Evolution of Cosmic Structure

Lecture 7:
Peculiar velocities and cosmological Inference

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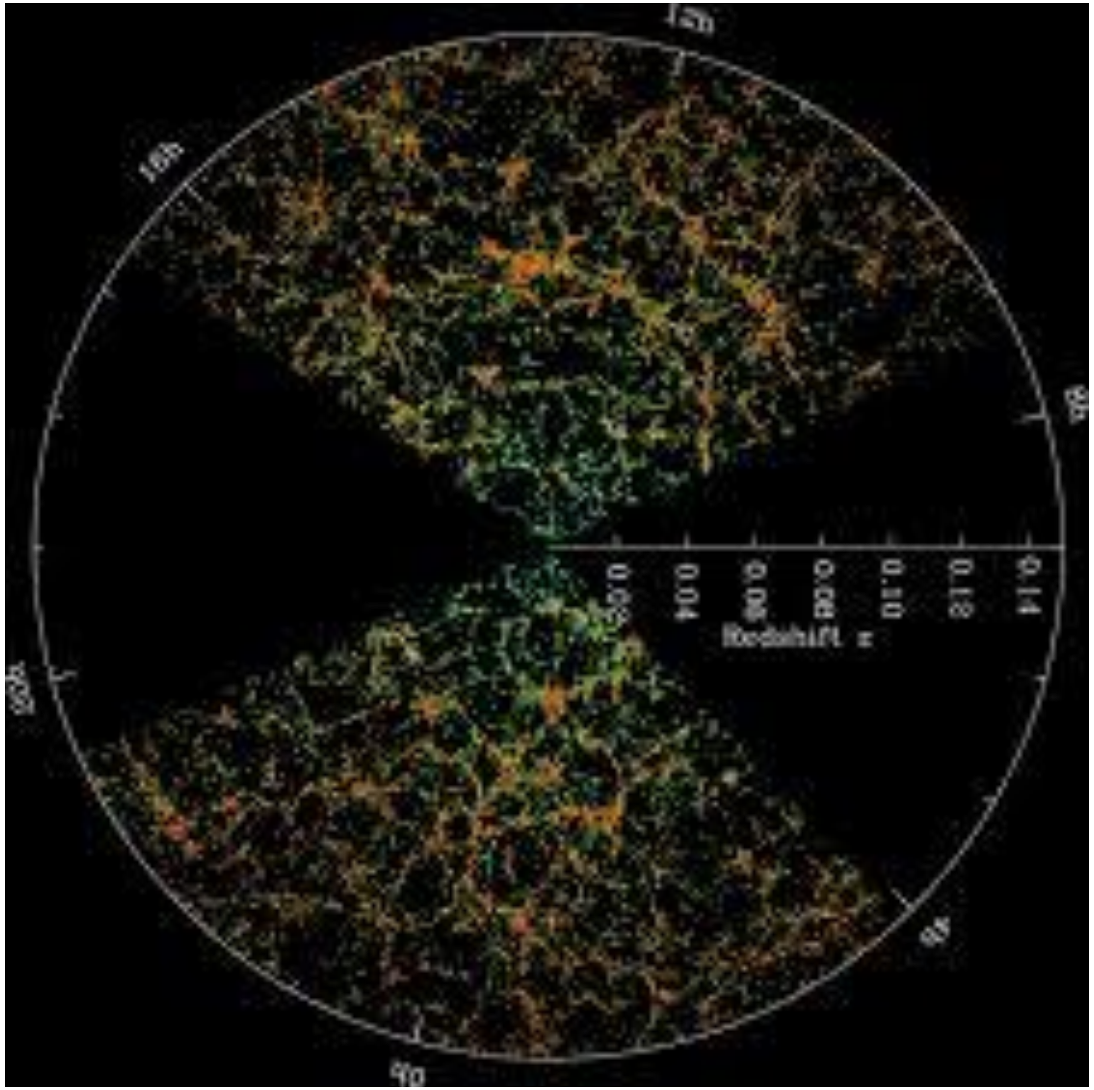
Learning Outcomes

What are peculiar velocities? How do they trace cosmological parameters? What are the methods to measure them?

- What are peculiar velocities?
- What distance probes do we use to infer them
- How are the velocity and density fields related?
- Redshift space distortions

Recap of Galaxy Clustering



Recap of Galaxy Clustering

1.Traditionally, cosmological constraints from galaxy clustering have been quoted in terms of constraints on the amplitude of mass fluctuations σ(z, R)

$$\sigma^2(R) = \int \frac{d^3k}{(2\pi)^3} P_{\rm lin}(k) \left|W(kR)\right|^2$$

Where Δ^2 is the dimensionless power spectrum, W is the window function and k is the wavenumber.

