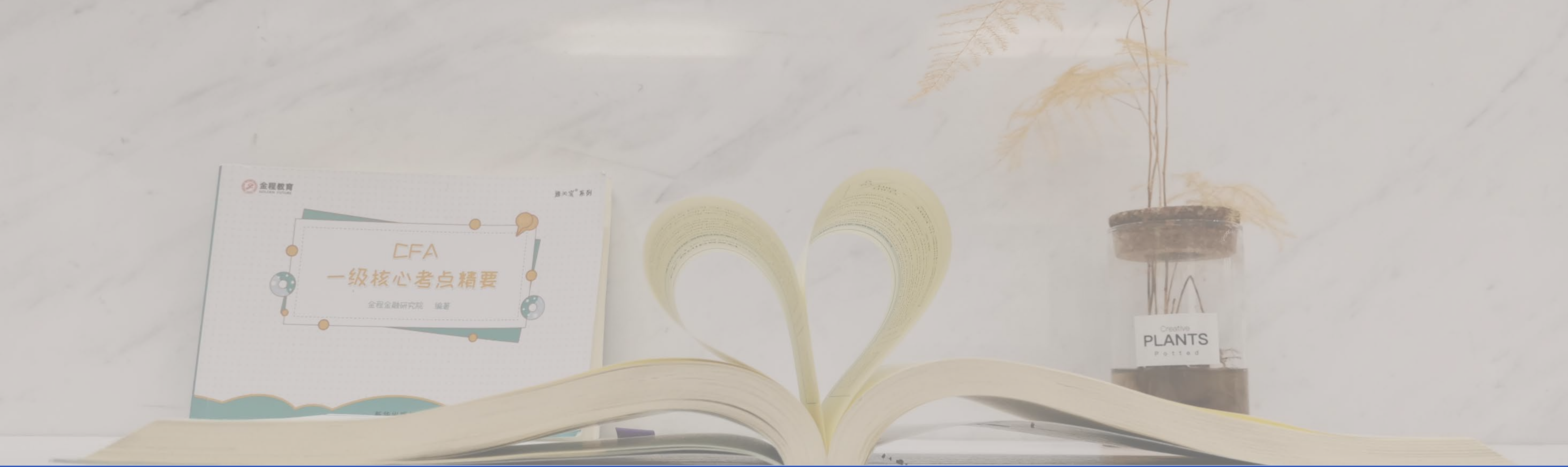


Derivatives

CFA一级培训项目

讲师：Jcy Ji

Jcy Ji

10年授课，5000+授课课时

学位证书

- 金程教育资深培训师，CFA持证人，FRM持证人，香港大学MBA。

工作背景

- 曾就职于国内龙头券商，十五年资本市场投资经验，资深个人投资者。市场少有的同时具备“深厚理论基础+丰富实战经验”的投资人。其系统的理论、落地的实战、前沿的案例、广受众多学员和粉丝的好评与喜爱。

服务客户

- 中国银行、建设银行、工商银行、中国进出口银行、杭州联合银行、国泰君安证券、太平洋保险、苏州元禾、平安集团等。

主编出版

- 参与金程CFA项目各类参考书目的编写工作，包括翻译CFA协会官方参考书《企业理财》，《国际财务报告分析》，金程CFA中文Notes等。

Topic Weightings in CFA Level I

Topics	Weights (%)
Quantitative Methods	8-12
Economics	8-12
Financial Statement Analysis	13-17
Corporate Issuers	8-12
Equity	10-12
Fixed Income	10-12
Derivatives	5-8
Alternative Investments	5-8
Portfolio Management	5-8
Ethical and Professional Standards	15-20

Derivatives

1. Derivative Instrument and Derivative Market Features
2. Forward Commitment and Contingent Claim Features and Instruments
3. Derivative Benefits, Risks, and Issuer and Investor Uses
4. Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives
5. Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities
6. Pricing and Valuation of Futures Contracts
7. Pricing and Valuation of Interest Rates and Other Swaps
8. Pricing and Valuation of Options
9. Option Replication Using Put–Call Parity
10. Valuing a Derivative Using a One-Period Binomial Model

中文精读

1. 衍生工具和衍生市场特征
2. 远期承诺和或有索赔权
3. 衍生产品收益、风险、发行人和投资者
4. 套利、复制和套利定价衍生品的成本
5. 远期合约的定价与估值
6. 期货合约的定价与估价
7. 利率和其他互换的定价与估值
8. 期权的定价与估价
9. 利用买卖权平价公式复制期权
10. 使用单期二叉树对期权估值

Framework

Module

Derivative Instrument and Derivative Market Features

1. Derivative Instruments
2. Derivative Markets

Derivative Instruments

- Define a derivative and describe basic features of a derivative instrument

Definition of Derivative Contract

- **A derivative contract is:**
  - a financial instrument (contract) that derives its performance from the performance of an underlying asset.
  - **Buy or Sell Something**
    - ✓ Buy or Sell **now**.
    - ✓ Buy or Sell sometime **in the future**.
  - Example
    - ✓ 3 month later → $3/bottle → purchase water.
    - ✓ 3 month later → $15/share → purchase stock.
    - ✓ 3 month later → 4% interest rate → borrow $1million.
    - ✓ 3 month later → 6.5CNY/USD → exchange CNY.
- **Tips**
  - Contracts.
  - Hedge risk vs. Speculate.
  - Derives its performance from the performance of an underlying asset.

Derivative Underlyings

Asset Class	Examples Sample	Uses
Equities	• Individual stocks • Equity indexes • **Equity price volatility**	• Change exposure profile (Investors) • Employee compensation (Issuers)
Interest rates	• Sovereign bonds (domestic) • Market reference rates (MRR)	• Change duration exposure (Investors) • Alter debt exposure profile (Issuers)
Foreign exchange	• Sovereign bonds (foreign) • Market exchange rates	• Manage global portfolio risks (Investors) • Manage global trade risks (Issuers)
Commodities	• **Soft and hard commodities** • Commodity indexes	• Manage operating risks (Consumers/Producers) • Portfolio diversification (Investors)
Credit	• Individual refence entities • Credit indexes	• Portfolio diversification (Investors) • Manage credit risk (Financial Intermediaries)
Other	• Weather • Cryptocurrencies • Longevity	• Manage operating risks (Issuers) • Manage portfolio risks (Investors)

Example

Derivative Underlyings

- Identify which example corresponds to each derivative underlying type.

A. Soft commodities	1. Aluminum futures
B. Hard commodities	2. SOFR futures
C. Neither soft nor hard commodities	3. Soybean options

- **Solution**

1→B. Aluminum futures are an example of a metals contract, which is a derivative with a hard commodity underlying.

2→C. SOFR futures are an example of an interest rate contract, not a commodity-based derivative contract.

3→A. Soybean options are an example of a derivative contract with an agricultural, or soft, commodity underlying.

Summary

Derivative Instrument and Derivative Market Features

Derivative Instruments

Definition of Derivative Contract

Derivative Underlyings

Module

Forward Commitment and Contingent Claim Features and Instruments

1. Forward Commitment and Contingent Claim

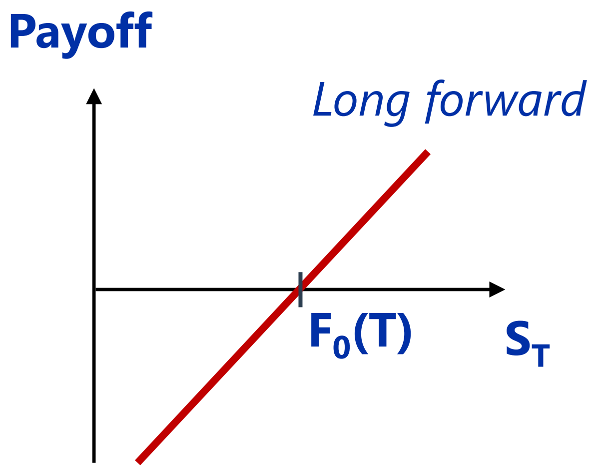
Forward Commitment and Contingent Claim

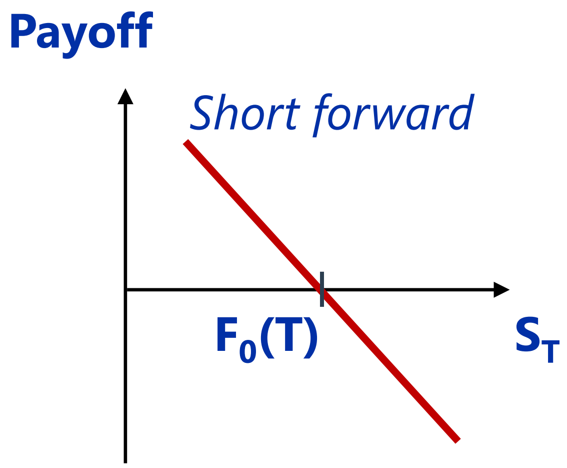
- Define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics
- Determine the value at expiration and profit from a long or a short position in a call or put option
- Contrast forward commitments with contingent claims

Four Main Derivatives

Forward contract	Futures contract	Swap contract	Option contract

- **Forward contract**
  - A **forward contract** is a **private agreement** that obligates one party to buy and the other party to sell a specific quantity/contract size of a specific underlying asset, at a **set price**, at a **future date**.
  - If the future price of the underlying assets increase, the buyer (long position) has a gain, and the seller (short position) has a loss.





Four Main Derivatives

Forward contract	**Futures contract**	**Swap contract**	**Option contract**

- **A Futures contract** is a specialized version of a forward contract that has been standardized and that trades on a futures exchange.
  - A forward contract.
  - Are standardized.
  - Exchanged-traded.
  - Are regulated.
  - Guaranteed by the exchange through the clearinghouse.
  - Daily settlement for gains and losses.

Four Main Derivatives

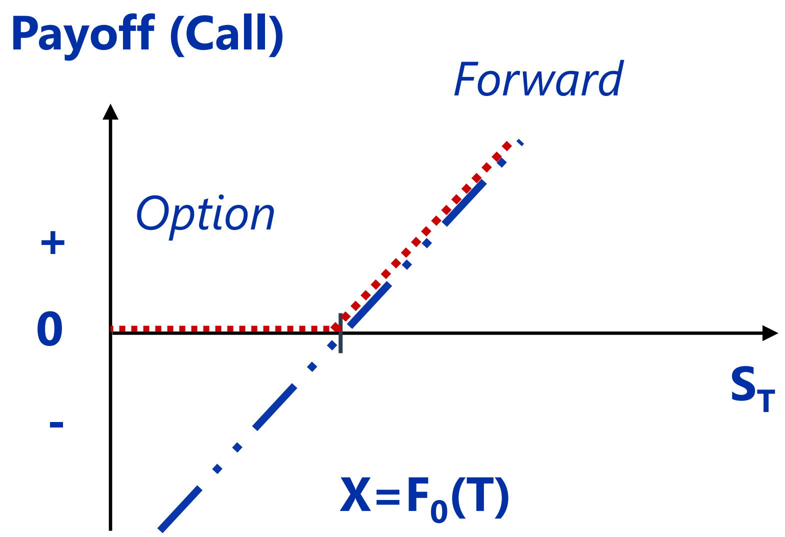
Forward contract	**Futures contract**	**Swap contract**	**Option contract**

- **A swap contract** is a series of forward contracts**.**
  - Exchange a series of cash flows.
  - Are customized.
  - Traded in OTC market.
  - Are not regulated.
  - Default risk.

Four Main Derivatives

Forward contract | **Futures contract** | **Swap contract** | **Option contract**

- **An option contract**
  - The owner has **the right**, but not **the obligation** to conduct a transaction.
  - *Right and obligations are not equal in option contract, so the long position needs to pay the option premium.*



Four Main Derivatives

Forward contract	Futures contract	Swap contract	Option contract

- **Basic characteristics of options**
  - An option to buy an asset at a particular price is termed a call option.

Buyer of a call	Right to buy	
Seller of a call		Obligation to sell

  - An option to sell an asset at a particular price is termed a put option.

Buyer of a put	Right to sell	
Seller of a put		Obligation to buy

Classification

- **According to contract features: Firm commitment & Contingent claim**
  - **Firm commitment** (linear derivatives, symmetric payoff profile): a pre-determined amount is agreed to be exchanged at settlement.
    - ✓ forward contracts;
    - ✓ futures contracts;
    - ✓ swaps involving a periodic exchange of cash flows.
  - **Contingent claim** (non-linear derivatives, asymmetric payoff profile): one of the counterparties determines whether and when the trade will settle.
    - ✓ An **option** is the primary contingent claim.
    - ✓ An **embedded derivative** is a derivative within an underlying, such as a callable, puttable, or convertible bond.

Summary

Forward Commitment and Contingent Claim Features and Instruments

Forward Commitment and Contingent Claim

Classification

Four Main Derivatives

Summary

Module: Forward Commitment and Contingent Claim Features and Instruments

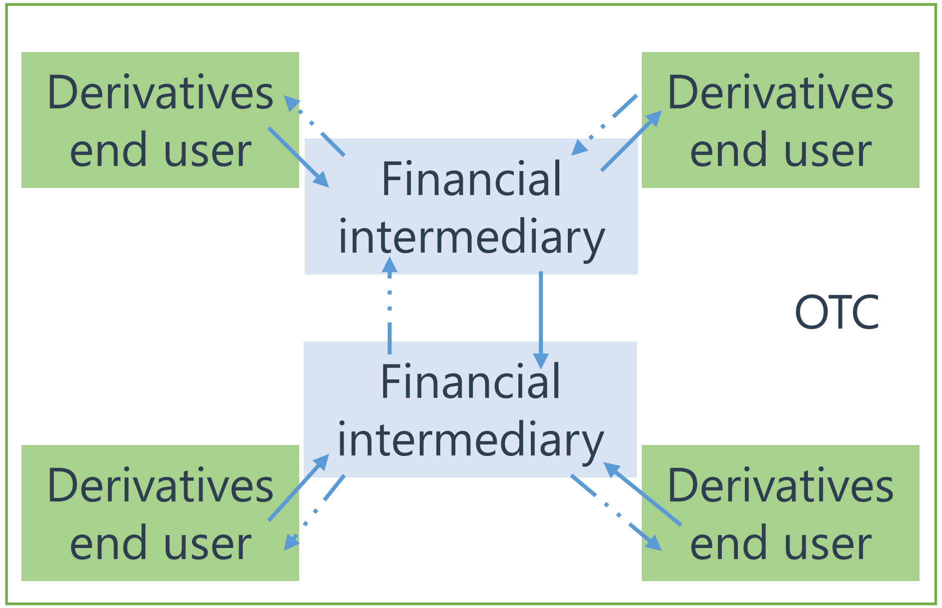
Forward Commitment and Contingent Claim

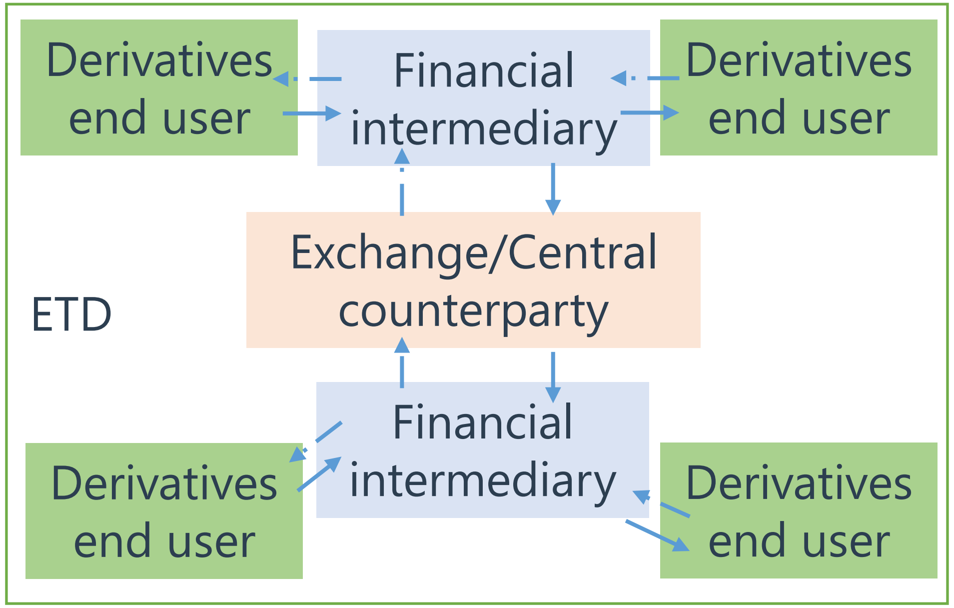
Derivative Markets

- Describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets

Classification

- According to trading place: ETD & OTC
  - **Exchange-traded derivative** (ETD) market: a place where traders can meet to arrange their traded.
  - **Over-the-counter** (OTC) traded derivative market: OTC markets can be formal organizations, such as NASDAQ, or informal networks of parties (dealer/market makers) that buy from and sell to one another, as in the US fixed-income market.





Differences between ETD and OTC

- **Differences**

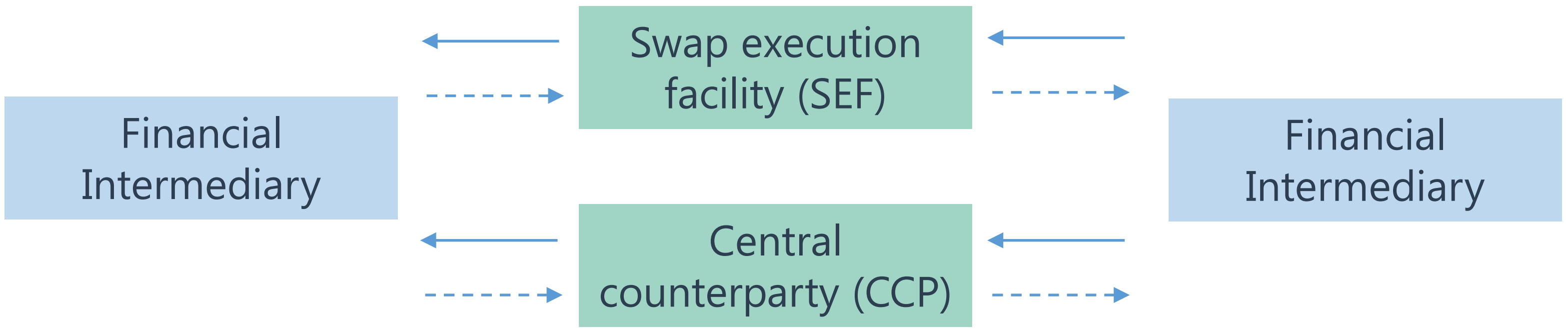
ETD	OTC
Standardized→ Liquid	Customized/Specific needs
Backed by a clearinghouse	Trade with counterparty (default risk)
Trade in a physical exchange	Not traded in organized markets
Regulated	Unregulated

- **Standardization leads to an efficient clearing and settlement process.**
  - Clearing is the exchange's process of verifying the execution of a transaction and recording the participants.
  - Settlement involves the payment of final amounts and/or delivery of securities or physical commodities between the counterparties based upon exchange rules.
- After 2008, for most **OTC derivatives**, a central clearing mandate requires that a **central counterparty (CCP)** assume the credit risk between derivative counterparties, one of which is typically a financial intermediary.

Differences between ETD and OTC

- After 2008, for most **OTC derivatives**, a central clearing mandate requires that a **central counterparty (CCP)** assume the credit risk between derivative counterparties, one of which is typically a financial intermediary.
- CCPs provide clearing and settlement for most derivative contracts.
- The systemic credit risk transfer from financial intermediaries to CCPs also leads to **centralization and concentration of risks**.

