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DEFINITIONS OF HISTORICAL VOLATILITY ESTIMATES

4 September 2007
Version: 1.00

Methods

Method 1 – Close-to-Close

$\sigma^2_{CtC} = \frac{1}{(N-1)\delta t} \sum_{n=1}^{N} \left[\ln\left(\frac{S_n}{S_{n-1}}\right) - \left(\frac{1}{N} \sum_{n=1}^{N} \ln\left(\frac{S_n}{S_{n-1}}\right)\right)\right]^2$, where observations are made after every interval of length δt, i.e. $S_n = S(t_0 + n\delta t)$

This is the standard estimator for the square of the volatility parameter, for a time-series. It is an unbiased estimator on geometric Brownian motions, with constant volatility and constant drift. σ_{CtC} is the annualized standard of the log returns.

Method 2 – Close-to-Close with Risk-Neutral Adjustment

$$\sigma^2_{CtC,RNA} = \frac{1}{(N-1)\delta t} \sum_{n=1}^{N} \left(\ln\left(\frac{S_n}{S_{n-1}}\right) - (r-d)\delta t \right)^2$$

This is the standard estimator for the square of the volatility parameter, in finance. If used to hedge a variance swap, it gives zero expected Profit-and-Loss.

Recently, a new class of estimators for volatility have come to the fore, which use intraday information, such as highs and lows, to increase the efficiency of the estimation. On simulated Geometric Brownian Motion processes, these methods have efficiencies which are ~8 times greater than the standard estimators, above. They do depend on some features of Brownian motions, however, and they are not as dominant on processes with time-varying drift, or jumps. Please see the article by Brandt and Kindlay, June 2005, “Estimating Historical Volatility”, for more details.

Let

H_n be the high price on the n th day

L_n be the low price on the n th day

O_n be the opening price on the n th day

C_n be the closing price on the n th day

$u_n = \ln H_n - \ln O_n$ be the normalized high price on the n th day

$d_n = \ln L_n - \ln O_n$ be the normalized low price on the n th day

$o_n = \ln O_n - \ln C_{n-1}$ be the normalized open price on the n th day

$c_n = \ln C_n - \ln O_n$ be the normalized close price on the n th day

Method 3 – Parkinson

The Parkinson method was the first method to use intraday high and low data to improve the efficiency of the volatility estimate.

$$\sigma_P^2 = \frac{1}{4N \ln 2} \sum_{n=1}^{N} (u_n - d_n)^2$$

Method 4 – Garman-Klass

The Garman-Klass method also uses intra-day high and low data, and has superior efficiency to the Parkinson method, and is maximally efficient when the process is geometric Brownian motion with zero drift, a good assumption as long as the process mean over the process standard deviation is small over the time horizon, often a good assumption for daily series.

$$\sigma_{GK}^2 = \frac{0.511}{N} \sum_{n=1}^{N} (u_n - d_n)^2 - \frac{0.019}{N} \sum_{n=1}^{N} \left[c_n (u_n + d_n) - 2u_n d_n \right] - \frac{0.383}{N} \sum_{n=1}^{N} c_n{}^2$$

However, there are periods when the drift dominates the volatility, periods of strong trends. The Parkinson and Garman-Klass methods will over-estimate volatility in these periods. Rogers and Satchell have devised an estimator that is accurate even when non-zero drift is present.

Method 5 – Rogers-Satchell

$$\sigma_P^2 = \frac{1}{N}\sum_{n=1}^{N}\left[u_n(u_n - c_n) + d_n(d_n - c_n)\right]$$

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