Evolution of Cosmic Structure

Lecture 10: Numerical simulations of structure growth

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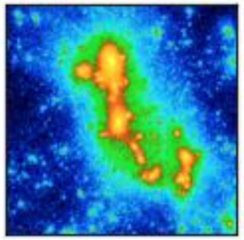
Numerical studies of structure formation

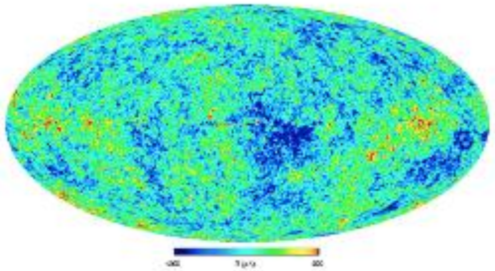
What techniques are used to simulate the development of cosmic structure? How can complex baryon physics be incorporated? What light do the results cast on the evolution of structure?

- The role of simulations
- N-body simulations and dark matter halos
- Adding gas - hydrodynamics
- Incorporating baryon physics



Cosmological model

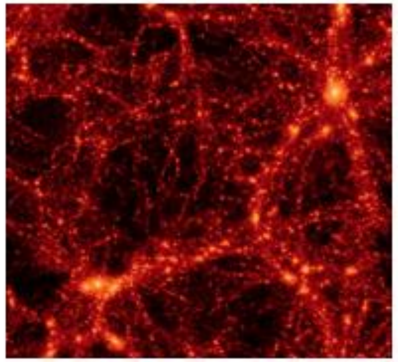
(Ω_m, Ω_Λ, h); dark matter



Primordial fluctuations

$\delta\rho/\rho$**(M, t)**

Well established



Dark matter halos

(N-body simulations)

Well understood

Gas processes

(cooling, star formation, feedback)

Gasdynamic simulations

Semi-analytics

Galaxy formation/evolution

DIFFICULT!

N-body simulations

- (ignoring baryonic physics) the dynamics is given by Newtonian gravity on an expanding background

$$\frac{d\vec{v}}{dt} = -H\vec{v} - \frac{\vec{\nabla}\delta\phi}{a}$$

- the main computational task is calculating the gravitational forces

$$-\frac{\vec{\nabla}\delta\phi}{a}\Big|_{\vec{x}=\vec{x}_j} = \vec{F}_j = \frac{Gm_j}{a^2}\sum_{i\neq j}\frac{m_i(\vec{x}_i - \vec{x}_j)}{|\vec{x}_i - \vec{x}_j|^3}$$

- for N particles, need to calculate N(N-1) ~ N^2 forces

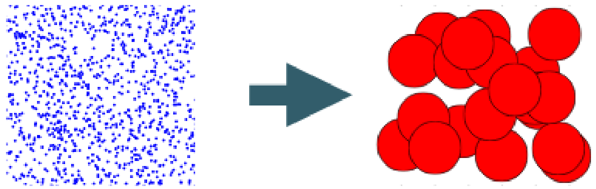
➡ for large N computationally very expensive (e.g. 300 billion particles, highly efficient code ~20 flop/interaction, worlds fastest supercomputer -> 2 years for single computation of forces for particles, need 1000s of timesteps)

N-body simulations

The N-body approach

- What do we mean by simulation particles?
- Most of the mass in the Universe is in the form of dark matter
  - e.g. weakly interacting massive particles (WIMPs) may have a mass of ~100 GeV/c^2

    ➡ 10^{12} solar mass galaxy halo consists of 10^{67} dark matter particles
- can only afford to represent if by ~10^2 - 10^9 particles
- Does representing ~10^{60} dark matter particles by 1 simulation particle have unwanted side effects?

