Mathematical Insights into Cicada Prime Periodicity

Jack Bogner

2023-11-29

Abstract

This paper investigates why periodical cicadas in nature consistently exhibit 13- and 17-year life cycles—both prime numbers—while other prime-numbered cycles such as 11 and 19 are absent. Building on the assumption that cicadas must avoid aligning their emergences with the birth years of predators, we explore the evolutionary advantages of specific periodicities using mathematical modeling. We incorporate findings from Lončarić (2015), whose predator–prey alignment framework demonstrates how cicadas emerging during predator birth seasons may suffer population loss. Using this framework, we evaluate the relative safety of different cycles, particularly focusing on why 13 and 17 may be evolutionarily optimal while 11 and 19 are not observed in nature. We conclude with a set of conjectures addressing the apparent instability or evolutionary disfavor of these unobserved cycles and highlight possible directions for further ecological and mathematical investigation.

Introduction

Why do periodic cicadas exhibit periodicity in primes? Further, why specifically 13 and 17, which are not only primes, but twin primes. The largely accepted concept is that cicada periodicity exists in primes as a survival strategy [Nature Education, 2010], in tandem with predator satiation. The logic is that when cicadas emerge in prime periods, the frequency that cicadas emerge when their predators are born is lowest. But why is this important? A great model from 2015 [Lončarić, 2015] visualizes exactly why cicadas emerging while predators are born may be detrimental to a brood's survival. The model describes the micro-evolution of predators, considering their generation length, and season of emergence, and the findings of this model will support one of our key assumptions.
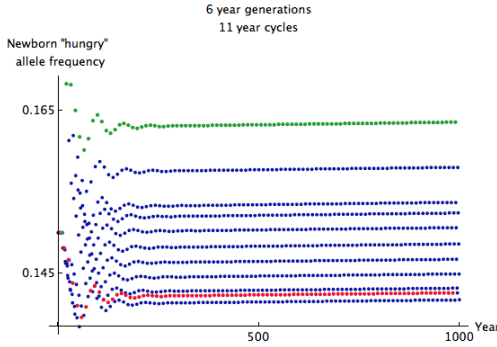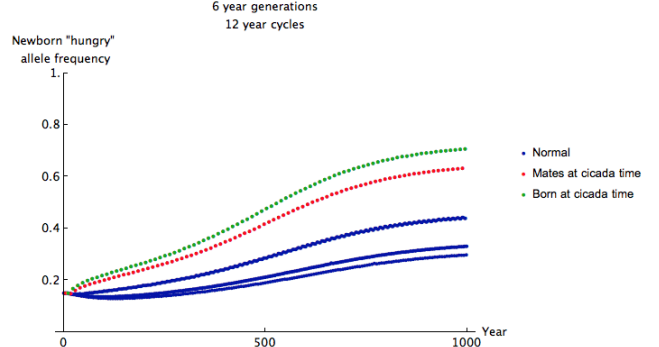

Fig 1: 11-Cycle w/ 6-Cycle

Fig 2: 12-Cycle w/ 6-Cycle

These figures highlight two important trends. First, when the predator and cicada cycles are co-prime, the resulting allele frequency remains low and stable over time. In contrast, when one cycle is a divisor of the other, the allele frequency increases significantly. Second, in both cases, the allele frequency peaks when predator births occur near cicada emergences. These observed dynamics form the basis for the assumptions used throughout our model.

Assumptions

1. All organisms emerge in the same season

This includes predators and prey: all predator offspring will be born in the season that cicadas emerge.

2. Predators only reproduce once

3. Predators of different cycles have an equivalent catch rates

Predators of different cycles eat the same amount of prey. This means considering two 3-cycle predators is logically equivalent to considering one 3-cycle predator that eats twice as much prey.

4. Declared cycles remain constant

In the real world these cycles would be in constant evolutionary flux, which is part of what makes the system so complicated. To simplify our model we will assume the cycles are constant and not changing.

