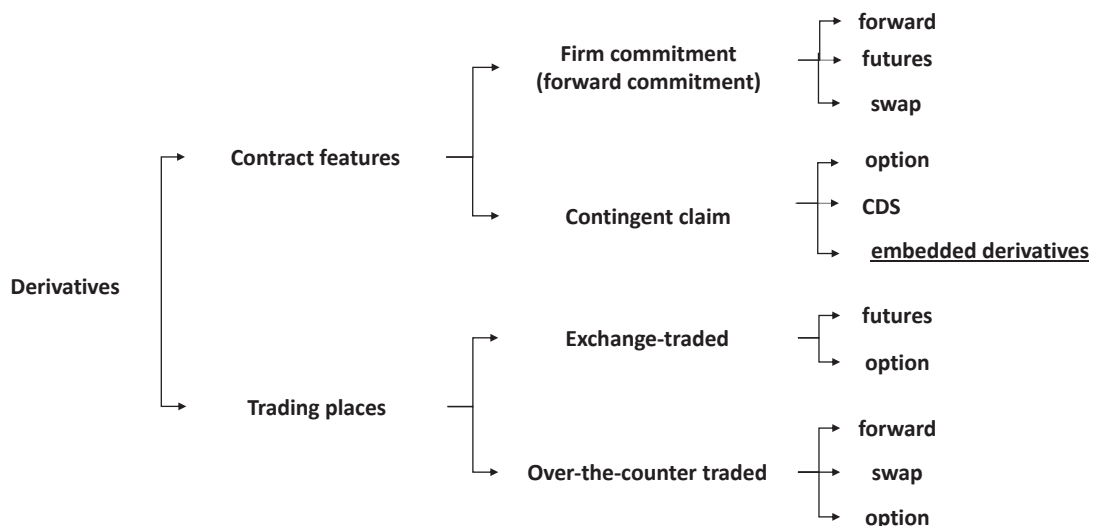| ETD	OTC
Standardized→ Liquid	Customized/Specific needs
Backed by a clearinghouse	Trade with counterparty (default risk)
Trade in a physical exchange	Not traded in organized markets
Regulated	Unregulated

Effect of Central Clearing of OTC Derivatives

- The advent of **derivatives central clearing, has created futures-like margining requirements for OTC derivative dealers** who buy and sell forwards to derivatives end users. Dealers who are required to post cash or highly liquid securities to a central counterparty often impose similar requirements on derivatives end users.

Margin requirements for centrally cleared OTC Derivatives

Classification Based On Contract Type

Derivatives
- Contract features
  - Firm commitment (forward commitment)
    - forward
    - futures
    - swap
  - Contingent claim
    - option
    - CDS
    - embedded derivatives
- Trading places
  - Exchange-traded
    - futures
    - option
  - Over-the-counter traded
    - forward
    - swap
    - option

Module

Derivative Benefits, Risks, and Issuer and Investor Uses

1. Benefits and Risks of Derivative Instruments
2. Use of derivatives Issuers & Investors

Benefits of Derivative Instruments

Purpose	Description
Risk Allocation, Transfer, and Management	✓ Allocate, trade, and/or manage underlying exposure without trading the underlying. ✓ Create exposures unavailable in cash markets.
Information Discovery	✓ Deliver expected price in the future as well as expected risk of underlying.
Operational Advantages	✓ Reduced cash outlay, lower transaction costs versus the underlying, increased liquidity and ability to "short".
Market Efficiency	✓ Less costly to exploit arbitrage opportunities or mispricing.

Risks of Derivative Instruments

Risk	Description
Greater Potential for Speculative Use	✓ High degree of implicit leverage may increase the likelihood of financial distress.
Lack of Transparency	✓ Derivatives add portfolio complexity and may create an exposure profile that is not well understood.
Basis Risk	✓ Potential divergence between the expected value of a derivative instrument versus an underlying or hedged transaction.
Liquidity Risk	✓ Potential divergence between the cash flow timing of a derivative instrument versus an underlying or hedged transaction.
Counterparty Credit Risk	✓ Derivative instruments often give rise to counterparty credit exposure, resulting from differences in the current price versus the expected future settlement price.
Destabilization and Systemic Risk	✓ Excessive risk taking and use of leverage may contribute to market stress, e.g., 2008 financial crisis.