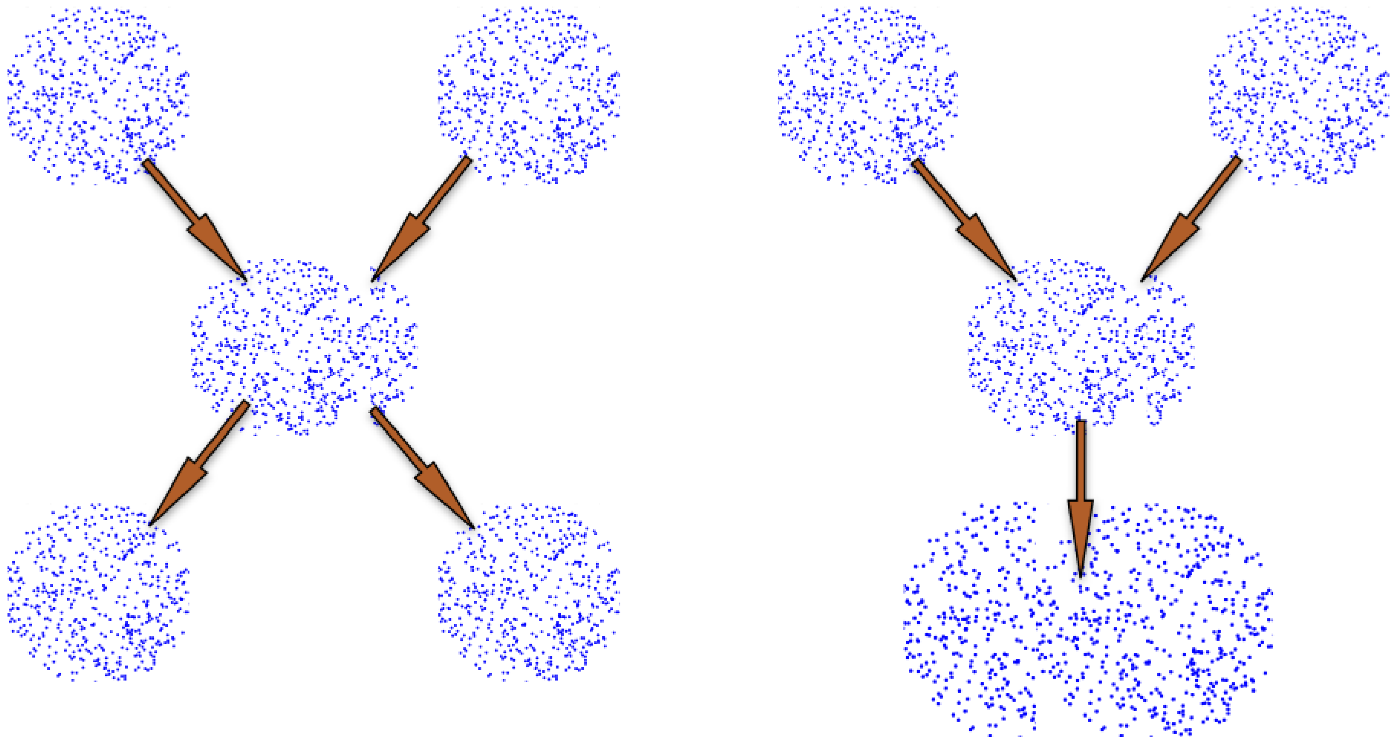


N-body simulations

Gravitational softening

dark matter in galaxies and galaxy clusters should be collisionless

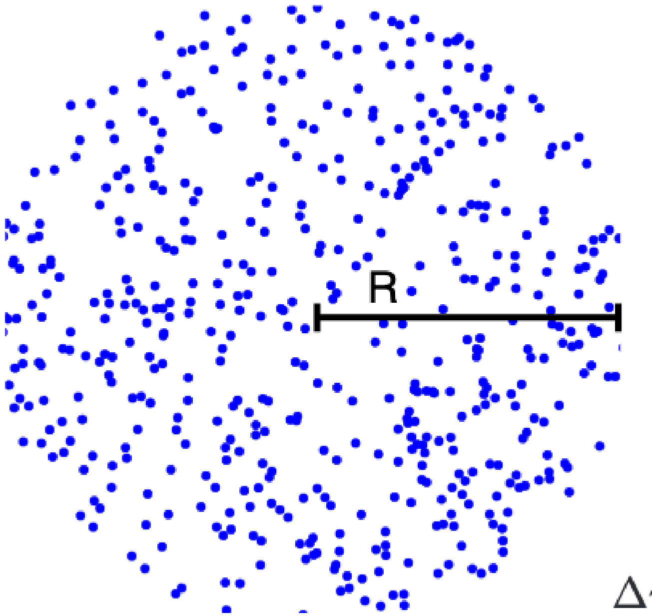
but may become collisional in simulations with a Newtonian force law



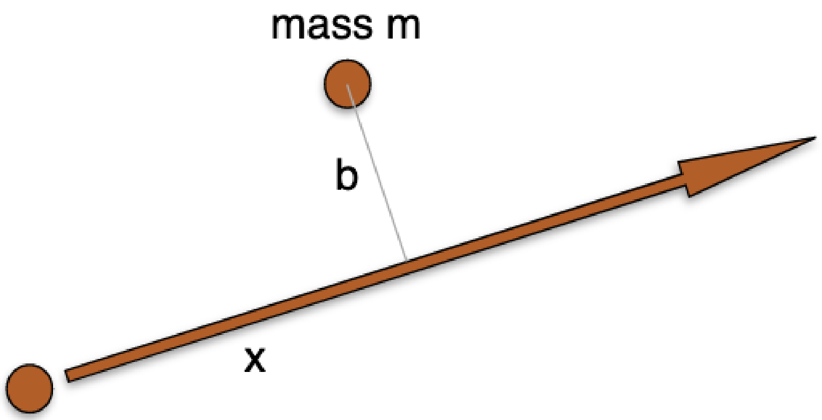
N-body simulations

Relaxation time of an N-body system

- time scale on which two body processes play a role in a N-body system



self-gravitating N-body system



$$\Delta v_\perp = \frac{1}{m}\int F_\perp dt = \int \frac{Gm}{x^2+b^2}\frac{b}{\sqrt{x^2+b^2}}\frac{dx}{v} = \frac{2Gm}{bv}$$

N-body simulations

Relaxation time of an N-body system

- typical velocity $$v^2 \approx \frac{GNm}{R}$$
- using this we get
$$b_{\min} \approx \frac{2Gm}{v^2} \approx \frac{2R}{N}$$
and
$$(\Delta v_\perp)^2 = \frac{8G^2m^2}{R^2v^2} N \ln\left(\frac{b_{\max}}{b_{\min}}\right) = \frac{8v^2}{N} \ln\left(\frac{N}{2}\right)$$ (per crossing time)
- the two-body relaxation time is then
$$t_{\text{relax}} \approx t_{\text{cross}} \frac{v^2}{(\Delta v_\perp)^2} \approx t_{\text{cross}} \frac{N}{8\ln\left(\frac{N}{2}\right)}$$

N-body simulations

Gravitational softening

- for collisionless systems we need to ensure:

  simulated time << relaxation time

  - prevent large angle scattering
  - prevent formation of bound particle pairs

- want to integrate equations of motion with low-order scheme and reasonably large timesteps

➡ need to soften force law on small scales:

$$\vec{F}_j = \frac{G m_j}{a^2} \sum \frac{m_i(\vec{x}_i - \vec{x}_j)}{((\vec{x}_i - \vec{x}_j)^2 + \epsilon^2)^{\frac{3}{2}}}$$

gravitational softening

N-body simulations

Choice of Gravitational softening

- too small softening:
    - system may become collisional
    - time integration more expensive
    - artificial heating
- too large softening:
    - loss of spatial resolution in the simulations
- typical value in cosmological simulations:
    - ~2% to 4% of the mean-interparticle distance $(V/N)^{1/3}$

N-body simulations

Requirements for (competitive) N-body simulations

- want large **N** to have high resolution (otherwise small objects are not resolved) and large volume (otherwise no representative volume and no rare objects like galaxy clusters, also the fundamental mode goes non-linear at low-z)
- need efficient self-gravity algorithms with scaling close to ~N (and not N^2)
- need to be able to run it efficiently in parallel on 1000s of CPU cores
- should be memory and communication efficient
- should automatically adapt the size of the timestep to the relevant dynamical time

Overview of self-gravity algorithms

- direct summation ~ N^2 -> not competitive in cosmological runs
- particle-mesh codes
- tree codes

rarely used alone nowadays

- tree particle-mesh codes (e.g. used in the GADGET code)
- multigrid relaxation (e.g. used in the RAMSES code)
- fast multipole codes, …

Halo Finding methods

- Can search for haloes using different particle information:
  - Positions (configuration space): 3D
  - Positions & velocities (configuration & velocity/phase space): 3D + 3D
  - Positions, velocities & time (phase space & time domain): 7D

Easiest methods using position:

Friends of Friends (FoF)

