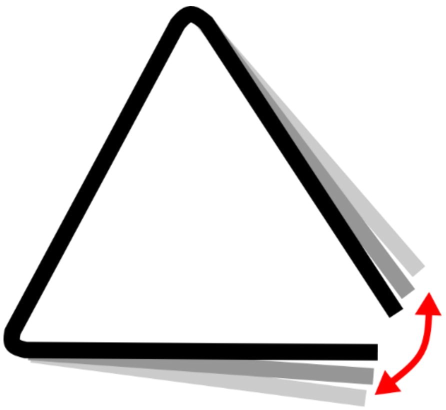

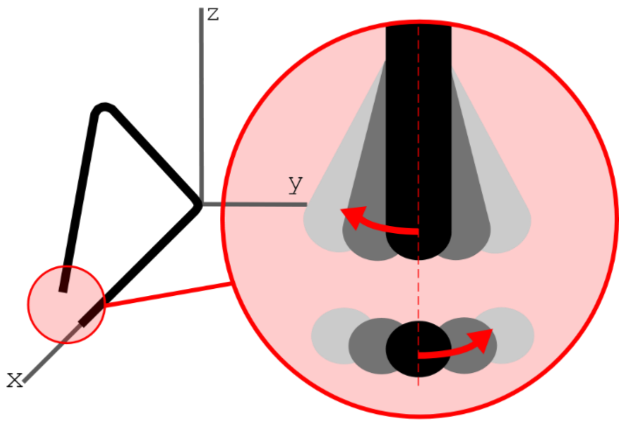

The complex sound wave of the triangle: visualised through programming

By

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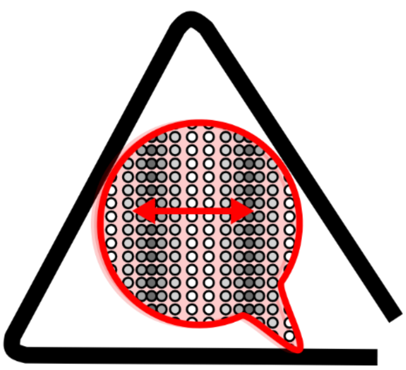

Abstract - Regardless of how complex the triangle is or isn't to play, the sound it produces is mathematically intriguing due to the sheer number of individual overlapping frequencies simultaneously produced by the instrument. This document explores, what a triangle is, how it produces sound & then describes how real-world audio recordings were broken down using software to describe the sound mathematically, thus showcasing the complex nature of the hundreds of frequencies which when combined create the unique "shimmer" of the triangle. How humans psychoacoustically perceive sound is also explored to understand how the wave produced is interpreted by the auditory system.

Introduction - The triangle is often disregarded as nothing more than a simple instrument that requires no skill to play or a mere child's toy. Very few triangle solos are written (although a few are) & even more infrequently are triangle players asked to take a bow at the end of a musical number; the few times it does happen it is accompanied by giggles & laughter from the audience.

NPR correspondent Tamara Keith on the Weekend "*Edition Saturday*" radio show summed up how complex the triangle can be to play: *"The participatory journalist George Plimpton did many things. He boxed against Sugar Ray Robinson. He tried his hand as a high wire circus performer. But he said the scariest thing he ever did was play the triangle with the New York Philharmonic."*[1]

History - The triangle most probably evolved from simpler proto-instruments which in their most basic form resembled banging two crude metal rods together. The Egyptian Sistrum is believed to be another early relative of the triangle[2]. Earlier iterations of the instrument seem to have also been adorned with metal rings on its lower leg[3] which would allow the triangle to produce a multiplicity of sounds when struck. First made in the 16th-century modern triangles are usually equilateral; however, older models could be isosceles triangles & feature curled ends. The instrument was popularised in western music by composers such as Mozart, Haydn & Beethoven. The triangle was made prominent by Franz Liszt's Piano Concerto No.1 in which the instrument even has a solo part dubbed "triangle concerto"[4].