- **Issuers** predominantly use derivatives to <u>offset or **hedge** market-based underlying exposures</u> incidental to their commercial <u>operations and financing activities</u>.
  - O Hedge accounting allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility.
  - O **Hedge accounting Types:** Cash Flow, fair value, net investment
- **Investors** use derivatives to **replicate a cash market strategy**, **hedge a fund's value against adverse movements in underlying** or **modify or add exposures using derivatives**.
- **Investor vs. Issuers**
  - O Investors are <u>less focused</u> than issuers on hedge accounting treatment,
    - ✓ as an investment fund's derivative position is typically marked to market each day and included in the daily net asset value (NAV) of the portfolio or fund.
  - O Investors <u>transact more frequently</u> in standardized and highly liquid <u>exchange-traded derivative markets</u> than do issuers.

Module

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Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

1. Arbitrage, Replication, and Risk Neutrality
2. Cost of Carry in Pricing Derivatives

Arbitrage and No-Arbitrage Rule

- **Arbitrage involves** earning an immediate gain with no future liabilities.
- **Arbitrage opportunities arise when:**
  - O Two assets with identical future cash flows trade at different prices.
- **The way of arbitrage:**
  - O Sell high, buy low.
- **Law of one price (no arbitrage rule):**
  - O The condition in a financial market in which two equivalent financial instruments or combinations of financial instruments can sell for only one price.

Replication

- **Replication** is a strategy in which a derivative's cash flow stream may be **recreated** using a combination of long or short positions in an underlying asset and borrowing or lending cash.
- Replication is typically used to mirror or offset a derivative position when the law of one price holds, and no riskless arbitrage profit opportunities exist.
  - To replicate a risk-free asset, we can purchase an asset at today's spot price, and simultaneously enter into a forward commitment to sell the asset at the forward price.
  - To replicate a long forward position , we can borrow at the risk-free rate and buy the underlying asset at today's spot price. At time T, sell the asset at the spot price. Repay the loan principal and interest.
  - To replicate a short forward position, we can short sell the asset at t = 0 and lend proceeds of the asset sale at the risk-free rate. At time T, buy back the asset at the spot price.

Arbitrage and No-Arbitrage Rule

- **Arbitrage involves** earning over the risk-free rate with no risk.
- **Arbitrage opportunities arise when:**
  - An asset with a known future price does not trade at the present value of its future price determined using an appropriate discount rate.
- **The ways of arbitrage:**
  - If asset provides higher interest rate, borrow money and invest today, then sell it in the future.
  - If asset provides lower interest rate, short sell the asset, saving at risk free rate, buy back in the future.
- **Certain payoff:**
  - The expected payoff of the derivative can be discounted at the risk-free rate. And should yield the risk-free rate of return, if it generates certain payoffs.

Arbitrage and Replication

分类	➢ Cash-and-Carry Arbitrage: the Forward Contract is Overpriced
	   ● $FP > S_0 \times (1+R_f)^T$
	➢ Reverse Cash-and-Carry Arbitrage: the Forward Contract is Under-priced
	   ● $FP < S_0 \times (1+R_f)^T$

限制
★
	➢ Limits to arbitrage
	   ● Transaction costs.
	   ● Borrow unlimited amounts of money at risk-free rate.
	   ● Transactions require additional capital to maintain position.
	   ● Gains from an offsetting position might not be liquid.
	   ● One position can not be perfect hedged in practice.

Module

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

1. Forward
2. Basics of Forward Pricing
3. Pricing & Valuation of Different Forward Contracts

Forward Contract

| Forward contract | Futures contract | Swap contract | Option contract |

- **Forward contract**
  - A **forward contract** is a **private agreement** that obligates one party to buy and the other party to sell a specific quantity/contract size of a specific underlying asset, at a **set price**, at a **future date**.
  - If the future price of the underlying assets increase, the buyer (long position) has a gain, and the seller (short position) has a loss.
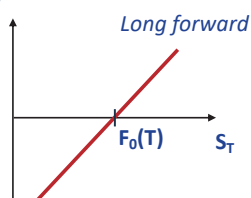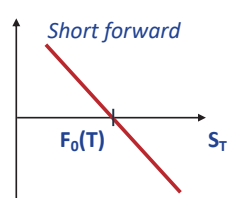
Forward Contract Settlement

- **Settling a forward contract prior to expiration.**
  - Enter into an opposite forward contract with an expiration date equal to the time remaining on the original contract.
- **Settling** the outstanding contracts (**open interest**) at expiration.
  - **Physical settlement:** deliver an actual asset, has storage cost, mostly used in commodity forward.
  - **Cash settlement:** the party that has a position with negative value is obligated to pay that amount to the other party, mostly used in financial forward.
    - ✓ These forward contracts also called *non-deliverable forwards (NDFs), cash-settled forwards, or contracts for differences*.

