Evolution of Cosmic Structure
Lecture 2 - Cosmological background

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Code:

Course Content

Introduction – Overview [1 lecture] Background + Universe before decoupling [2 lectures] Linear growth of perturbations [2 lectures]	Background and linear growth
Galaxy clustering and peculiar velocities [2 lectures] Non-linear evolution of structure [2 lectures] Numerical studies of structure formation [1 lecture]	Observational probes of linear growth and non-linear evolution
Observations of the distant Universe [2 lectures] Reionisation [1 lecture] Galaxy formation and galaxy properties [3 lectures] Host galaxies of supernovae [1 lecture]	Galaxy Formation and Evolution across cosmic time
Groups and clusters of galaxies [2 lectures] Fate of the Universe/Galaxies [1 lecture]	Galaxy groups, clusters and our fate

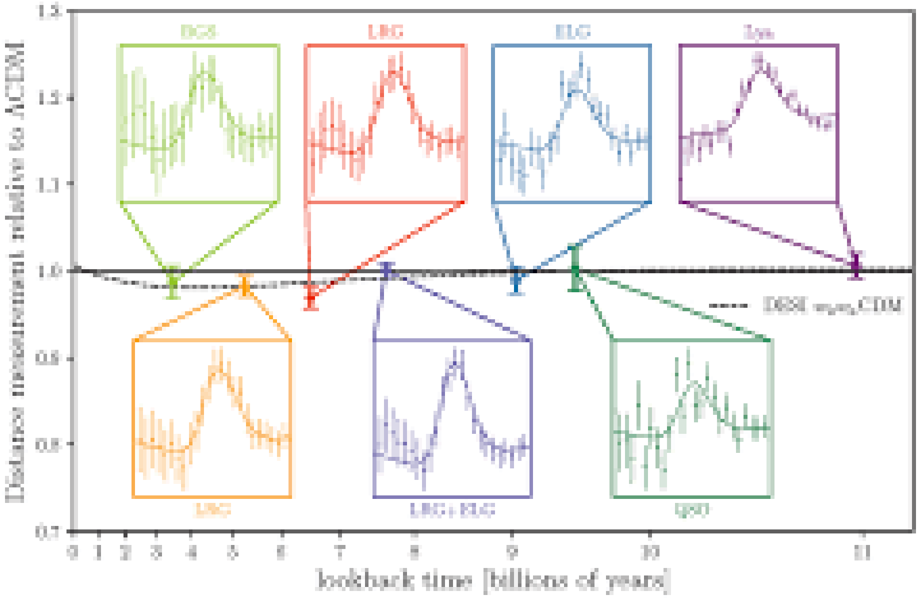
Learning Outcomes

What is known about the initial fluctuations from which cosmic structures have developed? How do we compute distances in the universe?

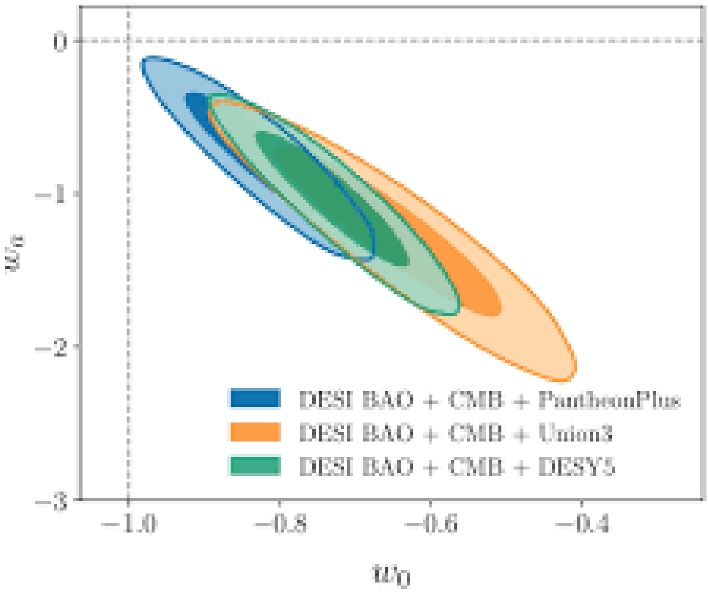
- Review of essential background cosmology
- Inflation and the horizon
- Seed fluctuations in the Universe at z~1000: evidence from the CMB
- Recombination and the origin of the CMB
- The cause of the CMB temperature fluctuations