Construction - Triangles commonly vary in size from 4 to 10 inches (Approx. 10-25 cm). Modern triangles tend to be equilateral & made of a variety of metals such as steel, brass or beryllium copper). The material properties will affect the sound being emitted, so a percussionist may have a selection of varied triangles.

Triangles with smooth surfaces & a solid construction tend to have a better sound than chrome-plated or ones with serrated edges. The triangle is struck with a beater (usually metal). Further research could easily be conducted into the material properties and how they vary the emitted sound, however, this is not investigated here.

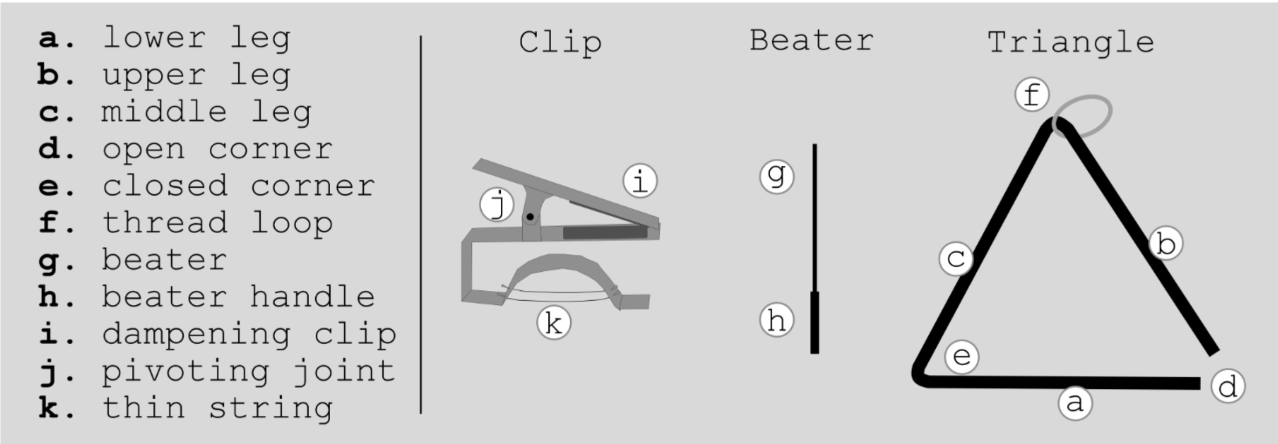

Figure 1 - A triangle, beater & clip labelled

Playing the triangle[1] - A triangle can be held in a few ways: it may be balanced directly on the percussionists, index finger, it may be suspended from a thread & held; alternatively it can be held in the hand using a clip or a clip can be used to secure it to a stand. A percussionist will often hold a triangle prominently at eye level to be able to see the triangle as well as the conductor whilst playing. If it is being clipped to a stand, this needs to be held as rigidly as possible in order not to "wobble", & the strings must have as low as possible mass per unit length in order to not be massive enough to transfer the oscillations to the stand. String used is often so light, very commonly a redundant loop will also be present in case one fails as depicted on the clip in Fig.1(k). Whichever way the instrument is held the idea is to allow it to vibrate & oscillate freely to create a rich variety of frequencies.

However, if needed a percussionist also has the option to muffle the sound at any time by placing a small amount of pressure on the upper corner they are holding to dampen the oscillations. A percussionist can sound the triangle as needed by striking it in a variety of different ways[2] & may hit the triangle in different locations at different speeds for varied effects.

For example, the percussionist may be required to strike the triangle on one of the outside legs Fig.1(a,b,c) or if a quick beat is required; repeating up & down motion of the beater inside the closed corner Fig.1(e). Most commonly the triangle is played by firmly striking the middle leg Fig.1(c) near the closed corner Fig.1(e). Some percussionists will "warm" their triangle by tapping it gently before a performance to avoid a slight clicking sound that can be generated from the first few strokes[5]. Such is the incredible amount of care taken to play a seemingly simple instrument.