Density Peak Finder

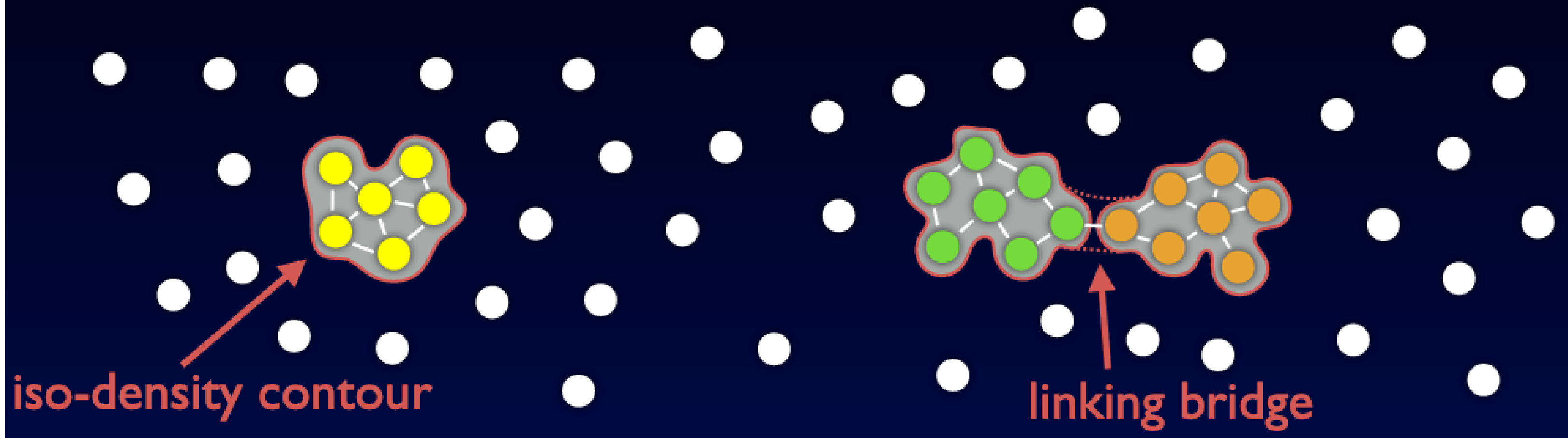
More advanced methods also use the velocity information to ‘unbind’ particles: adaptive/hierarchival FOF, BDM, SUBFIND, AHF..

Halo Finding methods

Friends-of-Friends

- Uses only particle positions to group spatially close particles:

  $|\vec{x}_i - \vec{x}_j| \leq b\,\Delta x = b\,B/\sqrt[3]{N}$, B = Boxsize, N = # of particles, b ≈ 0.2

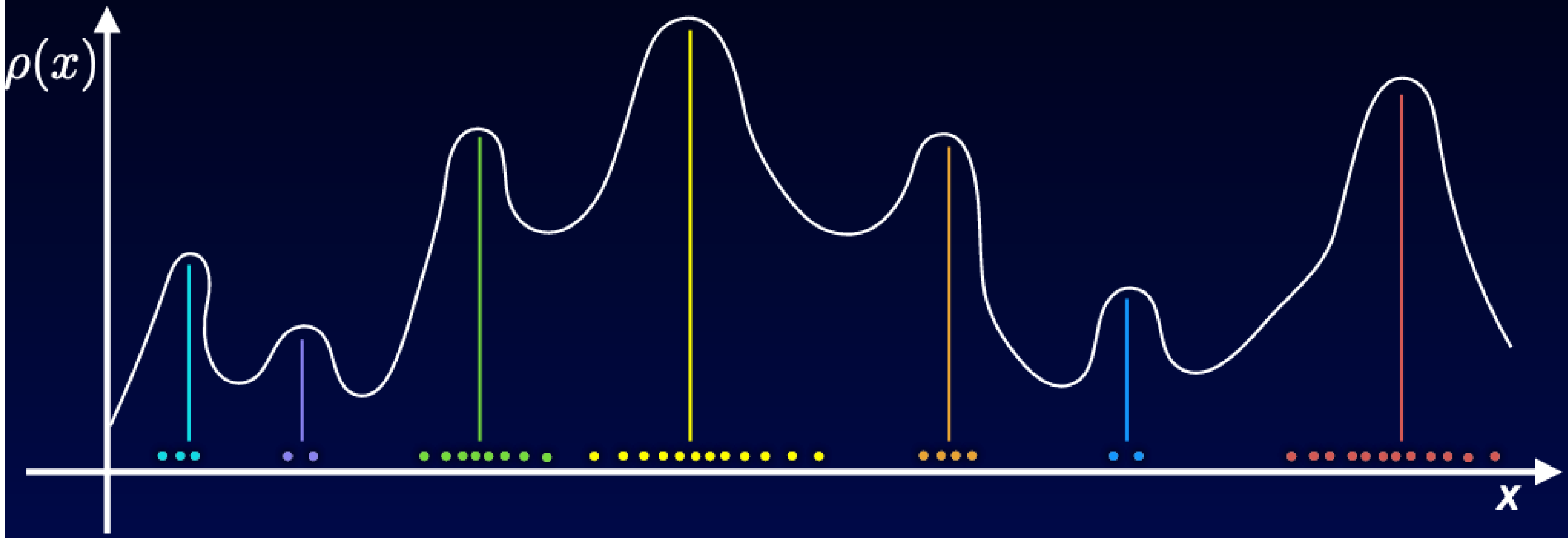


- Advantages: fast, arbitrary halo shapes
- Disadvantages: no subhaloes, danger of linking bridges