Modern Studies of Large-Scale Structure

DESI experiment at Mayall Telescope, Arizona



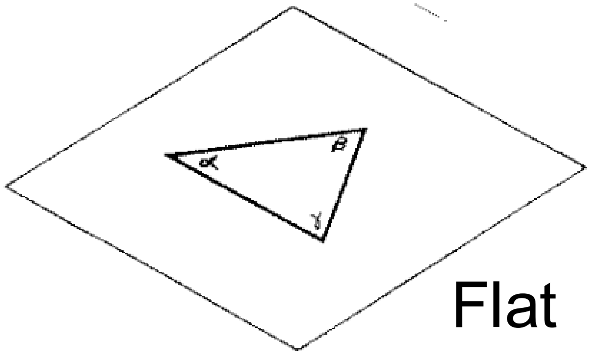
Baryonic Acoustic Oscillations measured at different redshifts -> corresponding to lookback time



Inferring properties of dark energy. Present day equation of state (x-axis) and time-dependence (y-axis)

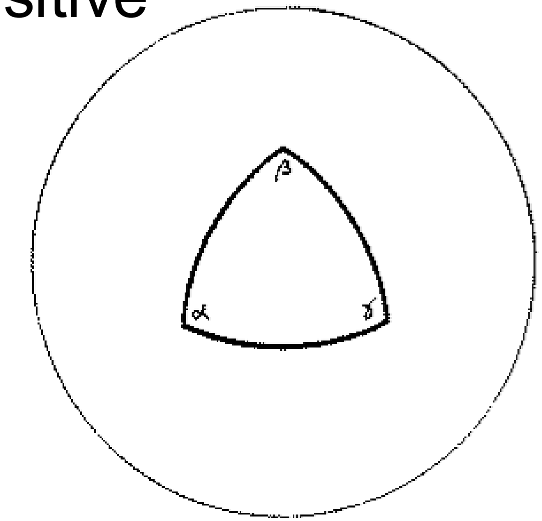
Essential background cosmology

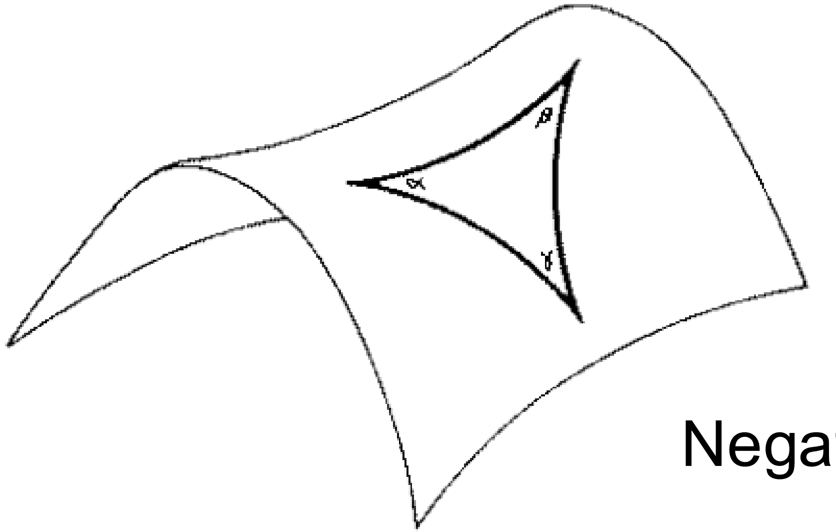
See (i) your notes from Observational Cosmology, (ii) Ryden Chapters 3-7.



Flat

Positive





Negative

Essential background cosmology

The space-time interval, ds, to an event in a uniform, isotropic universe is given (see Ryden §3.3) by the Robertson-Walker metric:

$$ds^2 = -c^2dt^2 + a(t)^2[dr^2 + S_k^2(r)(d\theta^2 + \sin^2\theta \; d\phi^2)]$$

where $S_k(r)$ depends on the curvature:

$$S_k(r) = r \qquad \text{(for } k = 0\text{; flat)}$$

$$S_k(r) = R_0 \sin\left[\frac{r}{R_0}\right] \qquad \text{(for } k = +1\text{; closed)}$$

$$S_k(r) = R_0 \sinh\left[\frac{r}{R_0}\right] \qquad \text{(for } k = -1\text{; open)}$$

and r, θ and ϕ are the comoving coordinates. $a(t)$ is the scale factor at universal time t, and R_0 is the radius of curvature of the universe.