How a triangle produces sound – The triangle is an idiophone[3] & primarily creates sound by vibrating in three distinct ways as shown in Fig.2.

When the triangle is struck with a beater by design it will begin to oscillate in a wide variety of ways creating thousands of frequencies that will eventually superimpose to create a sound of indeterminate pitch. The pitch is indeterminate (or incredibly hard to determine) because unlike many stringed or wind instruments, which emit just a single note, that may be a fundamental frequency with two or three overtones superimposed, the triangle literally emits an entire spectrum of wavelengths & frequencies which with relative ease can be visualised in the form of a spectrogram by using software.

Fig.3 shows a 6 second audio recording from striking a triangle a single time[4]. The microphone was able to pick up oscillations ranging from below 1kHz (lowest recorded was ~49Hz) to beyond the realms of human hearing over 20 kHz[6]. Analysing the data further, it can be seen that the strongest (highest volume) frequencies occured at ~9kHz, ~12kHz & ~16kHz, however, these oscillations die down quickly (~2s) & longer-lasting tones of ~4kHz last for just over 5 seconds.

However, not all these frequencies are created equally & it is possible to investigate where these oscillations come from. The triangle primarily vibrates in 3 ways, all of which are discussed in the following paragraphs & the frequencies produced can be analysed.

1 This section is by no means a comprehensive guide on playing the triangle. It is merely here so that a reader with no knowledge of the triangle & how it is played can get enough of an understanding to comprehend the contents of this document.
2 (Stronger vs softer hits, or glancing strikes vs direct "square" hits).
3 The entire instrument vibrates to create sound.
4 The x-axis represents The frequency of the waves, the y-axis measures time; the sound intensity in dB is shown by the colour as denoted by the legend on the right. The sound was downloaded from: https://bigsoundbank.com/detail-1689-triangle-3.html.

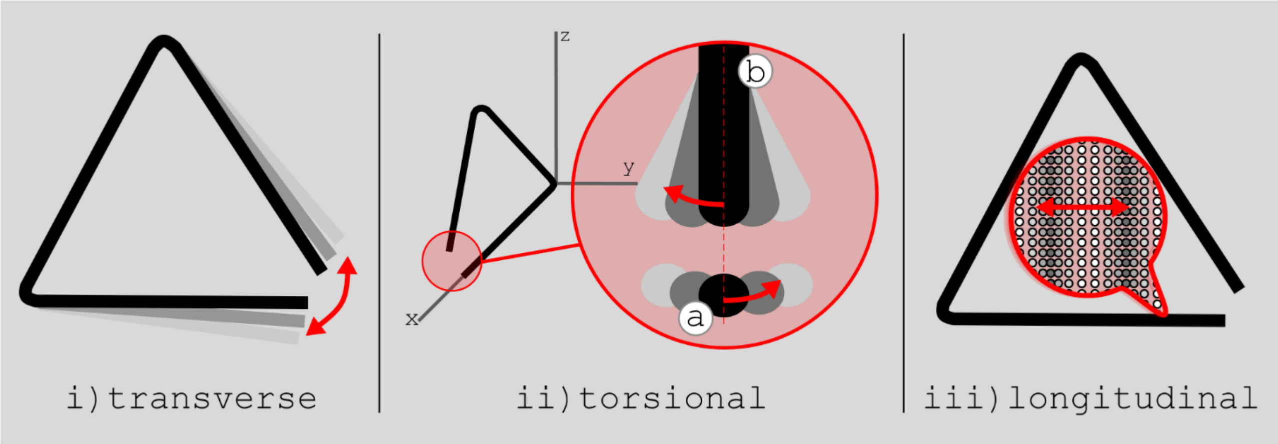

Figure 2 - Modes of vibration of the triangle

Transverse **–** The first forms of vibration to consider are the transverse ones, which occur when the open end of the triangle Fig. 1 (d) open & close in SHM in the plane of the triangle as depicted in Fig.2(i). Striking the triangle in the middle leg Fig.1(c), furthest away from either end is the easiest way to excite just the transverse oscillations described.[5]