Halo Finding methods

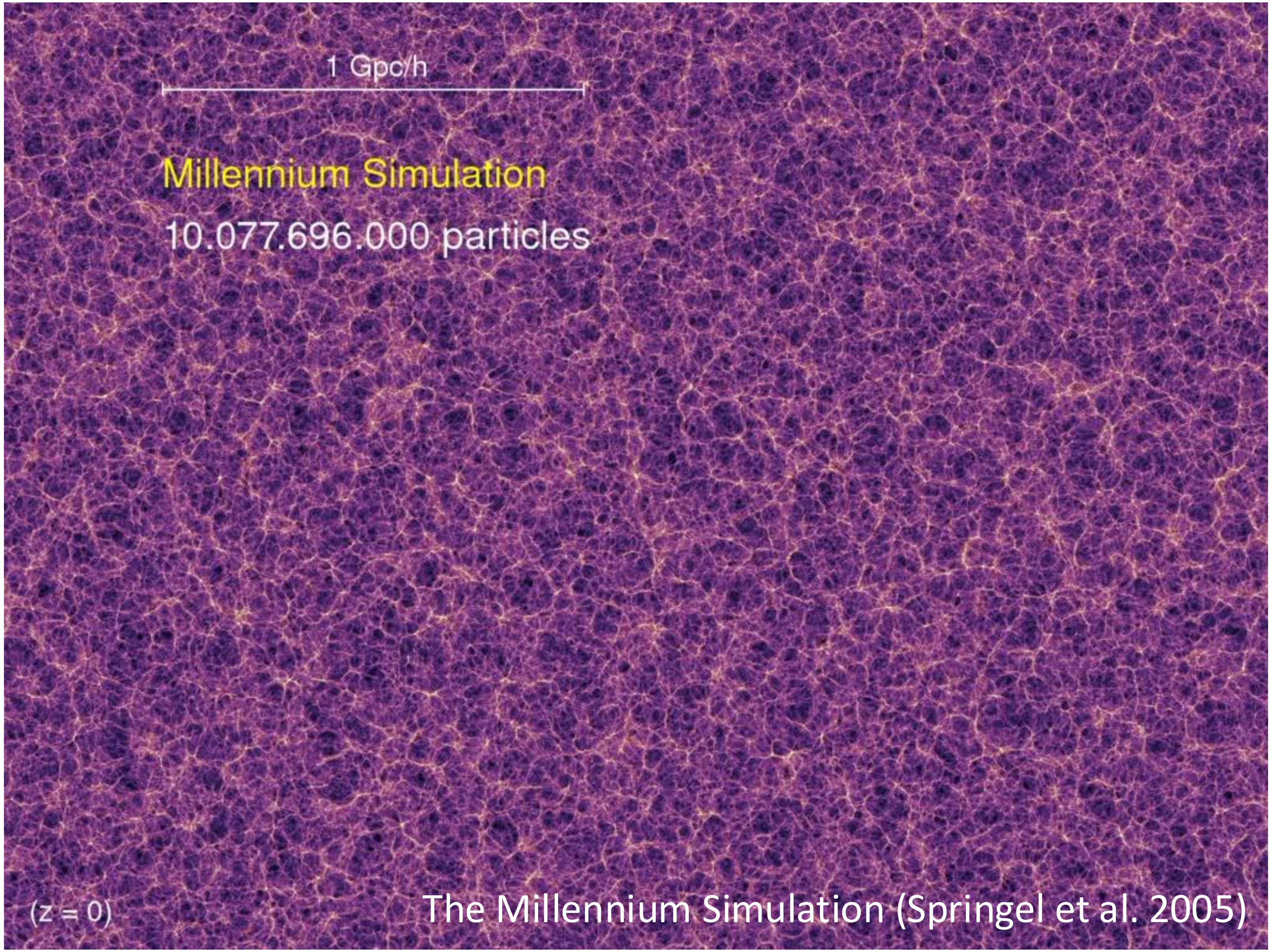
Density Peak Finders

- Smooth density field and locate peaks
- Collect particles around peaks



- Advantages: haloes clearly separated, can identify subhaloes
- Disadvantages: usually spherical shapes, need to smooth density field

N-body: The Millennium Simulation



The Millennium Simulation (Springel et al. 2005)

Dark matter halos

Early numerical studies soon revealed striking similarities in the density profiles of dark matter halos.

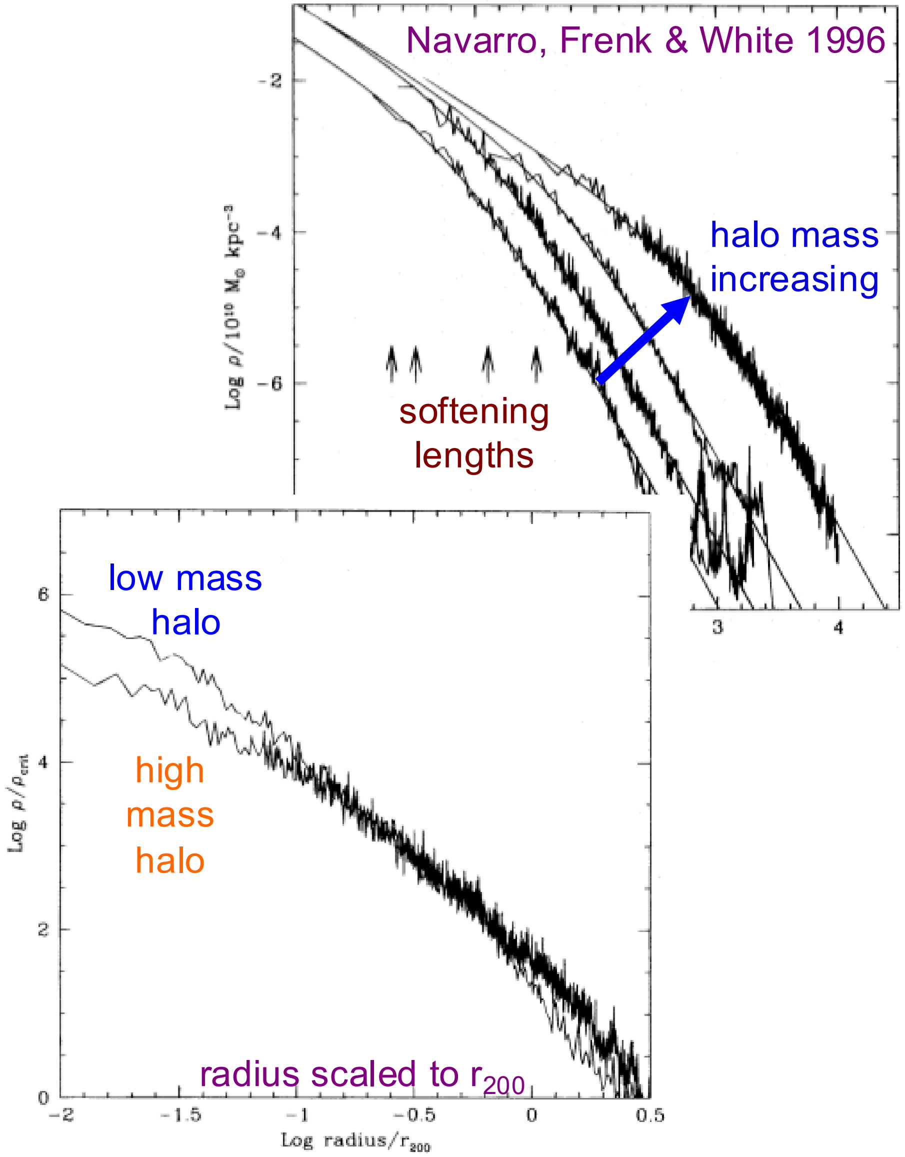
Navarro, Frenk & White (1996) studied a large sample of dark matter haloes and found the profiles of most halos were well fitted by the functional form:

$$\rho(r) = \frac{4\rho_s}{(r/r_s)(1+r/r_s)^2}$$

where ρ_s is the density at the scale radius r_s. This has consequently become known as an NFW profile.

They also found that when these profiles are almost self-similar when scaled to a characteristic overdensity radius, often taken (motivated by the top-hat collapse model) to be r_{200}, such that the mean interior density is $200\rho_c$ – i.e.

$$\frac{M_{200}}{4\pi r_{200}^3/3} = 200\rho_c$$ where M_{200} is the mass interior to r_{200}.