- **Pricing** a forward contract is the process of determining the **no-arbitrage price** that will make the value of the contract be zero to both sides at the initiation of the contract.

$$FP = S_0 \times (1 + R_f)^T + FVC_T - FVB_T$$

$$FP = (S_0 - PVB_0 + PVC_0) \times (1 + R_f)^T$$

- **Cost of carry** is the net of the **costs** and **benefits** related to owning an underlying asset for a specific period.
  - Carrying Costs
    - ✓ Costs of storage: incurred in owning commodity. E.g., corn, gold, etc.
    - ✓ The risk-free rate, R_f denotes the opportunity cost of carrying the asset, whether the long investor borrows to finance the asset.
  - Carrying Benefits
    - ✓ Monetary benefits: dividends, coupons, interest, etc.
    - ✓ Non-monetary benefits: **convenience yield**, a non-cash benefit of holding a physical commodity versus a derivative.

Cost of Carry of Underlying Assets

Asset Class	Examples	Benefits (i)	Costs (r, c)
Asset without Cash Flows	• Non-dividend-paying stock	• None	• Risk-free rate
Equities	• Dividend-paying stocks • Equity indexes	• Dividend • Dividend yield	• Risk-free rate
Foreign Exchange	• Sovereign bonds (foreign)	• None	• Difference between foreign and domestic risk-free rates $(r_f - r_d)$
Commodities	• Market exchange rates • Soft and hard commodities	• Convenience yield	• Risk-free rate • Storage cost
Interest Rates	• Commodity indexes • Sovereign bonds (domestic) • Market reference rates	• Interest income	• Risk-free rate
Credit	• Single reference entity • Credit indexes	• Credit spread	• Risk-free rate

Forward Rate Agreement (FRA)

概念 ★	➢ **An FRA can be viewed as a forward contract to borrow/lend money at a certain rate at some future date.** • Long position → Borrow • Short position → Lend
标的 ★	➢ The underlying is a hypothetical deposit of a notional amount in the future at a market reference rate (MRR) that is fixed at contract inception (t = 0). • the Secured Overnight Financing Rate (SOFR) ; the euro short-term rate (€STR) ; the Sterling Overnight Index Average (SONIA). • survey-based Libor rates used as reference rates in the past have been replaced by rates based on a daily average of observed market transaction rates.

Forward Pricing and Valuation

		Pricing → T=0	Valuation → T=t
公式 ★	T-bill forwards	$FP = S_0 \times (1 + R_f)^T$	$V_{\text{long}} = S_t - \dfrac{FP}{(1 + R_f)^{T-t}}$
	Dividend-paying stock	$FP = (S_0 - PVD_0) \times (1 + R_f)^T$ $FP = S_0 e^{(r+c-i)\times T}$	$V_{long} = (S_t - PVD_t) - \dfrac{FP}{(1 + R_f)^{T-t}}$
	Coupon bonds	$FP = (S_0 - PVC_0) \times (1 + R_f)^T$	$V_{\text{long}} = (S_t - PVC_t) - \dfrac{FP}{(1 + R_f)^{T-t}}$
	Interest rate forward	$(1 + IFR_{B-A})^{B-A} = \dfrac{(1 + Z_A)^A}{(1 + Z_B)^B}$	$(\text{Notional principal}) \left[\dfrac{(\text{Floating rate at settlement} - \text{forward rate})\left[\frac{\text{days}}{360}\right]}{1 + \text{Floating rate at settlement}\left[\frac{\text{days}}{360}\right]} \right]$
	Foreign exchange forward	$FP_{f/d} = S_{f/d} \times \dfrac{1 + r_f}{1 + r_d}$ $FP_{f/d} = S_{f/d} \times e^{(r_f - r_d) \times T}$	-

Module
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Pricing and Valuation of Futures Contracts

1. Futures
2. Pricing and Valuation of Futures

Futures Contract

Forward contract	Futures contract	Swap contract	Option contract

- **A Futures contract** is a specialized version of a forward contract that has been standardized and that trades on a futures exchange.
  - A forward contract.
  - Are standardized.
  - Exchanged-traded.
  - Are regulated.
  - Guaranteed by the exchange through the clearinghouse.
  - Daily settlement for gains and losses.

