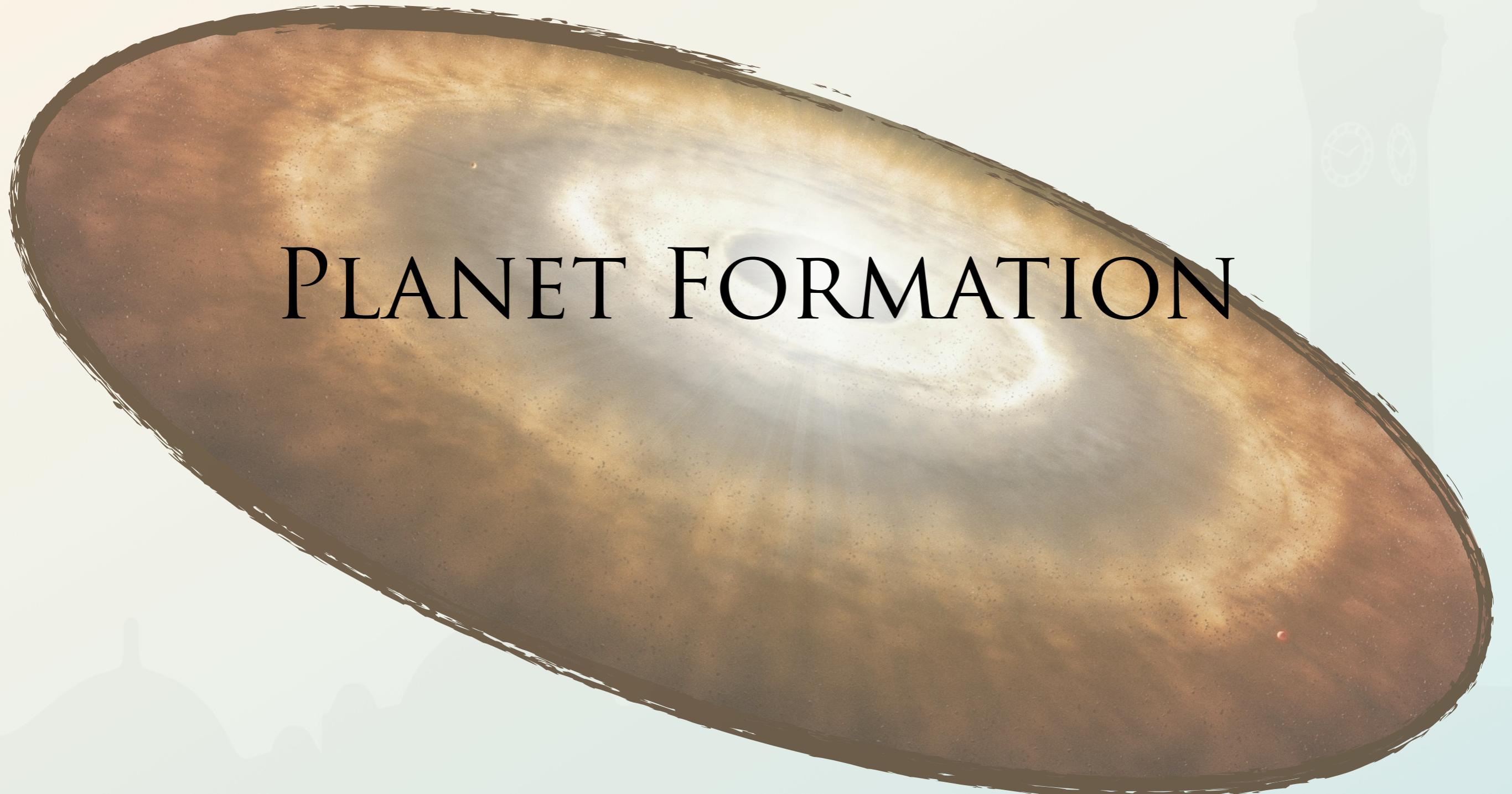




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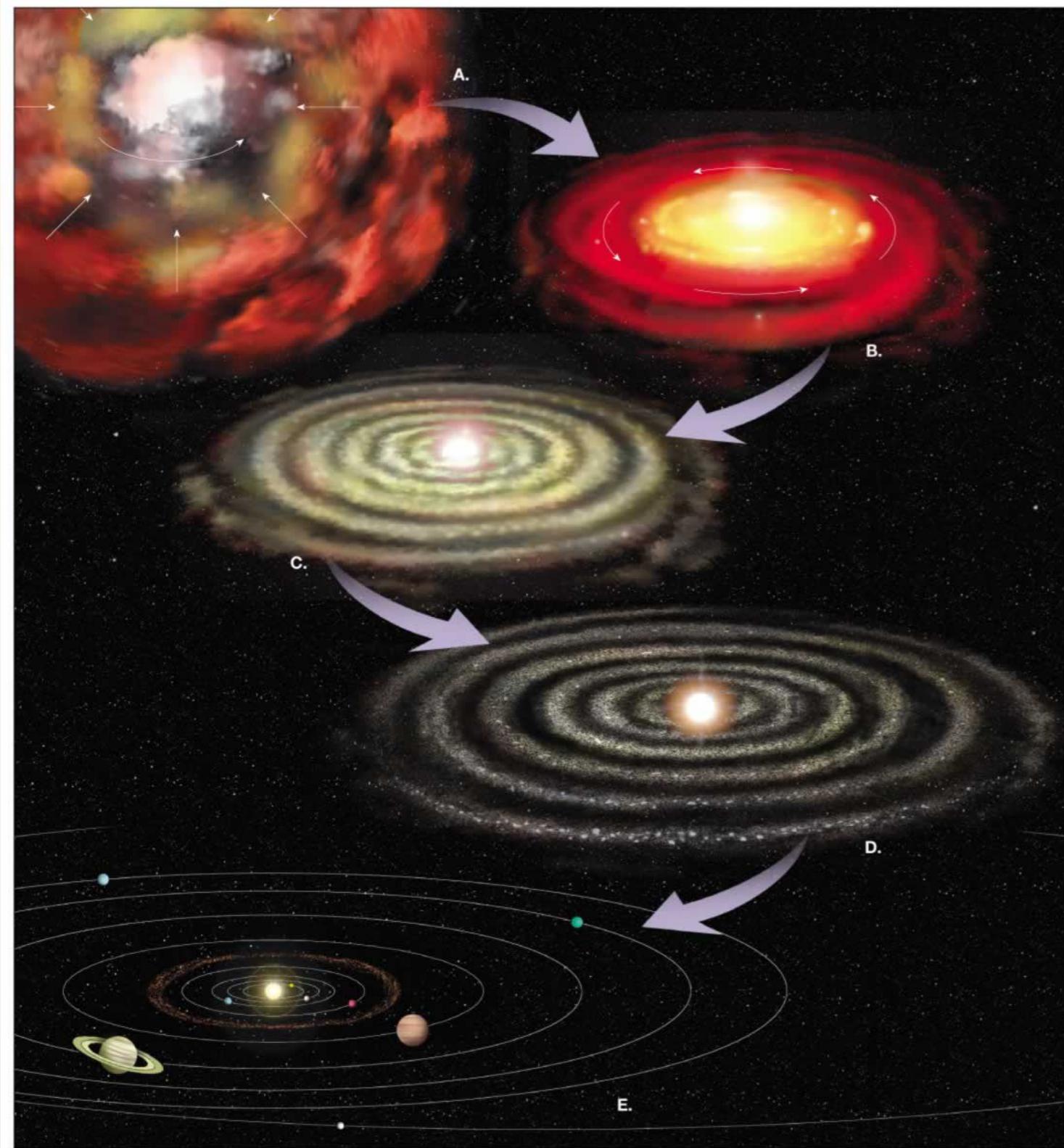


PLANET FORMATION

KEY CONCEPTS

- Cloud collapse
- Orbital angular momentum - Keplerian velocity
- Jean mass / Jeans Length
- Understanding gap formation & core-accretion
- Disc Scale height
- Ice-lines
- Equilibrium temperature
- Scaling relations
- Escape velocity & sound speed
- Hill Sphere & isolation mass
- Direct collapse / gravitational instability