Adding gas -hydrodynamics

The Euler equations

- The equations of hydrodynamics can be written in terms of conserved quantities

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad \text{mass conservation}$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) + \nabla P = \mathbf{0} \qquad \text{momentum conservation}$$

$$\frac{\partial \rho e_{\rm tot}}{\partial t} + \nabla \cdot ((\rho e_{\rm tot} + P)\mathbf{u}) = 0 \qquad \text{energy conservation}$$

with $e_{\rm tot} = e + \frac{u^2}{2}$ and e = internal energy per unit mass

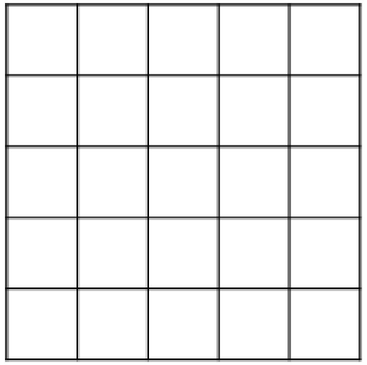
$$P = (\gamma - 1)\rho e \qquad \text{equation of state}$$

Adding gas -hydrodynamics

Eulerian vs Lagrangian methods

Eulerian methods

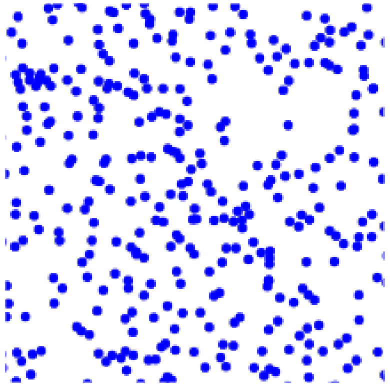
discretize space
(finite-volume scheme)



use a grid fixed in space

Lagrangian methods

discretize mass



use particles for the gas (like in n-body) which move with the flow

Adding gas -hydrodynamics

What simulation method would be better for simulating gas being stripped via ram pressure from a galaxy moving through a galaxy cluster?

Assume a cluster with a temperature of $T = 3 \times 10^7 K$ and the relative velocity between the ICM and the galaxy is $900\ km\ s^{-1}$.

$$M = \frac{u}{c}$$

$$c = \sqrt{\frac{5k_B T}{3\mu m_p}}$$

c = $640\ km\ s^{-1}$ so, $M = 1.4$

As this is clearly a shock, it would be better to use a Eulerian method.

Incorporating baryon physics

- The largest *hydrodynamic* simulations being carried out today have ~10^{8-9} resolution elements (cells or particles). For *uniform* resolution this implies a dynamic range of only 10^3 for 3D simulations. Even with adaptive resolution (such as adaptive mesh refinement, or SPH), **the dynamic range that can be probed presently is only ~10^{5-6}** (e.g. a 100 Mpc box with 1 kpc resolution).
- **Many physical processes relevant for galaxy formation occur on much smaller scales** (AU to pc). Simulators often refer to this as "sub-grid physics".
- Fortunately, some of these processes are well understood (e.g., stellar evolution, radiative cooling).
- For those not well understood we either use empirical prescriptions (e.g. star formation) or simple models (e.g. supernova feedback) and explore sensitivity to the parameters.

Incorporating baryon physics - examples

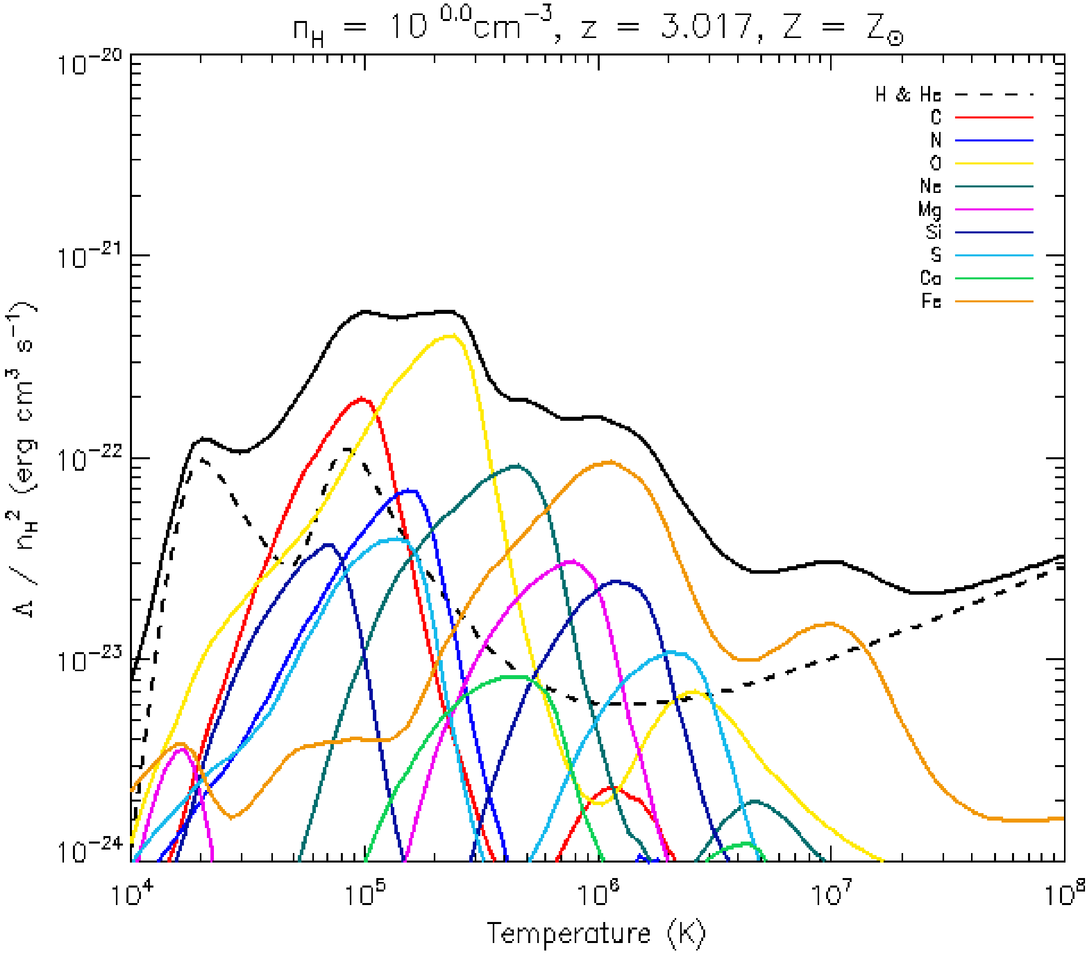
Cooling

- Dissipation of energy by radiative cooling plays a crucial role in galaxy formation
- In the absence of photoionization we have collisional ionization equilibrium (CIE)
- However, the cosmic UV/X-ray background can have a large effect on gas cooling
- Common approximations:
  - H and He included with optically thin photo-ionization from the background
  - Metal cooling ignored altogether or treated assuming CIE and solar relative abundances

Incorporating baryon physics - examples

Cooling

Example of a cooling function when metals and photoionization are included.