Futures

	Forwards	Futures
Comparison Forwards ★★	Private contracts	Exchange-traded
	Unique customized contracts	Standardized contracts
	Little or no regulation	Regulated
	Default risk is present	Guaranteed by clearinghouse
	Settlement at maturity	Daily settlement(mark to market)
	No margin deposit required	Margin required and adjusted
	Forward price is constant	Futures price fluctuates daily

风险控制 ★★	Margin 保证金制度	➢ Initial margin ➢ Maintenance margin ➢ Variation margin: 补交到Initial Margin的部分
	Daily price limit 涨跌停制度（控制价格波动）	
	Marking to market （逐日盯市）	

Futures Contract

	Futures margin	Equity margin
Comparison with Equity Margin ★	As pledge, control default risk	Borrow capital, has leverage
	Cash outflow	Cash inflow
	No interest paid	Loan, interest paid needed
	Back to initial margin	Back to maintenance margin
Daily price limit 概念	➢ Limit on the extent of price movement from the settlement price of the previous trading day. ● Establish a band ● Circuit breaker	
Marking to market 概念	➢ The margin requirement of a futures contract is low because at the end of every day there is a daily settlement process called marking to market.	

Futures Pricing and Valuation

- **Prices of Futures vs. Forward Contracts**
  - Daily settlement and margin requirements give rise to different cash flow patterns between futures and forwards, resulting in a pricing difference which depends on both the correlation between interest rates and futures prices and interest rate volatility.

If the correlation between the **futures prices** and **interest rates** is:	Investors will…
positive. ($\rho_{FP\&int} > 0$)	prefer to go long in a futures contract
zero. ($\rho_{FP\&int} = 0$)	have no preference
negative. ($\rho_{FP\&int} < 0$)	prefer to go long in a forward contract

- There is a price difference between interest rate futures and forward rate agreements (FRAs) due to **convexity bias** which arises given the difference in price changes for interest rate futures versus forward contracts.

Interest Rate Forwards versus Futures

- There is a price difference between interest rate futures and forward rate agreements (FRAs) due to **convexity bias** which arises given the difference in price changes for interest rate futures versus forward contracts.

Convexity Bias
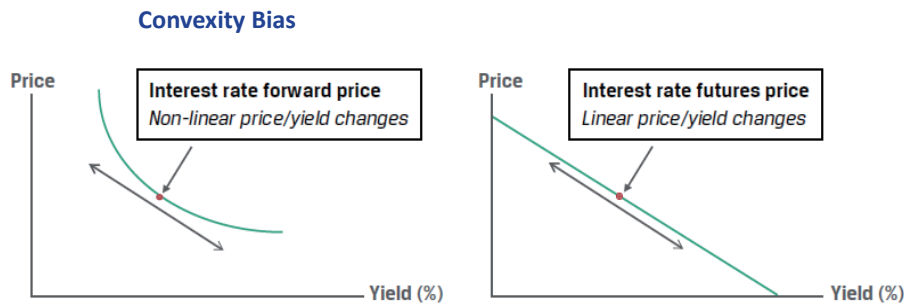
Interest Rate Futures

- The futures price for short-term interest rate futures is given by (100 - yield), where yield is expressed in percentage terms.

$$f_{A,B-A} = 100 - \left(100 \times MRR_{A,B-A}\right)$$

- where $f_{A,B-A}$ represents the futures price for a market reference rate for $B - A$ periods that begins in A periods ($MRR_{A,B-A}$)

- The interest rate exposure profile for long and short futures contracts are as follows:
  - Long futures contract (lender): Gains as prices rise, future MRR falls
  - Short futures contract (borrower): Gains as prices fall, future MRR rises

Interest Rate Futures versus FRAs		
Contract Type	**Gains from Rising MRR**	**Gains from Falling MRR**
Interest rate futures	Short futures contract	Long futures contract
Forward rate agreement	Long FRA: FRA fixed-rate payer (FRA floating-rate receiver)	Short FRA: FRA floating-rate payer (FRA fixed-rate receiver)

Futures Pricing and Valuation

- **Valuation of Futures Contracts**
  - Positions:
    - ✓ Long futures contract (lender): Gains as prices rise
    - ✓ Short futures contract (borrower): Gains as prices fall
  - The value of a futures contract is zero at contract inception.
  - The daily settlement of futures gains and losses via a margin account <u>resets the futures contract value to zero</u> at the current futures price $f_t(T)$. This process continues until contract maturity and the futures price converge to the spot price, S_T.
  - Between the times at which the contract is marked to market, the value can be different from zero.

