

## Graded Homework 3

 $\in$        $\subseteq$        $\subset$        $\cap$        $\cup$        $\mathcal{R}$        $\emptyset$        $\mathcal{U}$ 

1. Let  $S = \{2, 4, \{3, 4\}, 5, \{6\}, 8\}$  be a set.

(a) Mark true or false for each of the following:

i.  $\{2\} \in S$

ii.  $\{2\} \subset S$

iii.  $\{6\} \in S$

iv.  $\{6\} \subset S$

v.  $\{\{3, 4\}\} \subset S$

(b) Give a partition of  $S$ .

A. i. true

ii. false

iii. false

iv. true

v. true

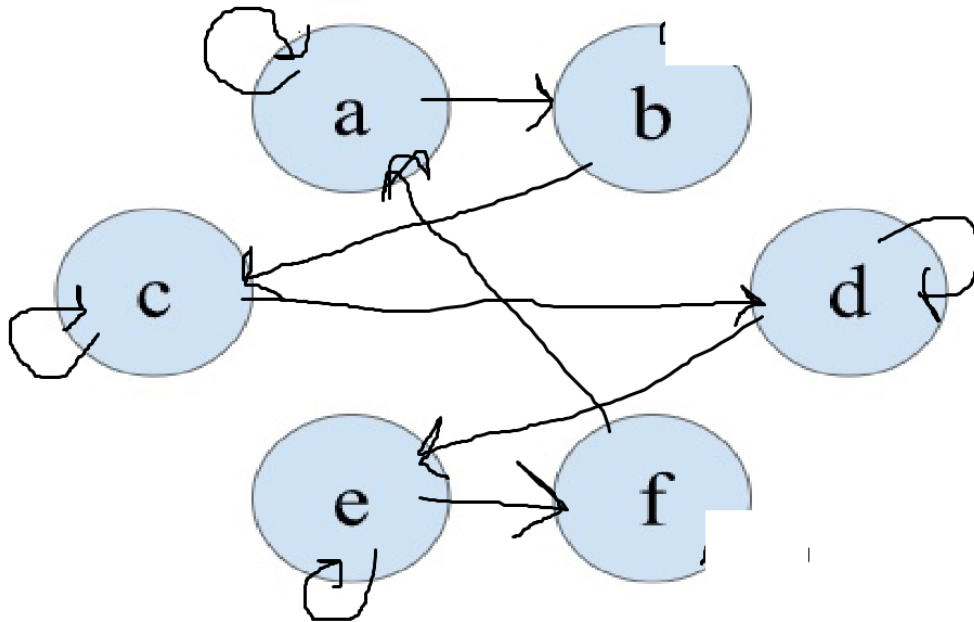
b.  $S_1 = \{2, 4, 5, 8\}$

$S_2 = \{3, 4\}$

$S_3 = \{6\}$

2. Let  $f$  be a function from the set  $A$  to the set  $B$ .
- (a) Say that the size of  $A$  is 6 and the size of  $B$  is 10. Is it possible for  $f$  to be onto? Is it possible for  $f$  to be one-to-one? Why or why not?
  - (b) Say that the size of  $A$  is 8 and the size of  $B$  is 5. Is it possible for  $f$  to be onto? Is it possible for  $f$  to be one-to-one? Why or why not?
  - (c) If we know that  $f$  is a bijection, what can we assume about the size of  $A$  and the size of  $B$ ?
- a) No, it is not possible for  $f$  to be onto. Yes, it is possible for  $f$  to be one-to-one. It can't be onto because not every element of  $A$  can map to every element of  $B$ . It can be one-to-one because every element of  $A$  can map to some element of  $B$ .
  - b) It is possible for  $f$  to be onto, but it cannot be one-to-one. Every element of  $A$  can map to some element of  $B$ , but it's not one-to-one because not every element of  $A$  has an exclusive  $B$ .
  - c) We can assume that set  $A$  and set  $B$  have the same size, because every  $A$  would have to map to exactly one  $B$ .

3. Let  $X = \{a, b, c, d, e, f\}$  be a set. Create a binary relation on  $X$  that is reflexive and antisymmetric.  
Represent your relation as a matrix and a digraph.



	A	B	C	D	E	F
A	1	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	1	1	0	0
D	0	0	0	1	1	0
E	0	0	0	0	1	1
F	1	0	0	0	0	0

4. Let  $Z = \{a, b, c, d, e, f, g, h, i, j\}$  be a set, and define a relation on  $Z$  as  $b > a$  if and only if  $b$  is after  $a$  in the English alphabet. The English alphabet, in order, is included below for your reference.

Find the minimal and maximal elements of the poset  $(Z, >)$ .

$$\text{Min}(Z) = j$$

$$\text{Max}(Z) = a$$

5. Determine if the following relation is an equivalence relation:  $x$  and  $y$  are integers and  $xRy$  if  $x-y = 3m$  for some integer  $m$ . Justify your reasoning.

Yes this would be an equivalence relation,

Reflexive:  $xRx$  is true because if  $x-x=0$  and 0 is divisible by all numbers including 3

Symmetric:  $yRx$  is true because  $y-x = -3m$ , which is also divisible by 3

And Transitive: if you let  $y-z = 3k$  where  $k$  is an integer and you add the equations together you will get

$x-z = 3(m+k) = xRz$ , which proves its transitive.