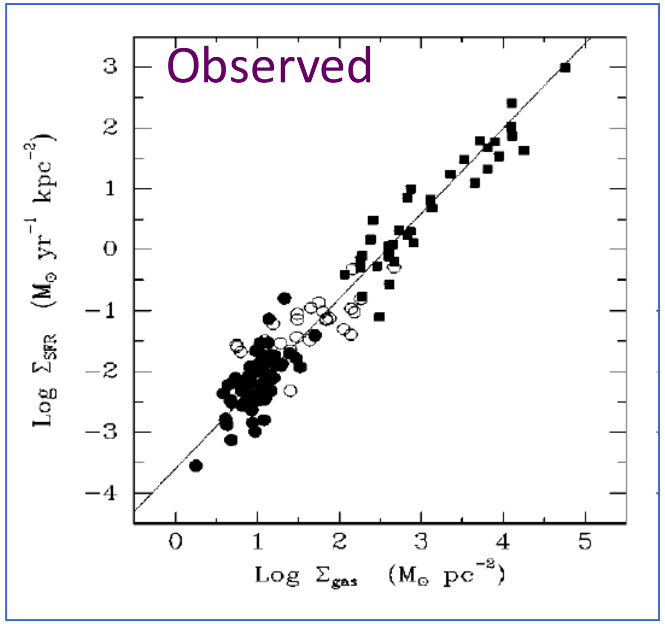


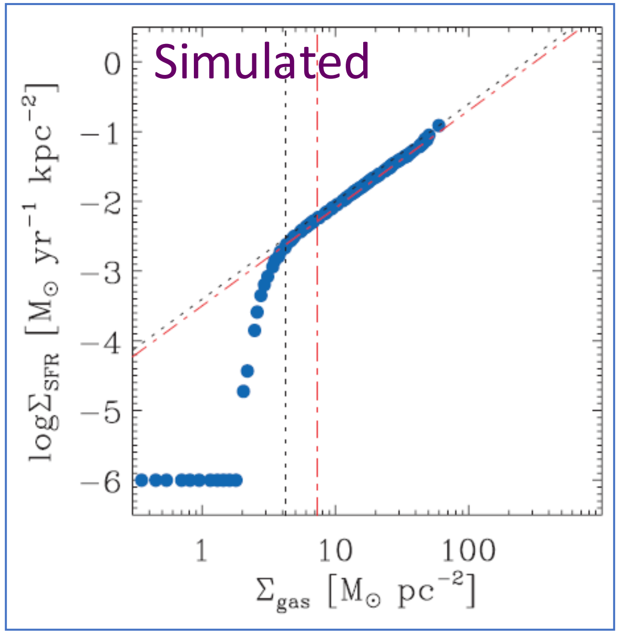
Wiersma, Schaye & Smith, 2009

Incorporating baryon physics - examples

Star formation

- Star formation is a potentially very nasty problem (magnetohydrodynamics + radiative transfer). Fortunately, nature has been kind...
- A simple relationship is observed between the surface density of gas in a galactic disk, and the star formation rate (SFR) per unit area – Kennicutt-Schmidt law
- This can be simply inserted into simulations, to estimate SFR.
- An `initial mass function' (IMF) for the stars formed must be assumed.





Incorporating baryon physics - examples

Cosmic feedback – Supernova, AGN, etc.

- In the absence of heating processes, it is found that most of the gas in cosmological simulations cools at high z, when the Universe is dense, and would form stars. In practice, only 10-15% of the baryonic matter is found in stellar form.
- In reality, this is prevented by *cosmic feedback* – the process whereby stars and AGN pump energy back into gas which surrounds them, preventing it from cooling.
- Since stellar evolution is fairly well understood, we can predict how many supernovae should follow after star formation with a given IMF.
- The total energy released from each supernova is believed to be about 10^{44}J, but it is very uncertain how this couples with surrounding gas. One simple assumption is that it simply heats surrounding gas (but how much gas?), another is that it is transmitted in the form of *kinetic* energy, driving a wind. These different assumptions can lead to very different consequences.
- For example, heating dense gas will simply cause it to radiate profusely, so that the supernova energy escapes from the galaxy with little effect.

Backup Slides

N-body simulations

Time integration

- How to do the time integration?
  - simplest way - Euler integration: $x_{n+1} = x_n + v_n \Delta t$
    $v_{n+1} = v_n + a_n \Delta t$

    only first order accurate
    -> would need much more time steps for comparable accuracy
  - leap-frog (used in many codes):

$$
\begin{aligned}
v_{n+1/2} &= v_n + a_n \Delta t/2 \\
x_{n+1} &= x_n + v_{n+1/2} \Delta t \\
v_{n+1} &= v_{n+1/2} + a_{n+1} \Delta t/2
\end{aligned}
$$

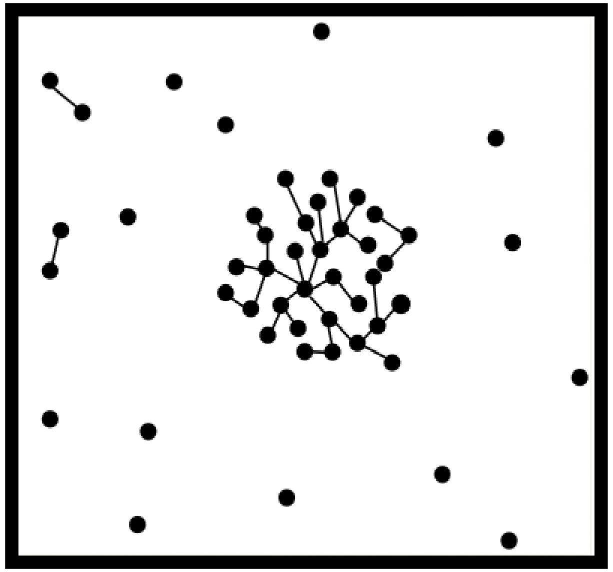
second order accurate

- adaptive timesteps: shortest dynamical timescale changes over time -> code should adapt to it, e.g. $\Delta t \propto 1/\sqrt{a}$
- individual timesteps: dynamical timescale depends on environment (large in low density regions, small in halo centers)