With homogeneity and isotropy, the geometry is completely contained within $a(t), k$ and R_0

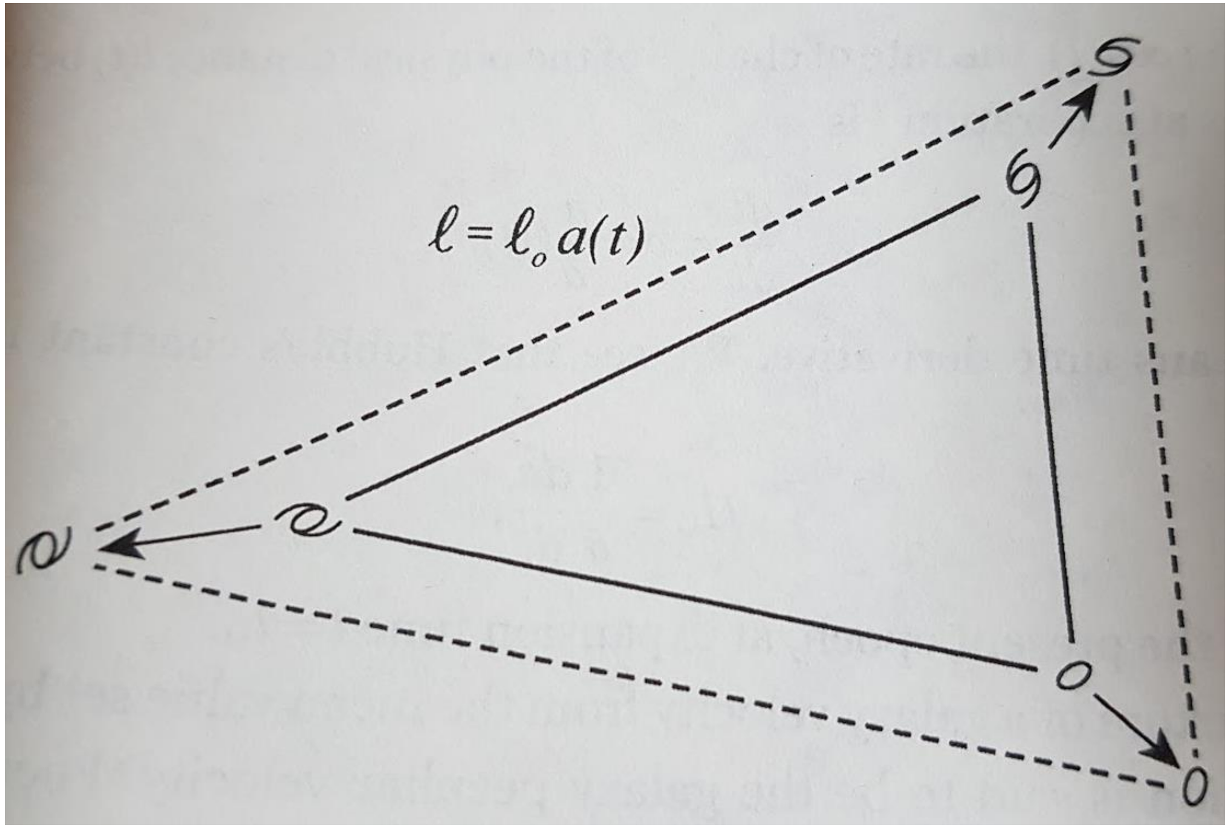
Definition 1.1. Homogeneity. A system is homogeneous if it is invariant under translations $X^a \longmapsto X^a + A^a$.

Definition 1.2. Isotropy. A system is isotropic if it is invariant under rotations. It looks the same in all directions.

Recall that a photon path through this space-time interval gives $ds^2 = 0$.

The scale factor of a universe

Universe is capitalized when it is our Universe, but lower case when it is a hypothetical universe.