Calculating Cycle Similarity

Z will be defined as the similarity between two numbers, and will be constructed of two main components: Z_{div}, Z_{diff}. The components, and Z as a whole are a function of the length of reproductive cycles of the predators and prey (C_{Pred}, C_{Prey}). We will allow C_{Pred} and C_{Prey} to change, and collect a range-normalized matrix, $\hat{Z}$, of size C_{Prey} by C_{Pred}. The formula for $Z(C_{Prey}, C_{Pred})$ is as follows:

$$Z = Z_{div} + Z_{diff} = \frac{1}{\omega_1 - \tau(C_{Pred}, C_{Prey})} + \frac{1}{\omega_2 + |C_{Prey} - C_{Pred}|}$$

Z_{div} compares the number of similar divisors between the numbers. The function $\tau(C_{Pred}, C_{Prey})$ counts the number of shared divisors between the cycles.

Z_{diff} compares the difference between the numbers. The closer the cycles are, the larger Z_{diff} is. This models the idea that if predator and prey generations have similar length generations, predators will be more inclined to eat the prey each generation, and potentially more inclined to adapt their cycle with respect to the prey's cycle.

ω_1, ω_2 are the "weights" to be assigned to the respective components. It is important to include these weights because it is unlikely that the two components carry the same weight. Using Fig 1 and 2 we can infer it is likely that $\omega_1 < \omega_2$ since co-prime cycles quickly achieved stability, and a predator cycle that divides the prey cycle leads to very high allele frequencies.

Creating an Environment and Choosing Weights

We can create an environment by comparing a predators cycle with potential prey cycles. To help visualize the system we will allow for one degree of freedom, that is an unknown predator in the environment, which has a cycle between 1 and 20 years. So our most simple environment would be comparing prey cycles of 1 thru 20 years against predator cycles of 1 thru 20 years. To add a predator to the environment, we can compare the new predator's known cycle to our potential prey cycles, and sum this with our simple environment before normalizing the values. We can repeat this until we have our desired environment, but Z values mustn't be normalized until after the last summation. Let us create an environment with two known predators with cycles three and four, and compare a few weight ratios to determine which ratio best describes our system.
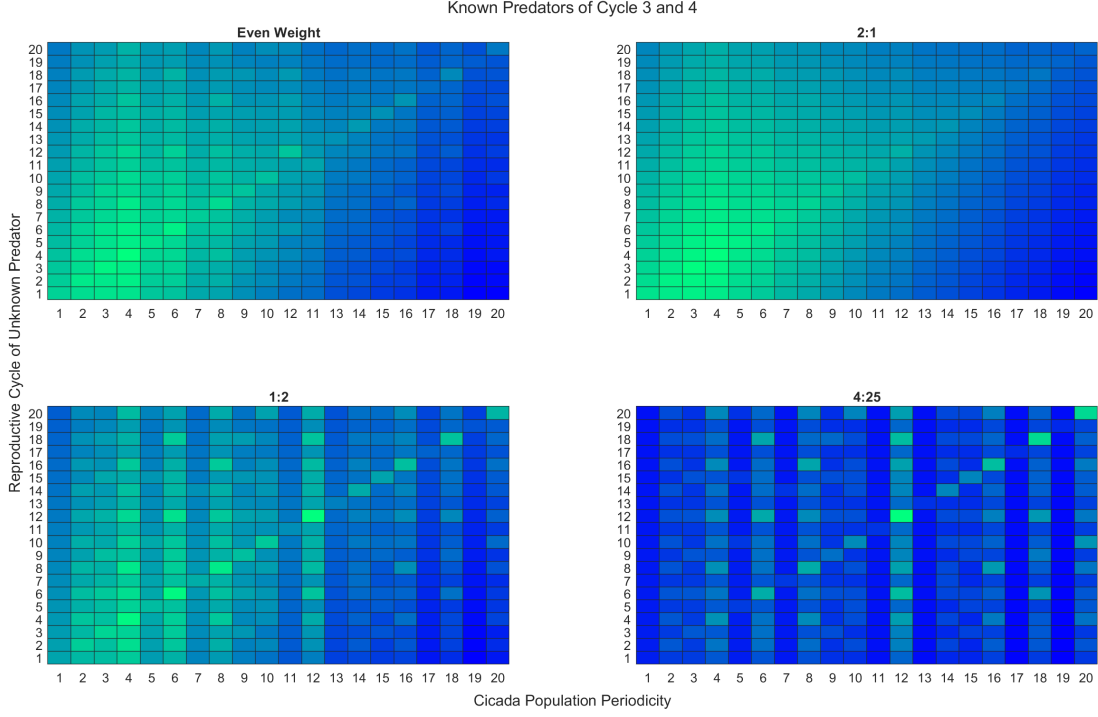
Fig. 3: Different Weight Ratios