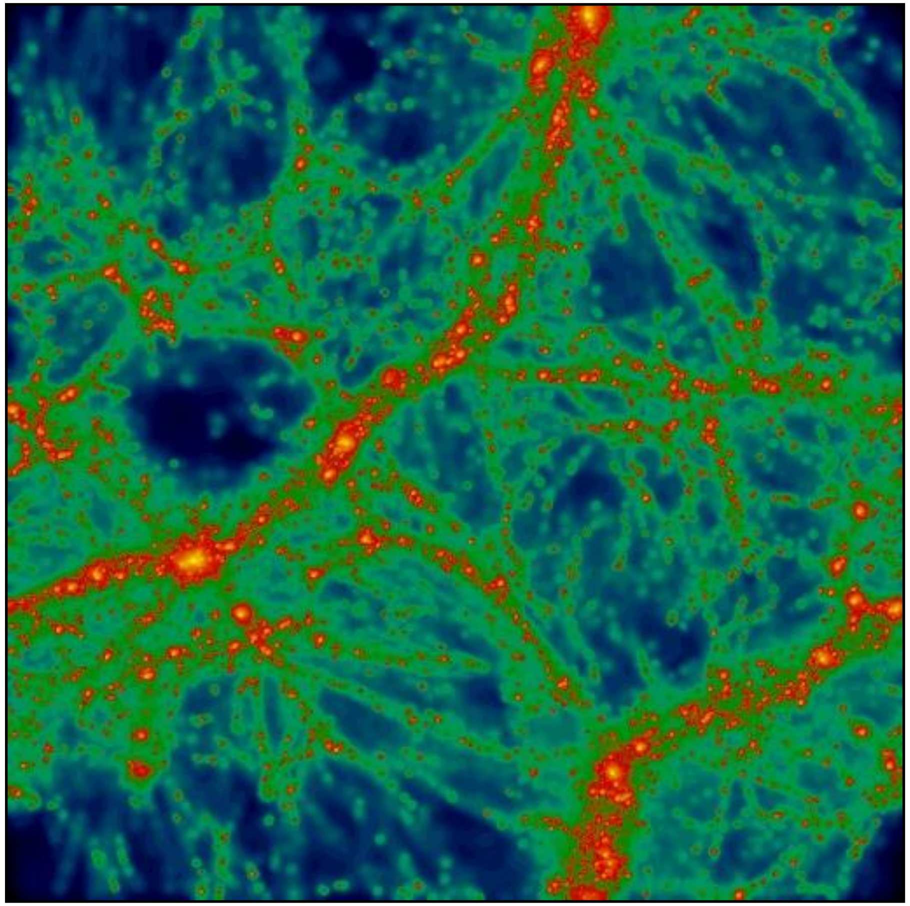
N-body: The Millennium Simulation

- Run in 2005 by The Virgo Consortium (www.virgo.dur.ac.uk) using 512 processors at the Computing Centre of the Max-Planck Society, using 350,000 processor hours of CPU time, or 28 days of wall-clock time
- Cubic region $500h^{-1}$Mpc on a side (where h=H/100 km $s^{-1}Mpc^{-1}$)
- $N = 2160^3 \cong 1.0078 \times 10^{10}$ particles, with mass $8.6 \times 10^8\, h^{-1}\, M_\odot$
- Initial conditions at z = 127 created by displacing dark matter particles with a Gaussian random field and ΛCDM linear power spectrum
- Used the GADGET-2 code, incorporating a TreePM force calculation algorithm
- Friend-of-friends linking to establish dark matter halos in each of 64 output snapshots