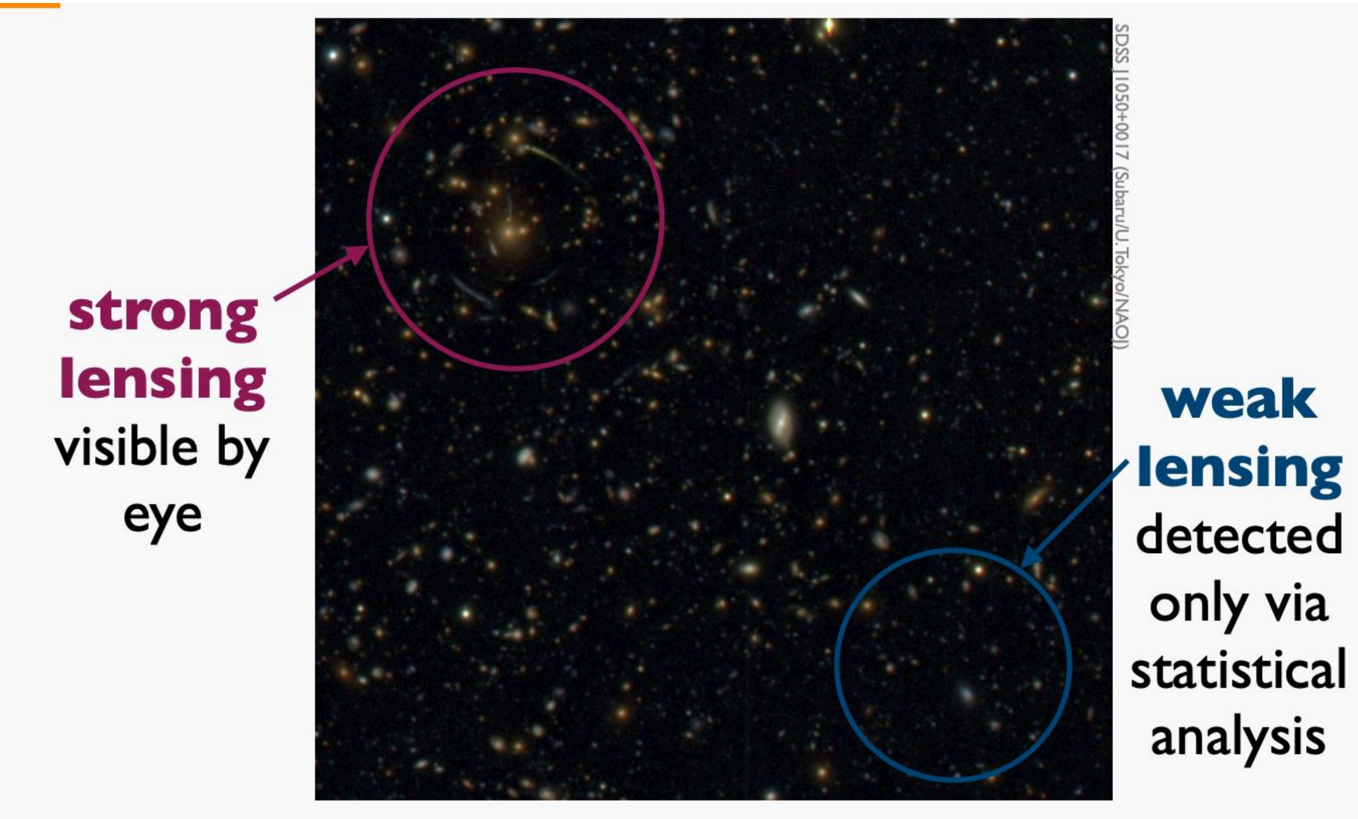
Conventionally, σ is measured at z = 0 and R = 8 h-1Mpc, hence, the parameter of interest is σ8.

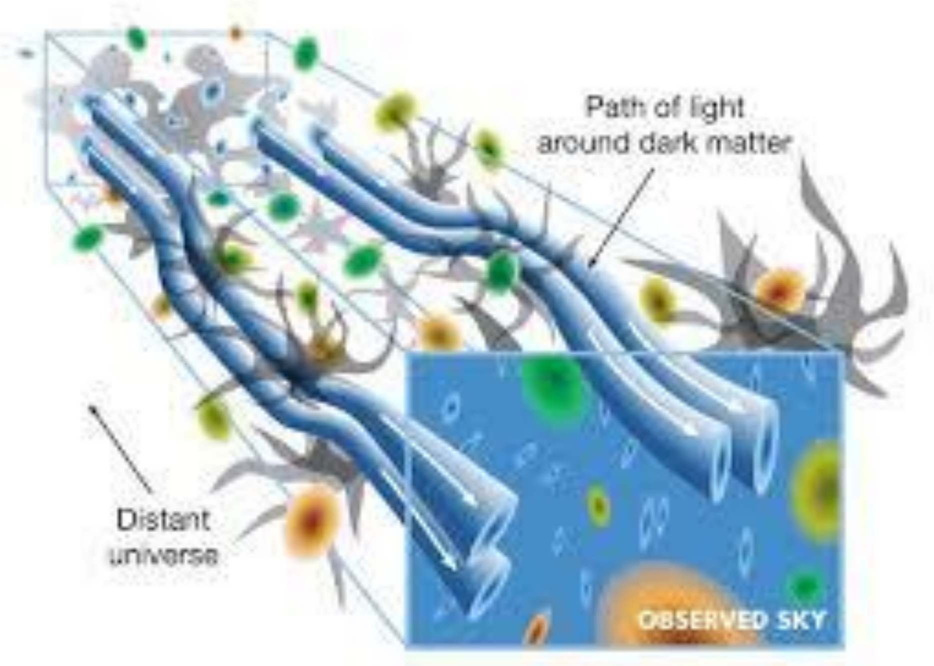
Recap of Galaxy Clustering

- Galaxy clustering traces the galaxy overdensity and not directly the matter density. The two are related as δm = bg δ_g. The relationship between the galaxy power spectrum
and the matter power spectrum is consequently