Recall: Z is a relative measure of danger of emergence at a certain prey cycle, given a predator cycle. In Figure 4 we see how the distribution of danger changes with different weight ratios. As previously discussed, it is likely that Z_{div} carries more weight than Z_{diff}, and so in order to construct an environment we must choose weights that reflect this. Examining the four panels of figure 4, we believe a weight ratio of 1:2 is ideal for this system. At this ratio we see the expected prime behavior, and smaller primes remain more dangerous than large primes. Specifically we will choose $\omega_1 = 10$, and $\omega_2 = 20$.

With these weights we will construct an environment to simulate cicada emergence. This environment will be called "EnvA" for "Environment A." and will contain a set amount of predators, and set weights. We will consider five predator species, with unique cycles of 1 year through 5 years. That is to say, we have a predator that reproduces every year, one that reproduces every two years, one every three, etc. Soon we will model cicada populations as a function of Z in this environment, but first, what does our $\hat{Z}$ look like for this environment? This environment does not have a degree of freedom, meaning it does not have an unknown or changing predator involved, so $\hat{Z}$ is a vector instead of a matrix. In a bit we will play around with adding a degree of freedom, so lets look at $\hat{Z}$ with and without a degree of freedom.
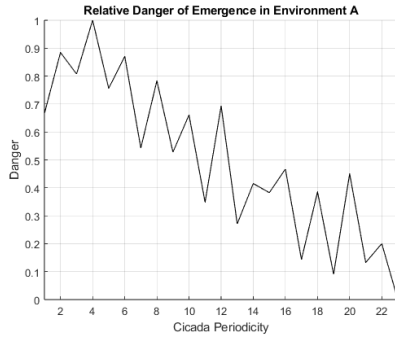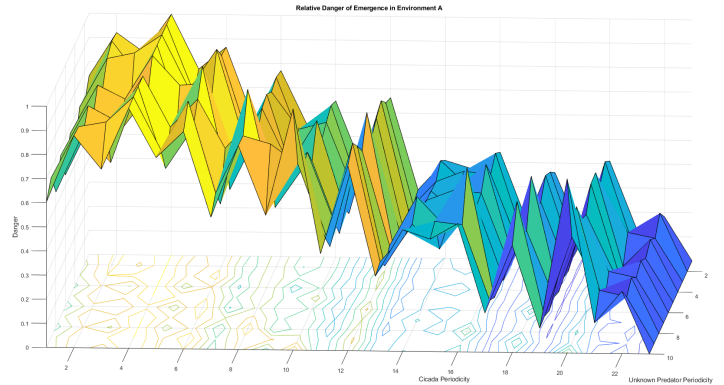

Fig. 4: Z in EnvA

Fig. 5: Z in EnvA with Degree of Freedom

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Graphing Population as a Function of Periodicity

We can consider the population of cicadas at the next emergence as a function of the population at the prior emergence, and survival rate, which is a function of cyclic overlap with predators. The equation is as follows:

$$N_{t+1} = N_t * b * (1 - \delta - \frac{Z}{b})$$

δ is the expected death rate given all predators do not anticipate emergence, and should be interpreted as the minimum death rate of the prey population.

To dissect this equation a tad, we first consider the survival rate: $(1 - \delta - \frac{Z}{b})$, which adheres to the necessary condition that $1 > (\delta + \frac{Z}{b})$. Then multiplying our initial population by survival rate yields the population that survives until reproduction. We then multiply this by the number of eggs laid by one individual to find the population of the next generation.