Consider an expanding universe, if galaxies are moving with the expansion, in the so-called 'Hubble Flow', then homogeneity and isotropy suggest their positions from one time to another can be described just by a normalization change

This normalization change is encapsulated by the scale factor a(t). It is defined so that the scale factor today is 1.

$$a(t = now) = a_0 = 1$$

So, for an expanding universe,

$$a\,(t < t_{now}\,) < 1$$

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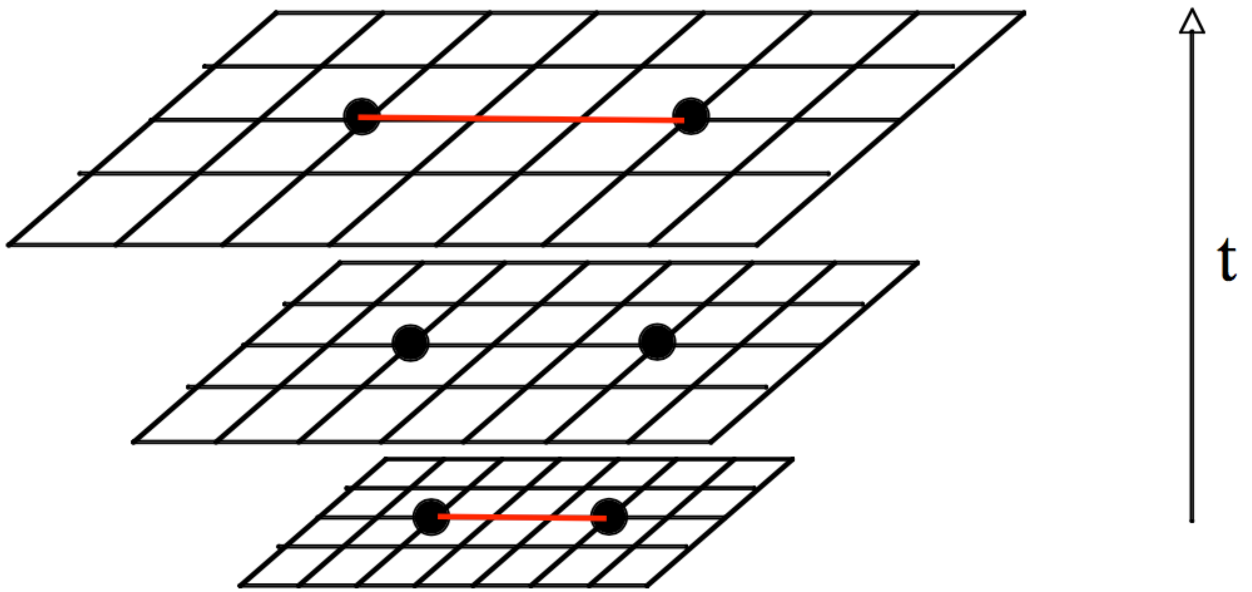

scale factor a(t): size of the grid (varies with time as universe expands)

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Essential background cosmology

Distances

With the above definition of the comoving coordinates, the proper distance $d_p(t)$ to an object (measured along a geodesic at time t) is simply