    V (long) = current futures price – futures price at the last mark-to-market time.

Module

Pricing and Valuation of Interest Rates and Other Swaps

1. Swap
2. Pricing and valuation of swaps

Swaps Contract

Forward contract	Futures contract	Swap contract	Option contract

- **A swap contract** is a series of forward contracts**.**
  - Exchange a series of cash flows.
  - are customized, traded in OTC market, are not regulated, have default risks.
- **Interest Rate Swaps**
  - The **plain vanilla interest rate swap** involves trading <u>fixed interest rate payments</u> for <u>floating-rate payment</u> (paying fixed and receiving floating).
    - ✓ Counterparties: fixed-rate payer and floating-rate payer.

Pay-fixed side → Paying fixed rate → Pay-floating side
Pay-floating side → Paying floating rate → Pay-fixed side
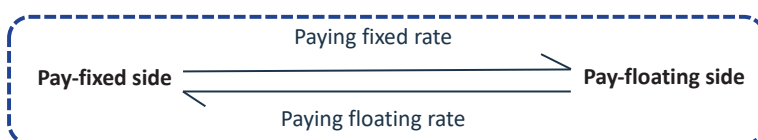
Swap Pricing and Valuation

- **Pricing a plain vanilla swap**
  - Since a floating-rate bond has a value equal to its par value at initiation, what we will do is to find a fixed-rate bond with a value equal to the same par value at initiation.
    - ✓ $1 = C \times d_1 + C \times d_2 + C \times d_3 + \ldots\ldots + C \times d_n + 1 \times d_n$

    $$C = \frac{1 - d_n}{d_1 + d_2 + \cdots + d_n}$$

      - ☐ C is the fixed coupon amount when the par value is equal to 1(also known as **fixed swap rate, or par swap rate**)
      - ☐ d_n is the discount factor, which is the present value of $1 in n periods
      - ☐ C is a periodic rate, and you must annualize it to get the annual swap rate.
- **Valuation formula:**

$$V_{Fixed-rate\,payer} = \sum [C_t(fixed\,swap\,rate) - C_0(fixed\,swap\,rate)] \times d_i$$

# Swap Pricing & Valuation through Replication

- **Equivalence of swaps to bonds**
  - An **interest rate swap** is identical to issuing a fixed-rate bond and using the proceeds to buy a floating-rate bond.
  - A **currency swap** is identical to issuing a fixed- or floating-rate bond in one currency, converting the proceeds to another currency, and using the proceeds to buy a floating- or fixed-rate bond in another currency.
  - An **equity swap** is identical to issuing a fixed- or floating-rate bond and using the proceeds to buy a stock or an index.
- **Equivalence of swaps to forward contracts (FRA)**
  - A forward contract is an agreement to exchange future cash flows once, so a swap can be viewed as a series of forward contracts.

Swap vs. Forward

- **Different FRA fixed rates** usually exist for different times to maturity.
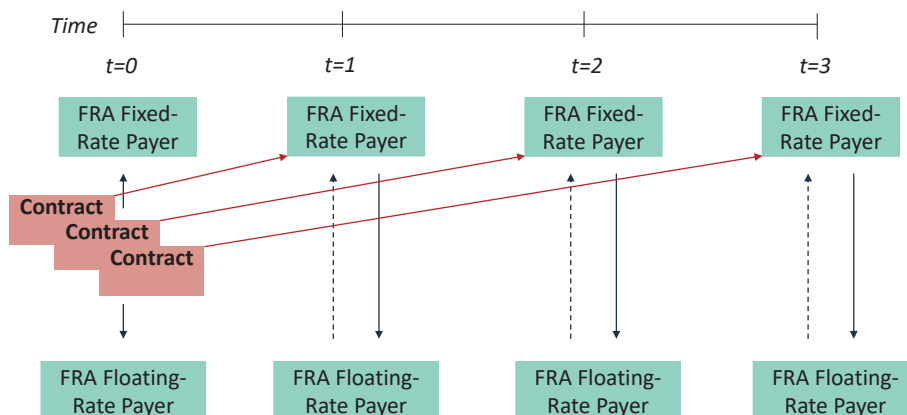
Swap vs. Forward