With this formula we may consider initial conditions and graph the populations of broods of prey with given cycles. To choose our initial conditions, we consider that female cicadas lay 200-400 eggs on average. We will assume half of the population is female, and divide average expected eggs laid by two, to account for males in the population. Thus we will consider the average eggs laid by a cicada to be 150. Further we will assume that only 1/100 cicadas survive to age of reproduction, making $\delta = 0.99$. Lastly we will choose an initial brood size of 100,000. This is smaller than brood sizes we see today, but we will only consider the populations over 50 generations.
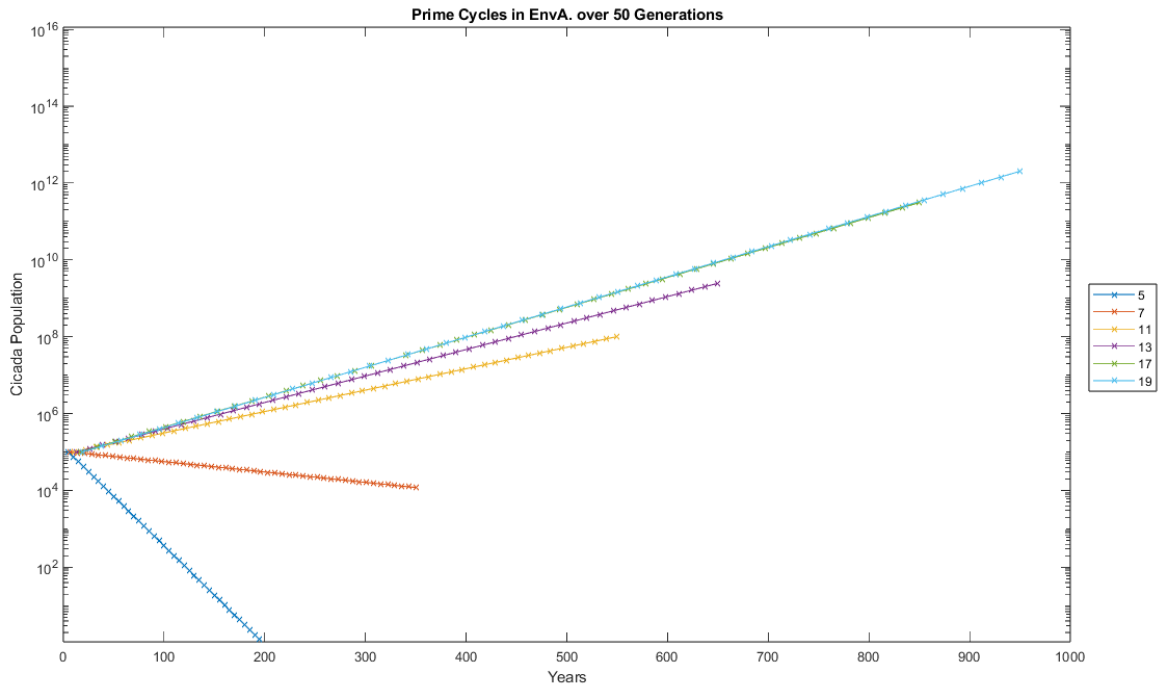

Fig. 6: Population of Prime Cycles in EnvA

Consider Assumption 3. Now suppose we know the predator of cycle 3 in EnvA eats three times as many cicadas as the others. In the eyes of our model, this means three times the danger from 3-cycle predators. Since EnvA has one 3-cycle predator, we can add two more to EnvA and observe an environment with three 3-cycle predators, or equivalently, one 3-cycle predator which is three times as dangerous. We can do the same for 5-cycle predators, and compare how a dangerous predator impacts a population.
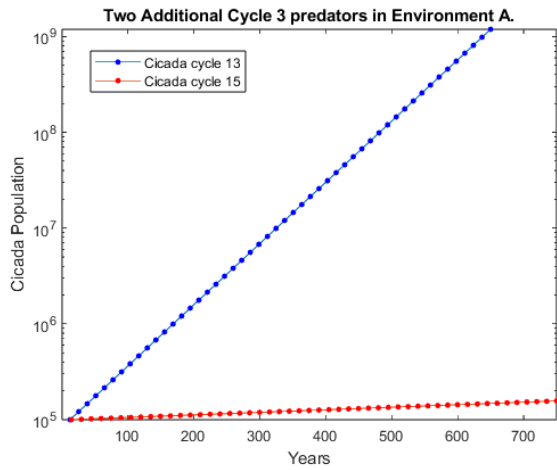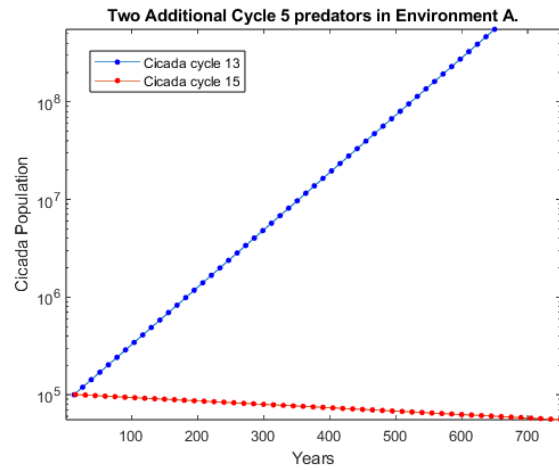
Fig. 7: EnvA + Two Cycle 3 Predators