$$P_{gg}(k,z) = b^2(k,z)P(k,z),$$

Gravitational Lensing and cosmology



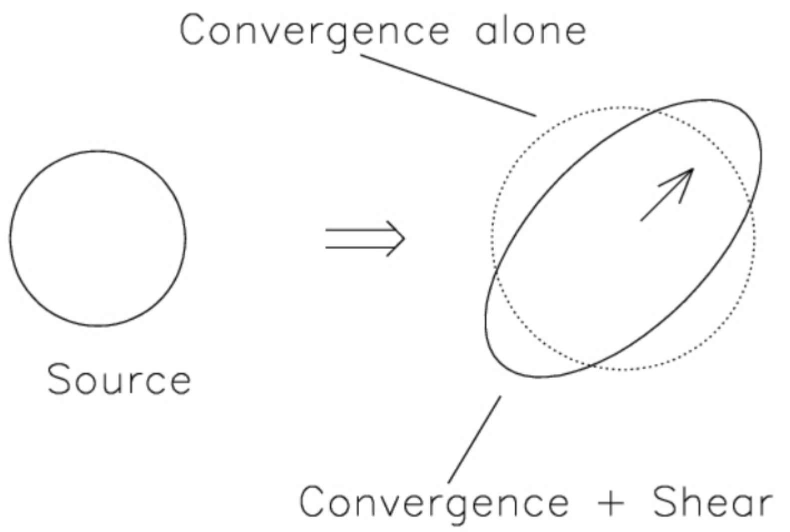


Lensing occurs in different regimes. Here strong lensing is a rare phenomenon, seen as the arcs of the lensed background source. Weak Lensing is an imprint of the dark matter on the shapes of galaxies

Quantities of interest in Lensing

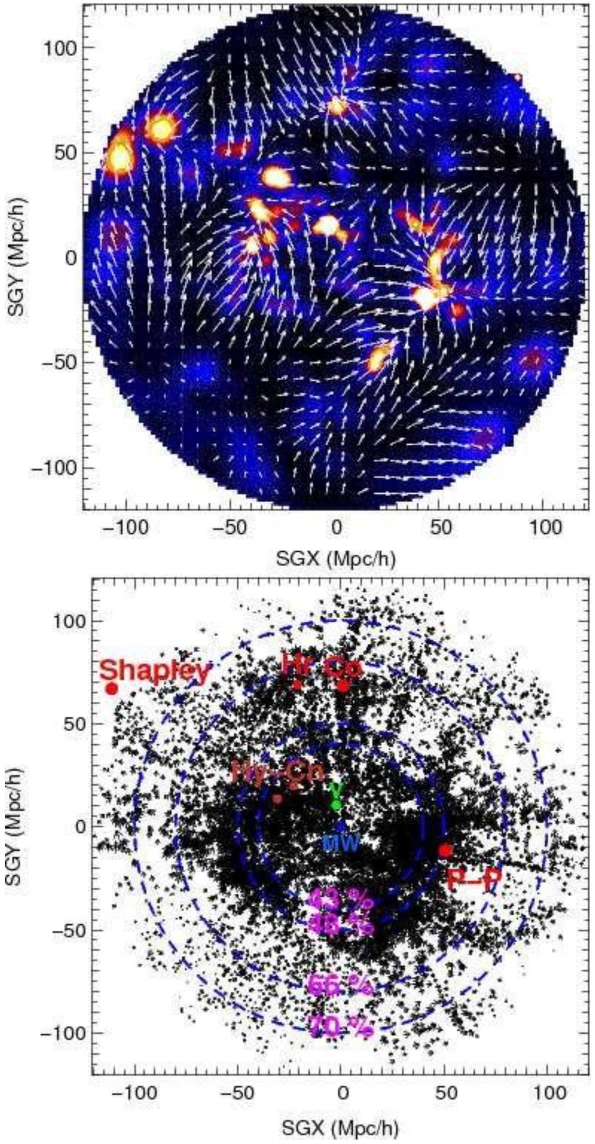
$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$= (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}$$



What are peculiar velocities?

Deviations from the Hubble flow due to the gravitational attraction of the matter density field



From: Guillhelm Lavaux

Understanding in terms of coordinates

We can understand peculiar velocities in terms of relating distances and velocities between physical and co-moving coordinates

Recall the relation between proper and physical distance

$$\mathbf{r} = a(t)\mathbf{x}\,.$$

Where a(t) is the scale factor at time t. Differentiating wrt t to get the proper velocity

$$\mathbf{u} = \frac{d\mathbf{r}}{dt} = \dot{a}\mathbf{x} + a\mathbf{v}(\mathbf{x}, t) = \left(\frac{\dot{a}}{a}\right)\mathbf{r} + a\mathbf{v}(\mathbf{r}/a, t)\,,$$

This equation looks like the Hubble law, except with an additional term on the rhs. This term a**v**, as we have already discussed, is due to the gravitational influence of local overdensities. Every galaxy will deviate from the Hubble law to some extent.