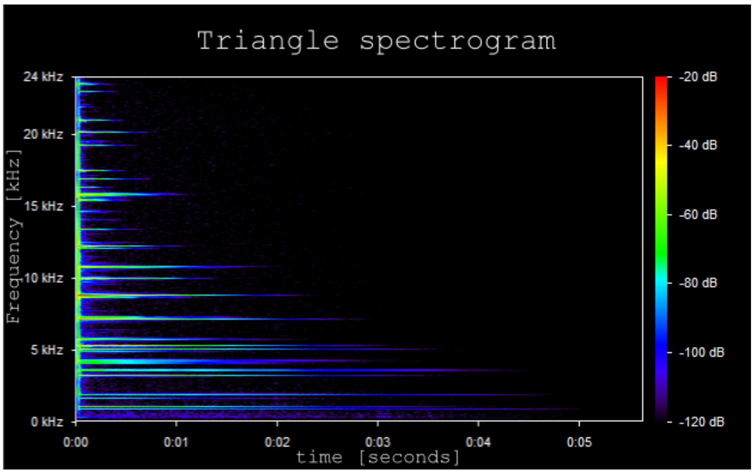

Figure 3 - Triangle spectrogram (software: spek.cc)

Torsional – The second key mode of vibration of the triangle occurs when the legs at the open corner Fig.1(d) twist away from each other out of the plane of the triangle as shown in Fig.2(ii). Torsional vibrations can be excited by striking the triangle at a right angle to the plane of the triangle.

Longitudinal – Lastly longitudinal waves are considered. These act like compression waves which propagate (longitudinally) through the entire length of the metal bar Fig.2(iii). If the triangle is struck it is very hard to excite these forms of oscillations without also exciting the other modes of vibration. The most straight forward way to excite these waves whilst limiting other unwanted excitations is to rapidly "stroke" away from the bar with two fingers which have been made to be sticky with a substance such as resin as was originally done for Kundt tube experiments.[7] Further research could also be conducted into how the bends of metal rod affect the longitudinal waves, however, for now, the triangle is treated like a steel bar.

As the ends of the rod are free to oscillate, a standing wave can be set up that would mirror the standing wave patterns of a tube open at both ends[8], [9], this means that displacement antinodes (& pressure nodes) can be found at each end of the rod. The formula for the frequency of the standing waves inside a tube open at both ends is given by: $f_n = \frac{nv}{2L}$ [10] where n=1 represents the fundamental frequency, n=2 is used for the first overtone & so on. Since a 4" triangle was used for the sound recordings used to generate Figs. 4-5 & the speed of sound in steel is $5{,}960\ ms^{-1}$ [11] the standing wave frequencies are calculated & matched up to the real world results.

Table 1 shows from left column to right: number of the harmonic, expected Hz value calculated as described in the preceding paragraph, closest measured frequency exported from software & percentage difference between calculated & expected waves.

n	Calculated (Hz)	Closest Measured (Hz)	Difference
1	9,776.90	9,776.07	-0.0085%
2	19,553.81	19,552.15	-0.0085%
3	29,330.71	NA	NA

Table 1 – Longitudinal standing waves calculated vs measured

The measured fundamental frequency & first overtone are incredibly close to the expected values with an identical difference, potentially due to an inexact steel bar length, or impurities in the steel affecting the speed of sound through the material (since both frequencies were created with the same strike, other variables such as human error would be identical for all rows. The second overtone (3rd harmonic) had no comparable value as the highest exported frequency was of ~22kHz.

When striking the triangle normally (without attempting to excite any particular vibration, rather attempting to achieve a desired sound) it should be expected that all three forms of the above-described modes of vibrations would be excited, therefore whilst the professionally recorded & played triangle sound downloaded, allow for the analysis of the whole sound wave, separate recordings of the triangle were made by striking the triangle as described above to showcase how the different modes of vibrations excited different frequencies. These are showcased in Figs 4 & 5.