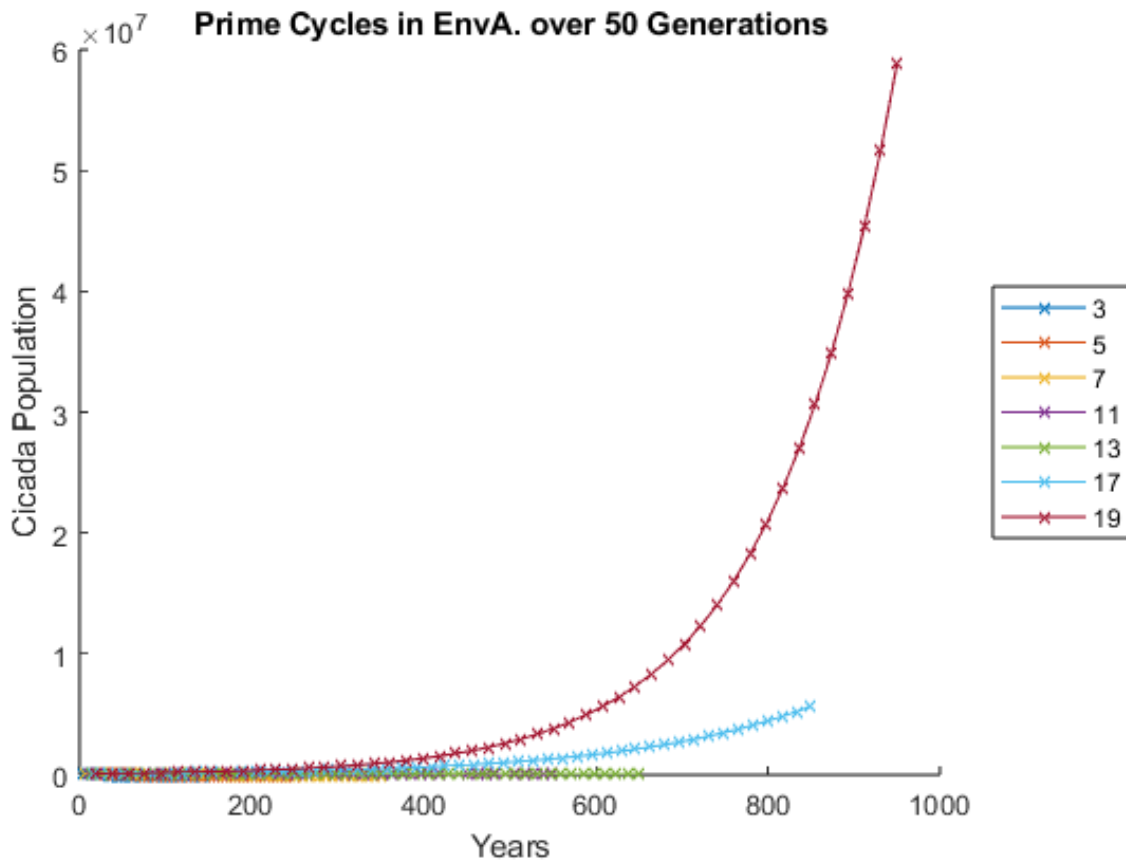Fig. 8: EnvA + Two Cycle 5 Predators

Fig. 9: Population of Prime Cycles in EnvA over 50 Generations

We see that a dangerous 3-cycle predator is not as detrimental to a population of cicadas with a period of 15, as an equally dangerous 5-cycle predator. Environment A is small and simple enough for cicadas of period 15 to survive and grow as a population. However, simple fluctuations in EnvA can force broods into decline. The prime cycles avoid the repercussions of these fluctuations, whereas composite odd numbers like 15, may be temporarily be safer than periods of 14 or 16. So we can consider period 15 to be an unstable point of equilibrium. To visualize which points are stable and which are unstable we can collapse the x-axis on Fig. 5.
