Calculus Notes

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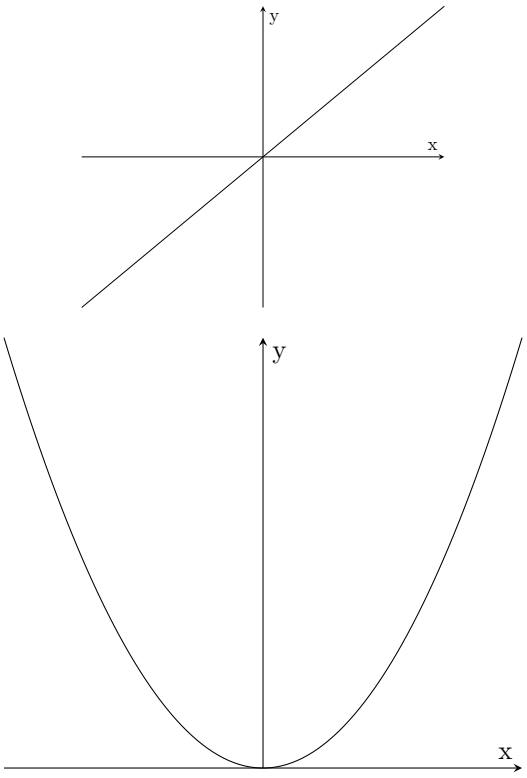
Part I
Single Variable Calculus

1 Differentiation

1.1 The Derivative



$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\begin{aligned}\frac{d}{dx}(x^n) &= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x^n + nx^{n-1}\Delta x + \frac{1}{2}n(n-1)x^{n-2}\Delta x^2 + \dots) - x^n}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{nx^{n-1}\Delta x + \frac{1}{2}n(n-1)x^{n-2}\Delta x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} nx^{n-1} + \frac{1}{2}n(n-1)x^{n-2}\Delta x \\ &= nx^{n-1}\end{aligned}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

1.2 The Chain Rule

$$\frac{d}{dx}f[g(x)] = \frac{df}{dg}\frac{dg}{dx}$$

1.3 The Product Rule

$$\frac{d}{dx}f(x)g(x) = \frac{df}{dx}g(x) + f(x)\frac{dg}{dx}$$

$$\frac{d}{dx}f(x)g(x)h(x) = \frac{df}{dx}g(x)h(x) + f(x)\frac{dg}{dx}h(x) + f(x)g(x)\frac{dh}{dx}$$

$$\begin{aligned}\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{df}{dx}\left(\frac{1}{g(x)}\right) + f(x)\frac{d}{dx}\left(\frac{1}{g(x)}\right) \\ &= \frac{f'(x)}{g(x)} + f(x)\left(-\frac{g'(x)}{(g(x))^2}\right) \\ &= \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{(g(x))^2} \\ &= \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\end{aligned}$$

2 Integration

2.1 The Antiderivative

$$I = \int_a^b f(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + c, \int \frac{1}{x}dx = ln(x) + c$$

2.2 Integration by Substitution

2.3 Integration by Parts

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$$

$$uv = \int u\frac{dv}{dx}dx + \int \frac{du}{dx}vdx$$

$$\int u\frac{dv}{dx}dx = uv - \int \frac{du}{dx}vdx$$

2.4 Surfaces and Volumes of Revolution

$$\Delta S \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$S = \int_a^b 2\pi y ds$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$V = \int_a^b \pi y^2 dx$$

$$1 \times 2 = 3, \qquad 2 \times 3 = 1, \qquad 3 \times 1 = 2$$

Part II
Multi Variable Calculus

3 Partial Differentiation

4 Multiple Integrals

5 Vector Calculus