Exciting the triangle as described made two points clear:

1) It is very difficult to excite only one of these modes of vibration in isolation so there is always some overlap.
2) Each mode of vibration exhibits multiple frequencies that are only present in that mode of oscillation but not others. This can be seen in Figs. 4 & 5. In Fig 4 the triangle was hit as described under the subheading ***Transverse***. Fig 5 showcases the frequencies as described by striking the triangle under the subheading **Longitudinal.**

Clear distinct signature frequencies can be seen in each graph caused by the frequencies excited due to the method of striking. The "transverse" method of striking (which is one common method used to produce a desirable sound) creates a much fuller mid-range between 2-6kHz, which humans are more sensitive to as shown by Fig 6. There is also a clear signature peak at around ~1.25kHz.

The "longitudinal" method (Fig. 5) has many similarities however it has very different dB levels between the ~1-4kHz ranges it also has more pronounced peaks past the ~10khz mark & a fuller low range between 200-1,000Hz.

[5] It is virtually impossible to excite only one form of the three major vibrations in an effort to isolate them, the methods described simply attempt to maximise the vibrations being isolated whilst attempting to minimise those not being focused on.

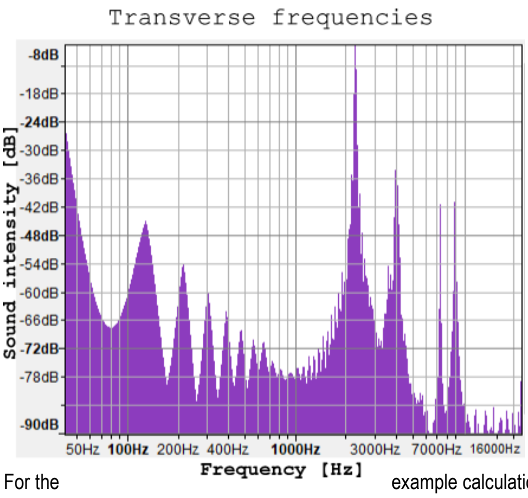

Figure 4 - Transverse frequencies (software: audacityteam.org)

This information is outlined simply to showcase the complex array of the frequencies that superimpose to create the final sound wave that is eventually heard & where they originate. When sound waves are described the expected wave equation takes the form of: $y(x,t) = A\sin(kx - \omega t + \phi)$, where y(x,t) is the longitudinal displacement of the particles[6] from its equilibrium position at position x & time t, A is the maximum displacement or amplitude, ω is the angular frequency in radians per second, & k is the wavenumber. ϕ is the phase angle, however, here it is assumed that the phase angle is zero as all component oscillations are created simultaneously when the beater strikes the triangle. This may however be an interesting area of further research. (we are assuming that y=0 is true at x=0 & t=0)

The intensity level in dB & frequency in Hz can be exported from the audio program audacity. The equation for intensity level $dB = 10 \times \log_{10}\left(\frac{I}{I_0}\right)$ [12] can be re-arranged to convert out intensity level (logarithmic) in dB into linear intensity in $[Wm^{-2}]$ & this can finally be converted into pressure level in Pa which is proportional to longitudinal displacement y(x,t), so plots will be of P(x,t) instead to save further calculations (where $I_0 = 10^{-12}\,Wm^{-2}$, or the reference intensity/threshold of human hearing). Intensity can be converted into pressure by rearranging the equation: $I = \frac{P^2}{2\rho v}$ [13]. ω & k can be calculated from each frequency by rearranging the equations: $\omega = 2\pi f$ & $k = \frac{2\pi}{\lambda}$ & $\lambda = \frac{v}{f}$. Below is a brief example:

For the example calculation an arbirarty frequency is exported from the audio software; the component of the sound wave with frequency 4,078.1 Hz which has an intensity level of 85.8dB.