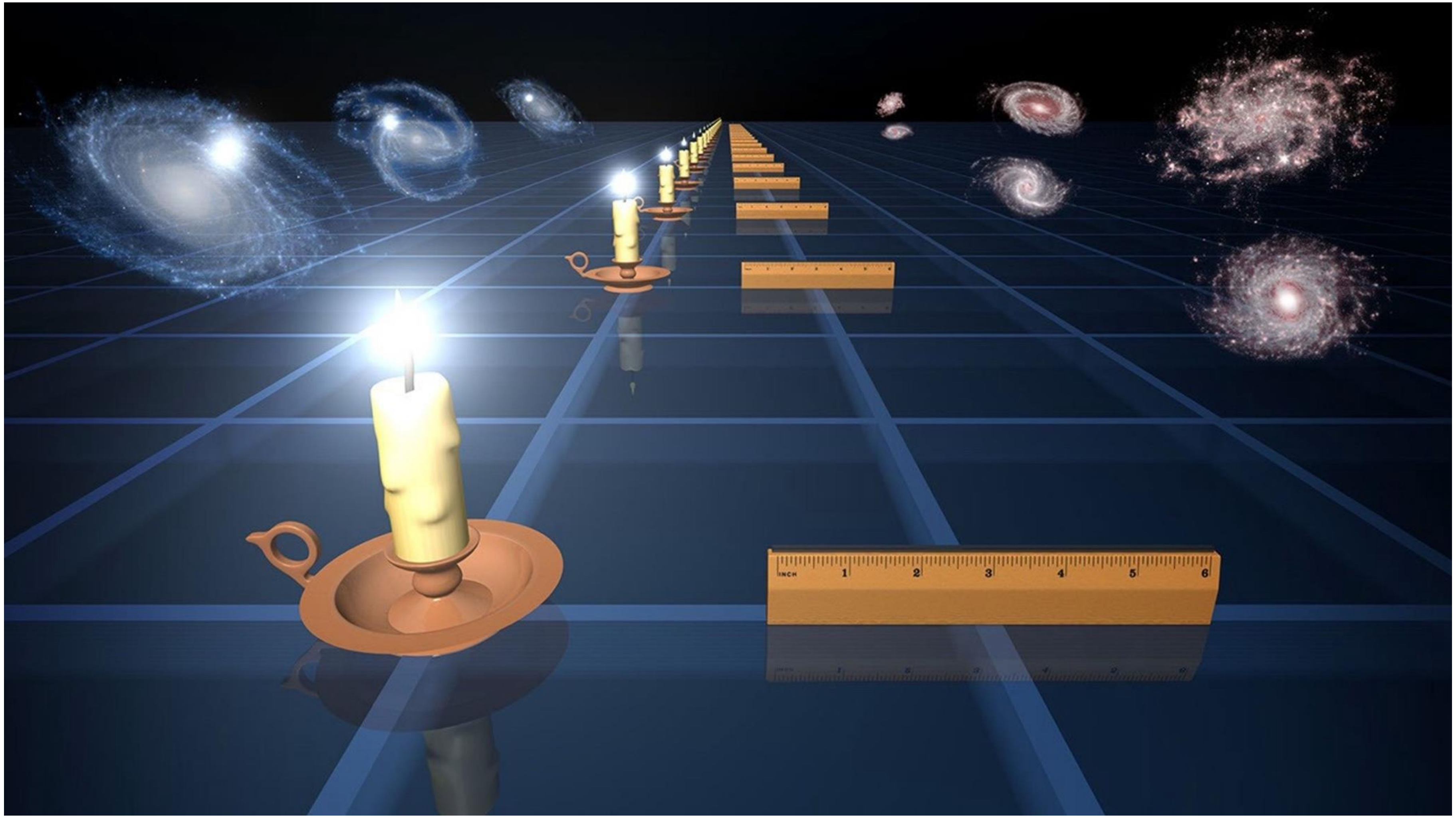
$$d_p(t) = a(t)\int_0^r dr \; = a(t)\, r$$

For a flat universe ($k = 0$), the luminosity distance is $d_L = d_p(t_0)(\,1 + z)$

And the angular diameter distance is $d_A = d_p(t_0)/(\,1 + z)$

Recall that the 'proper distance' is not measurable. Proper distance is the instantaneous separation between two points in space, measured at a specific moment in time, taking into account the current value of the scale factor $a(t)$

Standard Candles and Rulers



Essential background cosmology

Horizon

The horizon at time t is a sphere of comoving radius within which the material inside is causally connected. The horizon is

$$r_{hor}(t) = c \int_0^t \frac{dt'}{a(t')}$$

The corresponding proper distance can be calculated by multiplying by $a(t)$

$$d_{hor}(t) = a(t)\, r_{hor}(t)$$

Since the horizon will depend on the scale factor, and it’s evolution over the measured time, it depends on how background cosmology or its evolution. (We will do a direct calculation.)

The dynamics of the universe

Important parameters to characterize the properties and kinematics of the universe

The **Friedmann equation**:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8}{3}\pi G \rho = -\frac{k}{a^2}$$

$$\boxed{H = \frac{\dot{a}}{a}\,.}$$

Hubble parameter govern the rate of expansion

The **acceleration equation:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right)$$

The **fluid equation:**

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right) = 0$$

$$\boxed{q_0 := -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a},}$$

The **equation of state:**

$$P = w\,\rho$$

Deceleration parameter govern the rate of change of expansion

Essential background cosmology

Omega

The density ρ can be partitioned into contributions from matter (dark and baryonic), radiation and vacuum energy, each with its own equation of state:

w=0 for non-relativistic matter, 1/3 for radiation and -1 for vacuum energy.

$$P = w\rho$$

$$\rho_c = \frac{3H^2}{8\pi G}$$

We compare the density evolutions relative to the critical density of the universe. This is a time-varying quantity (via the Hubble parameter).

The ratio of each density to the critical value gives its contribution to the density parameter $\Omega = \Omega_m + \Omega_r + \Omega_v$.

Note that the vacuum contribution ρ_v is related to Λ, by $\rho_v = \Lambda/8\pi G$.