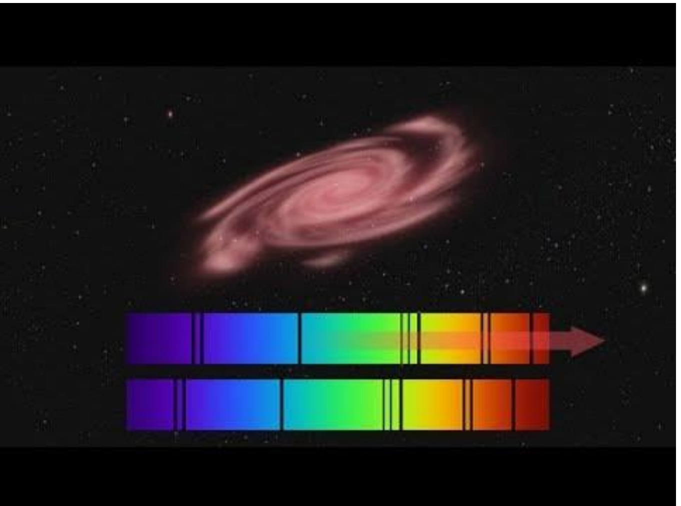
What are peculiar velocities?

$$(1 + z_{obs}) = (1 + z_{cos})(1 + v_{pec}/c)$$

Measured (e.g. from host galaxy spectrum)

From expansion of the universe

Peculiar motion



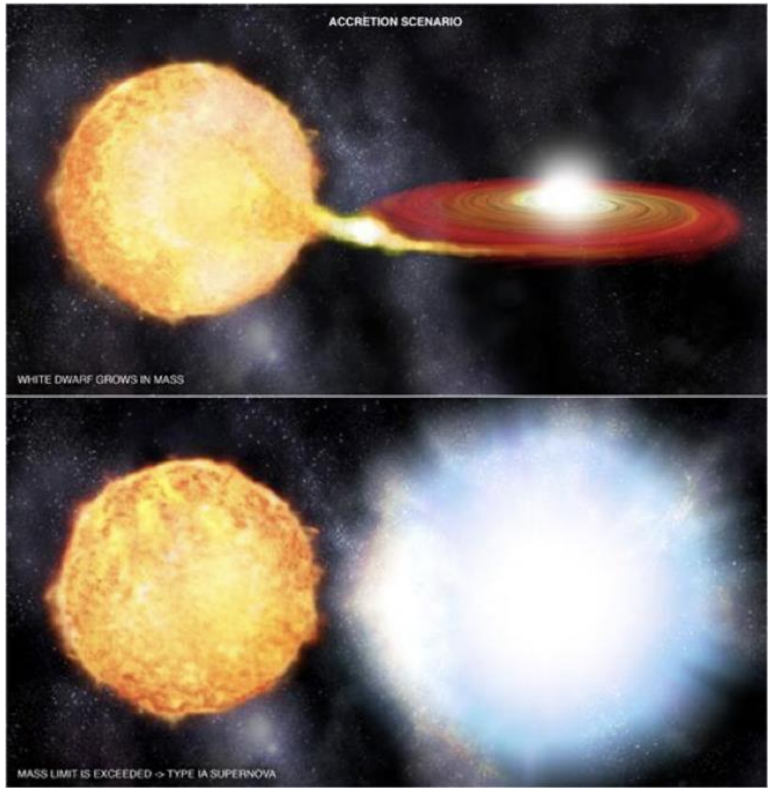
$cz_{cos} \sim H_0 D_L$ (to first order)

Type Ia supernovae as distance indicators

$$cz_{cos} = H_0\, D_L$$

It requires a precise "distance indicator". A distance indicator is any astrophysical object or collection of objects which has an intrinsic property which can be considered uniform across the population. In this aspect, e.g. a Type Ia supernova is a distance indicator as the luminosity is to within ~0.1 mag the same across the sample after certain corrections are made.