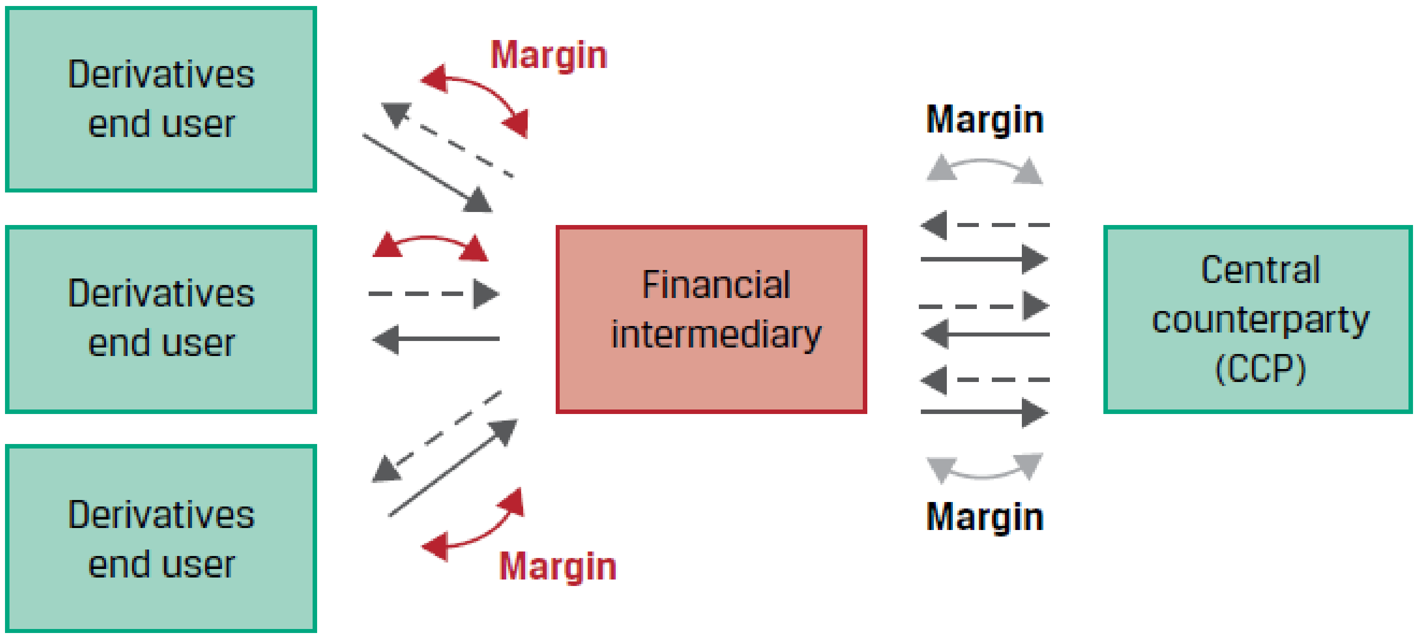
CCP replaces (novates) existing trade, acting as new counterparty to both financial intermediaries



Effect of Central Clearing of OTC Derivatives

- The advent of **derivatives central clearing, has created futures-like margining requirements for OTC derivative dealers** who buy and sell forwards to derivatives end users. Dealers who are required to post cash or highly liquid securities to a central counterparty often impose similar requirements on derivatives end users.

Margin requirements for centrally cleared OTC Derivatives



Classification

- Derivatives
  - Contract features
    - Firm commitment (forward commitment)
      - forward
      - futures
      - swap
    - Contingent claim
      - option
      - CDS
      - embedded derivatives
  - Trading places
    - Exchange-traded
      - futures
      - option
    - Over-the-counter traded
      - forward
      - swap
      - option

Derivative Markets

Baywhite Financial LLC 80% Principal Protected Structured Note	
Description:	The Baywhite Financial LLC 80% Principal Protected Structured Note (“the Note”) is linked to the performance of the S&P 500 Health Care Select Sector Index (SIXV).
Issuer:	Baywhite Financial LLC
Start Date:	[Today]
Maturity Date:	[Six months from Start Date]
Issuance Price:	102% of Face Value
Face Value:	Sold in a minimum denomination of USD1,000 and multiple units thereof
Payment at Maturity:	At maturity, you will receive a cash payment, for each USD1,000 principal amount note, of USD800 plus the Additional Amount, which may be zero.
Partial Principal Protection Percentage:	80% Principal Protection (20% Principal at Risk)
Additional Amount:	At maturity, you will receive the greater of 100% of the returns on the S&P 500 Health Care Select Sector Index (SIXV) in excess of 5% above the current spot price of the SIXV or zero.

Derivative Markets

- Which of the following statements best contrasts the credit risk of the Baywhite Financial LLC Structured Note with the counterparty credit risk of an investor entering into the embedded exchange-traded derivative on a stand-alone basis?

A. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 20% of the note’s face value, as the remaining 80% is principal protected. An investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of a financial intermediary.

B. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 80% of the note’s face value, as the remaining 20% is associated with the embedded derivative. An investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of a financial intermediary.

C. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 100% of the note’s face value, while an investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of an exchange and its clearinghouse.

Derivative Markets(cont.)

- **Solution: C.**
  - The investor assumes the credit risk of Baywhite Financial LLC for the full value of the structured note as the structured note issuer. Under the purchased exchange-traded SIXV call option, the investor faces the risk of the exchange and its clearinghouse, which provides a guarantee of contract settlement backed by the exchange insurance fund.

Derivative Markets

- Montau AG is a German capital goods producer that manufactures its products domestically and delivers its products to clients globally. Montau's global sales manager shares the following draft commercial contract with his Treasury team:

Montau AG Commercial Export Contract

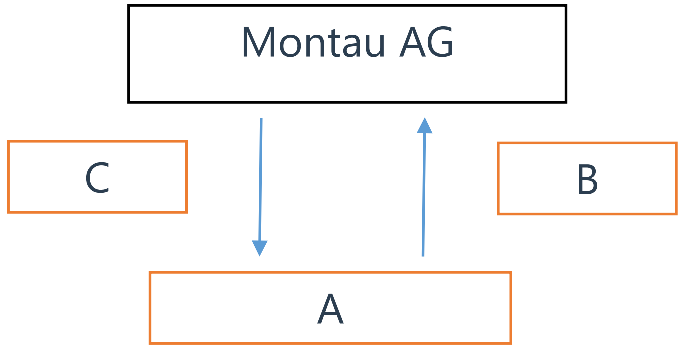
Contract Date:	[Today]
Goods Seller:	Montau AG, Frankfurt, Germany
Goods Buyer:	Jeon Inc. , Seoul, Korea
Description of Goods:	A-Series Laser Cutting Machine
Quantity:	One
Delivery Terms:	Freight on Board (FOB) , Busan Korea with all shipping, tax and delivery costs payable by Goods Buyer
Delivery Date:	(75 Days from Contract Date)
Payment Terms:	100% of Contract Price payable by Goods Buyer to Good Seller on Delivery Date
Contract Price:	KRW650,000,000

Example

Derivative Markets(cont.)

- Identify A, B, and C in the correct order in the following diagram, as in Exhibit 1, for the derivative to hedge Montau's financial risk under the commercial transaction.

Exhibit 1



A. A: Financial intermediary, B: KRW650,000,000, C: Fixed EUR amount

B. A: Jeon Inc., B: KRW650,000,000, C: Fixed EUR amount

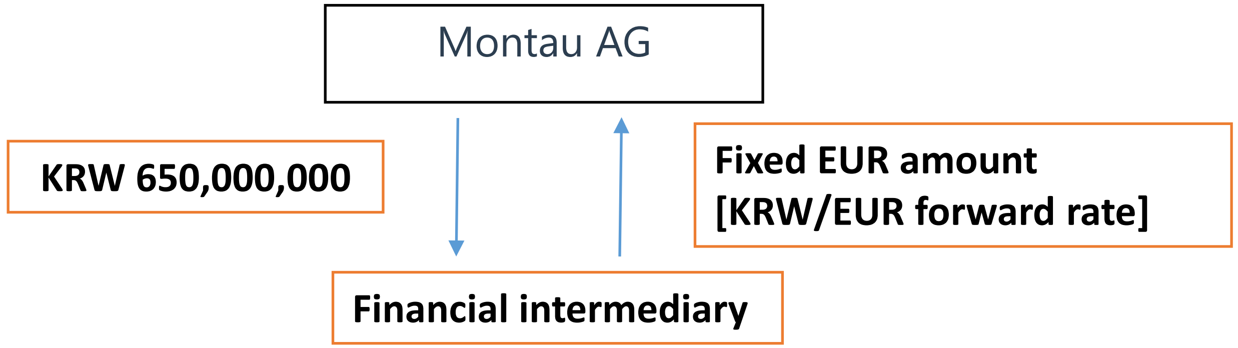
C. A: Financial intermediary, B: Fixed EUR amount, C: KRW650,000,000.

Example

Derivative Markets

- **Solution: C.**

Exhibit 1



Derivative Markets

- Which of the following statements about the most appropriate derivative market to hedge Montau AG's financial risk under the commercial contract is most accurate?
  A. The OTC market is most appropriate for Montau, as it is able to customize the contract to match its desired risk exposure profile.
  B. The ETD market is most appropriate for Montau, as it offers a standardized and transparent contract to match its desired risk exposure profile.
  C. Both the ETD and OTC markets are appropriate for Montau AG to hedge its financial risk under the transaction, so it should choose the market with the best price.

- **Solution: A.**
  The OTC market is most appropriate for Montau, as OTC contracts may be customized to match Montau's desired risk exposure profile. This is important to end users seeking to hedge a specific underlying exposure based upon non-standard terms. Montau would be unlikely to find an ETD contract under B that matches the exact size and maturity date of its desired hedge, which also makes C incorrect.

Summary

Derivative Instrument and Derivative Market Features

Derivative Markets

Classification

Differences between ETD and OTC

Effect of Central Clearing of OTC Derivatives

Summary

Module: Derivative Instrument and Derivative Market Features

Derivative Instruments

Derivative Markets

Module

Derivative Benefits, Risks, and Issuer and Investor Uses

1. Benefits and Risks of Derivative Instruments
2. Use of derivatives Issuers & Investors

Benefits and Risks of Derivative Instruments

- Describe benefits and risks of derivative instruments

Benefits of Derivative Instruments

Purpose	Description
Risk Allocation, Transfer, and Management	• Allocate, trade, and/or manage underlying exposure without trading the underlying. • Create exposures unavailable in cash markets.
Information Discovery	• Deliver expected price in the future as well as expected risk of underlying
Operational Advantages	• Reduced cash outlay, lower transaction costs versus the underlying, increased liquidity and ability to “short”
Market Efficiency	• Less costly to exploit arbitrage opportunities or mispricing

Risks of Derivative Instruments

Risk	Description
Greater Potential for Speculative Use	• High degree of **implicit leverage** may increase the likelihood of financial distress.
Lack of Transparency	• Derivatives add portfolio complexity and may create an exposure profile that is not well understood.
Basis Risk	• Potential divergence between the expected value of a derivative instrument versus an underlying or hedged transaction, e.g., Issuer CDS spread v.s. actual bond
Liquidity Risk	• Potential divergence between the cash flow timing of a derivative instrument versus an underlying or hedged transaction.
Counterparty Credit Risk	• Derivative instruments often give rise to counterparty credit exposure, resulting from differences in the current price versus the expected future settlement price.
Destabilization and Systemic Risk	• Excessive risk taking and use of leverage may contribute to market stress, e.g., 2008 financial crisis.

Example

- Which of the following statements *most accurately* describes a derivative security? A derivative:
  A. always increases risk.
  B. has no expiration date.
  C. has a payoff based on an asset value or interest rate.

- **Solution: C.**

- Which of the following statements about exchange-traded derivatives is *least accurate*? Exchange-traded derivatives:
  A. are liquid.
  B. are standardized contracts.
  C. carry significant default risk.

- **Solution: C.**

Example

Purchase of Spot Gold versus a Gold Futures Contract

- Procam Investments enters a futures contract to buy 100 ounces of gold at a futures price [f_0(T)] of USD1,792.13 per ounce that expires in three months and must post USD4,950 in initial margin as required by the exchange. The spot gold price (S_0) at the time Procam enters the futures contract is USD1,770 per ounce. Assume that Procam is able to borrow funds from a financial intermediary at a rate of 5% per year. Contrast the expected opportunity cost of the initial margin for the three-month futures contract with a cash purchase of 100 ounces of gold for the same three-month period.

Example(cont.)

- **Solution:**

1. Procam borrows the USD4,950 initial margin from a financial intermediary at 5% for three months.
   - ✓ = (USD4,950 × 0.05)/4 = USD61.88
2. Procam borrows USD177,000 (USD1,770 × 100 ounces) for the spot gold purchase from a financial intermediary at 5% for three months.
   - ✓ = (USD177,000 × 0.05)/4 = USD2,212.50
3. Procam gains exposure to USD177,000 in underlying gold price risk with just USD61.88, implying a leverage ratio of 2,860.

Summary

Derivative Benefits, Risks, and Issuer and Investor Uses

Benefits and Risks of Derivative Instruments

Benefits of Derivative Instruments

Risks of Derivative Instruments

Use of derivatives Issuers & Investors

- Compare the use of derivatives among issuers and investors

Issuer use of derivatives

- **Issuers** predominantly use derivatives to offset or **hedge** market-based underlying exposures incidental to their commercial operations and financing activities.
- **Hedge accounting** allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility.

Hedge accounting Types		**Description Examples**
Cash Flow	Absorbs variable cash flow of floating-rate asset or liability (forecasted transaction)	✓ Interest rate swap to a fixed rate for floating-rate debt ✓ FX forward to hedge forecasted sales
Fair Value	Offsets fluctuation in fair value of an asset or liability	✓ Interest rate swap to a floating rate for fixed-rate debt ✓ Commodity future to hedge inventory
Net Investment	Designated as offsetting the FX risk of the equity of a foreign operation	✓ Currency swap ✓ Currency forward

Example

- Describe hedge accounting treatment.

- **Solution:**
  - Hedge accounting allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility.

Example

- Montau AG will deliver a laser cutting machine for KRW650,000,000 in 75 days and has hedged its FX exposure by agreeing to sell the KRW it will receive and purchase EUR upon delivery in an over-the-counter FX forward with a financial intermediary.
- Montau agrees to a KRW/EUR forward exchange rate [$F_0(T)$] of 1,350 (i.e.,1,350 KRW = 1 EUR), at which it will sell 650,000,000 KRW and receive 481,481 EUR (650,000,000/1,350) in 75 days. The production manager estimates the machine's cost to be €430,000. Montau's Treasury manager compiles the following profit margin scenarios at different KRW/EUR spot rates (ST):

Spot KRW/EUR (s_T)	Unhedged EUR Proceeds	Unhedged Profit Margin	Hedged EUR Proceeds	Hedged Profit Margin
1,525	€426,230	−1%	€481,481	11%
1,400	€464,286	7%	€481,481	11%
1,280	€507,813	15%	€481,481	11%
1,225	€530,612	19%	€481,481	11%

Investor use of derivatives

- **Investors use derivatives to**
  - **replicate a cash market strategy;**
    - ✓ greater liquidity and reduced capital required to trade derivatives.
  - **hedge** a fund's value against adverse movements in underlying;
    - ✓ derivative hedges enable investors to isolate certain underlying exposures in the investment process while retaining a position in others.
  - or **modify or add exposures using derivatives**, which in some cases are unavailable in cash markets.
    - ✓ Finally, the flexibility to take short positions or to increase or otherwise modify exposure using derivatives beyond cash alternatives is an attractive feature for portfolio managers targeting excess returns by using a variety of strategies.
- An investment fund's prospectus typically specifies which derivative instruments may be used within a fund and for which purpose.

Use of derivatives among issuers & investors

- **Investor vs. Issuers**
  - Investors are less focused than issuers on hedge accounting treatment,
    - ✓ as an investment fund's derivative position is typically marked to market each day and included in the daily net asset value (NAV) of the portfolio or fund.
  - Investors transact more frequently in standardized and highly liquid exchange-traded derivative markets than do issuers.

Example

- Match these derivative market participants to the following statements:

A. Investors	1. They use derivatives to offset or hedge market-based underlying exposures incidental to their commercial operations and financing activities.
B. Both issuers and investors	2. They tend to transact more frequently in exchange-traded derivative markets.
C. Issuers	3. They use derivatives to change their exposure to an underlying asset price without transacting in the cash market.

- **Solution**
  - **1.** The correct answer is C. Issuers use derivatives to offset or hedge market-based underlying exposures incidental to their commercial operations and financing activities.
  - **2.** The correct answer is A. Investors tend to transact more frequently in exchange-traded derivative markets.
  - **3.** The correct answer is B. Both issuers and investors use derivatives to change their exposure to an underlying asset price without transacting n the cash market.

Summary

Derivative Benefits, Risks, and Issuer and Investor Uses

Use of derivatives Issuers & Investors

Issuer use of derivatives

Investor use of derivatives

Use of derivatives among issuers & investors

Summary

Module: Derivative Benefits, Risks, and Issuer and Investor Uses

Benefits and Risks of Derivative Instruments

Use of derivatives Issuers & Investors

Module

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

1. Arbitrage, Replication, and Risk Neutrality
2. Cost of Carry in Pricing Derivatives

Arbitrage, Replication, and Risk Neutrality

- Explain how the concepts of arbitrage and replication are used in pricing derivatives

Arbitrage and No-Arbitrage Rule

- **Arbitrage involves** earning over the risk-free rate with no risk or earning an immediate gain with no future liabilities.
- **Arbitrage opportunities arise when:**
  - Two assets with identical future cash flows trade at different prices
  - An asset with a known future price does not trade at the present value of its future price determined using an appropriate discount rate.
- **The way of arbitrage: sell high, buy low.**
- **Law of one price (no arbitrage rule):** the condition in a financial market in which two equivalent financial instruments or combinations of financial instruments can sell for only one price.

Replication and Risk Neutrality

- **Replication** is a strategy in which a derivative's cash flow stream may be recreated using a combination of long or short positions in an underlying asset and borrowing or lending cash.
  - E.g., In order to long a call option, we can borrow at the risk-free rate and long the underlying; in order to long a forward, we can long (for a call) and short (for a put).
- **Risk neutrality**
  - Risk-neutral investors are willing to buy risky investments for which they expect to earn only the risk-free rate. They do not expect to earn a premium for bearing risk.
  - The expected payoff of the derivative can be discounted at the risk-free rate. And should yield the risk-free rate of return, if it generates certain payoffs.

Summary

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Arbitrage, Replication, and Risk Neutrality

Arbitrage and No-Arbitrage Rule

Replication and Risk Neutrality

Module

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

1. Forward
2. Basics of Forward Pricing
3. Pricing & Valuation of Different Forward Contracts

Forward

- Define forward contracts and its basic characteristics
- Explain how forward rates are determined for an underlying with a term structure and describe their uses

Forward Contract

- **Definition:** A **forward contract** is a bilateral contract that obligates one party to buy and the other party to sell a specific quantity of an underlying asset, at a set price, on a specific date in the future.
- **Long and short forward position**
  - **Long:** buy underlying.
  - **Short:** sell underlying.
  - No payments will be made at the inception of a forward contract. So, both parties of a forward contract is exposed to potential default risk.