- In contrast, A standard interest rate swap has a **constant fixed rate** over its life, which includes multiple periods.
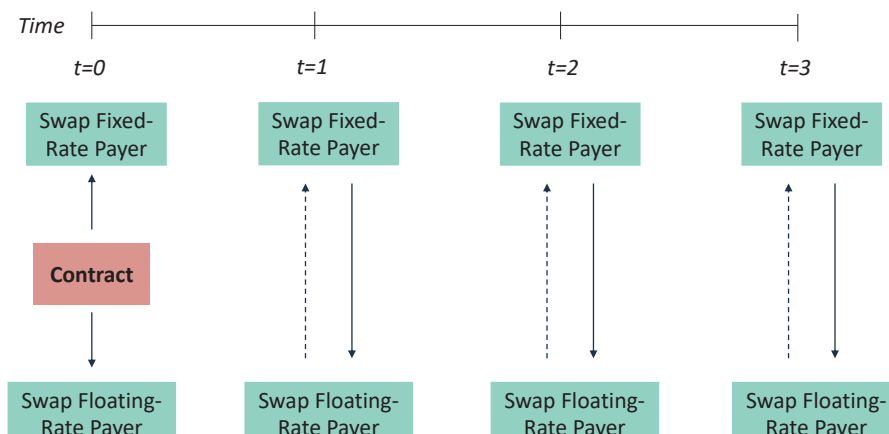
Pricing and Valuation of Options

1. Basic concepts of options
2. Option payoff and profit
3. Option Valuation

Option Contract

Forward contract	Futures contract	Swap contract	Option contract

● **An option contract**
  ○ The owner has **the right**, but not **the obligation** to conduct a transaction.
  ○ *Right and obligations are not equal in option contract, so the long position needs to pay the option premium.*

Payoff (Call)

Forward

Option

+

0

-

S_T

$X=F_0(T)$
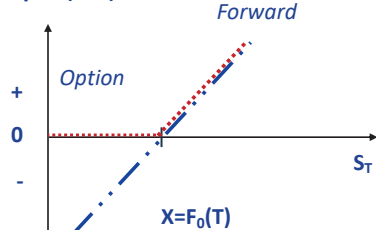
Call option: to buy an asset at a particular price.		
Buyer of a call	Right to buy	
Seller of a call		Obligation to sell
Put option: to sell an asset at a particular price		
Buyer of a put	Right to sell	
Seller of a put		Obligation to buy

Credit derivatives

● **Credit derivatives:** Based on a credit underlying, or the default risk of a single debt issuer or a group of debt issuers in an index.

● **Credit default swaps (CDS):** Allow an investor to manage the risk of loss from issuer default separately from a cash bond.
  ○ CDS credit spreads depend on the probability of default (POD) and the loss given default (LGD).
  ○ Issuer's CDS spread ↓, Protection Seller faces an MTM gain.
  ○ Contingent payment upon credit event= LGD (%) x Notional

Periodic Payments under a Credit Default Swap

Protection Buyer (Short Risk)		Protection Seller (Long Risk)

Fixed CDS Spread

Contingent Payment Upon Credit Event
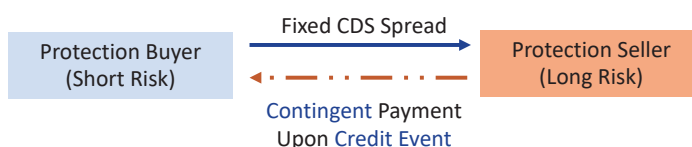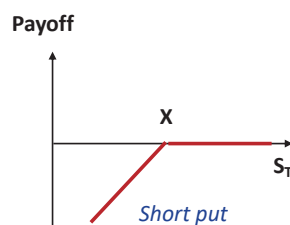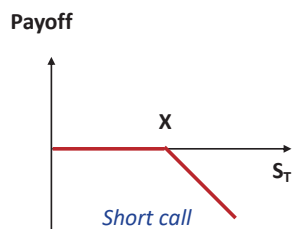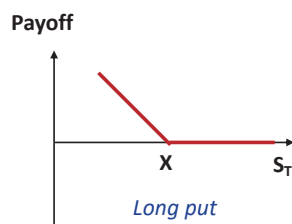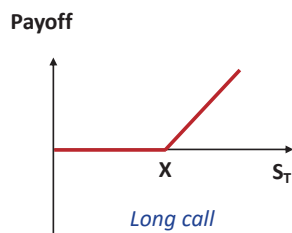
Payoff and Profits