The equation for intensity level (dB) can be rearranged to give intensity I (Wm^{-2}):

$$I = I_0 10^{\frac{dB}{10}} = 10^{-12} \times 10^{\frac{85.5}{10}} = 0.0035\ Wm^{-2}$$

The speed of sound in air is $\sim 344 ms^{-1}$ at 20C[14] & the density of air to be $1.225\ kgm^{-3}$ [15] the density and pressure equation can be rearranged so that:

$$P = \sqrt{2I\rho v} = \sqrt{2 \times 0.0035 \times 1.225 \times 344} = 1.72\ Pa\ (2\ d.p.)$$

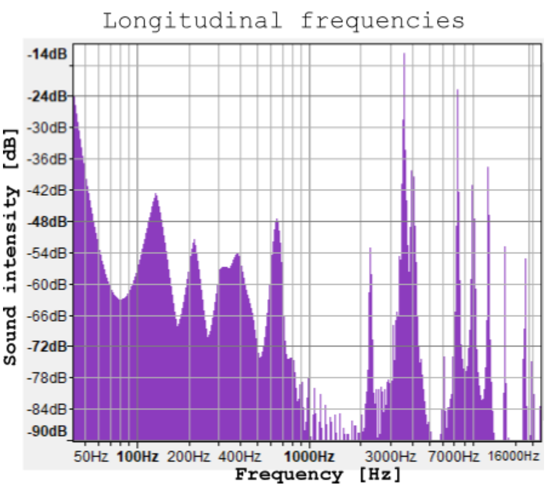

Figure 5 - Longitudinal frequencies (software: audacityteam.org)

Again the speed of sound in air is used to calculate the wavelength of the sound wave:

$$\lambda = \frac{v}{f} = \frac{344}{4078.1} = 0.08m\ (2\ d.p.)$$

From this, the wave number k can be calculated: $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = 74.49\ radm^{-1}(2\ d.p.)$ & finally, angular frequency can be calculated: $\omega = 2\pi f = 2\pi \times 4078.1 = 25623.46\ rads^{-1}\ (2\ d.p)$. Now the required I information is available to build the pressure equation as a function of x and t for this frequency:

$$P(x,t) = 1.72\sin(74.49x - 25623.46t)$$

However, this is just the first of 511 different frequencies that the software extracted from the downloaded 5 second triangle audio recording; to truly see what a sound wave emitted from a triangle looks like all these equations need to be summed together as described by the following equation:

$$P(x.t) = \sum_{i}^{511} A_i \sin(k_i x - \omega_i t)$$

Adding 511 sine waves of varying amplitude, frequency & wavenumber would be a near-impossible task, fortunately, it can be managed easily with code. Using R programming a simple script[7] was created that performs all the calculations mentioned in the above example, cycles through all the frequencies & intensity levels exported for the triangle sound & sums them together. The resultant function can then be plotted assuming x=0 (the moment the sound waves emanate from the triangle) therefore the second term of the sine function kx is always equal to zero.

Looking at Fig.6 it can start to be seen why the sound from a triangle is often described as unpitched or of indeterminate pitch. There are so many overlapping frequencies that create an incredibly unique signature, unlike almost any instrument. The three graphs all represent the same wave at different magnifications, the y-axis represents the pressure waves reaching the microphone at a time prescribed by the x-axis. Whilst the y-axis of these three graphs are similar, the x-axes (time), however, are vastly different deliberately to showcase some interesting results.

High detail – The leftmost graph (Fig. 6) represents a very zoomed-in view of the wave. The entirety of the time axis represents less than 2ms. Here a fairly regular sinusoidal pattern with varying amplitudes can be seen. The period of these oscillations (peak to peak) is of around $115\mu s$ (obtained from visual inspection) & therefore has a frequency of approximately ~8,695Hz. Using $\lambda = \frac{v}{f}$ the approximate wavelength can be calculated to be ~0.04m, this is the base frequency of the wave.