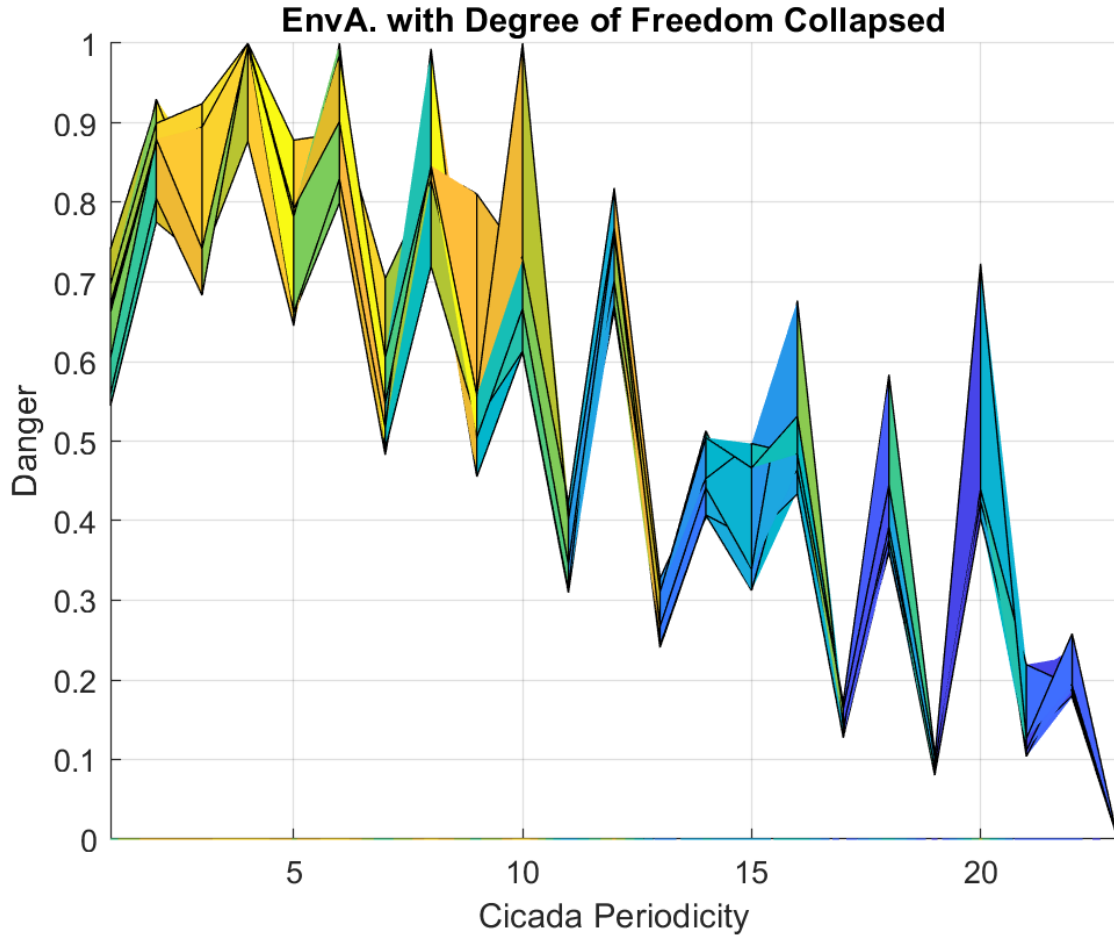
Fig. 10: EnvA with DoF, and collapsed X-axis

Discussion

Using mathematics to investigate the periodicity of cicadas presents several challenges, particularly due to the biological complexity of their life cycles and interactions with predators. This paper relies on a set of strong assumptions to model population dynamics, most notably the idea that cicadas evolve to minimize overlap between their emergence and the birth years of predators. While this assumption is a simplification, it is grounded in one of the most widely accepted theories of cicada periodicity—predator avoidance through prime-numbered cycles.