Matter – only universe

Thus, we can solve several things about the matter only universe

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

And the density goes with time as

$$\rho(t) = \frac{\rho_0}{a^3} = \frac{\rho_0 {t_0}^2}{t^2}$$

And we can find the evolution of the Hubble parameter

$$H = \frac{\dot{a}}{a} = \frac{2}{3t}$$

Radiation-only universe

Similar techniques can be used to look at the radiation only universe (w = 1/3), and we find

$$\rho \propto \frac{1}{a^4}$$

And so the scale factor changes as

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

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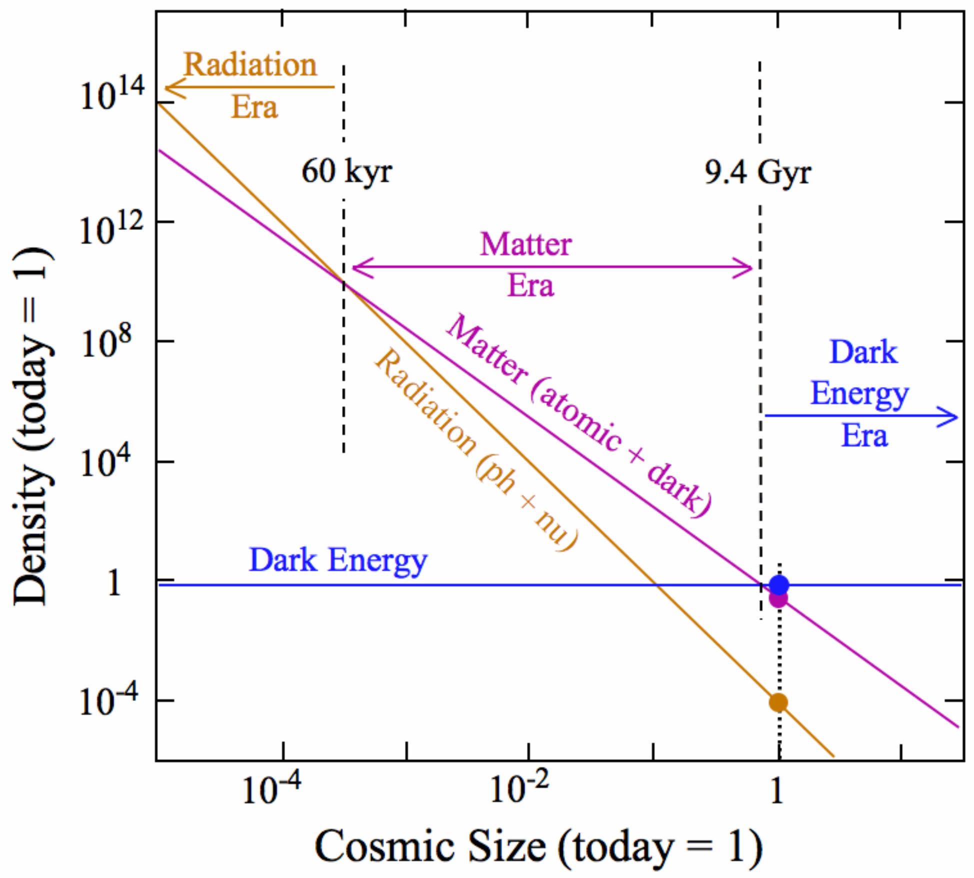
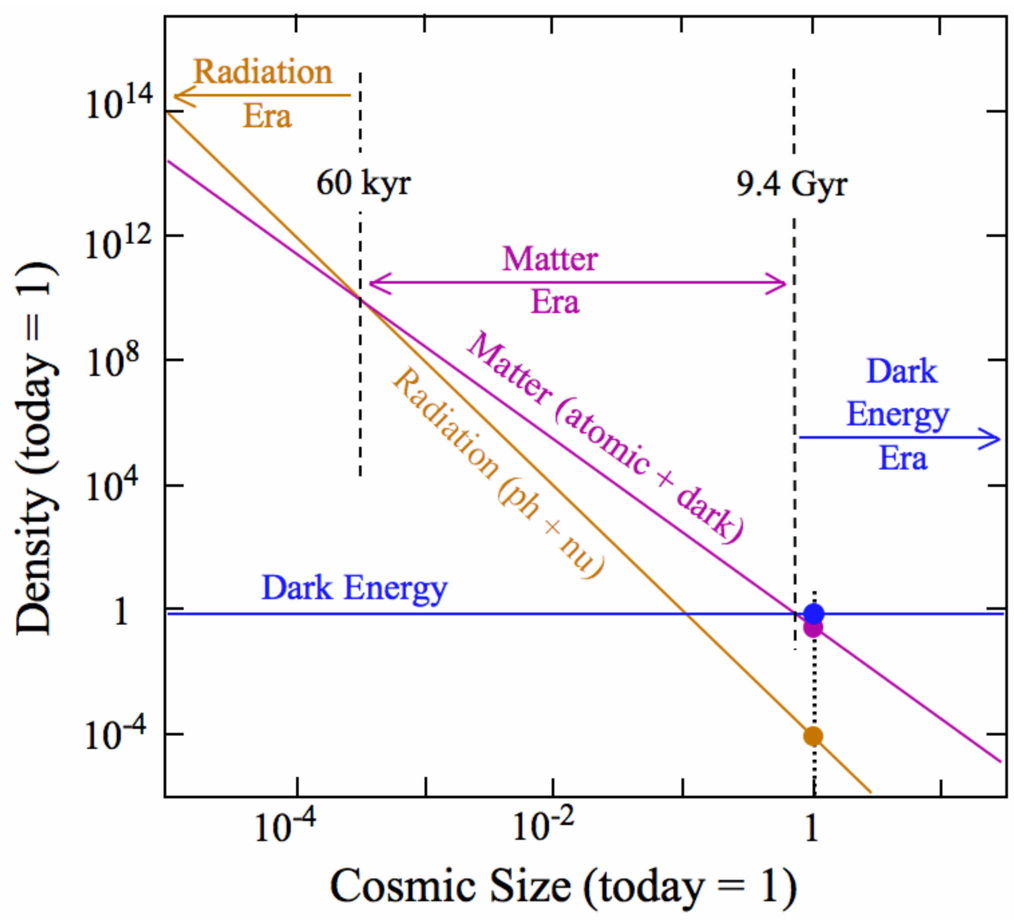
Essential background cosmology