Medium detail – In this medium detail graph (Fig.6) the entirety of the time axis is around 50 times that of the first graph at ~100ms. The peaks and troughs of seemingly random varied amplitude of the first graph are contained within a reverberating envelope, which grows before dying down again every 23ms (obtained via visual inspection). The frequency of the envelope can be calculated to be ~43hz. The distance between peeks is around ~8m. Interestingly the peaks in the graph can be seen to be growing as the shape slowly tends towards yet another peak in another repeating pattern, which is investigated in the next graph.

Outer envelope (low resolution) – The rightmost graph (Fig.6) now contains the entire 5s recording of the triangle on the x-axis. The overarching shape of the outermost envelope produced by the superposition of the many frequencies can be seen. Here the peaks seemingly occur ever half-second, with a frequency of ~2Hz. Almost like a beat frequency, this should be detectable by human ears[16] as an increase & decrease in amplitude (intensity/volume) every half second. The distance between the peaks here would be of the order of hundreds of meters (~172m).

6 Particles of the medium transporting the soud waves
7 Full code available here: https://github.com/JackDCornwall/VWO_Triangle_sound

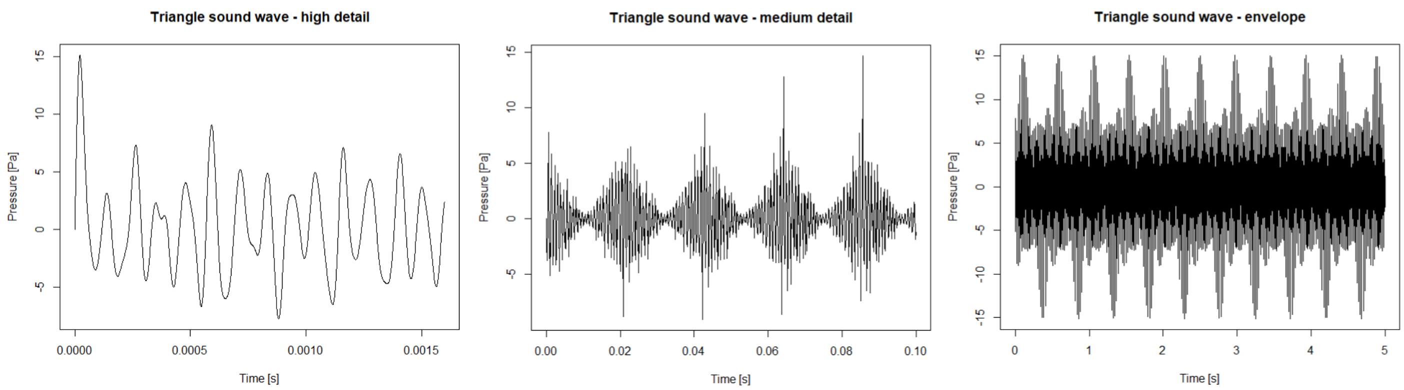

Figure 6 - Triangle sound wave

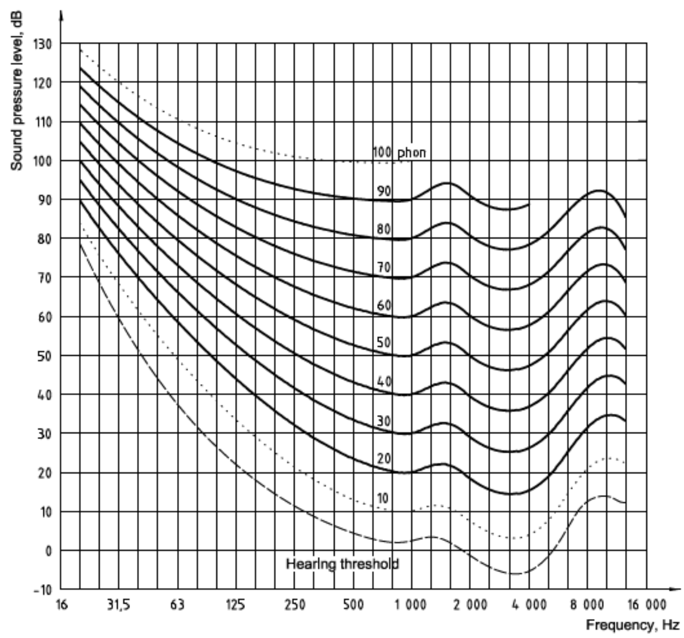

Figure 7 - Equal loudness curve BS ISO 226:2003 [12]