Forward Contract

- **Forward contracts classification**
  - Commodity forward contract;
  - Financial forward contract.
- **Purposes of trading forward contracts**
  - Hedge risk: Lock the cost in the future, but not sure to make money; Have default risk.
  - Speculation: gambling the price movement.
- **Characteristics of Forward contracts**
  - Each party are exposed to **default risk** (or **counterparty risk**);
  - Zero-sum game.

Forward Contract Settlement

- **Settling** the outstanding contracts (**open interest**) at expiration.
  - **Physical settlement:** deliver an actual asset, has storage cost, mostly used in commodity forward.
  - **Cash settlement:** the party that has a position with negative value is obligated to pay that amount to the other party, mostly used in financial forward. Having same economic effect, these forward contracts is called non-deliverable forwards (NDFs), cash-settled forwards, or contracts for differences.
- **Settling a forward contract prior to expiration.**
  - Entering into an opposite forward contract: with an expiration date equal to the time remaining on the original contract.

Example

Forward

- Determine the correct answers to fill in the blanks: An oil producer enters a derivative contract with an investor to sell 1,000 barrels of oil in two months at a forward price of $64 per barrel. If the spot oil price at maturity is $58.50 per barrel, the investor realizes a _________ at maturity equal to_________.

- **Solution:**
  - The oil forward price, $F_0(T)$, under the contract equals $64 per barrel. At contract maturity, the spot oil price (S_T) is $58.50 per barrel.
  - Investor payoff per barrel:
  - $[S_T - F_0(T)]$ = $58.50 – $64.00 = –$5.50 per barrel.
  - Total amount the investor pays the oil producer to settle the forward contract for 1,000 barrels at maturity: 1,000 × $5.50 = $5,500.
  - An oil producer enters a derivative contract with an investor to sell 1,000 barrels of oil in two months at a forward price of $64 per barrel. If the spot oil price at maturity is $58.50 per barrel, the investor realizes a *loss* at maturity equal to *$5,500*.

Forward Rate Agreements

- The underlying is a hypothetical deposit of a notional amount in the future at a **market reference rate (MRR)** that is fixed at contract inception (t = 0).
  - The Secured Overnight Financing Rate (SOFR) is an overnight cash borrowing rate collateralized by US Treasuries.
  - Other MRRs include the euro short-term rate (€STR) and the Sterling Overnight Index Average (SONIA).
  - Survey-based Libor rates used as reference rates in the past have been replaced by rates based on a daily average of observed market transaction rates.
- A forward rate agreement (FRA) is a forward contract in which counterparties agree to apply a specific interest rate to a future period.
  - **The long position**: is the party that would **borrow** the money.
  - **The short position**: is the party that would **lend** the money.

Forward Pricing and Valuation – FRA

- **LIBOR, Euribor, and FRAs（cont.）**
  - **Settlement process**: settle in cash, but no actual loan is made at the settlement date.
  - **Payoff (**qualitative analysis**)**
    - ✓ If the reference rate at the expiration date is above the specified contract rate, the long will receive cash payment from the short;
    - ✓ If the reference rate at the expiration date is below the contract rate, the short will receive cash payment from the long.
  - **Payoff (**quantitative analysis**)**

$$\boxed{(\text{Notional principal})\left[\frac{(\text{Floating rate at settlement-forward rate})\left[\dfrac{\text{days}}{360}\right]}{1+\text{Floating rate at settlement}\left[\dfrac{\text{days}}{360}\right]}\right]}$$

Example

Forward Rate Agreements

- A trader observes one-year and two-year zero-coupon bonds that yield 4% and 5%, respectively, and would like to protect herself against a rise in one-year rates a year from now. Explain the position she should take in the FRA contract to achieve this objective and the forward interest rate she should expect on the contract.

- **Solution:**
  - A forward rate agreement involves an underlying hypothetical future deposit at a market reference rate fixed at contract inception (t = 0). To protect against higher rates, the trader should enter into a fixed-rate payer FRA in order to realize a gain if one-year spot rates one year from now exceed the current forward rate. A long FRA position, or fixed-rate payer, agrees to pay interest based on the fixed rate and receives interest based on a variable market reference rate determined at settlement.
    - ✓ $(1 + z_1) \times (1 + IFR_{1,1}) = (1 + z_2)^2$
    - ✓ $(1.04) \times (1 + IFR_{1,1}) = (1.05)^2$
    - ✓ $IFR_{1,1} = 6.0096\%$

Example

Forward Rate Agreements

- If $MRR_{B\text{-}A}$ in three months' time sets at 2.15% and we assume a 90-dayinterest period, calculate the net payment of FRA.

Yangzi Bank CNY Forward Rate Agreement Term Sheet	
Start Date:	[Today]
Maturity Date:	[Three months from Start Date]
Notional Principal:	CNY 100,000,000
Fixed-Rate Payer:	Yangzi Bank
Fixed Rate:	2.24299% on a quarterly actual/360 basis
Floating-Rate Payer:	[Financial Intermediary]
Floating Rate:	Three-month CNY MRR on a quarterly actual/360 basis
Payment Date:	Maturity Date
Business Days:	Shanghai
Documentation:	ISDA Agreement and credit terms acceptable to both parties

Forward Rate Agreements(cont.)

- **Solution:**
  - Net Payment = ($MRR_{B\text{-}A}$ − $IFR_{A,B\text{-}A}$) × Notional Principal × Period
    - ✓ (2.15% − 2.24299%) × CNY100,000,000 × 90/360 = −CNY23,247.50
  - This is the net payment amount at the end of six months. However, the FRA settles at the *beginning* rather than the *end* of the interest period, which is three months in our example. We must therefore calculate the *present value* of the settlement amount to the beginning of the period during which the reference rate applies, using *MRRB-A* as the discount rate:
  - Cash Settlement (PV): −CNY23,247.50/(1 + 0.0215/4) = −CNY23,123.21

Summary

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Forward

Forward Contract

Forward Contract Settlement

Forward Rate Agreements

Forward Pricing and Valuation – FRA

Basics of Forward Pricing

- Explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration

Basics of Forward Pricing

- The **price** is the predetermined price in the contract that the long should pay to the short to buy the underlying asset at the settlement date.
  - Generally positive;
  - Constant over time.

Forwards Pricing: No-Arbitrage Principle

- **Cash-and-Carry Arbitrage** When the forward contract is overpriced
  - If FP $> S_0 \times (1 + R_f)^T$

At initiation	At settlement date
• Borrow S_0 at the risk-free rate • Use the money to buy the underlying bond • Short a forward contract	• Deliver the underlying to the long to get FP from the long • Repay the loan amount of $S_0 \times (1 + R_f)^T$
	Profit$= FP - S_0 \times (1 + R_f)^T$

Forwards Pricing: No-Arbitrage Principle

- **Reverse Cash-and-Carry Arbitrage** when the forward contract is underpriced
  - If $FP < S_0 \times (1 + R_f)^T$

At initiation	At settlement date
• Short sell the underlying bond to get S_0 • Invest S_0 at the risk-free rate • Long a forward contract	• Pay the short FP to get the underlying bond • Close out the short position by delivering the bond • Receive investment proceeds
	$Profit = S_0 \times \left(1 + R_f\right)^T - FP$

Example

Forwards Pricing: No-Arbitrage Principle

- An investor observes that the spot price, S_0, of an underlying asset with no additional costs or benefits exceeds its known future price discounted at the risk-free rate, $S_T(1 + r)^{-T}$. Describe and justify an arbitrage strategy that generates a riskless profit for the investor.

- **Solution:**
  - Since the spot price of the underlying asset exceeds the known future price discounted at the risk-free rate ($S_0 > F_0(T)(1 + r)^{-T}$)
  - At time $t = 0$, the investor:
    - ✓ Sells the underlying asset short in the spot market at S_0
    - ✓ Simultaneously enters a long forward contract at $F_0(T)$
    - ✓ Lends $S0$ at the risk-free rate r to receive $S_0(1 + r)^T$ at time T.
  - At time $t = T$, the investor:
    - ✓ Settles the long forward position and receives $S_T – F_0(T)$
    - ✓ Offsets the short underlying asset position at S_T, and
    - ✓ Retains $S_0(1 + r)^T – F_0(T)$ as a riskless profit regardless of the underlying spot price at time T.

Basics of Forward Valuation

- **Valuation** of a forward contract means determining the **payoff** of the contract to the long (or short) position at some time **during** the life of the contract.
  - Zero at initiate;
  - Change over time.
- $V_{long} = S_t - \frac{FP}{(1+\mathrm{R_f})^{T-t}}$
- $V_{short} = -V_{long} = \frac{FP}{(1+\mathrm{R_f})^{T-t}} - S_t$

Replication and Risk Neutrality

- **Replication** is a strategy in which a derivative's cash flow stream may be recreated using a combination of long or short positions in an underlying asset and borrowing or lending cash.
  - E.g., In order to long a call option, we can borrow at the risk-free rate and long the underlying; in order to long a forward, we can long (for a call) and short (for a put).
- **Risk neutrality**
  - Risk-neutral investors are willing to buy risky investments for which they expect to earn only the risk-free rate. They do not expect to earn a premium for bearing risk.
  - The expected payoff of the derivative can be discounted at the risk-free rate. And should yield the risk-free rate of return, if it generates certain payoffs.

Example

- Formulate a replication strategy for a three-month short forward commitment for 1,000 shares of a non-dividend-paying stock.

- **Solution:**
  - The replication strategy for a three-month short forward commitment on a non-dividend-paying stock involves the short sale of 1,000 shares of stock at $t = 0$ and investment of proceeds at the risk-free rate, r. At time $t = T$, the short sale is covered at S_T, and under the no-arbitrage condition of $F_0(T) = S_0(1 + r)^T$, the return is equal to $F_0(T) - S_T$ for both the short forward and the replication strategy.

Example

- Identify which of the following activities corresponds to which replication strategy

1.Sell the asset short for S_0 at $t = 0$, and lend proceeds of the asset sale at the risk-free rate, r. At time $t = T$, buy back the asset at the spot price, S_T.	A.Risk-free trade replication
2.Purchase an asset at today's spot price (S_0), and simultaneously enter into a forward commitment to sell the asset at the forward price, $F_0(T)$.	B.Long forward replication
3.Borrow at the risk-free rate, r, and buy the underlying asset at today's spot price (S_0). At time T, sell the asset at the spot price (S_T). Repay the loan principal and interest ($S_0(1 + r)^T$) at time T.	C.Short forward replication

Example(cont.)

- **Solution:**
  - 1. C is correct. The short sale of an asset at time $t = 0$ creates a position that replicates a short forward, as the return is equal to $F_0(T) – S_T$ for both a short forward and the replication strategy.
  - 2. A is correct. This long cash, short derivative position earns the risk-free rate r as long as the no-arbitrage condition ($F_0(T) = S_0(1 + r)^T$) holds. If the investor instead borrows at the risk-free rate, r, to purchase the underlying asset at $S0$, the return is zero.
  - 3. B is correct. This strategy returns $S_T – F_0(T)$, as in the case of a long forward commitment, with $F_0(T) = S_0(1 + r)^T$ under the no-arbitrage condition.

Example

Forwards Pricing: No-Arbitrage Principle

- Calculate the arbitrage profit if a spot asset with no additional costs or benefits trades at a spot price of 100, the three-month forward price for the underlying asset is 102, and the risk-free rate is 5%.

- **Solution:**
  - The forward price, $F_0(T) = S_0(1 + r)^T$, at which no-arbitrage opportunities would exist is 101.23 (= $100(1.05)^{0.25}$).
  - With an observed forward price of 102, the arbitrage opportunity would be to sell the forward contract and buy the underlying, borrowing at the risk-free rate to fund the purchase. The arbitrage profit is the difference between the observed forward price and the no-arbitrage forward price of 0.77 (= 102 – 101.23).

Summary

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Basics of Forward Pricing

Forwards Pricing: No-Arbitrage Principle

Basics of Forward Valuation

Cost of Carry in Pricing Derivatives

- Explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset

Cost of Carry of Underlying Assets

- **Pricing** a forward contract is the process of determining the **no-arbitrage price** that will make the value of the contract be zero to both sides at the initiation of the contract.

$$\mathrm{FP} = S_0 \times \left(1 + R_f\right)^T \boxed{+FVC_{\boldsymbol{T}} - FVB_T}$$

$$\mathrm{FP} = (S_0 \boxed{- PVB_{\mathbf{0}} + PVC_{\mathbf{0}}}) \times \left(1 + R_f\right)^T$$

- **Cost of carry** is the net of the **costs** and **benefits** related to owning an underlying asset for a specific period.
  - Carrying Costs
    - ✓ Costs of storage: incurred in owning commodity. E.g., corn, gold, etc.
    - ✓ The risk-free rate, Rf, denotes the opportunity cost of carrying the asset, whether or not the long investor borrows to finance the asset.
  - Carrying Benefits
    - ✓ Monetary benefits: dividends, coupons, interest, etc.
    - ✓ Non-monetary benefits: **convenience yield:** a non-cash benefit of holding a physical commodity versus a derivative.

Cost of Carry of Underlying Assets

Asset Class	Examples	Benefits (i)	Costs (r, c)
Asset without Cash Flows	• Non-dividend-paying stock	• None	• Risk-free rate
Equities	• Dividend-paying stocks • Equity indexes	• Dividend • Dividend yield	• Risk-free rate
Foreign Exchange	• Sovereign bonds (foreign)	• None	• Difference between foreign and domestic risk-free rates ($r_f - r_d$)
Commodities	• Market exchange rates • Soft and hard commodities	• Convenience yield	• Risk-free rate • Storage cost
Interest Rates	• Commodity indexes • Sovereign bonds (domestic) • Market reference rates	• Interest income	• Risk-free rate
Credit	• Single reference entity • Credit indexes	• Credit spread	• Risk-free rate

Example

Cost of Carry of Underlying Assets

- Which of the following does not represent a benefit of holding an asset?

  A. The convenience yield.

  B. An optimistic expected outlook for the asset.

  C. Dividends if the asset is a stock or interest if the asset is a bond.

- **Solution: B.**

Summary

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Cost of Carry in Pricing Derivatives

Cost of Carry of Underlying Assets

Summary

Module: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Arbitrage, Replication, and Risk Neutrality

Cost of Carry in Pricing Derivatives

Pricing & Valuation of Different Forward Contracts

- Explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration

T-Bill Forward Contracts

- **T-bill (zero-coupon bond) forward contracts**
  - buy a T-bill today at the spot price (S_0) and short a T-month T-bill forward contract at the forward price (FP) .

$$\mathrm{FP} = \mathrm{S}_0 \times (1 + \mathrm{R}_\mathrm{f})^\mathrm{T}$$

- Forward values of long position at initiation, during the contract life, and at expiration are shown below.

Time	Forward Contract Valuation
t=0	Zero, because the contract is priced to prevent arbitrage.
t=t	$V_{long} = S_t - \dfrac{FP}{(1+\mathrm{R}_\mathrm{f})^{T-t}}$ $\quad V_{short} = -V_{long} = \dfrac{FP}{(1+\mathrm{R}_\mathrm{f})^{T-t}} - S_t$
t=T	$S_T - FP$

Equity & Bond Forward Contracts

- **Equity forward contracts on a dividend-paying stock**
  - Price:

$$FP = (S_0 - PVD_0) \times (1 + R_f)^T \text{ or } FP = S_0 e^{(r+c-i)\times T} \text{ (continuous compounding)}$$

  - Value:

$$V_{long} = (S_t - PVD_t) - \frac{FP}{(1+R_f)^{T-t}}$$

- **Bond forward contracts on a coupon bond**
  - Similar to dividend-paying stocks, but the cash flows are coupons.
  - Price:

$$FP = (S_0 - PVC_0) \times (1 + R_f)^T$$

  - Value:

$$V_{long} = (S_t - PVC_t) - \frac{FP}{(1+R_f)^{T-t}}$$

Example

Forward Contract Pricing and Valuation

- Assuming a forward contract with 100 days until maturity on a stock, the stock price is $45 and expected to pay dividend of $0.3 in 20 days, and $0.5 in 75 days. The risk-free rate is 4%. Calculate the no-arbitrage forward price.

  - **Solution:**

✓ $PVD = \frac{\$0.3}{1.04^{20/365}} + \frac{\$0.5}{1.04^{75/365}} = \$0.795343$

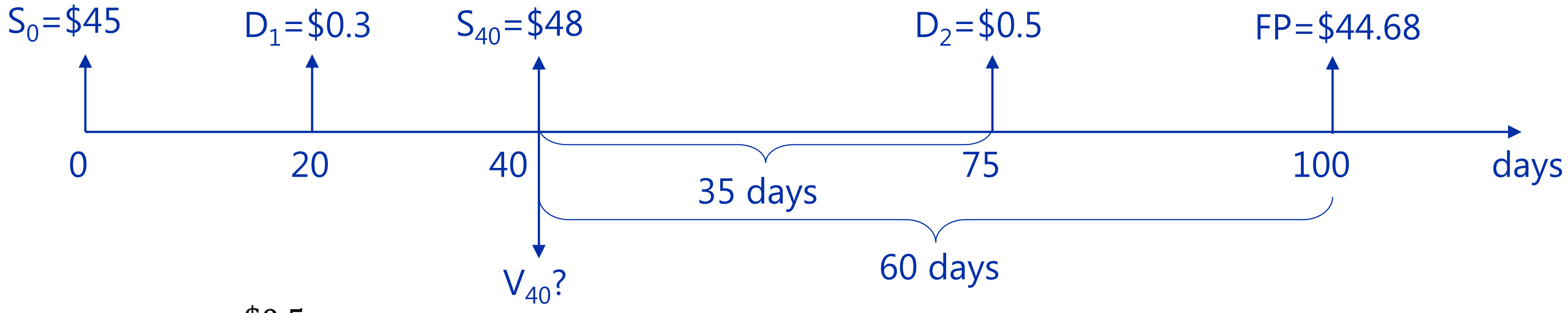
✓ $FP = (\$45 - \$0.795343) \times 1.04^{100/365} = \$44.68$

Example

Forward Contract Pricing and Valuation

- After 40 days, the stock price changed to \$48. Calculate the valuation of the forward contract.

- **Solution:**
  - There's only one dividend remaining (in 35 days) before the contract matures (in 60 days) as shown below, so:



  - $PVD_{40} = \frac{\$0.5}{1.04^{35/365}} = \$0.498123$
  - $V_{40}\ (long\ position) = (\ \$48 - \$0.498123) - \frac{\$44.68}{1.04^{60/365}} = \$3.11$

Example

Equity Forward Contracts

- Stock Index Futures with Dividend Yield
  - The Viswan Family Office (VFO) would like to enter into a three-month forward commitment contract to purchase the NIFTY 50 benchmark Indian stock market index traded on the National Stock Exchange. The spot NIFTY 50 index price is INR15,200, the index dividend yield is 2.2%, and the Indian rupee risk-free rate is 4%. Use Equation 6 (with c = 0) to solve for the forward price:
    - ✓ $F_0(T) = S_0 e^{(r+c-i)T} = 15{,}200e^{(0.04 - 0.022)0.25}$ = INR15,268.55.

Interest Rate Forward Contracts

- **Interest rate forward contracts - example 1**
  - Spot rate or **zero rate**: e.g., 1-yr rate Z_A=2.396%, 2-yr rate Z_B=3.4179%.
  - **Implied forward rate** (IFR), E.g., **breakeven reinvestment rate,** $IFR_{1,1}$=4.4536%, is the interest rate for a period in the future at which an investor earns the same return from (no-arbitrage condition) .