Evolution of matter, radiation and vacuum contributions

The energy density of each component evolves differently with a (as a^{-3}, a^{-4} and a^{0} respectively).

Essential background cosmology



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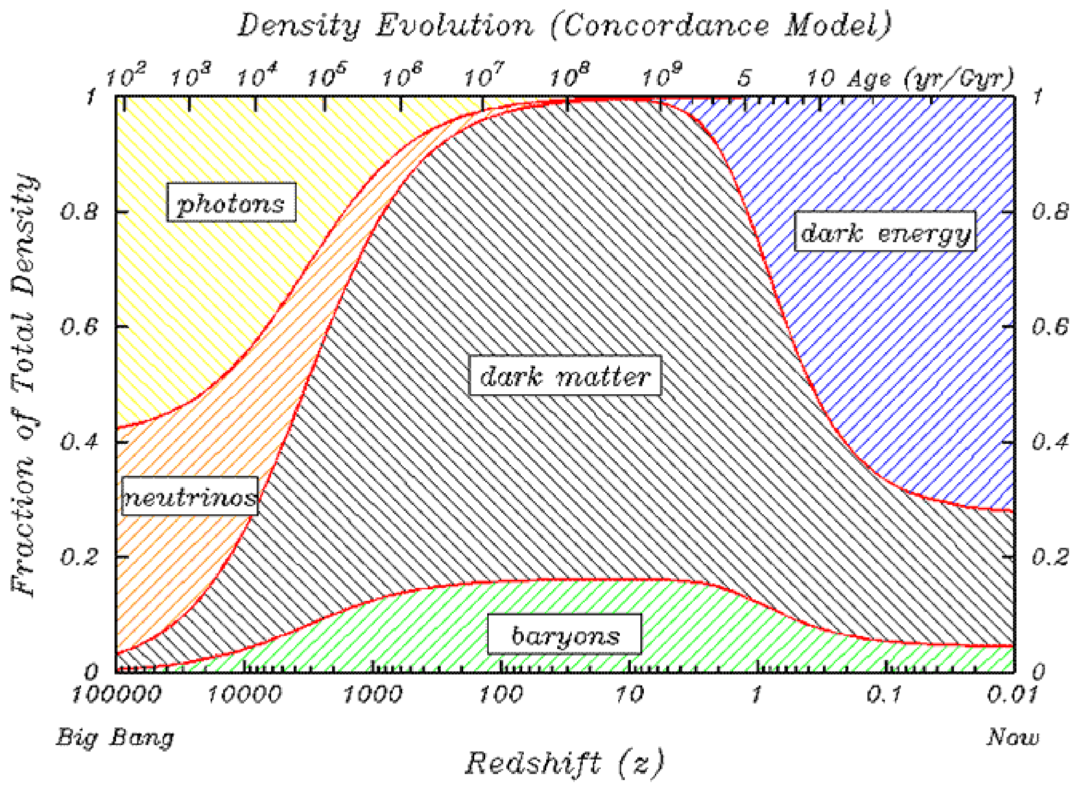