Setting up a standing wave – Whilst the longitudinal standing wave inside the steel rod that makes up a triangle has been discussed above, it is worth considering what room size may be required for creating standing waves with the triangle. A standing wave is one whose peak amplitude varies with time statically with respect to space (the peaks do not translate in the direction of propagation). These are created when waves reflect between two surfaces such as walls & the reflected wave constructively interferes at antinodes whilst destructively interfering at nodes. This results in the nodes, not moving and the antinodes simply oscillating up & down (side to side for longitudinal waves) rather than along with the wave.

As pressure antinodes occur at hard walls, the equation to calculate the room dimensions required for a standing wave can be obtained by rearranging $f_0 = \frac{v}{2L}$ [17] to give $L = \frac{v}{2f_0}$; the speed of sound in air is $344ms^{-1}$ & the frequency of our triangle wave is 8,695Hz it is calculated that the room must be of the order of 0.198m ($\lambda/2$), very small indeed.

One suggested avenue of further research would be to look at how the low & medium detail envelope waves would react when being placed into rooms of the order of magnitude required to set up standing waves using the much lower envelope frequencies, these would be ~4m for the 43Hz envelope (a much more plausible room size) & ~86m for the outermost 2Hz envelope, which is very much of the order of magnitude of some opera halls. This suggests that the overall sound of the instrument may be dependent on the size of the room it is played in.

How humans hear sound – The sound wave has been very accurately described mathematically, however, humans are more sensitive to certain frequencies than others because of the makeup of the auditory system. Sound travels through the air as longitudinal waves, compressing & rarefracting until it hits the ear & is funnelled into the ear canal. Here it is picked up by the eardrum which vibrates in response to the changes in pressure, the eardrum passes these vibrations mechanically to the ossicles[8] which in turn pass the vibrations on to the fluid of the inner ear & picked up by small hairs connected to nerve cells & transmitted as impulses which the brain interprets as sound. However, due to a wide variety of factors, the auditory system is better at picking up certain frequencies than others. This is shown by Fig.7. The equal loudness curve shows the hearing threshold a sound needs to reach before humans start to register sounds. It is fair to say that whilst humans can hear very low frequencies (0-500 Hz) these need to be many orders of magnitude more intense before they compare to a sound in the 1-10kHz range. So it is possible mathematically to depict the sound wave from a triangle, it isn't a fair representation of what humans will hear, instead certain frequencies such as the very high 10kHz frequencies discussed under the heading **High detail** would be more apparent than the low periodic "beat" hum discussed under headings **High detail** & **Medium detail**.

Conclusion - With so many overlapping frequencies across the whole range of the human hearing spectrum being superimposed to make one remarkably complex sound it is no wonder that the sound from the triangle is described as of indeterminate pitch. The base frequency of the triangle was calculated to be 8,695Hz, which online tools[9] confirm this to correspond to the note C#9 minus 34 cents. This base frequency was found to reverberate inside two separate nestled envelopes of differing frequencies making the sound signature of the triangle even more complex. The code was simply used as a tool to sew the individual frequencies together into an overarching picture & mathematical description of the sound. The study also investigated the standing waves inside the steel of the instrument & looked at how standing waves might be set up in a room before finally looking at how humans would interpret this fascinating sound. So in conclusion whilst striking a triangle may seem simple, it requires rather complex and intricate mathematics just to attempt to visualise & understand the sound being produced.

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