Z_B

Time

t=0 t=1 t=2

Z_A IFR_{B-A}

$$(1+Z_B)^2 = (1+Z_A)^1 \times (1+IFR_{B-A})^1$$

- **Interest rate forward contracts - example 2**

0 1 3 9

S_3 S_9

$$\left(1+S_{0,3}\times\frac{3}{12}\right)\times\left(1+FRA_0\times\frac{6}{12}\right)=1+S_{0,9}\times\frac{9}{12}$$

Example

Interest Rate Forward Contracts

- Using the one-year and two-year zero rates, calculate the "1y1y" implied forward rate ($IFR_{1,1}$).

Years to Maturity	Zero Rate
1	2.3960%
2	3.4197%
3	4.0005%

- **Solution:**
  - USD100 × $(1 + z_1) \times (1 + IFR_{1,1})$ = USD100 × $(1 + z_2)^2$
  - USD100 × (1.02396) × $(1 + IFR_{1,1})$ = USD100 × $(1.034197)^2$
  - IFR1,1 = 4.4536%

  - General equation:
    - ✓ $(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$

Foreign Exchange Forward Contracts

- **Foreign exchange forward Contracts (FX forward contracts)**
  - Price:

    ✓ $\frac{FP_{f/d}}{S_{f/d}} = \frac{1+r_f}{1+r_d}$; $FP_{f/d} = S_{f/d} \times \frac{1+r_f}{1+r_d}$

    ✓ $FP_{f/d} = S_{f/d} \times e^{(r_f - r_d)\times T}$

- **Example 6**
- Assume the current AUD/USD spot price is 1.3335. The Australian dollar is the price currency or foreign currency, and the US dollar is the base or domestic currency (AUD1.3335 = USD1). The six-month Australian dollar risk-free rate is 0.05%, and the six-month US dollar risk-free rate is 0.2%.
- Solution:
  - $S_{f/d} = S_{AUD/USD} = 1.3335$; $r_f = r_{AUD} = 0.05\%$; $r_d = r_{AUD} = 0.02\%$; T=0.5
  - $FP_{f/d} = S_{AUD/USD} \times e^{(r_f - r_d)\times T} = 1.3325 = 1.3335 \times e^{(0.05\% - 0.2\%)\times 0.5}$

Foreign Exchange Forward Contracts

- At any given time, *t*, the MTM value of the FX forward is the difference between the *current* spot FX price ($S_{t,f/d}$) and the present value of the forward price discounted by the *current* difference in risk-free rates ($r_f - r_d$) for the remaining period through maturity:
  - $Vt(T) = S_{t,\,f/d} - F_{0,\,f/d(T)}e^{-(rf-rd)(T-t)}$

- Rook Point Investors LLC has entered into a long one-year USD/EUR forward contract. That is, it has agreed to purchase EUR1,000,000 in exchange for USD1,201,000 in one year. At time *t* = 0 when the contract is initiated, the USD/ EUR spot exchange rate is 1.192 (i.e., USD1.192 = EUR1), the one-year USD risk-free rate is 0.50%, and the one-year EUR risk-free rate is −0.25%.
- Describe the MTM impact on the FX forward contract from Rook Point's perspective if the one-year USD risk-free rate instantaneously rises by 0.25% once the contract is initiated, with other details unchanged.

$V_t(T) = S_{0,USD/EUR} - F_{0,USD/EUR}(T)e^{-(r_{USD} - r_{EUR})T}$

$= 1.192 - 1.201e^{-(0.0075\ +\ 0.0025)}$

$= 0.00295$ USD/EUR

Foreign Exchange Forward Contracts

- We see that it is the risk-free interest rate *differential* $(r_f - r_d)$, rather than the absolute level of interest rates, that determines the spot versus forward FX price relationship.
  - For example, in previous Example, the Australian dollar risk-free rate is 0.15% below the US dollar rate $(r_f - r_d < 0)$. Borrowing at the higher US dollar rate and lending at the lower Australian dollar rate results in a no-arbitrage forward price at which *fewer* Australian dollars are required to purchase USD1 in the future, so the Australian dollar is said to trade at a *premium* in the forward market versus the US dollar.

FX Forward vs. Spot Price Relationship

Interest Rate Differential	Forward vs. Spot Price	Foreign Currency Forward	FX Forward Premium/ Discount
$(r_f - r_d) > 0$	$F_{0,f/d}(T) > S_{0,f/d}$	Discount	Premium
$(r_f - r_d) < 0$	$F_{0,f/d}(T) < S_{0,f/d}$	Premium	Discount
$(r_f - r_d) = 0$	$F_{0,f/d}(T) = S_{0,f/d}$	Neither a premium nor a discount	Neither a premium nor a discount

Example

Foreign Exchange Forward Contracts

- The analyst observes a current USD/EUR spot exchange rate of 1.192 (that is, USD1.192 = EUR1), a US dollar risk-free rate of 0.5%, and a euro risk-free rate of –0.25%. Which of the following statements describes the action Rook Point can take today to earn a riskless profit if the one-year USD/EUR forward rate is observed to be 1.201?

  A. No arbitrage opportunity exists, because the observed one-year USD/ EUR FX forward rate equals the no-arbitrage rate capturing the net effect of the domestic versus foreign risk-free rate for one year.

  B. Since the no-arbitrage one-year USD/EUR forward rate is 1.195, Rook Point should borrow in euros and buy US dollars today, simultaneously selling US dollars against euros one year forward.

  C. Since the no-arbitrage one-year USD/EUR forward rate is 1.195, Rook Point should borrow in US dollars and buy euros today, simultaneously buying US dollars and selling euros one year forward.

Example

Foreign Exchange Forward Contracts(cont.)

- **Solution:**
  - A is correct. Recall from Equation that the spot versus forward relationship for a foreign exchange may be shown as follows:
    - ✓ $F_{0,f/d}(T) = S_{0,f/d}\, e\text{^}(r_f – r_d)T$.
  - In the specific case of USD/EUR, the equation may be rewritten as
    - ✓ $F_{0,USD/EUR}(T) = S_{0,USD/EUR}\, e\text{^}(r_{USD} – r_{EUR})T$.
  - If S_0 = 1.192, r_f = 0.5%, and r_d = –0.25%, the no-arbitrage forward price in one year equals 1.201 (= $1.192e^{(0.005\,+\,0.0025)}$) and no riskless profit opportunity exists.

Example

Foreign Exchange Forward Contracts

- A Baywhite client has entered into a long six-month MXN/USD FX forward contract—that is, an agreement to sell MXN and buy USD. The MXN/USD spot exchange rate at inception is 19.8248 (MXN19.8248 = USD1), the six-month MXN risk-free rate is 4.25%, and the six-month USD risk-free rate is 0.5%. Baywhite's market strategist predicts that the Mexican central bank (Banco de Mexico) will surprise the market with a 50 bp short-term rate cut at its upcoming meeting. Which of the following statements best describes how the client's existing FX forward contract will be impacted if this prediction is realized and other parameters remain unchanged?

  A. The lower interest rate differential between MXN and USD will cause the MXN/USD contract forward rate to be adjusted downward.

  B. The client will realize an MTM gain on the FX forward contract due to the decline in the MXN versus USD interest rate differential.

  C. The lower interest rate differential between MXN and USD will cause the client to realize an MTM loss on the MXN/USD forward contract.

Example

Foreign Exchange Forward Contracts(cont.)

- **Solution: C.**
  - A decline in the interest rate differential between MXN and USD will cause the client to realize an MTM loss on the MXN/USD forward contract, while B states that this decline will result in an MTM gain. A is incorrect as the forward price, $F_0(T)$, is not adjusted during the contract life. Specifically, the original MXN/USD forward exchange rate at inception is equal to 20.20 (= $19.8248e^{(.0425 - 0.005) \times 0.5}$). If the MXN rate were to decline by 50 bps immediately after the contract is agreed, a new MXN/USD forward contract would be at a forward exchange rate of 20.15 (= $19.8248e^{(.0375 - 0.005) \times 0.5}$).
  - The MXN would weaken or depreciate against the USD. Since the MXN seller has locked in a forward sale at the original 20.20 versus the new 20.15 rate, the seller's MTM loss is equal to 0.05, or MXN50,000 per MXN1,000,000 (= 0.05 × 1,000,000) notional amount.

Summary

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Pricing & Valuation of Different Forward Contracts

T-Bill Forward Contracts

Equity & Bond Forward Contracts

Interest Rate Forward Contracts

Foreign Exchange Forward Contracts

Summary

Module: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Forward

Basics of Forward Pricing

Pricing & Valuation of Different Forward Contracts

Module

Pricing and Valuation of Futures Contracts

1. Futures
2. Pricing and Valuation of Futures

Futures

- Define futures contracts and its basic characteristics
- Compare the value and price of forward and futures contracts

Futures Contracts

- **Definition**
  - A **futures contract** is an agreement that **obligates** one party to buy and the other party to sell a specific quantity of an underlying asset, at a set price, at a future date.
- **Similarity with forward contract**
  - Both are settled with assets delivered or in cash;
    - ✓ **Deliverable contracts** obligate the long to buy and the short to sell a certain quantity of an asset for a certain price on a specified future date.
    - ✓ **Cash settlement** contracts are settled by paying the contract value in cash on the expiration date.

Difference with Forward

- **Difference with forward**

Forwards	Futures
Private contracts	Exchange-traded
Unique customized contracts	Standardized contracts
Little or no regulation	Regulated
Default risk is present	Guaranteed by clearinghouse
Settlement at maturity	Daily settlement (mark to market)
No margin deposit required	Margin required and adjusted
Forward price is constant	Futures price fluctuates daily

Risk Control of Futures Contract

- **Risk control of Futures contract**
  - Margin;
  - Daily Price Limit;
  - Marking to market.

Risk Control of Futures Contract

- **Risk control of Futures contract**
  - **Margin**
    - ✓ **Initial margin**: The first deposit is called the initial margin. Initial margin must be posted before any trading takes place;
    - ✓ **Maintenance margin**: is the amount of money that each participant must maintain in the account after the trade is initiated. If the margin balance is lower than the maintenance margin, the trader will get a **margin call**;
    - ✓ **Variation margin**: used to bring the margin balance back up to the **initial margin level.**

Example

Risk Control of Futures Contract

- **Initial margin=$5/contract, maintenance margin=$2/contract, long 20 contract**

Day	Beginning balance	Funds deposited	Futures price	Price change	Gain/Loss	Ending balance
0	0	100	82			100
1	100	0	84	2	40	140
2	140	0	78	-6	-120	20
3	20	80	73	-5	-100	0
4	0	100	79	6	120	220
5	220	0	82	3	60	280
6	280	0	84	2	40	320

Risk Control of Futures Contract

- **Risk control of Futures contract**
  - **Margin**: difference between futures margin and equity margin.

	Futures margin	**Equity margin**
Purpose	As pledge, control default risk	Borrow capital, has leverage
Cash flow direction	Outflow	Inflow
Interest paid	No interest paid	Loan, interest paid needed
Replenish margin	Back to initial margin	Back to maintenance margin

Risk Control of Futures Contract

- A futures trader must keep the money in the margin account above the:

A. initial margin requirement.

B. variation margin requirement.

C. maintenance margin requirement.

- **Solution: C.**

Risk Control of Futures Contract

- **Risk control of Futures contract**
  - **Daily Price Limit**
    - ✓ Establish a **band** relative to the previous day's settlement price within which all trades must occur.
    - ✓ **Circuit breaker:** pause intraday trading for a brief period if a price limit is reached.
  - **Marking to market**: The margin requirement of a futures contract is low because at the end of every day there is a daily settlement process called marking to market.

Example

Futures Contracts

- Which of the following statements about futures contracts is **least correct?**
  A. The futures clearinghouse allows traders to reverse their positions without having to contact the other side of the initial trade.
  B. To safeguard the clearinghouse, the exchange requires traders to post margin and settle their accounts on a weekly basis.
  C. Offsetting trades rather than exchanges for physicals are used to close most futures contracts.

- **Solution: B.**

Example

Futures Contracts

- Which of the following **occurs** in the daily settlement of futures contracts?

  A. Initial margin deposits are refunded to the two parties.

  B. Gains and losses are reported to other market participants.

  C. Losses are charged to one party and gains credited to the other.

- **Solution: C.**

Example

Risk Control of Futures Contract

- **Describe the mark-to-market process for a futures contract.**

- **Solution:**
  - The exchange clearinghouse determines an average of the final futures prices of the day and designates that price as the end-of-day settlement price.
  - The daily settlement of gains and losses takes place via each counterparty’s futures margin account.

Summary

Pricing and Valuation of Futures Contracts

Futures

Futures Contract

Difference with Forward

Risk Control of Futures Contract

Pricing and Valuation of Futures

- Compare the value and price of forward and futures contracts
- Explain why forward and futures prices differ

Futures Pricing and Valuation

- **Prices of Futures vs. Forward Contracts**
  - Daily settlement and margin requirements give rise to different cash flow patterns between futures and forwards, resulting in a pricing difference which depends on both the correlation between interest rates and futures prices and interest rate volatility.

If the correlation between the **futures prices** and **interest rates** is:	**Investors will…**
positive. ($\rho_{FP\&int} > 0$)	prefer to go long in a futures contract
zero. ($\rho_{FP\&int} = 0$)	have no preference
negative. ($\rho_{FP\&int} < 0$)	prefer to go long in a forward contract

- There is a price difference between interest rate futures and forward rate agreements (FRAs) due to **convexity bias** which arises given the difference in price changes for interest rate futures versus forward contracts.

Futures Pricing and Valuation

- **Valuation of Futures Contracts**
  - Positions:
    - ✓ Long futures contract (lender): Gains as prices rise
    - ✓ Short futures contract (borrower): Gains as prices fall
  - The value of a futures contract is zero at contract inception.
  - The daily settlement of futures gains and losses via a margin account <u>resets the futures contract value to zero</u> at the current futures price $f_t(T)$. This process continues until contract maturity and the futures price converge to the spot price, S_T.
  - Between the times at which the contract is marked to market, the value can be different from zero.

    V (long) = current futures price − futures price at the last mark-to-market time.

Example

Futures vs. Forward Price & Value over Time

- A forward commitment to buy 100 ounces of gold at a price of $1,778.76 per ounce in 91 days, with a risk-free rate of 2% and no gold storage cost. As per futures exchange daily settlement rules, the contract buyer and seller must post an initial cash margin of $4,950 per gold contract (100 ounces) and maintain a maintenance margin of $4,500 per contract.
- Consider the first day of trading, where the spot gold price, (S_0), is $1,770 per ounce and the opening gold futures price, $f_0(T)$, is $1,778.76 per ounce.
  - Assume that the gold futures price, $f_1(T)$, falls by $5 on the first trading day to $1,773.76 and the spot price, S_1, ends the day at a no arbitrage equivalent of $1,765.12 (= $1,773.76(1.02)$^{-90/365}$).
  - Procam realizes a $500 MTM loss (= $5 per ounce × 100 ounces) deducted from its margin account, leaving Procam with $4,450.
  - The MTM value of Procam's futures contract resets to zero at the futures closing price of $1,773.76 per ounce.
  - Since Procam's margin account balance has fallen below the $4,500 maintenance level, it must deposit $500 to return the balance to the $4,950 initial margin.

Example

Futures vs. Forward Price & Value over Time

- Using the same details, we compare the futures and forward price and value over two trading days. Assume that day two trading opens at day one's closing spot and futures prices. The following table shows the comparison:

Beginning of Day 2 Trading				
Contract Type	**Contract Price**	**Contract MTM**	**Realized MTM**	**Margin Deposit**
Forward	$F_0(T)$ = \$177,876	–\$498	\$0	\$0
Futures	$f_1(T)$ = \$177,376	\$0	–\$500	\$4,950

- The forward MTM contract value, $V_t(T)$, equals the difference between the current spot price, S_1 = \$1,765.12, and the present value of the original forward price, $PV_t[F_0(T)]$, here with 90 days remaining to maturity, $T - t = 90/365$ or 0.24657:
  - $V_t(T) = \$1,765.12 - \$1,778.76(1.02)^{-0.24657} = \$4.98$ per ounce.

Interest Rate Futures

- The futures price for short-term interest rate futures is given by (100 - yield), where yield is expressed in percentage terms.

$$f_{\boldsymbol{A,B-A}} = 100 - \left(100 \times MRR_{A,B-A}\right)$$

  - where $f_{\boldsymbol{A,B-A}}$ represents the futures price for a market reference rate for $\boldsymbol{B-A}$ periods that begins in A periods ($MRR_{\boldsymbol{A,B-A}}$)

- **An inverse price/yield relationship**

  - The implied three-month MRR rate in three months' time (where A = 3m, B = 6m, B − A = 3m) of an interest rate futures contract is trading at a price of 98.25.
  - Solution:
    - ✓ $f_{3m,3m}$: $98.25 = 100 - (100 \times MRR_{A,B-A})$
    - ✓ $MRR_{3m,3m} = 1.75\%$.

Interest Rate Futures

- The interest rate exposure profile for long and short futures contracts are as follows:
  - Long futures contract (lender): Gains as prices rise, future MRR falls
  - Short futures contract (borrower): Gains as prices fall, future MRR rises

Interest Rate Futures versus FRAs

Contract Type	Gains from Rising MRR	Gains from Falling MRR
Interest rate futures	Short futures contract	Long futures contract
Forward rate agreement	Long FRA: FRA fixed-rate payer (FRA floating-rate receiver)	Short FRA: FRA floating-rate payer (FRA fixed-rate receiver)

- Example:

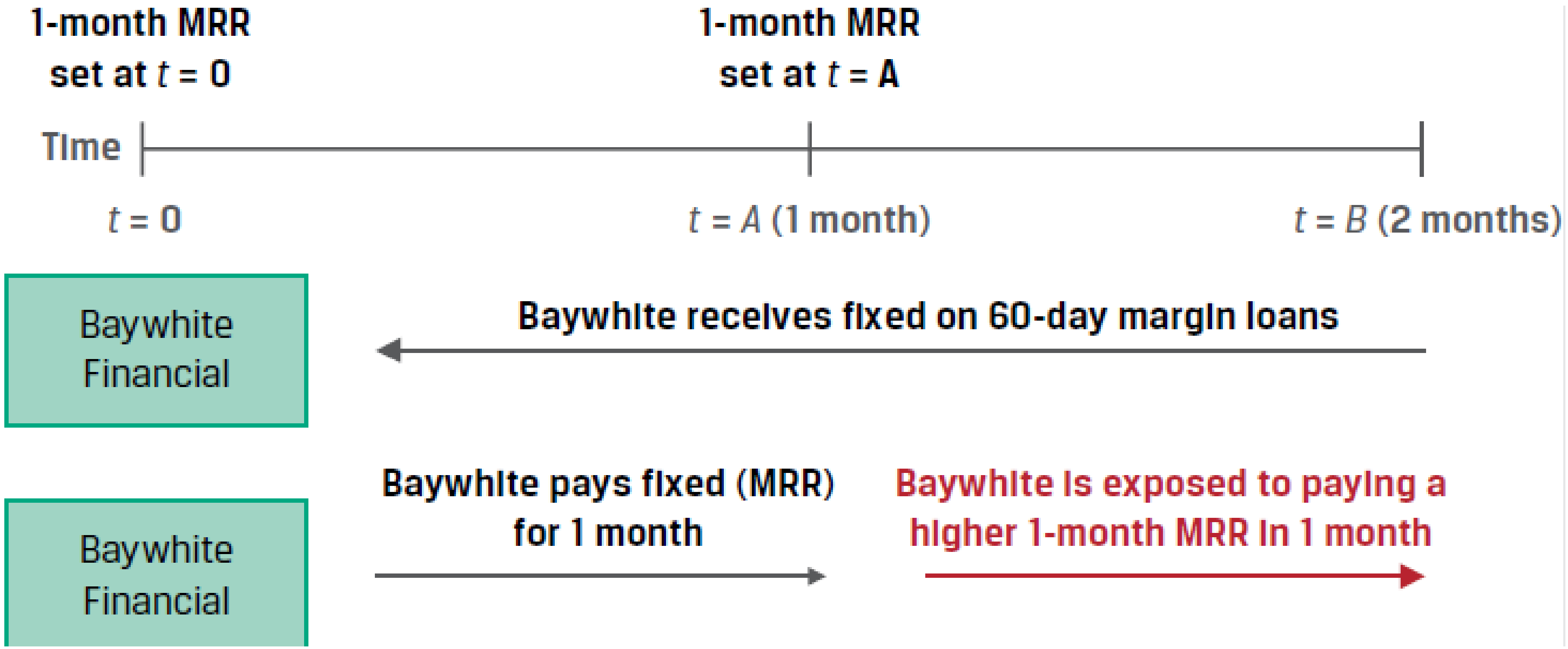
- Identify the following statement as true or false and justify your answer: An FRA fixed-rate receiver (floating-rate payer) position is equivalent to a long interest rate futures contract on MRR, as both positions realize a gain as MRR falls below the initial fixed rate.
- **Solution:** The statement is true.

