

# Calculus Notes

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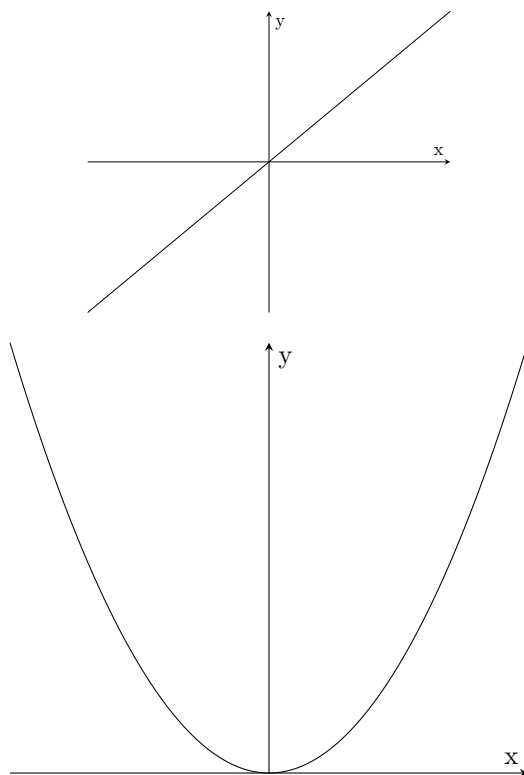
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## Part I

# Single Variable Calculus

## 1 Differentiation

### 1.1 The Derivative



$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}$$
$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\begin{aligned}
\frac{d}{dx}(x^n) &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^n - x^n}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{(x^n + nx^{n-1}\Delta x + \frac{1}{2}n(n-1)x^{n-2}\Delta x^2 + \dots) - x^n}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} \frac{nx^{n-1}\Delta x + \frac{1}{2}n(n-1)x^{n-2}\Delta x^2}{\Delta x} \\
&= \lim_{\Delta x \rightarrow 0} nx^{n-1} + \frac{1}{2}n(n-1)x^{n-2}\Delta x \\
&= nx^{n-1}
\end{aligned}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

## 1.2 The Chain Rule

$$\frac{d}{dx}f[g(x)] = \frac{df}{dg} \frac{dg}{dx}$$

## 1.3 The Product Rule

$$\frac{d}{dx}f(x)g(x) = \frac{df}{dx}g(x) + f(x)\frac{dg}{dx}$$

$$\frac{d}{dx}f(x)g(x)h(x) = \frac{df}{dx}g(x)h(x) + f(x)\frac{dg}{dx}h(x) + f(x)g(x)\frac{dh}{dx}$$

$$\begin{aligned}
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) &= \frac{df}{dx} \left( \frac{1}{g(x)} \right) + f(x) \frac{d}{dx} \left( \frac{1}{g(x)} \right) \\
&= \frac{f'(x)}{g(x)} + f(x) \left( -\frac{g'(x)}{(g(x))^2} \right) \\
&= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{(g(x))^2} \\
&= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}
\end{aligned}$$

# 2 Integration

## 2.1 The Antiderivative

$$I = \int_a^b f(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c, \int \frac{1}{x} dx = \ln(x) + c$$

## 2.2 Integration by Substitution

## 2.3 Integration by Parts

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$$

$$uv = \int u \frac{dv}{dx} dx + \int \frac{du}{dx} v dx$$

$$\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx$$

## 2.4 Surfaces and Volumes of Revolution

$$\Delta S \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$S = \int_a^b 2\pi y ds$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$V = \int_a^b \pi y^2 dx$$

$$1 \times 2 = 3, \quad 2 \times 3 = 1, \quad 3 \times 1 = 2$$

## Part II

# Multi Variable Calculus

## 3 Partial Differentiation

## 4 Multiple Integrals

## 5 Vector Calculus