This means these are not "standard" but "standardizable" candles instead



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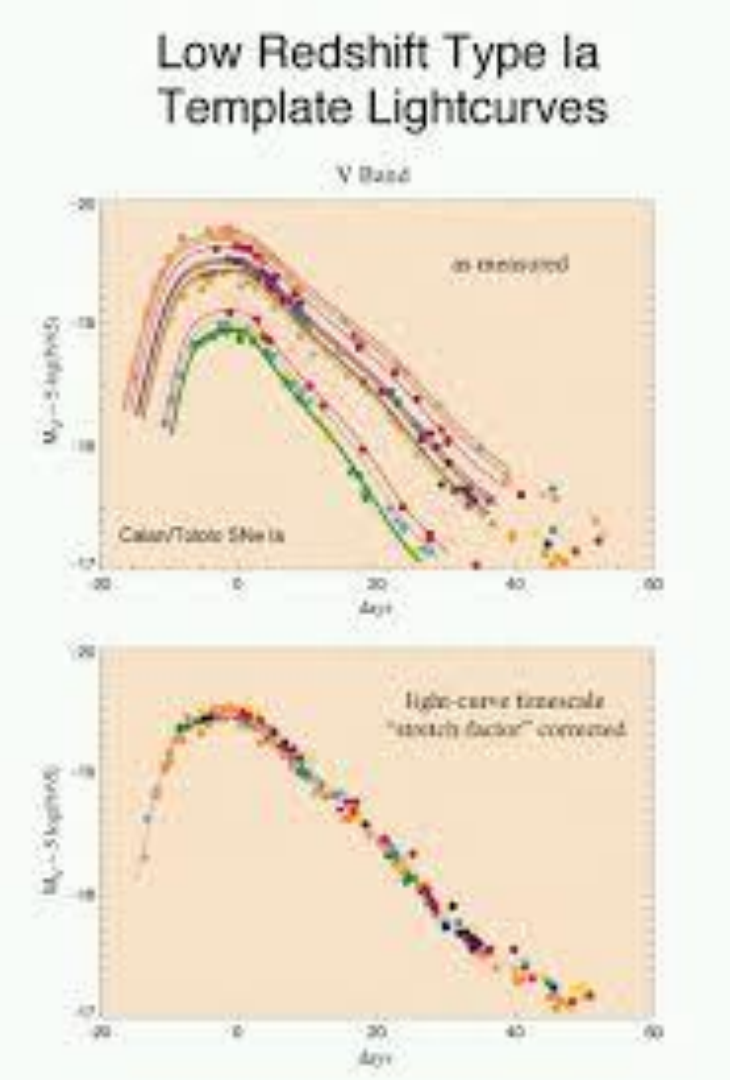
$$\mu = m_B + \alpha x_1 - \beta c - M - \delta_{\mathrm{bias}} + \delta_{\mathrm{host}}$$

Apparent magnitude (uncorrected)   lightcurve shape.   colour   Absolute Magnitude   selection   host galaxy (pt. 4)

Standardisation of Type Ia supernovae

mB + α(x1)



Accurate distances are required for inferring the cosmological component of the redshift for inferring peculiar velocities. This requires that Type Ia supernovae are standardized such their peak luminosity (post-correction) has a small scatter

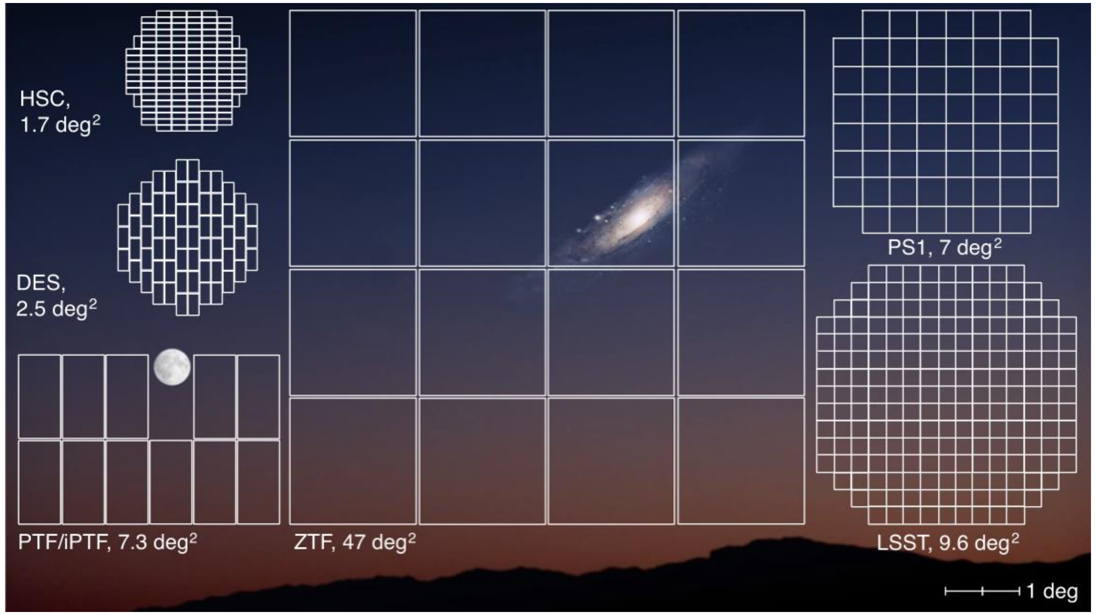
The first such correction was the width-luminosity relation, which is the relation between the peak luminosity and the shape of the lightcurve (flux evolution with time.