Interest Rate Futures

- Interest rate futures daily settlement occurs based on price changes, which translate into **futures contract basis point value (BPV)** as follows:
  - Futures Contract BPV = Notional Principal × 0.01% × Period.
- For example, assuming a $1,000,000 notional for three-month MRR of 2.21% for one quarter (or 90/360 days), the underlying deposit contract value would be:
  - $1,005,525 = $1,000,000 × [1 + (2.21% / 4)].
- Consider how a one basis point (0.01%) change in MRR affects contract value:
  - 1 bp increase (2.22%): $1,005,550 = $1,000,000 × [1 + (2.22% / 4)].
  - 1 bp decrease (2.20%): $1,005,500 = $1,000,000 × [1 + (2.20% / 4)].
- Both the increase and decrease in MRR by one basis point change the contract BPV by $25.

Interest Rate Futures

- Baywhite Financial offered 60-day margin loans at fixed rates to its clients and borrowed at a variable one-month MRR to finance the loans.



Example

Interest Rate Futures

- Baywhite faces the risk of higher MRR in one month's time ($MRR_{1,1}$), which would reduce the return on its fixed margin loans. In the prior example, Baywhite entered an FRA where it agreed to pay fixed one-month MRR and receive floating. If Baywhite were to use an interest rate futures contract instead, it would *sell* a futures contract on one-month MRR. The futures contract BPV for a $50,000,000 notional amount is:
  - Contract BPV = $50,000,000 × 0.01% × [1/12] = $416.67
- If Baywhite sells $f_{1,1}$ for $98.75 (or $MRR_{1,1}$ = 1.25%) and settles at maturity at a price of $97.75 ($MRR_{1,1}$ = 2.25%), it would expect to have a *cumulative* gain on the contract through maturity equal to $41,667 (= Contract BPV × 100 bps).

Interest Rate Forwards versus Futures

- An interest rate futures contract of $1,000,000 notional for three-month MRR of 2.21% for one quarter (or 90/360 days) has the underlying deposit contract value was:
  - $1,005,525 = $1,000,000 × [1 + (2.21% / 4)].
- The contract BPV was shown to be $25 (= $1,000,000 × 0.01% × [1/4]). Consider in contrast a $1,000,000 notional FRA on three-month MRR in three months' time with an identical 2.21% rate. The net payment on the FRA is based upon the difference between MRR and the implied forward rate (IFR):
  - Net Payment = $(\text{MRR}_{B-A} - \text{IFR}_{A,B-A})$ × Notional Principal × Period.
- For example, if the observed MRR in three months is 2.22% (+0.01%), the net payment *at maturity* would be $25 (= $1,000,000 × 0.01% × [1/4]). However, the settlement of an FRA is based upon the present value of the final cash flow discounted at MRR, so:
  - Cash Settlement (PV): $24.86 = $25 /(1 + 0.0222 / 4).

Interest Rate Forwards versus Futures

- If we increase the magnitude of the MRR change at settlement and compare these changes between a long interest rate futures position and a short receive-fixed (pay floating) FRA contract, we arrive at the following result:

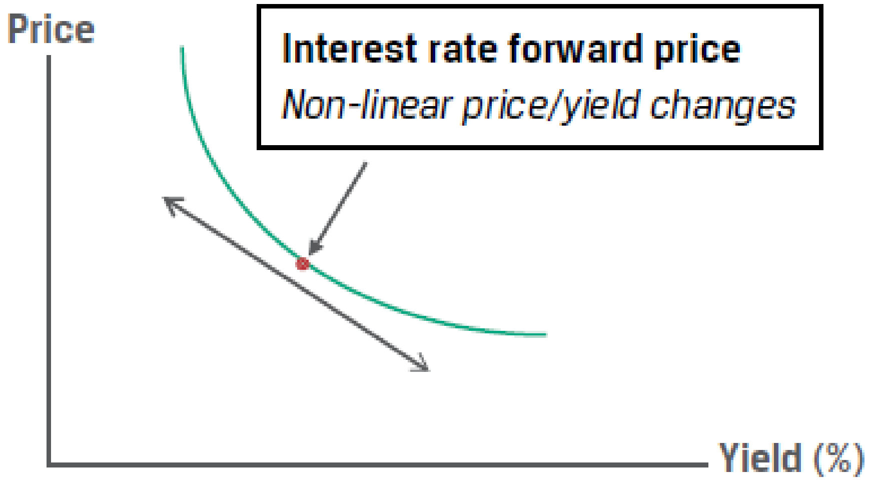
$MRR_{3m,3m}$	Short FRA Cash Settlement (PV)	Long Futures Settlement
2.01%	$497.50	$500
2.11%	$248.69	$250
2.21%	$0	$0
2.31%	($248.56)	($250)
2.41%	($497.01)	($500)

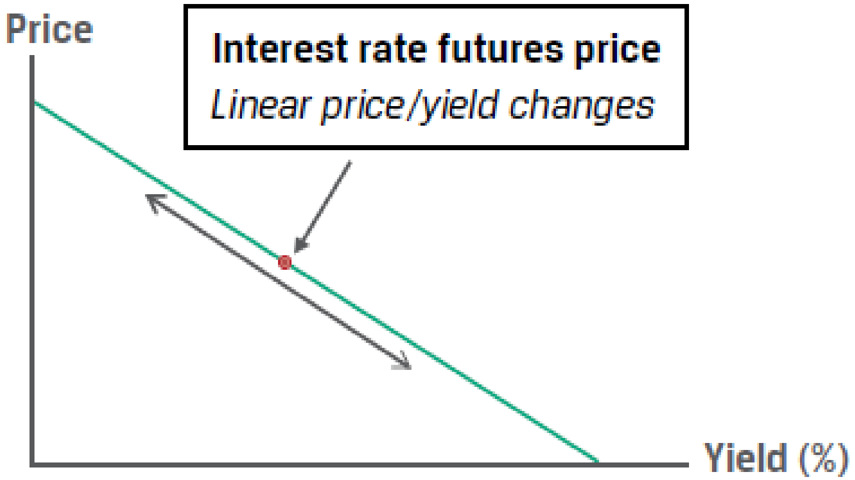
- Although the settlement values differ due to different conventions across these instruments, note that while **the futures contract has a fixed linear payoff profile for a given basis point change, the FRA settlement does not.**

Interest Rate Forwards versus Futures

- In the FRA contract in Example, **we see that the percentage price change is greater (in absolute value) when MRR falls than when it rises**. Although the difference here is very small due to the short forward period, note that this non-linear relationship is the **convexity property**, which characterizes fixed-income instruments from earlier lessons, as shown in Exhibit 5.

Convexity Bias





Example

Interest Rate Forwards versus Futures

- Identify the following statement as true or false and justify your answer:
  - The convexity bias between interest rate futures and interest rate forwards causes the percentage price change to be greater (in absolute value) when MRR rises than when it falls for a forward than for a futures contract.

- **Solution:**
  - The statement is false.
  - The convexity bias between interest rate futures and interest rate forwards causes the percentage price change to be greater (in absolute value) when MRR *falls* than when it *rises* for a forward contract, as opposed to a futures contract.

Interest Rate Futures

- An investor seeks to hedge its three-month MRR exposure on a £25,000,000 liability in two months and observes an implied forward rate today ($IFR_{2m,\ 3m}$) of 2.95%. Calculate the settlement amounts if the investor enters a long pay-fixed (receive floating) FRA and a short futures contract, and compare and interpret the results if $MRR_{2m,3m}$ settles at 3.25%.

- **Solution:**
  - Solve for the pay-fixed FRA Cash Settlement (PV) value as follows:
    - ✓ Net Payment = ($MRR_{B-A} - IFR_{A,B-A}$) × Notional Principal × Period
    - ✓ = £18,750 (= [3.25% − 2.95%] × £25,000,000 × [1/4]).
  - The present value based upon $MRR_{2m,5m}$ of 3.25% is £18,598.88 (= £18,750 / [1 + 0.0325/4]).
  - For the futures contract, contract BPV is equal to:
    - ✓ Contract BPV = £625 (= £25,000,000 × 0.01% × [1/4]).
  - For a 30-basis point increase in MRR (= 3.25% − 2.95%), the short futures contract will realize a price appreciation of £18,750 (= £625 × 30). Both contracts result in a gain from the investor's perspective as MRR rises. However, the futures settlement is larger due to the discounting of the FRA final payment to the settlement date.

Summary

Pricing and Valuation of Futures Contracts

Pricing and Valuation of Futures

Futures Pricing and Valuation

Futures versus Forward Price and Value over Time

Interest Rate Futures

Interest Rate Forwards versus Futures

Summary

Module: Pricing and Valuation of Futures Contracts

Futures

Pricing and Valuation of Futures

Module

Pricing and Valuation of Interest Rates and Other Swaps

1. Swap
2. Pricing and valuation of swaps

Swaps

- Define swap contracts and its basic characteristics
- Describe how swap contracts are similar to but different from a series of forward contracts
- Contrast the value and price of swaps

Characteristics of Swaps

- **Characteristics of Swap Contracts**
  - A **swap contract** obligates two parties to change a series of cash flows on periodic settlement dates over a certain time period.
  - **Similarity with forward contracts**
    - ✓ No payment required by either party at initiation except the principal values exchanged in currency swaps;
    - ✓ Custom instruments;
    - ✓ Traded in OTC markets (no secondary markets);
    - ✓ Much less regulated;
    - ✓ Subject to counterparty credit exposure;
    - ✓ Symmetric payoff profile.
- **Three types of swaps:** interest rate swap, currency swap and equity swap.

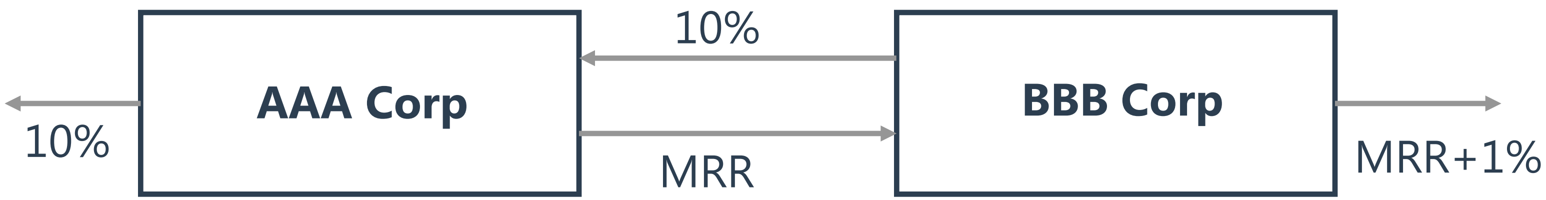
Interest Rate Swap

- **Interest Rate Swaps**
  - The **plain vanilla interest rate swap** involves trading fixed interest rate payments for floating-rate payment (paying fixed and receiving floating).
    - ✓ **Counterparties**: The parties involved in any swap agreement are called the counterparties.
    - ✓ **Pay-fixed side**: The counterparty that makes fixed-rate interest payment (constant) in exchange for variable interest rate (MRR).
    - ✓ **Pay-floating side**: The counterparty that makes variable-rate interest payment in exchange for fixed payment.

Interest Rate Swap

- **Interest Rate Swaps**
  - The Comparative Advantage Argument
    - ✓ AAA Corp: wants to borrow floating.
    - ✓ BBB Corp: wants to borrow fixed.

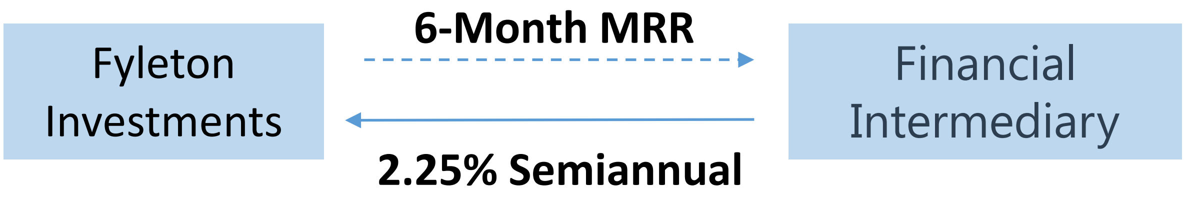
	Fixed	Floating
AAA Corp	10.00%	6-month MRR + 0.30%
BBB Corp	11.20%	6-month MRR + 1.00%



- ✓ AAA Corp: MRR, save 0.3%.
- ✓ BBB Corp: 11%, save 0.2%.

Example

- Fyleton Investments has entered a five-year, receive-fixed GBP200 million interest rate swap with a financial intermediary to increase the duration of its fixed-income portfolio. Under terms of the swap, Fyleton has agreed to receive a semiannual GBP fixed rate of 2.25% and pay six-month MRR.



- Calculate the first swap cash flow exchange if six-month MRR is set at 1.95%.
  - The financial intermediary owes Fyleton a fixed cash flow payment ofGBP2,250,000 (= GBP200 million × 0.0225/2).
  - Fyleton owes the financial intermediary a floating cash flow payment of GBP1,950,000 (= GBP 200 million × 0.0195/2).
  - The fixed and floating payments are netted against one another, and the net result is that the financial intermediary pays FyletonGBP300,000 (= GBP2,250,000 – GBP1,950,000).

Example

- **Identify the interest rate swap participants that correspond to the following statements:**

A. Fixed-rate payer	1. Makes a payment each interest period based on a market reference rate
B. Floating-rate payer	2. May face a positive or a negative mark to market over the life of an interest rate swap contract
C. Both a fixed-rate payer and a floating-rate payer	3. Receives a net payment on the swap for any interest period for which the market reference rate exceeds the fixed rate

- **Solution:**
  - 1. The correct answer is B. A floating-rate payer on a swap makes a payment each period based on a market reference rate.
  - 2. The correct answer is C. Both a fixed-rate payer and a floating-rate payer may face a positive MTM or negative MTM on a swap contract.
  - 3. The correct answer is A. A fixed-rate payer (also known as the floating-rate receiver) receives a net payment if the market reference rate exceeds the fixed rate for a given period.

Summary

Pricing and Valuation of Interest Rates and Other Swaps

Swap

Characteristics swap

Interest Rate Swap

Pricing and valuation of swaps

- Contrast the value and price of swaps

Swap Pricing

- A **swap contract** is an agreement between two parties to exchange a series of future cash flows.
- There are three kinds of swaps:
  - Interest rate swaps
    - ✓ A **plain vanilla swap** is an **interest rate swap** in which one party pays a fixed rate and the other pays a floating rate. The terms of the long and short are not used here, instead we say the fixed-rate payer and floating-rate (variable-rate) payer.
  - Currency swaps
  - Equity swaps
- Pricing of an interest rate swaps
  - The price is just the fixed rate (called the **swap rate**) that makes the contract value zero to both parties at initiation.
  - After some days the market situation changes, one party will make money and the other lose money. The contract value is no longer zero to both parties.

Example

- Calculate a three-period par swap rate (S_3)

Periods	Zero Rates
1	2.3960%
2	3.4197%
3	4.0005%

- **Solution:**
  - Firstly using zero rates to solve for IFRs, since the expected floating cash flows on the swap are the implied forward rates:

$$(1+Z_A)^A \times (1+IFR_{B-A})^{B-A} = (1+Z_B)^B$$

Solve for $IFR_{0,1} = 2.396\%$, $IFR_{1,1} = 4.4536\%$, $IFR_{2,1} = 5.1719\%$

  - then substitute the respective IFRs discounted by zero rates into the following equation to solve $S_3 = 3.9641\%$:

$$\frac{IFR_{0,1}}{(1+Z_1)} + \frac{IFR_{1,1}}{(1+Z_2)^2} + \frac{IFR_{2,1}}{(1+Z_3)^3} = \frac{S_1}{(1+Z_1)} + \frac{S_2}{(1+Z_2)^2} + \frac{S_3}{(1+Z_3)^3}$$

Example

- Based on the following floating rates, please calculate the par swap rate for an one-year swap with four-period settlements.

  R(90)=3% ; R(180)=4% ; R(270)=5% ; R(360)=6%

- **Solution:**

0	1	2	3	4
	f	f	f	1+f
	d_1=0.9926	d_2=0.9804	d_3=0.9639	d_3=0.9434
	$=1/(1+3\%\times\frac{90}{360})$	$=1/(1+4\%\times\frac{180}{360})$	$=1/(1+5\%\times\frac{270}{360})$	$=1/(1+6.0\%\times\frac{360}{360})$

$$f_{\text{periodic}} = \frac{1-d_4}{d_1+d_2+d_3+d_4} = \frac{1-0.9434}{3.8802} = 1.4588\%$$

$$f_{\text{annulized}} = 4f = 5.8351\%$$

Example

- At time t = 0, Ace observes the following zero rates over three periods:

Periods	Zero Rates	Periods	Zero Rates	Periods	Zero Rates
1	2.2727%	2	3.0323%	3	3.6355%

- Which of the following best describes how Ace arrives at a three-period par swap rate (s3)?

A. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we discount each zero rate back to the present using zero rates and solve for s3 to get 2.961%.

B. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we first solve for the implied forward rate per period using zero rates, then discount each implied forward rate back to the present using zero rates, and solve for s3 to get 3.605%.

C. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we first solve for the implied forward rate per period using zero rates, then discount each zero rate back to the present using implied forward rates, and solve for s3 to get 3.009%.