However, the limitations of this assumption become apparent when comparing model predictions to real-world observations. For instance, the model suggests that an 11-year cycle is highly stable (Fig. 10), yet no cicada broods with an 11-year periodicity have been observed in nature. Similarly, the model implies that a 19-year cycle would offer even greater protection than a 17-year cycle. This is counterintuitive given the widespread prevalence of 17-year broods compared to 13-year ones [University of Connecticut, 2023], which would seem to support the idea that longer cycles are generally more favorable.

These inconsistencies suggest that other ecological, genetic, or environmental constraints may influence which periodicities become evolutionarily stable. Factors such as developmental thresholds, habitat constraints, or diminishing returns on predator avoidance may contribute to the absence of 11- and 19-year broods, even if they appear viable in a theoretical model.

Ultimately, Figure 10 offers valuable insight into the relative safety of different cycles, but it does not fully explain why only 13- and 17-year periodicities are observed in nature. The findings emphasize the usefulness of mathematical modeling in forming hypotheses, while also underscoring the importance of integrating empirical and ecological data to fully understand cicada periodicity.

Conjectures

Conjecture 1: Insufficient Safety Among Smaller Cycles
Smaller cycles are inherently less safe, as evidenced by the long periods we observe in nature. It is possible an 11-cycle is not sufficiently safe, likely because it is simply too frequent. Part of what makes predator satiation work, is the fact that predators are unsuspecting. It could be that there is an advantage to having cycles greater than 12, since most predators will have reached their life expectancy after 12 years.

Conjecture 2: Sufficient Safety Among Larger Cycles
It is possible that 17 is sufficiently safe, and while 19 may be safer than 17, there is no reason to adopt 19 over 17. Both cycles preform very similarly, as shown in Fig 6, so it is possible that pressure to adopt a 19-year cycle is negligible since broods are able to thrive in a 17-cycle.

Conjecture 3: Instability of Cycles Adjacent to Highly Divisible Cycles
Keeping in line with the mathematical logic this model is constructed on, it is possible that cycles which are adjacent to cycles with a large amount of divisors is has a larger negative impact than our model suggests. For example, due to the close proximity to a 20 year cycle, a cicada with a 19 year cycle has a much higher probability of frequently encountering species with reproductive periods of 4 and 5, which tend to be larger animals capable of consuming more cicadas each emergence season. On the other hand, an 11 cycle could be more dangerous than a 13-cycle, due to its proximity to 10 and 12. The neighboring cycle divisors for an 11-cycle are 1,2,3,4,5,6 whereas the neighboring cycle divisors for a 13-cycle are 1,2,3,4,6,7. Cicada predators typically have a shorter reproductive cycle, with only large birds reaching 5+ years. For this reason, the 11-cycle, which neighbors cycles divisible by all predator reproductive cycles, could be more dangerous than a 13-cycle, which neighbors cycles divisible by most, but not all, predator reproductive cycles.

Conclusion

This model provides insight into why only 13- and 17-year periodic cycles are observed in cicadas. Based on both model outputs and natural observations, we can reasonably conclude that an 11-year cycle is not sufficiently safe for predator avoidance, a 13-year cycle is viable, and a 17-year cycle appears even more advantageous [University of Connecticut, 2023]. However, the model does not fully explain the absence of 11- and 19-year broods in nature, despite the apparent stability of an 11-cycle in our simulations and the seemingly superior protection offered by a 19-year cycle.

Conjectures 1 and 2 are grounded in ecological reasoning and consistent with observed emergence cycles, offering plausible explanations for the evolutionary favorability of longer, prime-numbered intervals. Conjecture 3, while more speculative, proposes that mathematical adjacency to highly divisible cycles may introduce unaccounted risks—raising an interesting direction for future exploration.

Ultimately, the absence of 11- and 19-year cicada broods suggests a more complex interplay of biological, ecological, and evolutionary factors than this model alone captures. While our approach, based on predator overlap and prime periodicity, offers a compelling framework, further research is needed to explore the role of genetic constraints, climate effects, and regional predator dynamics. This study serves as a foundation for understanding the mathematical and ecological trade-offs in cicada evolution, while highlighting the limits of purely theoretical modeling in capturing the full biological story.

References

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