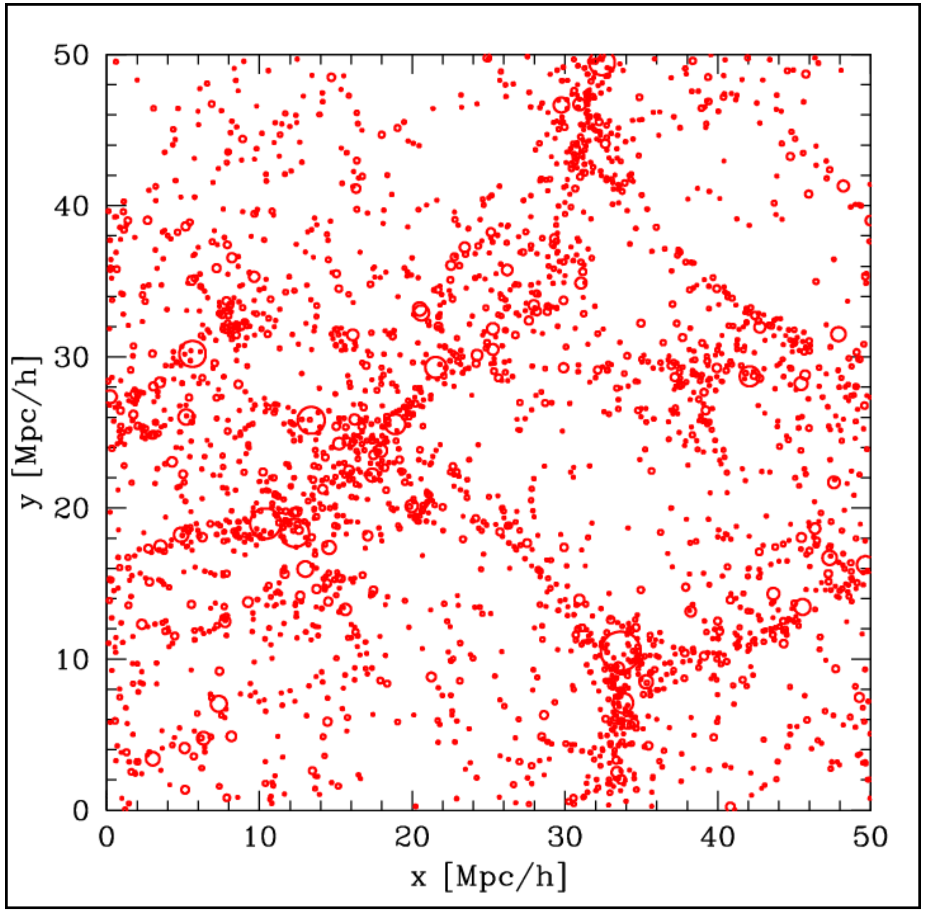
FoF linking of dark matter particles



Dark matter halos



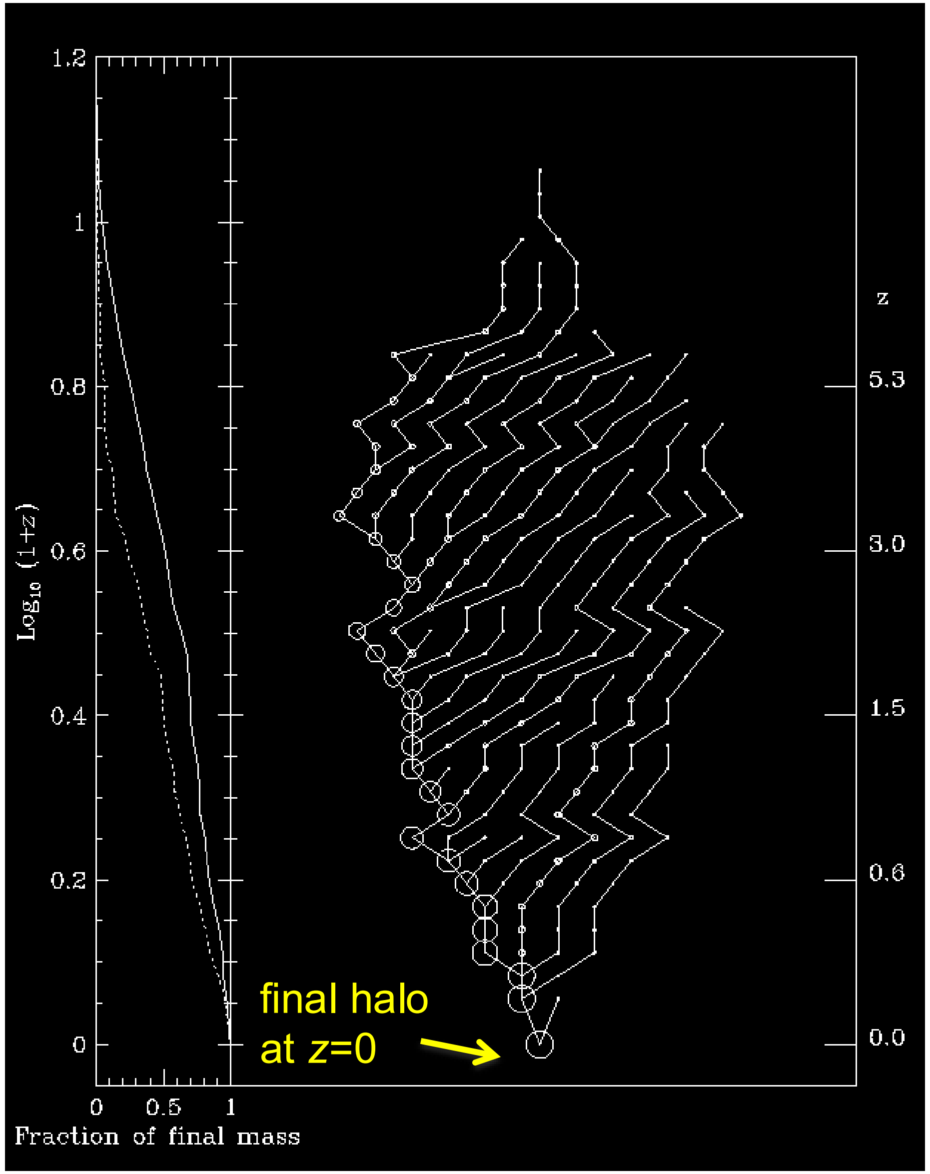
Dark matter particle density



Identified dark matter halos

N-body: The Millennium Simulation

- Within friend-of-friends haloes, dark matter subhaloes are identified, using an algorithm called SUBFIND
- Merger histories are calculated by tracking individually labeled dark matter particles to link different generations of halos and subhalos
- After determining all of the halo and subhalo links, these are traced backwards from *z* = 0 to form merger trees



N-body simulations

Relaxation time of an N-body system

- particles encountered with impact parameter between b and b+db during one crossing

$$dn \approx \frac{2\pi b\,\mathrm{d}b}{\pi R^2} N$$

- individual encounters add incoherently

$$(\Delta v_\perp)^2 = \int \left(\frac{2Gm}{bv}\right)^2 dn = \frac{8G^2m^2}{R^2v^2} N \int \frac{\mathrm{d}b}{b} = \frac{8G^2m^2}{R^2v^2} N \ln\left(\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\right)$$

(per crossing time)

$b_{\mathrm{max}} \approx R$ given by system size

b_{min} controls maximum deflection which should be ~ v $\quad \frac{2Gm}{b_{\mathrm{min}} v} \approx v \quad \rightarrow \quad b_{\mathrm{min}} \approx \frac{2Gm}{v^2}$

The problem of dynamic range

	Dream	**Reality 1**	**Reality 2**	**Reality 3**
Cosmic structures to simulate	stars to horizon	stars to galaxy	sub-galactic objects to large-scale structure	galaxy to cosmological scale
Spatial scales to resolve	0.1pc to 10^4Mpc 10^{15}-10^{26} m	0.1pc~100kpc 10^{15}-10^{21} m	few kpc ~ few 100Mpc 10^{20}-10^{25} m	few 10kpc to 3000Mpc 10^{21}-10^{26} m
Spatial dynamic range : initial conditions / force resolution**	10^{11}	10^6 / 10^5	10^5 / 10^4	10^5 / 10^4
Dynamic range in mass (# of simulation particles)	10^{33}	10^{15}	10^{12}	10^{12} (memory~40TB)

** Force resolution 10 times higher than the mean particle separation assumed

N-body simulations

Size of cosmological simulations over time

- computers double speed every 18 months (Moore's law)
- particle number in simulations doubles every 16-17 months
- only possible with algorithms that scale close to ~N (or N log(N))

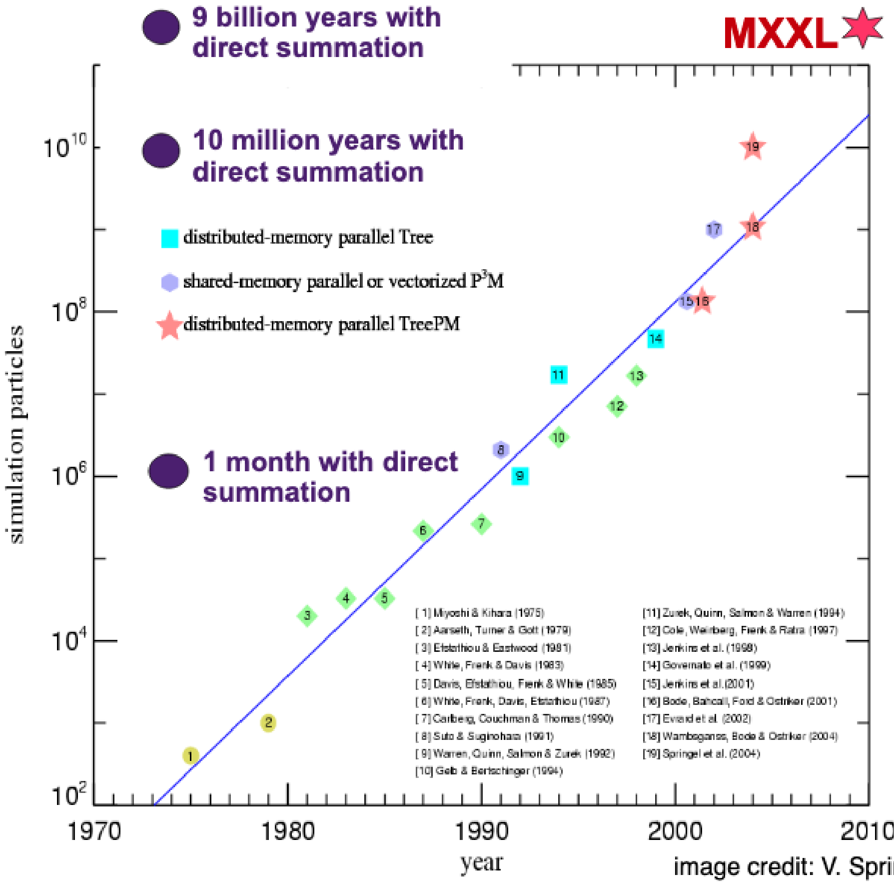


image credit: V. Springel

N-body simulations

Relaxation time of an N-body system (examples)

	N	tcross	trelax
star cluster	10^5	~1/2 Myr	~1/2 Gyr collisional
stars in galaxy	10^{11}	~0.01 / H_0	~5x10^6 / H_0 collisionless
dark matter in galaxy	10^{67}	~0.1 / H_0	~10^{63} / H_0 collisionless
galaxy in low-res simulation (without softening)	1000	~0.1 / H_0	~2 / H_0 somewhat collisional (but should be collisionless)

Hydrodynamics

Cosmological simulations