Example

- **Solution: B.**
  - Since the expected floating cash flows on the swap are the implied forward rates, we first use zero rates to solve for IFRs using Equation 1: $(1+Z_A)^A \times (1+IFR_{B-A})^{B-A} = (1+Z_B)^B$
  - We may solve for these rates as $IFR_{0,1}$ = 2.2727%, $IFR_{1,1}$ = 3.7975%, and $IFR_{2,1}$ = 4.8525%. We then substitute the respective IFRs discounted by zero rates into the following equation to solve for s_3:

$$\frac{IFR_{0,1}}{(1+Z_1)} + \frac{IFR_{1,1}}{(1+Z_2)^2} + \frac{IFR_{2,1}}{(1+Z_3)^3} = \frac{S_1}{(1+Z_1)} + \frac{S_2}{(1+Z_2)^2} + \frac{S_3}{(1+Z_3)^3}$$

  - Solving for the left-hand side of the equation, we get: $0.10159 = \frac{2.2727\%}{(1.022727)} + \frac{3.7975\%}{(1.030323)^2} + \frac{4.8525\%}{(1.036355)^3}$
  - Solving for the right-hand side, we get: $2.81819\ S_3 = \left[\frac{1}{(1.022727)} + \frac{1}{(1.030323)^2} + \frac{1}{(1.036355)^3}\right] \times S_3$

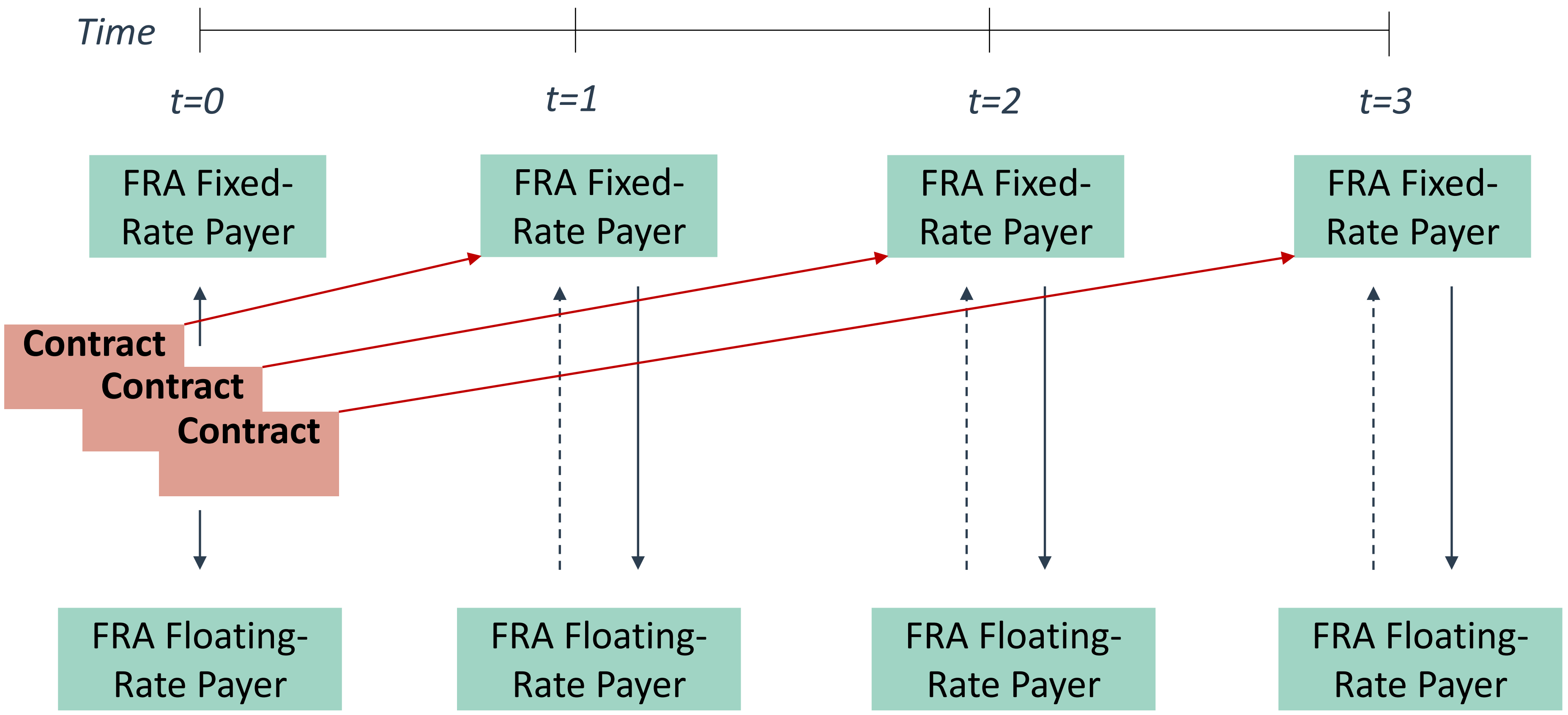
    s_3 = 3.605% = 0.10159 ÷ 2.81819.
  - A is incorrect, because it discounts zero rates, not IFRs, back to the present using zero rates, while C incorrectly discounts zero rates by the respective IFRs.

⟜ Swap Pricing & Valuation through Replication ⊸

- **Equivalence of swaps to bonds**
  - An **interest rate swap** is identical to issuing a fixed-rate bond and using the proceeds to buy a floating-rate bond.
  - A **currency swap** is identical to issuing a fixed- or floating-rate bond in one currency, converting the proceeds to another currency, and using the proceeds to buy a floating- or fixed-rate bond in another currency.
  - An **equity swap** is identical to issuing a fixed- or floating-rate bond and using the proceeds to buy a stock or an index.
- **Equivalence of swaps to forward contracts (FRA)**
  - A forward contract is an agreement to exchange future cash flows once, so a swap can be viewed as a series of forward contracts.

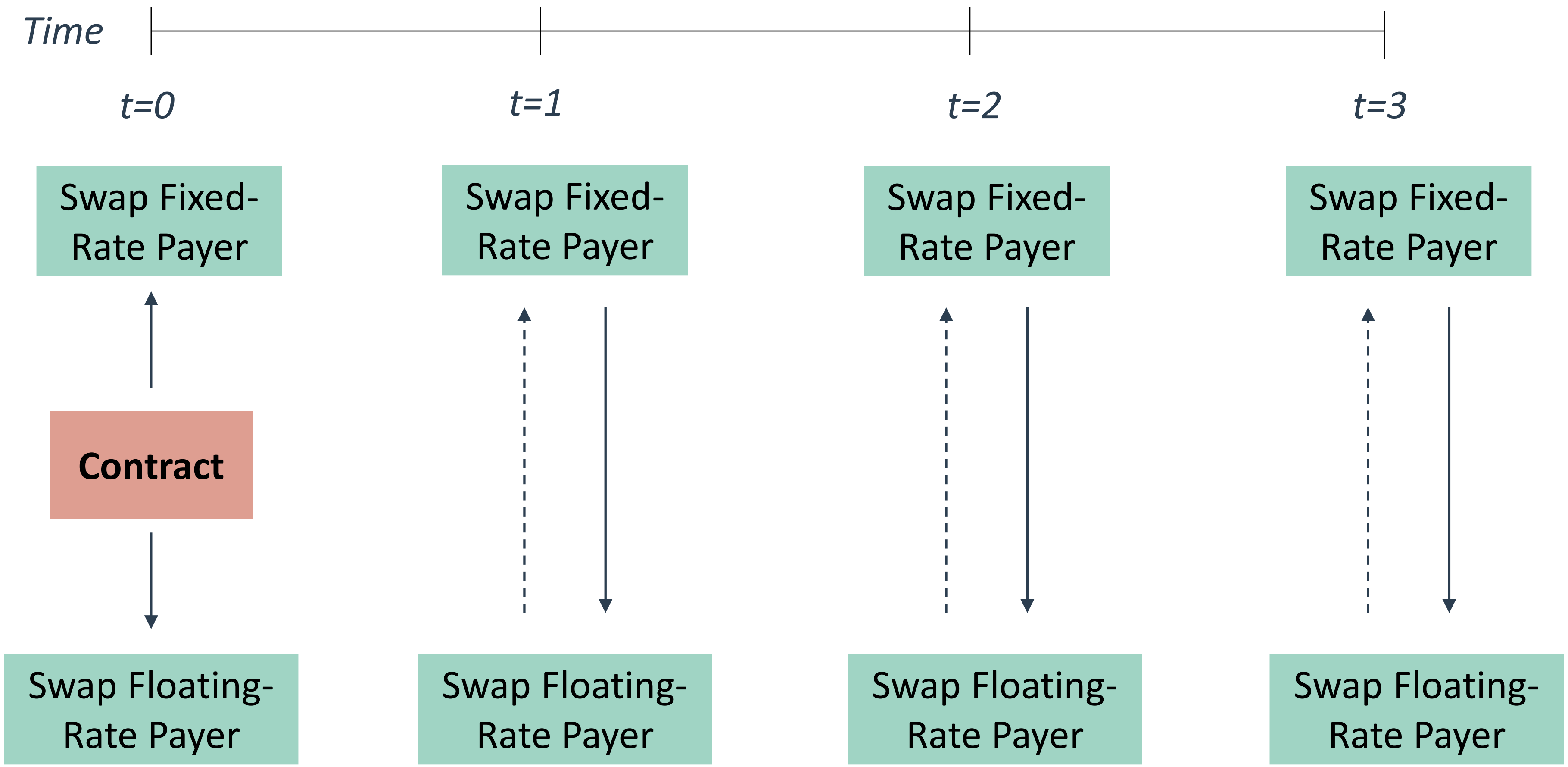
Swap vs. Forward

- **Different FRA fixed rates** usually exist for different times to maturity.



Swap vs. Forward

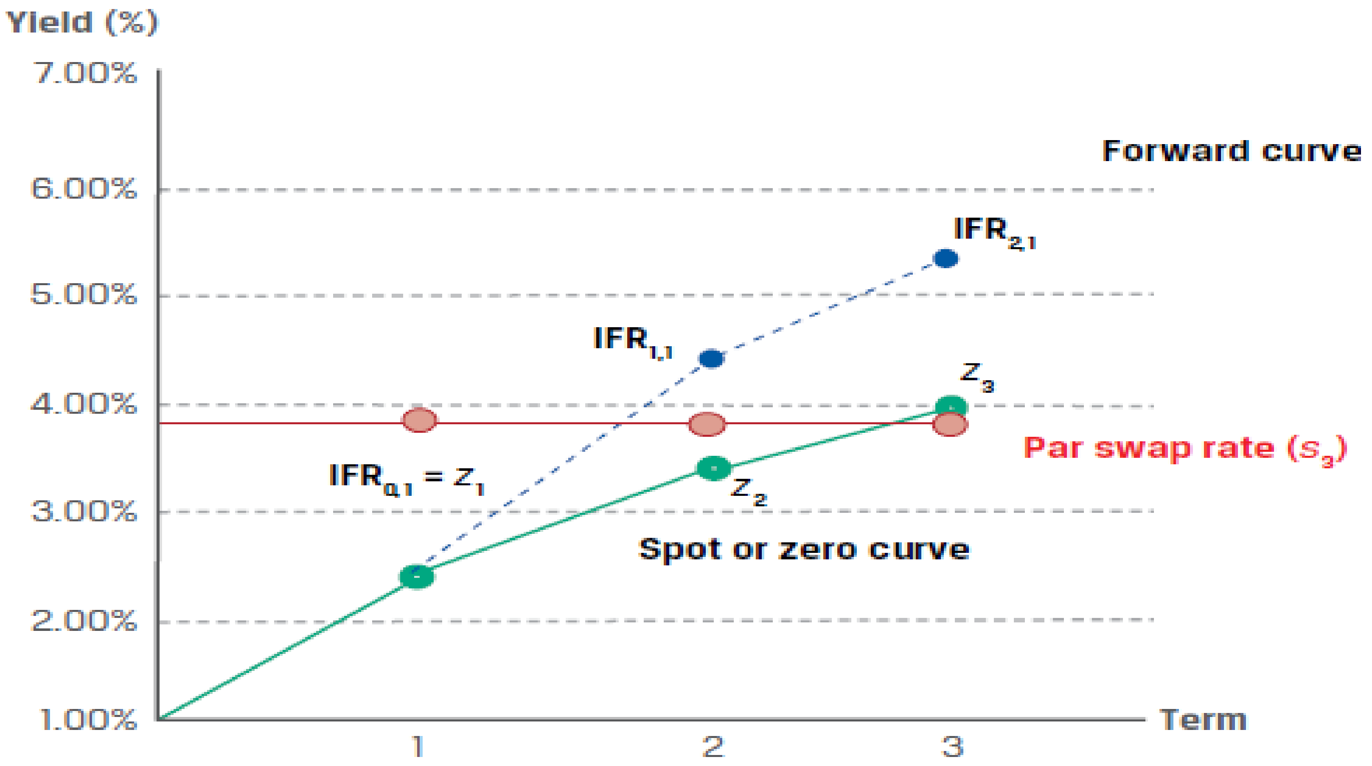
- In contrast, A standard interest rate swap has a **<u>constant fixed rate</u>** over its life, which includes multiple periods.



Swap vs. Forward

- The par swap rate is the fixed rate that equates the present value of all future expected floating cash flows to the present value of fixed cash flow
- For this reason, we may think of **the fixed swap rate as an internal rate of return on the implied forward rates** through the maturity of the swap as of t = 0.

Par swap rate, spot and forward curve



Swap Valuation

- For a swap with periodic exchanges, the current MRR is the "spot" price and the fixed swap rate, S_N, is the forward price. Restating this result for the fixed-rate payer on a swap, the periodic settlement value is:

$$\textit{Periodic settlement value} = (\textit{MRR} - S_N) \times \textit{Notional amount} \times \textit{Period}.$$

- The value of a swap on any settlement date equals the current settlement value (***Periodic settlement value***) plus the present value of all remaining future swap settlements.

Example

- Determine the correct answers to fill in the blanks: A rise in the expected forward rates after inception will __________ the present value of floating payments, causing a fixed-rate receiver to realize a(n) ____________ in MTM value on the swap contract.

- **Solution:**
  - A rise in the expected forward rates after inception will *increase* the present value of floating payments, causing a fixed-rate receiver to realize a *decline* in MTM value on the swap contract.

Example

- A swap is equivalent to a series of:
  A. forward contracts, each created at the swap price.
  B. long forward contracts, matched with short futures contracts.
  C. forward contracts ,each created at their appropriate forward prices.

- **Solution: A.**
  - Each implicit forward contract is said to be off-market, because it is created at the swap price, not the appropriate forward price, which would be the price created in the forward market.

- The price of a swap typically:
  A. is zero at initiation.
  B. fluctuates over the life of the contract.
  C. is obtained through a process of replication.

- **Solution: C.**

Summary

Pricing and Valuation of Interest Rates and Other Swaps

Pricing and valuation of swaps

Swap Pricing

Swap Pricing & Valuation through Replication

Swap vs. Forward

Swap Valuation

Summary

Module: Pricing & Valuation of Interest Rates and Other Swaps

Swap

Pricing and valuation of swaps

Module

Pricing and Valuation of Options

1. Basic concepts of options
2. Option payoff and profit
3. Option Valuation

Basic concepts of options

- Define options (calls and puts), and credit derivatives and compare their basic characteristics

Option

- **Basic Concepts**
  - **Definition of option:** An option is a derivative contract in which one party, the buyer, pays a sum of money to the other party, the seller or writer, and receives the right to either buy or sell an underlying asset at a fixed price either on a specific expiration date or at any time prior to the expiration date.
    - ✓ **Call option:** Long call & Short call.
    - ✓ **Put option:** Long put & Short put.
    - ✓ The seller or short position in an options contract is sometimes referred to as the writer of the option.
  - **Prices**
    - ✓ **Option premium:** option premium paid by the buyer of option;
    - ✓ **Exercise price:** Strike price (X) represents the exercise price specified in the contract.

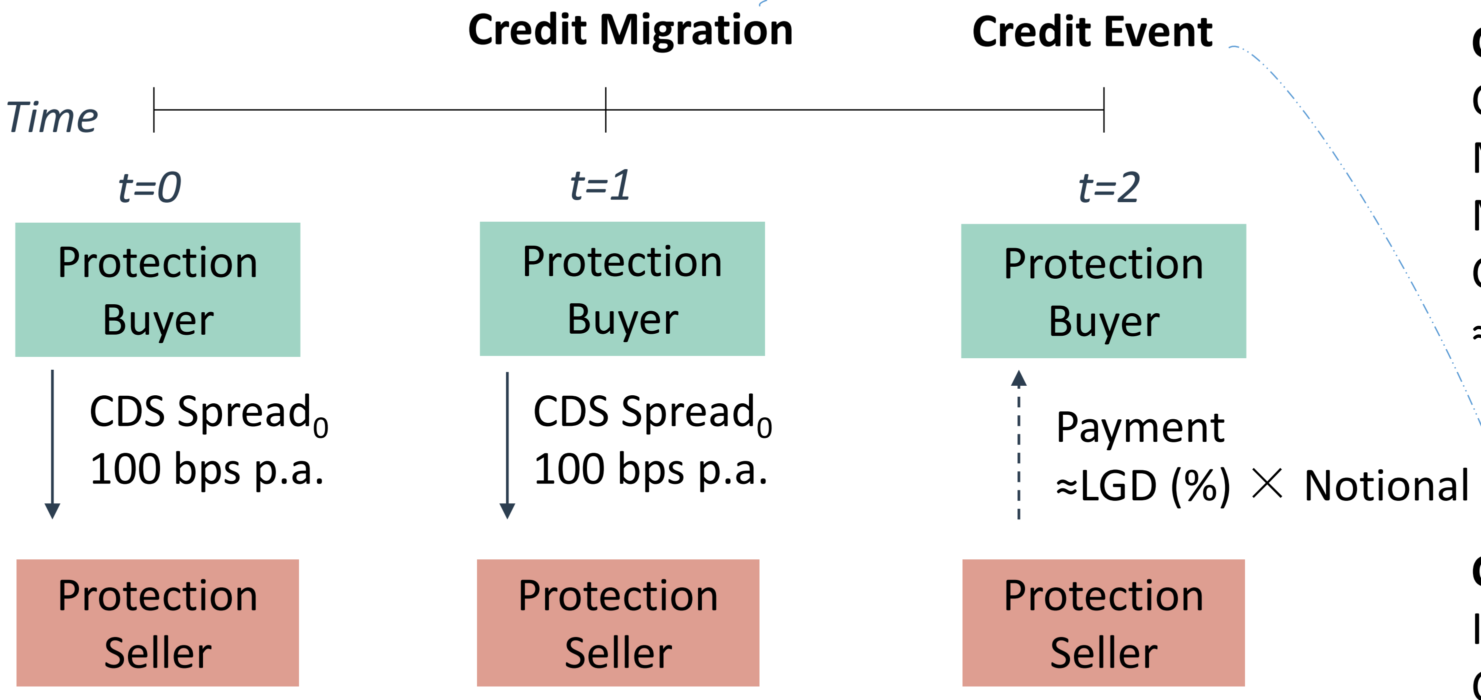
Credit derivatives

- **Credit derivatives:** based on a credit underlying, or the default risk of a single debt issuer or a group of debt issuers in an index.
- **Credit default swaps (CDS):** allow an investor to manage the risk of loss from issuer default separately from a cash bond.
  - CDS credit spreads depend on the probability of default (POD) and the loss given default (LGD)
  - Issuer's CDS spread ↓, Protection Seller faces an MTM gain
  - Contingent payment upon credit event= LGD x Notional

Periodic Payments under a Credit Default Swap

Example
CDS Contract with Credit Migration and Credit Event

- The protection buyer agrees to pay a fixed spread of 100 bps p.a. at t = 0 for the contract term. As the issuer's CDS spread widens to 250 bps p.a. at t = 1, the buyer gains on the CDS contract due to the low fixed spread paid.



Credit migration
CDS $Spread_1$ = 250 bps
MTM Gain to buyer
MTM Loss to seller
CDS MTM Change
$\approx \triangle$ CDS Spread $\times$ Notional $\times$ $EffDur_{CDS}$

Credit event
Issuer default/Failure to pay CDS
Contract terminates
Payment from seller to buyer ≈
LGD (%) $\times$ Notional

Example

- **Describe how a credit protection seller's position is similar to that of an underlying cash bond investment.**

- **Solution:**
  - A credit protection seller receives a periodic CDS spread payment in exchange for the contingent risk of payment to the buyer under an issuer credit event. A cash bond investor receives a periodic coupon that incorporates an issuer's credit spread in exchange for a potential loss if the issuer defaults.
  - Under the CDS contract and the cash bond, this potential payment or loss equals the LGD. The credit protection seller's position is therefore similar to that of a long risk position in the issuer's underlying bond.