- **Payoff for options (t=T)**
  - Long call: $c_T = Max(0, S_T - X)$
  - Short call: $c_T = -Max(0, S_T - X)$
  - Long put: $p_T = Max(0, X - S_T)$
  - Short put: $p_T = -Max(0, X - S_T)$
- **Profits for options (t=T)**
  - Long call: $c_T = Max(0, S_T - X) - c_0$
  - Short call: $c_T = -Max(0, S_T - X) + c_0$
  - Long put: $p_T = Max(0, X - S_T) - p_0$
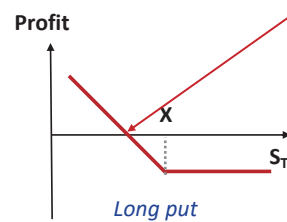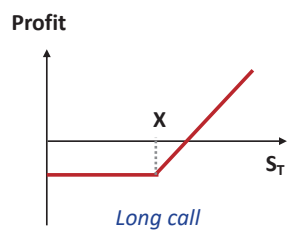 - Short put: $p_T = -Max(0, X - S_T) + p_0$

Option Payoff

- **Payoff**

Long call
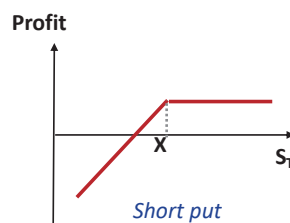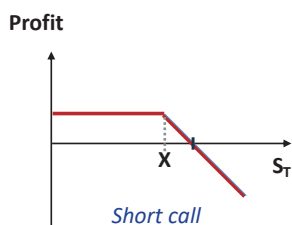
Long put

Short call

Short put

Option Profits

- **Gain/Loss**

Breakeven point:
A cash settlement of zero at maturity in the absence of transaction costs with zero profit.

Long call

Long put

Short call

Short put

Moneyness	➢ 如果能马上行权是否赚钱→定性分析long方是否赚钱		
	Moneyness	**Call option**	**Put Option**
	In-the-money	$S_t > X$	$S_t < X$
	At-the-money	$S_t = X$	$S_t = X$
	Out-of-the-money	$S_t < X$	$S_t > X$
Option value	➢ Option value = **exercise value**(or intrinsic value, current payoff) + time value ➢ **exercise value**(also the lower bound of the option value)		
	● Exercise value of call option: $max[0, S_t - X/(1 + r)^{(T-t)}]$ ● Exercise value of put option: $max[0, X/(1 + r)^{(T-t)} - S_t]$		
	➢ Higher bound of call option(S_t) and put option (X)		
	➢ **time value**		
	● Time value of call option: $c_t - max[0, S_t - X/(1 + r)^{(T-t)}]$ ● Time value of put option: $p_t - max[0, X/(1 + r)^{(T-t)} - S_t]$		

- That is, the current option price is equal to the sum of its exercise value and time value. As Exhibit 2 shows, the time value of an option is always positive but declines to zero at maturity, a process referred to as **time value decay**.

exercise value

time value
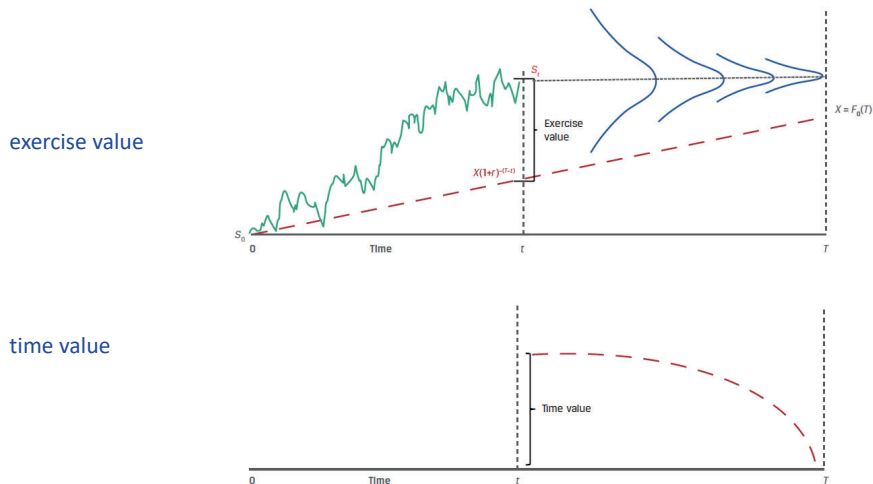
Factors Affect The Value Of An Option