Example: I

Consider the matter dominated cosmos with $P = 0$ and $\Lambda = 0$. Is this universe accelerating or decelerating?

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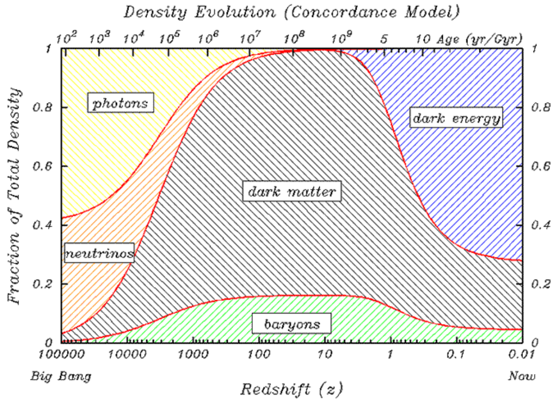
$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\dot{a} = \frac{2}{3}\left(\frac{t}{t_0}\right)^{-1/3}$$

$$\ddot{a} = \frac{-2}{9}\left(\frac{t}{t_0}\right)^{-4/3}$$

$$\boxed{q_0 := -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a},}$$



$q_0 = 1/2$

Therefore, a matter-dominated universe is decelerating.
At present-day (t=t0) our Universe is accelerating (q0 < 0), therefore, it cannot be matter dominated.

Dynamics of the universe

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\rho_{\rm tot} + \frac{\Lambda}{3} - \frac{k}{a^2}$$

$$\Leftrightarrow \quad \dot a^2 = \frac{8\pi}{3}\rho_r a^2 + \frac{8\pi}{3}\rho_m a^2 + \frac{\Lambda}{3}a^2 - k$$

$$\Leftrightarrow \quad \dot a^2 = \frac{K_r}{a^2} + \frac{K_m}{a} + \frac{\Lambda}{3}a^2 - k$$

$$\Leftrightarrow \quad \dot a^2 - \frac{K_r}{a^2} - \frac{K_m}{a} - \frac{\Lambda}{3}a^2 = -k$$

$$\boxed{\rho_r \sim \frac{1}{a^4} \quad \Rightarrow \quad K_r = \frac{8\pi}{3}\rho_r a^4 = \text{const.}}$$

$$\boxed{\rho_m \sim \frac{1}{a^3} \quad \Rightarrow \quad K_m = \frac{8\pi}{3}\rho_m a^3 = \text{const.}}$$

Analog of Newtonian equation KE+ PE = const

$$\boxed{\dot a^2 + V_{\rm eff}(a) = -k}$$

K = 0, expansion velocity asymptotically approaches zero.
K=1, universe expands and later contracts
K=-1, kinetic energy dominates, expansion never stops.

Shows that matter content (or energy density) are inextricably linked to the curvature of the universe. As we know from General Relativity.

Example: II

Starting from the Robertson-Walker Metric, calculate the proper radius (in kpc) of the horizon $3 \text{ X } 10^4$ years after the Big Bang. At this time, the Universe is still radiation dominated.

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But, along a light ray, we know that $ds^2 = 0$. So, this leaves $dr = \frac{c}{a(t)} dt$, which can be integrated to get the comoving horizon.

$$r_{hor} = \int_0^{r_{hor}} dr = c \int_0^t \frac{dt}{a(t)} \tag{2}$$

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But, since this is in the radiation-dominated era, we know that scale factor grows as $a(t) = At^{-1/2}$. Therefore,

$$r_{hor} = c \int_0^t \frac{dt}{a(t)} = \frac{2ct^{1/2}}{A} = \frac{2ct}{a(t)} \tag{3}$$

The proper radius of the horizon is $d_{hor} = ar_{hor} = 2ct$. Inserting t = 30,000 gives d_{hor} = 5.7 × 10^{20} m = 18.4 kpc.