Summary

Pricing and Valuation of Options

Basic concepts of options

Option

Credit derivatives

Option payoff and profit

- Determine the value at expiration and profit from a long or a short position in a call or put option

A Qualitative View

- **Moneyness: on the long position**
  - **Moneyness**
    - ✓ **In the money**: Immediate exercise would generate a positive payoff.
    - ✓ **At the money**: Immediate exercise would generate no payoff.
    - ✓ **Out of the money**: Immediate exercise would generate a negative payoff.
  - The following table summarizes the moneyness of options based on the stock's current price, S, and the option's exercise strike price, X.

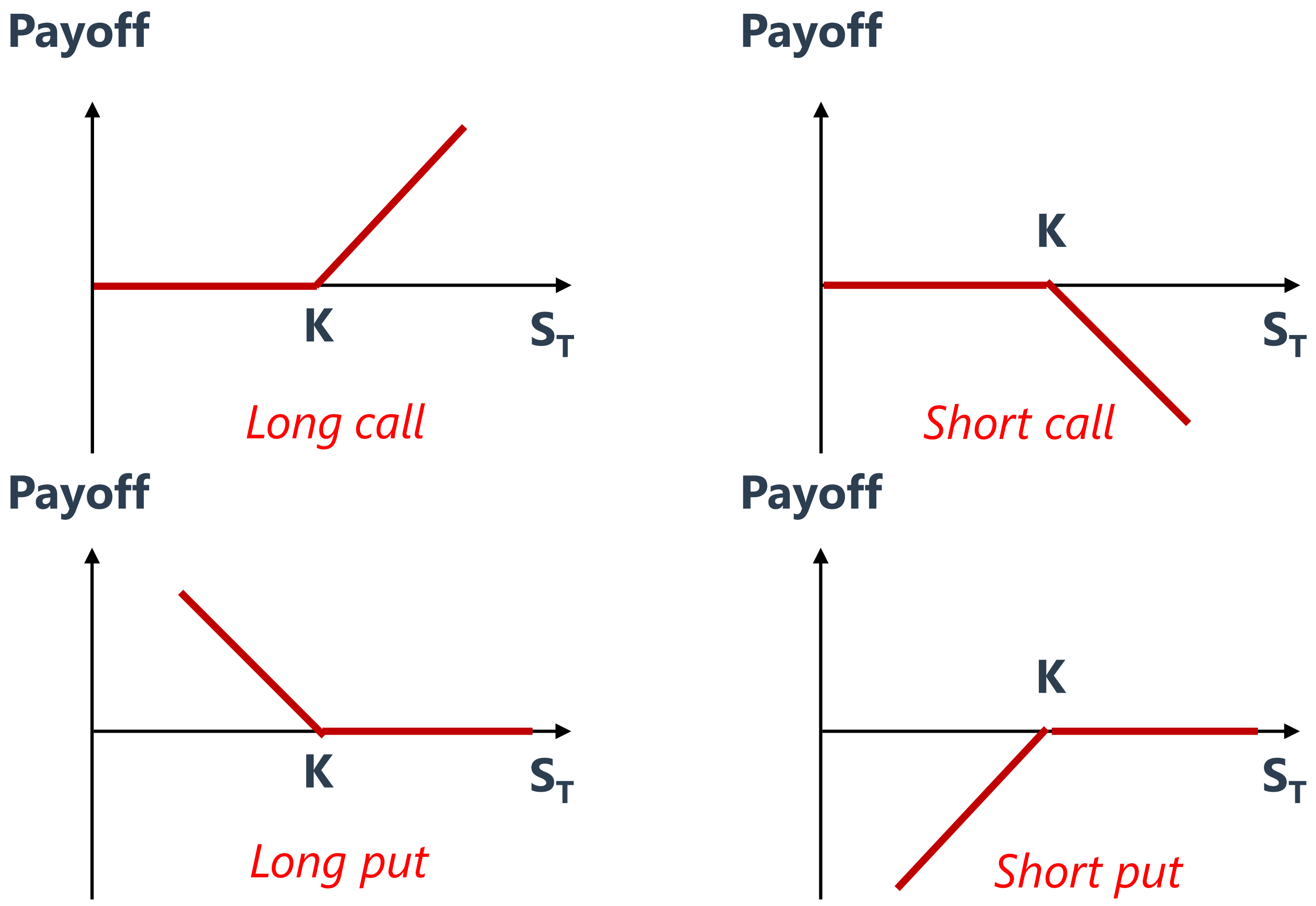
Moneyness	Call option	Put Option
In-the-money	$S > X$	$S < X$
At-the-money	$S = X$	$S = X$
Out-of-the-money	$S < X$	$S > X$

Payoff and Profits

- **Payoff for options (t=T)**
  - Long call: $c_T = Max(0, S_T - X)$
  - Short call: $c_T = -Max(0, S_T - X)$
  - Long put: $p_T = Max(0, X - S_T)$
  - Short put: $p_T = -Max(0, X - S_T)$
- **Profits for options (t=T)**
  - Long call: $c_T = Max(0, S_T - X) - c_0$
  - Short call: $c_T = -Max(0, S_T - X) + c_0$
  - Long put: $p_T = Max(0, X - S_T) - p_0$
  - Short put: $p_T = -Max(0, X - S_T) + p_0$

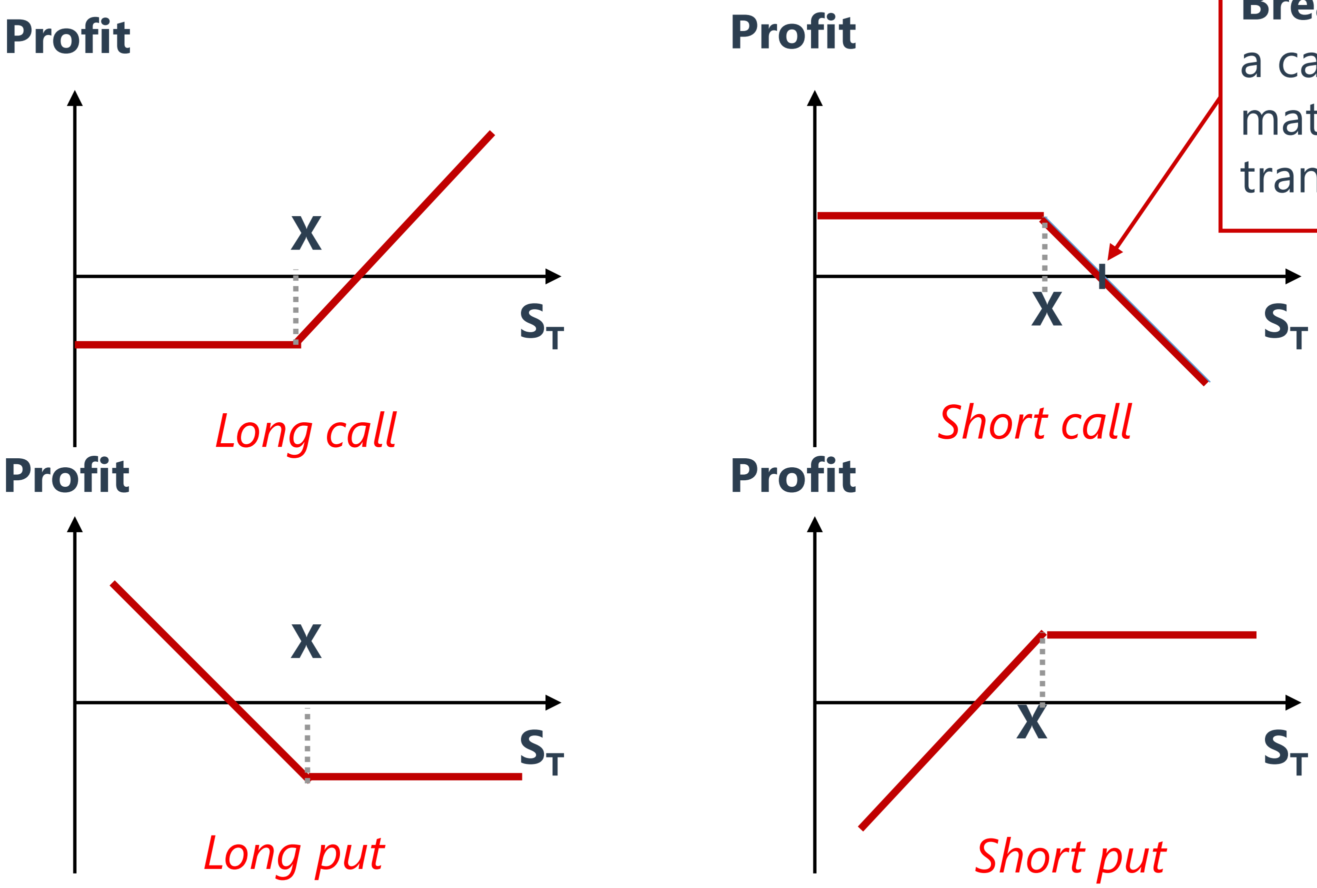
Option Payoff

- **Payoff**



Option Profits

- **Gain/Loss**



Breakeven point:
a cash settlement of zero at maturity in the absence of transaction costs with zero profit.

Forward commitments vs contingent claims

- **Long call VS. long forward, gives us the following relative profit profile between the forward and option:**
  - $S_T - F_0(T) > -c_0$ Forward profit exceeds option profit
  - $S_T - F_0(T) = -c_0$ Forward profit equals call option profit
  - $S_T - F_0(T) < -c_0$ Option profit exceeds forward profit

- **Short put VS. long forward, gives us the following relative profit profile between the forward and option:**
  - $S_T - F_0(T) > p_0$ Forward profit exceeds option profit
  - $S_T - F_0(T) = p_0$ Forward profit equals option profit
  - $S_T - F_0(T) < p_0$ Option profit exceeds forward profit

Example

- Biomian Limited is a Mumbai-based biotech company with common stock and listed futures and options on the National Stock Exchange (NSE). The Viswan Family Office (VFO) currently owns 10,000 Biomian common shares. VFO would like to reduce its long Biomian position and diversify its equity market exposure but will delay a cash sale of shares for tax reasons for six months.
- VFO's market strategist is considering a six-month call option strategy on the NIFTY 50 benchmark Indian stock market index to increase broad market equity exposure. The NIFTY 50 price today is INR15,200, and the strategist observes that a call option with a INR16,000 exercise price (X) is trading at a premium of INR1,500. Which of the following represents the payoff and profit of this strategy just prior to maturity if the NIFTY 50 is trading at INR16,500?
  - A. Payoff is INR500; profit is –INR1,000
  - B. Payoff is INR1,300; profit is INR800.
  - C. Payoff is INR1,300; profit is INR500.

Example

- **Solution: A.**
  - The profit is equal to $\Pi = \max(0, S_T - X) - c_0$, and the payoff is equal to $\max(0, S_T - X)$. The exercise price is INR16,000, and the spot price just prior to maturity is INR16,500, so $\Pi = -1{,}000$ [= (16,500 – 16,000) – 1,500], and the payoff is equal to INR500 [= (16,500 – 16,000)]

Summary

Pricing and Valuation of Options

Option payoff and profit

Payoff and Profits

Forward commitments vs contingent claims

Option Valuation

- Identify the factors that determine the value of an option and describe how each factor affects the value of an option

A Quantitative View

- **European options** are exercised **only at maturity**, while **American options** may be exercised **at any time** from contract inception until maturity.
- **Option value = intrinsic value + time value**
  - **Intrinsic Value/Exercise Value**: long position at t=t.
    - ✓ Exercise value of call option: $max[0, S_{\mathrm{t}} - X/(1+\mathrm{r})^{(T-t)}]$
    - ✓ Exercise value of put option: $max[0, X/(1+\mathrm{r})^{(T-t)} - S_{\mathrm{t}}]$
  - **Time Value**: the difference between the price of an option (called its premium) and its intrinsic value is due to its time value.
    - ✓ Before expiration: option value>intrinsic value
    - ✓ At expiration: option value=intrinsic value
- Price of the option is more **volatile** than prices of underlying stock.

Example

- Example1
  - Consider the earlier case of a one-year put option with an exercise price (X) of EUR 1,000, an initial price of EUR 990, and a risk-free rate of 1%. What is the exercise value of the option in six months if the spot price (S_t) equals EUR 950?
  - Use Equation to solve for Max (0, PV(X) − S_t):
    - ✓ $X(1 + r)^{-(T-t)} - S_t$ = EUR 1,000$(1.01)^{-0.5}$ − EUR 950 = EUR 45.04

- Example 2
  - Previous example showed that a one-year put option with an exercise price (X) of EUR 1,000 had an exercise value of EUR 45.04 with six months remaining to maturity when the spot price (S_t) was EUR 950. If we observe a current put option price (p_t) of EUR 50, what is the time value of the put option?
  - Use Equation to solve for p_t − Max (0, PV(X) − S_t):
    - ✓ $p_t - \max(0, X(1 + r)^{-(T-t)} - S_t)$ = EUR 50 − EUR 45.04 = EUR 4.96

Risk Factors on Option Valuation

- Factors affect the value of an option

Sensitivity Factor	Calls	Puts
Underlying price	Positively related	Negatively related
Volatility	Positively related	Positively related
Risk-free rate	Positively related	Negatively related
Time to expiration	Positively related	Positively related*
Strike price	Negatively related	Positively related
Payments on the underlying	Negatively related	Positively related
Carrying cost	Positively related	Negatively related

- * There is an exception to the general rule that European put option thetas are negative. The put value may increases as the option approaches maturity if the option is deep in-the-money and close to maturity.

Example

- An investor purchases an equity call option priced at CHF3 with an exercise price of CHF41. If at expiration of the option, the underlying is priced at CHF38, the profit for the investor's position is closest to:

  A. –CHF6.

  B. CHF0.

  C. –CHF3.

- **Solution: C.**

Example

- Consider a put option on Deter, Inc., with an exercise price of $45. The current stock price of Deter is $52. What is the intrinsic value of the put option, and is the put option at-the-money or out-of-the-money?

	Intrinsic Value	Moneyness
A.	$7	At-the-money
B.	$0	Out-of-the-money
C.	$0	At-the-money

- **Solution: B.**

Upper and Lower Bounds of Option

- The call buyer will not pay more for the right to purchase an underlying than the price of that underlying, which is the *upper* bound:
  - $\text{Max}[0, S_t - X(1+r)^{-(T-t)}] < c_t \le S_t$
  - $C_{t,\ \text{Lower bound}} = \text{Max}[0, S_t - X(1+r)^{-(T-t)}]$
  - $C_{t,\ \text{upper bound}} = S_t$

- A put option buyer will exercise only if the spot price, *ST*, is below *X* at maturity. The exercise price, *X*, therefore represents the *upper* bound on the put value. The *lower* bound is the present value of the exercise price minus the spot price or zero, whichever is greater:
  - $\text{Max}[0, X(1+r)^{-(T-t)} - S_t] < p_t \le X$
  - $p_{t,\ \text{Lower bound}} = \text{Max}[0, X(1+r)^{-(T-t)} - S_t]$
  - $p_{t,\ \text{upper bound}} = X$

Example

- Consider a one-year call option with an exercise price, *X*, of EUR 1,000. The underlying asset, *S0*, trades at EUR 990 at time *t* = 0 and the risk-free rate, *r*, is 1%. What are the no-arbitrage upper and lower bounds in six months' time if the underlying asset price, *St*, equals EUR 1,050?
- As the option buyer will exercise only if *ST* > *X* at *t* = *T*, the *lower* bound is equal to S_t – PV (*X*) or zero, whichever is greater:

$$C_{t,\ Lower\ bound}=Max(0, S_t-X(1+r)^{-(T-t)})$$

$$C_{t,\ Lower\ bound}=Max(0, EUR\ 1{,}050-EUR\ 1{,}000(1.01)^{-(0.5)})=Max(0, EUR\ 54.96)$$

  - The call buyer will not pay more than S_t for the right to purchase the underlying.

$$C_{t,\ upper\ bound}=S_t$$

$$C_{t,\ upper\ bound}=EUR\ 1{,}050$$

Example

- A put option seller receives a $5 premium for a put option sold on an underlying with an exercise price of $30. What is the option seller's maximum profit under the contract? What is the maximum loss under the contract?

- **Solution:**
  - A put option seller receives a $5 premium ($p_0$) for a put option sold on an underlying with an exercise price (X) of $30. The put option seller's profit is $\Pi = -\text{Max}[0, X - S_T] + p_0$.
  - If the option is unexercised, $-\text{Max}[0, X - S_T] = 0$. and the put seller earns $p_0 = \$5$.
  - If the option is exercised and $S_T = 0$, then $\Pi = -\text{Max}[0, 30 - 0] + 5 = -\$25$.
  - Therefore, the option seller's maximum profit under the contract is $5 and the maximum loss under the contract is $25.

Summary

Pricing and Valuation of Options

Option Valuations

A Qualitative View

Risk Factors on Option Valuation

Upper and Lower Bounds of Option

Summary

Module: Pricing and Valuation of Options

Basic concepts of options

Option payoff and profit

Option Valuation

Module

Option Replication Using Put–Call Parity

1. Put-call parity

Put-call parity

- Explain put–call parity for European options
- Explain put–call *forward* parity for European options

Proof of Price Parity through Replication

- A **fiduciary call** is a portfolio consisting of
  - A long position in a European call option with an exercise price of X that maturities in T years on a stock.
  - A long position in a pure-discount riskless bond that pays X in T years.
- The cost a fiduciary call is the cost of the call (C_0) plus the cost of the bond (the present value of X). Different payoffs to a **fiduciary call** are shown in the following table.

	$S_T \leq X$ (Call is out-of or at-the-money)	$S_T > X$ (Call is in-the-money)
Long call payoff	0	$S_T - X$
Long bond payoff	X	X
Total payoff	X	S_T

Proof of Price Parity through Replication

- A **protective put** is a portfolio consisting of
  - A long position in a European put option with an exercise price of X that maturities in T years on a stock.
  - A long position in the underlying stock with no cash payments and no carrying costs.
- The cost of a protective put is the cost of the put (P_0) plus the cost of the stock(S_0). Different payoffs to a **protective put** are shown in the following table.

	$\boldsymbol{S_T < X}$ (put is in-the-money)	$\boldsymbol{S_T \geq X}$ (put is out-of or at-the-money)
Long put payoff	$\mathrm{X} - S_T$	0
Long stock payoff	S_T	S_T
Total payoff	X	S_T

•— Price Parity of Options and their Underlying's —•

- **Put call parity**
  - $\boldsymbol{c + X/(1+R_f)^T = S + p}$ or $\boldsymbol{c + K/(1+R_f)^T = S + p}$

  - Positions replicating (examples):
    - ✓ Condition A $-s = -c + p - X/(1+R_f)^T$
    - ✓ Condition B $p = c + X/(1+R_f)^T - S$
    - ✓ Condition C $c = p + S - X/(1+R_f)^T$
    - ✓ Condition D $-p = -c + S - X/(1+R_f)^T$
    - ✓ Condition E $-c = -p + X/(1+R_f)^T - S$

Example

- 90-day European call and put options with a strike price of \$45 is priced at \$7.50 and \$3.70. The underlying is priced at \$48 and makes no cash payments during the life of the options. The risk-free rate is 5%. Calculate the no-arbitrage price of the call option, and illustrate how to earn an arbitrage profit.