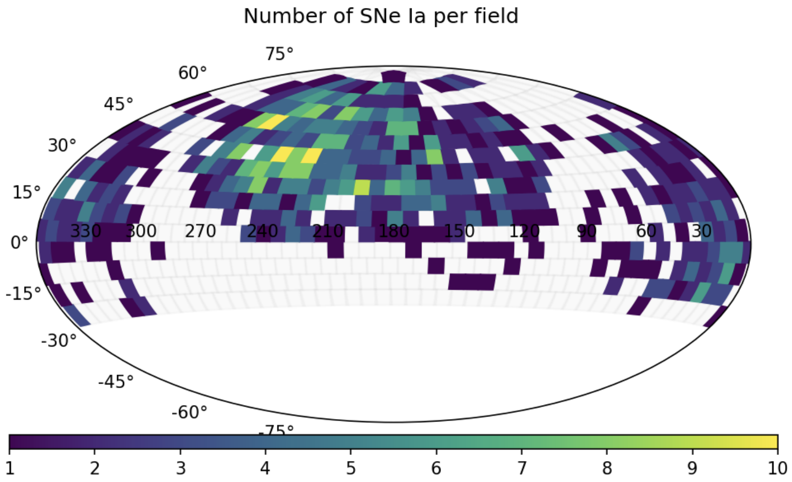
While it is purely empirical -> i.e. the models don’t play a role in using it, it can be interpreted physically.

Brighter → more radioactive material that decays
Broader → more material on top of the central engine, slower escape of photons

How do we measure them

Unlike dark energy inference, which requires a relative measurement of distances to high-redshift and low-redshift supernovae, for peculiar velocities we require low-z SNe across the sky. This is because the relative effect of peculiar velocities is more significant compared to the expansion of the universe more nearby





Dhawan et al. 2022

Ingredients to measure peculiar velocities on the sky:

- A good distance indicator (eg Type Ia supernovae)
- Large sample
- Sufficiently wide sky coverage

Cosmological Inference with PVs

We can now relate the peculiar motions of galaxies to the formation of structure via the theory of gravitational instability

$$\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Continuity equation

The Continuity equation ensures the conservation of mass and states that the growth of density in a volume is equivalent to the amount of matter entering the volume

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0\,,$$

Euler equation

The Euler equation states that gravitational forces and forces due to pressure are what induce velocity flows in the universe, and can be thought of as an application of Newton's second law

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho}\nabla P + \nabla \Phi = 0$$

Poisson equation

The Poisson equation describes the source of the gravitational potential Φ as being induced by the matter and energy content of the universe. Regions with higher matter density will produce stronger gravitational forces

$$\nabla^2 \Phi = 4\pi G \rho$$

Cosmological Inference with PVs

Combining the three equations, i.e. continuity, Euler and poisson we get the general equation governing the growth evolution

$$\frac{\partial^2 \delta}{\partial t^2} = 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} - \frac{\nabla^2 p}{\rho a^2} - 4\pi G\rho\delta$$

From continuity equation, assuming d << 1, i.e. linear regime

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad\qquad \frac{\partial \delta}{\partial t} = -\frac{1}{a}\nabla \cdot \mathbf{v}$$

As the growth equation is a second order differential equation the most general solution is

$$\delta(x, t) = \delta(x)\, D_1(t) + \delta(x)\, D_2(t)$$

Where D1 is the growing and D2 is the decaying mode. The growing mode dominates, therefore we can substitute the general solution into the continuity equation

$$\frac{\partial \delta}{\partial t} = \delta(\mathbf{x})\frac{dD_1}{dt} = -\frac{1}{a}\nabla \cdot \mathbf{v} \, .$$

Relating velocities and densities

The velocity associated with the growing density mode is:

$$\mathbf{v} = \frac{Hf\mathbf{g}}{4\pi G\rho} = \frac{2f\mathbf{g}}{3H\Omega}$$

Where Omega is the total density and g is the acceleration due to gravity. We can define the linear growth of structure as

$$f \equiv \frac{d\ln D_1(a)}{d\ln a} = \frac{1}{D_1(a)}\frac{d}{d\ln(a)}D_1(a)$$

We can re-write the relation between velocity and density as

$$\nabla\cdot\mathbf{v} = -aHf\delta(\mathbf{x},t)$$

Or in Fourier Space

$$v_i(\mathbf{k}) = iHf\frac{k_i}{k^2}\delta(\mathbf{k})$$

Integrating over all positions in 3-D space

$$v(\mathbf{r}) = \frac{H_0 af}{4\pi}\int d^3\mathbf{r}'\,\frac{\delta(\mathbf{r}')(\mathbf{r}'-\mathbf{r})}{|\mathbf{r}'-\mathbf{r}|^3}$$

Relates velocity and density via linear growth rate!

Growth rate of large scale structure

$$f(z) = \Omega_m(z)^\gamma ,$$

$$v(\mathbf{r}) = \frac{H_0 a f}{4\pi} \int d^3\mathbf{r}' \frac{\delta(\mathbf{r}')(\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}$$

$$\sigma^2(R) = \int \frac{d^3k}{(2\pi)^3} P_{\rm lin}(k) \left| W(kR) \right|^2$$

The combination of parameters f and σ_8 is commonly referred to as the normalised growth rate of large scale structure $f\sigma_8$, and the two are considered degenerate parameters. Intuitively, this can be understood by their effect on the velocity field. Larger values of f imply that density perturbations grow more quickly, resulting in more overdense regions, but this can also be explained by the rms fluctuation in the density field being larger. A way to disentangle the effects is to make measurements spanning a range of redshifts.