	Sensitivity Factor	Calls	Puts
因素 ★ ★	Underlying price	Positively related	Negatively related
	Volatility	Positively related	Positively related
	Risk-free rate	Positively related	Negatively related
	Time to expiration	Positively related	Positively related*
	Strike price	Negatively related	Positively related
	Payments on the underlying	Negatively related	Positively related
	Carrying cost	Positively related	Negatively related

* There is an exception to the general rule that European put option thetas are negative. The put value may increases as the option approaches maturity if the option is deep in-the-money and close to maturity.

Option Replication Using Put–Call Parity

1. Put-call parity

Put-call Parity

计算 ★	➢ c +X/(1+R_f)^T=S+ p 或 c +K/(1+R_f)^T=S + p
	
	 ➢ **V_portfolio A(fiduciary call)=V_portfolio B(protective put)**
	    ● **Portfolio A**(fiduciary call)=: **Long a European call option and a pure-discount riskless bond**
	    ● **Portfolio B**(protective put): Long a European **put option and a stock**

复制 ★	$-s = -c + p - X/(1 + R_f)^T$ $-c = -p + X/(1 + R_f)^T - S$ $p = c + X/(1 + R_f)^T - S$ $X/(1 + R_f)^T = -c + p - S$
Put-call Forward Parity	➢ $c_0 + \frac{X}{(1+R_f)^T}$ {Fiduciary call} $= p_0 + \frac{FP}{(1+R_f)^T}$ {Synthetic Protective Put}
公司 价值 （应用）	Solvency （Long asset Long a put） ➢ Debtholders receive D and are repaid in full. ➢ Shareholders receive the residual: $E_T = V_T - D$.
	Insolvency （Long bond Short a put） ➢ Debtholders have a priority claim on assets and receive $V_T<D$. ➢ Shareholders receive the residual, $E_T=0$.

Valuing a Derivative

Using a One-Period Binomial Model

1. Option pricing: one-period binomial model

Option Pricing-binomial Model

公式	➤ Value of an option: $c_0 = [\pi_u C_1^u + \pi_d C_1^d] \times \dfrac{1}{(1 + R_f)^T}$ ➤ Risk-neutral probability (π) of an up move is: $\pi_u = \dfrac{1 + R_f - R_d}{R_u - R_d}$
对冲组合	➤ Hedge ratio (h^*: *shares per option*) Call option: $h^* = \dfrac{c_1^u - c_1^d}{S_1^u - S_1^d}$ put option: $h^* = \dfrac{p_1^u - p_1^d}{S_1^u - S_1^d}$ ➤ A risk-free portfolio: • Call option: $V_0 = h^* S_0 + c_0 = PV(h^* S_1^u - c_1^u) = PV(h^* S_1^d - c_1^d)$ • Put option: $V_0 = h^* S_0 + p_0 = PV(h^* S_1^u + p_1^u) = PV(h^* S_1^d + p_1^d)$

问题反馈

- 如果您认为金程**课程讲义/题库/视频**或其他资料中**存在错误，欢迎您告诉我们，**所有提交的内容我们会在最快时间内核查并给与答复**。**
- **如何告诉我们？**
  - 将您发现的问题通过电子邮件告知我们，具体的内容包含：
    - ✓ 您的姓名或网校账号
    - ✓ 所在班级
    - ✓ 问题所在科目(若未知科目，请提供章节、知识点和页码）
    - ✓ 您对问题的详细描述和您的见解
  - 请发送电子邮件至: academic.support@gfedu.net
- **非常感谢您对金程教育的支持，您的每一次反馈都是我们成长的动力。**

求知若饥，谦卑若愚

Stay hungry, Stay foolish