- **Solution:**

  $C_0 = P_0 + S_0 - X/(1+R_f)^T$ = \$3.70 +\$48 − \$45/$1.05^{90/365}$ = \$7.24<\$7.5

  - Since the call is overpriced
    - ✓ we should sell the call for \$7.50 and buy the synthetic call for \$7.24.
    - ✓ To buy the synthetic call, buy the put for \$3.70, buy the underlying for \$48, and issue (sell short) a 90-day zero-coupon bond with a face value of \$45.
    - ✓ The transaction will generate an arbitrage profit of \$0.26 today.
  - As with all arbitrage trades, you want to "buy low and sell high." if put-call parity doesn't hold (if the cost of a fiduciary call does not equal the cost of a protective put), then you buy (go long in) the underpriced position and sell (go short) in the overpriced position.

Put – Call – Forward Parity

- The first portfolio (**Fiduciary call**) consist of:
  - a call option on the underlying with an exercise price of X.
  - a pure-discount bond that pays X at time T.
  - the cost of this portfolio is: $C_0 + \frac{X}{(1+R_f)^T}$
- The second portfolio (**Protective Put with a forward contract**) can be constructed by combining:
  - synthetic underlying position:
    - ✓ A pure-discount bond that pays X at time T.
    - ✓ A put option on the underlying with an exercise price of X.
  - a forward contract that is agreed to buy the underlying at FP at time T.
  - the cost of this portfolio is: $P_0 + \frac{FP}{(1+R_f)^T}$
- The payoff of the first portfolio at time T is identical to that of the second portfolio.

$$C_0 + \frac{X - FP}{(1 + R_f)^T} = P_0$$

Example

- Consider the Viswan Family Office example using a long forward and a risk-free bond, rather than a cash underlying position as in the prior example. Biomian shares trade at a price (*S*0) of INR295 per share. VFO is considering the purchase of a six-month put on Biomian shares at an exercise price (*X*) of INR265. If VFO's chief investment officer observes a traded six-month call option price of INR59 per share for the same INR265 exercise price, what should he expect to pay for the put option per share if the relevant risk-free rate is 4%?
- From Equation 5, the put–call forward parity relationship is：
  - $p_0 - c_0 = [X - F_0(T)](1 + r)^{-T}$.
- Substituting terms and solving for *F*0(*T*) = INR300.84 (= INR295(1.04)0.5),
  - p_0 – INR59 = (INR265 – INR300.84)$(1.04)^{-0.5}$.
  - p_0 = INR23.86.
- VFO should expect to pay a six-month put option premium of p_0 = INR23.86

Example

- Match the following statements about replication strategies with their associated derivative instrument(s)

1.At time t = 0, borrow at the risk-free rate and purchase the underlying at *S0*.	A. Both a call option and a put option replication strategy
2. The strategy requires adjustment over time as the likelihood of exercise changes.	B. A call option replication strategy
3. At time t = T, receive the loan repayment and purchase the underlying at *ST*.	C. A put option replication strategy if exercised

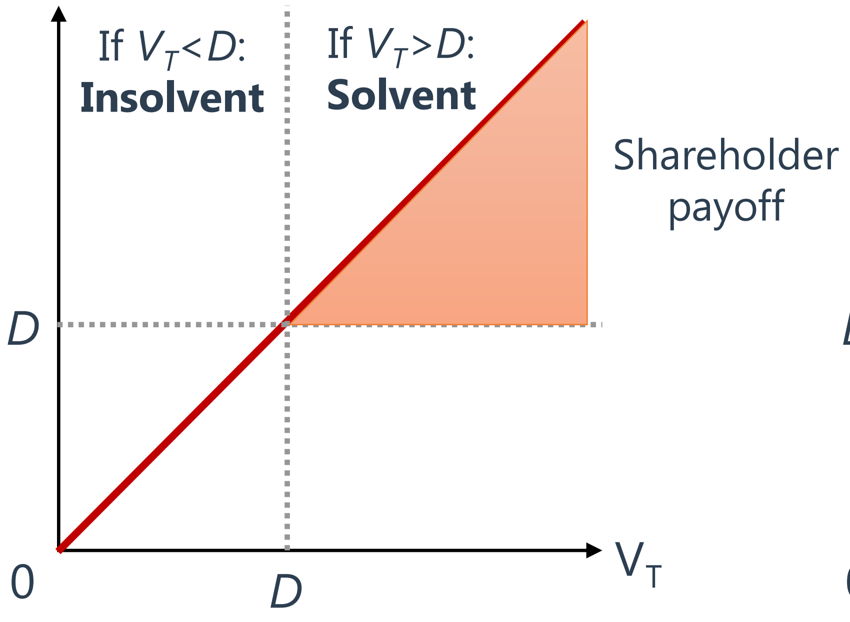
- **Solution:**
  - **1.** B is correct. At time t = 0, a call option replication strategy involves borrowing at the risk-free rate and purchasing the underlying.
  - **2.** A is correct. As both call and put options have a non-linear payoff profile, the replication strategy requires adjustment over time as the likelihood of exercise changes.
  - **3.** C is correct. A put option replication strategy if exercised involves receiving the loan repayment and purchasing the underlying at *ST* at time t = T.

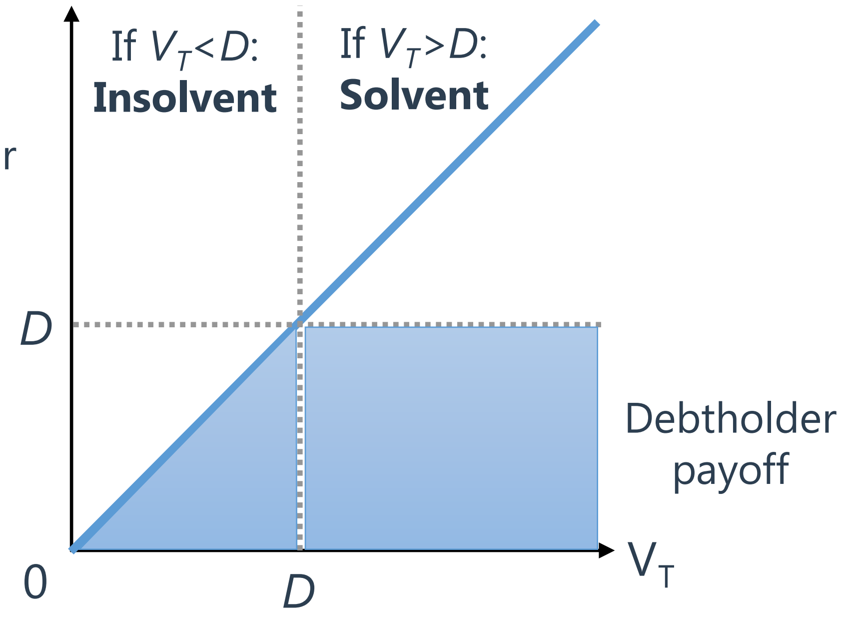
Put–call Parity Applications: Firm Value

- Assume ($t=0$) a firm with a market value (V_0) has a equity value E_0 and has access to borrowed capital in the form of zero-coupon debt with a face value (D). $V_0 = E_0 + PV(D)$.
- When the debt matures at T, the firm's debt and assets are distributed between shareholders and debtholders with two possible outcomes depending on the firm's value (V_T):
  - **Solvency:** If $V_T > D$, the firm is *solvent* and able to return capital to *both* its shareholders and debtholders.
    - ✓ Debtholders receive D and are repaid in full.
    - ✓ Shareholders receive the residual: $E_T = V_T - D$.
  - **Insolvency**: If $V_T < D$, the firm is insolvent. In the event of insolvency, shareholders receive nothing and debtholders are owed more than the value of the firm's assets. Debtholders therefore receive V_T to settle their debt claim of D at time T.
    - ✓ Debtholders have a priority claim on assets and receive $V_T < D$.
    - ✓ Shareholders receive the residual, $E_T = 0$.

Put–call Parity Applications: Firm Value

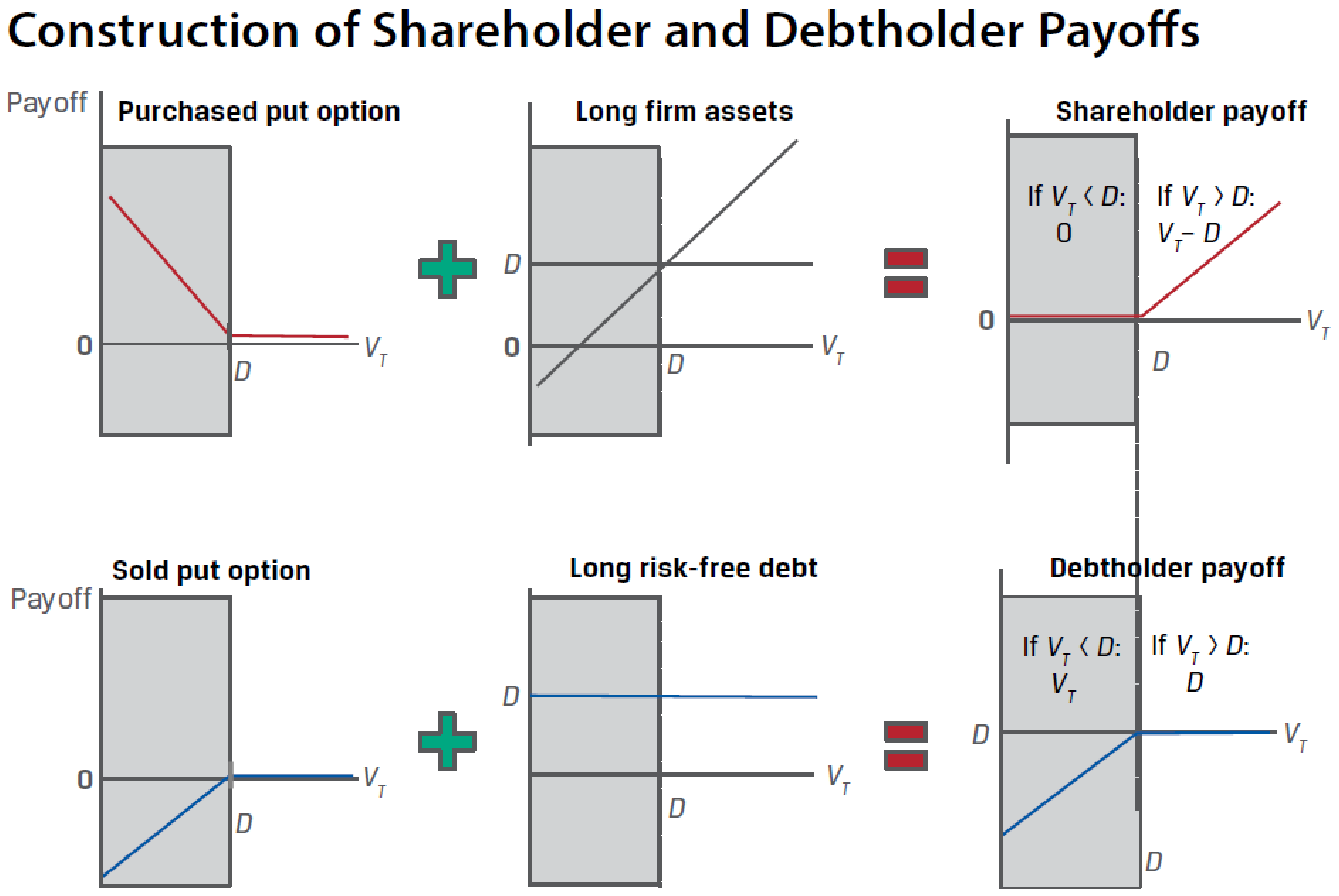
- **Firm value distribution between shareholders and debtholders**
  - Shareholder
    - ✓ Position: long asset, long put
    - ✓ Payoff: Max $(0, V_T - D)$
  - Debtholder
    - ✓ Position: long bond, short put
    - ✓ Payoff = Min (V_T, D) = $D - \max(0, D - V_T)$





Put–call Parity Applications: Firm Value

Shareholder and debtholder payoff at time T



- Consider these payoff profiles in terms of options:
  - Shareholders hold a long position in the underlying firm's assets (V_T) and have purchased a put option on firm value (V_T) with an exercise price of D; that is, max(0, $D - V_T$).
  - Debtholders hold a long position in a risk-free bond (D) and have sold a put option to shareholders on firm value (V_T) with an exercise price of D.

Example

- Based on put-call parity application, the firm value is least likely be described as:
  A. If the value of the firm (V_T) is below the face value of its debt outstanding, debtholders will receive less than the face value (D) to settle their debt claim.
  B. A debtholder's payoff is min(D, V_T) and equals the debt face value (D) minus a put option on firm value (V_T) with an exercise price of D.
  C. A debtholder's position may be considered similar to the long put option on firm value.

- **Solution: C.**
  - If the value of the firm (V_T) is below the face value of its debt outstanding, or V_T < D at time T, we say the firm is insolvent and debtholders receive less than the face value (D) to settle their debt claim.
  - Stated differently, a debtholder's payoff is min(D, V_T) = D – max(0, D – V_T) and equals the debt face value (D) minus a put option on firm value (V_T) with an exercise price of D, which represents a sold put on firm value.

Summary

Option Replication Using Put–Call Parity

Put-call parity

Proof of Price Parity through Replication

Price Parity of Options and their Underlying's

Put – Call – Forward Parity

Put–call Parity Applications: Firm Value

Summary

Module: Option Replication Using Put–Call Parity

Put-call parity

Module

Valuing a Derivative

Using a One-Period Binomial Model

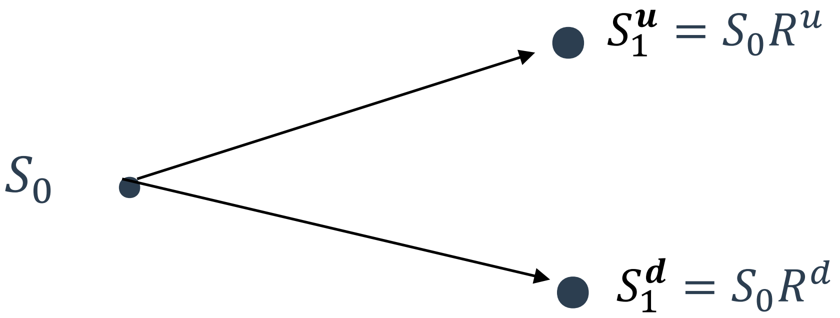
1. Option pricing: one-period binomial model

Option pricing: one-period binomial model

- Explain how to value a derivative using a one-period binomial model
- Describe the concept of risk neutrality in derivatives pricing

Option Pricing – Binomial Model

- A **binomial model** is for pricing options in which the underlying price can move to only one of two possible new prices.
- Valuing an option by using one binomial period:
  - The underlying stock price (S_0 at time 0) moves to two new prices at option expiration.
  - One period later, the stock price can move up to $S_1^{\boldsymbol{u}}$ or down to $S_1^{\boldsymbol{d}}$.
  - Identify a factor, R, as the up move on the stock and d as the down move. Thus, $S_1^{\boldsymbol{u}} = S_0 \times R^u$and $S_1^{\boldsymbol{d}} = S_0 \times R^d$. Assume that $R^u = 1/R^d$.



Option Pricing – Binomial Model

- **Risk-neutral probability (π)** of an increase in the underlying price is π_u ; risk-neutral probability (π) of a decrease in the underlying price is π_d = 1- π_u

$$\pi_u = \frac{1+R_f-R_d}{R_u-R_d}$$

- We start with a call option. If the stock goes up to $S_1^{\boldsymbol{u}}$, the call option will be worth $c_1^{\boldsymbol{u}}$. If the stock goes down to $S_1^{\boldsymbol{d}}$, the call option will be worth $c_1^{\boldsymbol{d}}$. We know that the value of a call option will be its intrinsic value on expiration date. Thus we get: $c_1^{\boldsymbol{u}}$=Max (0, $S_1^{\boldsymbol{u}}$−X); $c_1^{\boldsymbol{d}}$=Max (0, $S_1^{\boldsymbol{d}}$−X)

Value of an option: $c_0 = [\pi_u C_1^{\boldsymbol{u}} + \pi_d C_1^{\boldsymbol{d}}] \times \frac{1}{(1+R_f)^T}$

Example

Option Pricing – Binomial Model

- Calculate the value today of a 1-year call option on the stock with the strike price of \$20. The price of the stock is \$20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.

- **Solution:**
  - Step 1: Calculate the parameters:
    - ✓ u=1.25; d=1/u=0.8; S_u=20×1.25=25; S_d=20×0.8=16
    - ✓ C^+ = Max (0, 25−20) = 5; C^- = Max (0, 16−20) = 0
  - Step 2: Calculate risk-neutral probabilities:
    - ✓ π_u = (1+0.07−0.8)/(1.25−0.8) = 0.6
    - ✓ $\pi_d = 1 - \pi_u = 0.4$

Example

Option Pricing – Binomial Model

- Step 3: Draw the one-period binomial tree

π_u=0.6 $\qquad$ $S^+ = S_u$=25; c^+ = 5

(c_0=?)
S_0=20

π_d=0.4 $\qquad$ $S^- = S_d$=16; c^- = 0

today $\qquad$ year 1

- $C_0 = \frac{C^+ \times \pi_u + C^- \times \pi_d}{1+Rf} = \frac{5\times0.6+0\times0.4}{1+7\%}$ =2.8037

Hedge ratio

- **Hedge ratio (h^*: *shares per option*)**

  Call option: $h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$ put option: $h^* = \frac{p_1^u - p_1^d}{S_1^u - S_1^d}$

- **A risk-free portfolio:**
  - Call option: $V_0 = h^* S_0 - c_0 = PV(h^* S_1^u - c_1^u) = PV(h^* S_1^d - c_1^d)$
  - Put option: $V_0 = h^* S_0 + p_0 = PV(h^* S_1^u + p_1^u) = PV(h^* S_1^d + p_1^d)$

Example

- Calculate the value today of a 1-year call option on the stock with the strike price of $20. The price of the stock is $20 now, and the size of an up-move is 1.25. The risk-free rate is 7%.
- **Answer:**
  - Step 1: Calculate the parameters:
    - ✓ $u=1.25$; $d=1/u=0.8$; $S_u=20\times1.25=25$; $S_d=20\times0.8=16$
    - ✓ $c^+ = \text{Max}(0, 25-20) = 5$; $c^- = \text{Max}(0, 16-20) = 0$
  - Step 2: Calculate hedge ratio,:
    - ✓ $h=(5-0)/(25-16)=5/9$
  - Step 3: Calculate call option: $c_0=hS_0 + PV(-hS_1^- + c_1^-) = hS_0 + PV(-hS_1^+ + c_1^+)$
    - ✓ $c_0 = 5/9*20+(-5/9*25+5)/(1+7\%)=2.80$ or $c_0 = 5/9*20+(-5/9*16+0)/(1+7\%)=2.80$

Summary

Valuing a Derivative Using a One-Period Binomial Model

Option pricing: one-period binomial model

Option Pricing – Binomial Model

Hedge ratio

Summary

Module: Valuing a Derivative Using a One-Period Binomial Model

Option pricing: one-period binomial model

问题反馈

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    - ✓ 所在班级
    - ✓ 问题所在科目(若未知科目，请提供章节、知识点和页码）
    - ✓ 您对问题的详细描述和您的见解
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求知若饥，谦卑若愚

Stay hungry, Stay foolish