Relating Velocity and Matter Power Spectra

The matter power spectrum is defined by

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P_{\delta\delta}(k)$$

The velocity power spectrum is defined analogously:

$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P_{v_i v_j}(k)$$

Substituting the velocity–density relation:

$$v_i(\mathbf{k}) = iHf \frac{k_i}{k^2} \delta(\mathbf{k})$$

we obtain

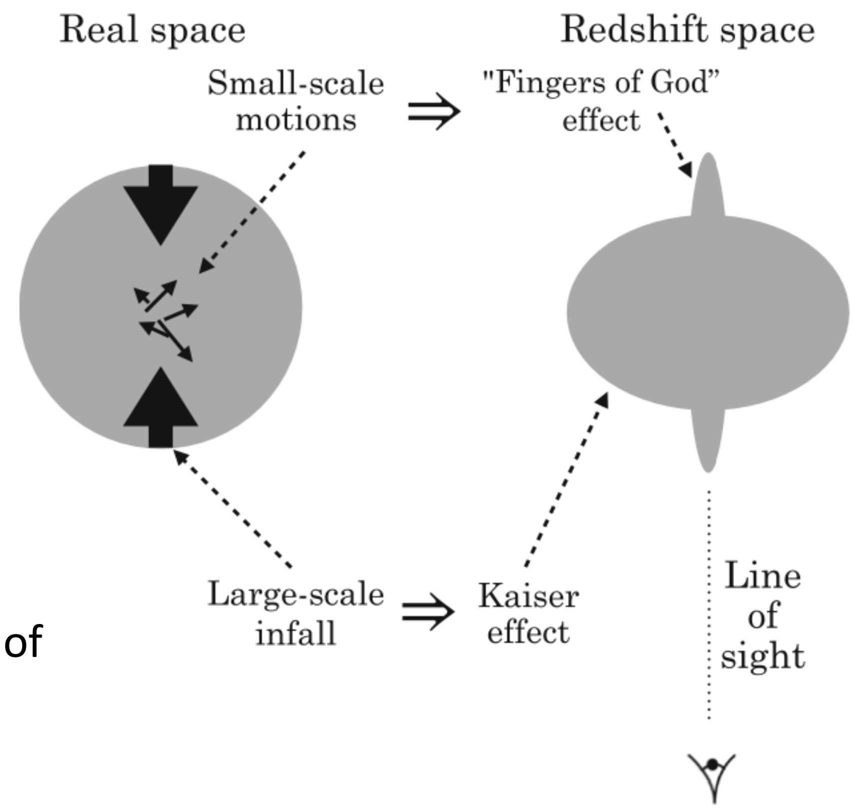
$$P_{v_i v_j}(k) = (Hf)^2 \frac{k_i k_j}{k^4} P_{\delta\delta}(k)$$

$$P_{vv}(k) \equiv \langle |\mathbf{v}(\mathbf{k})|^2 \rangle = (Hf)^2 \frac{1}{k^2} P_{\delta\delta}(k)$$

$$\boxed{P_{vv}(k) = (Hf)^2 \frac{P_{\delta\delta}(k)}{k^2}}$$

Redshift space distortions

A complementary approach to measure peculiar velocities and hence, fσ8 is redshift space distortions. This is a more “statistical” approach compared to using individual distances, e.g. with Type Ia supernovae

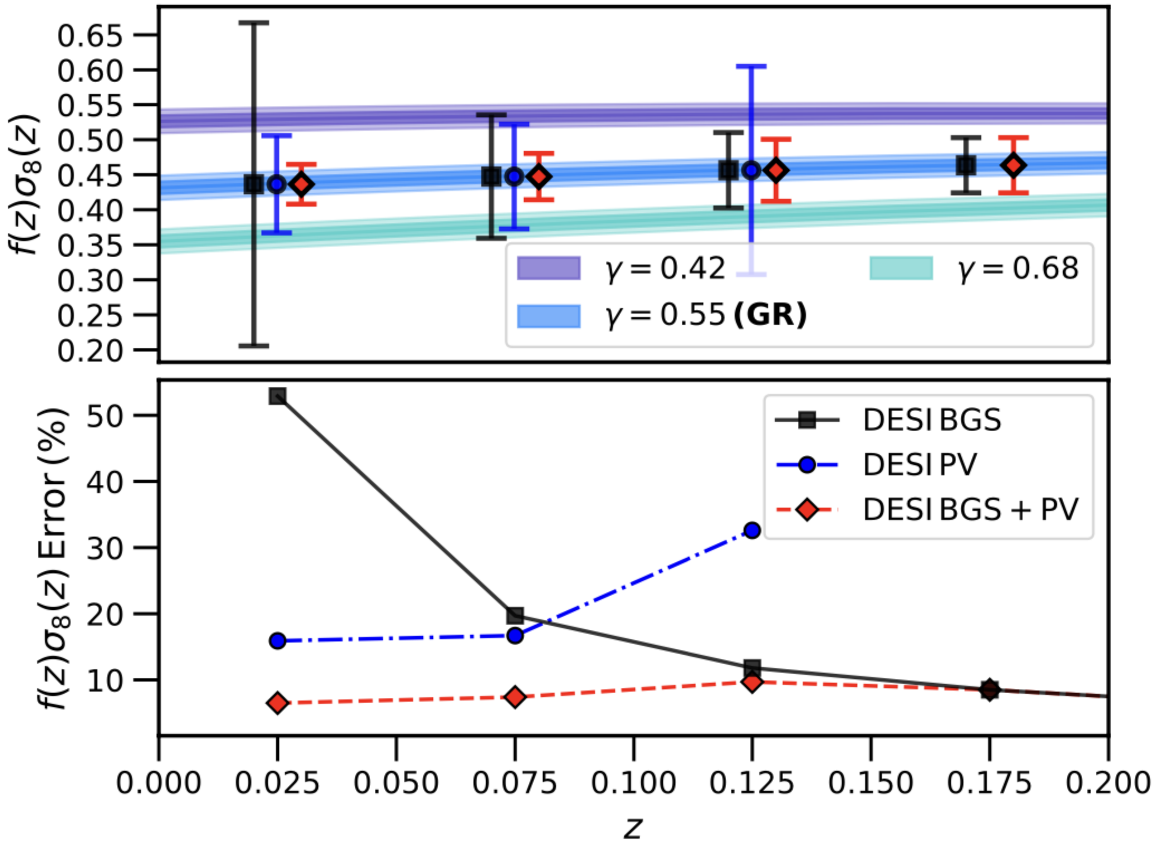


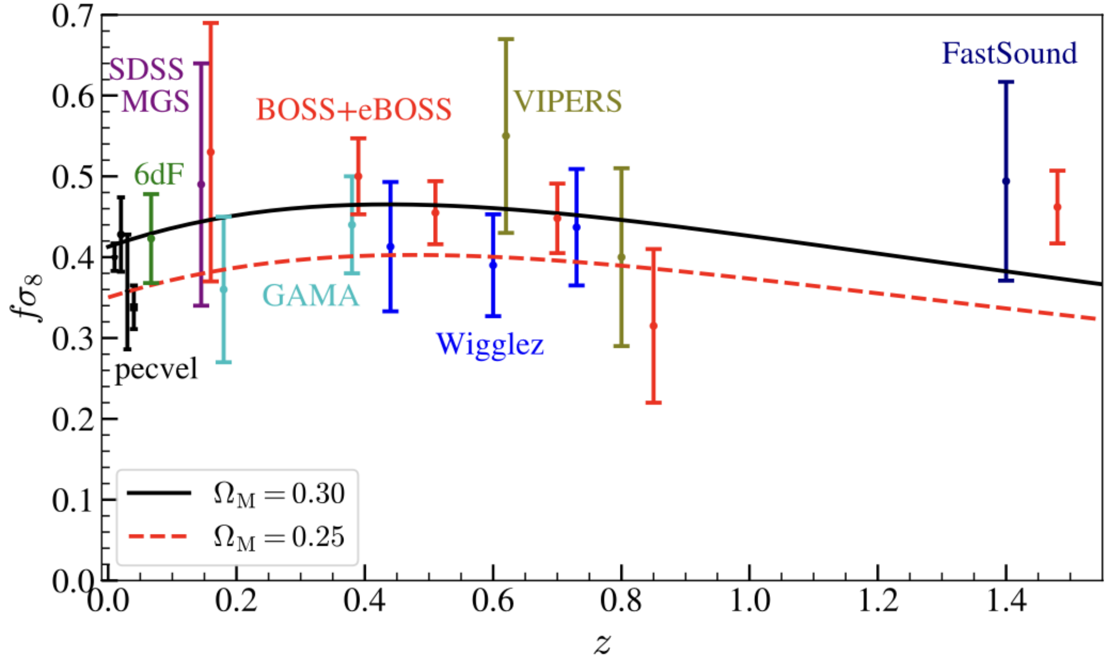
There are two principal effects:

(1) On large scales, velocity flows into large overdensities, “squishing” the appearance of the object along the line of sight; this is the Kaiser effect;
(2) On smaller scales, random motions introduce apparent elongation along the line of sight; this is the somewhat hyperbolically called “fingers of god” effect.

Motivations for peculiar velocity surveys

$$f(z) = \Omega_m(z)^\gamma \,,$$





Measurements of $f\sigma_8$ and systematics

While fs8 measurements are powerful for testing standard cosmology and measuring the growth of structure, there are systematic uncertainties. We discuss them below

1. Fails on non-linear scales

Solution: Gaussian smoothing (5-10h-1Mpc), excluding z <=0.01 SNe

2. Malmquist and selection biases

Solution: Selection effect simulations on the distances

3. Bias / tracer mismatch

Solution: Joint fits with redshift-space distortions, bias marginalisation

4. Survey Geometry and Sky Incompleteness

- Solution: Wiener filtering, Model directional selection effects, combining multiple surveys