BRIEF HISTORY OF PLANET FORMATION



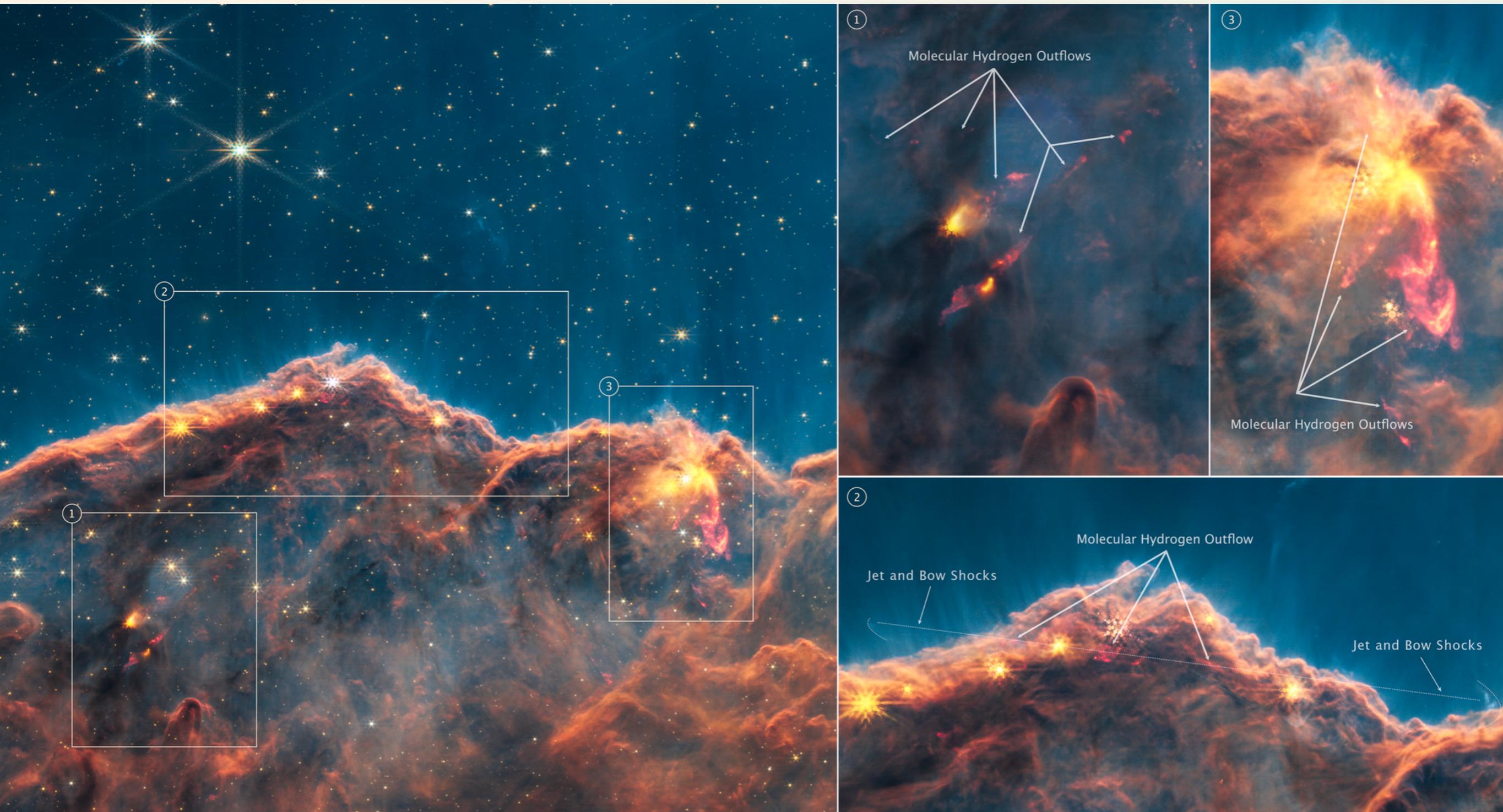
Part of a nebula collapses on itself.

The centre becomes a **protostar**

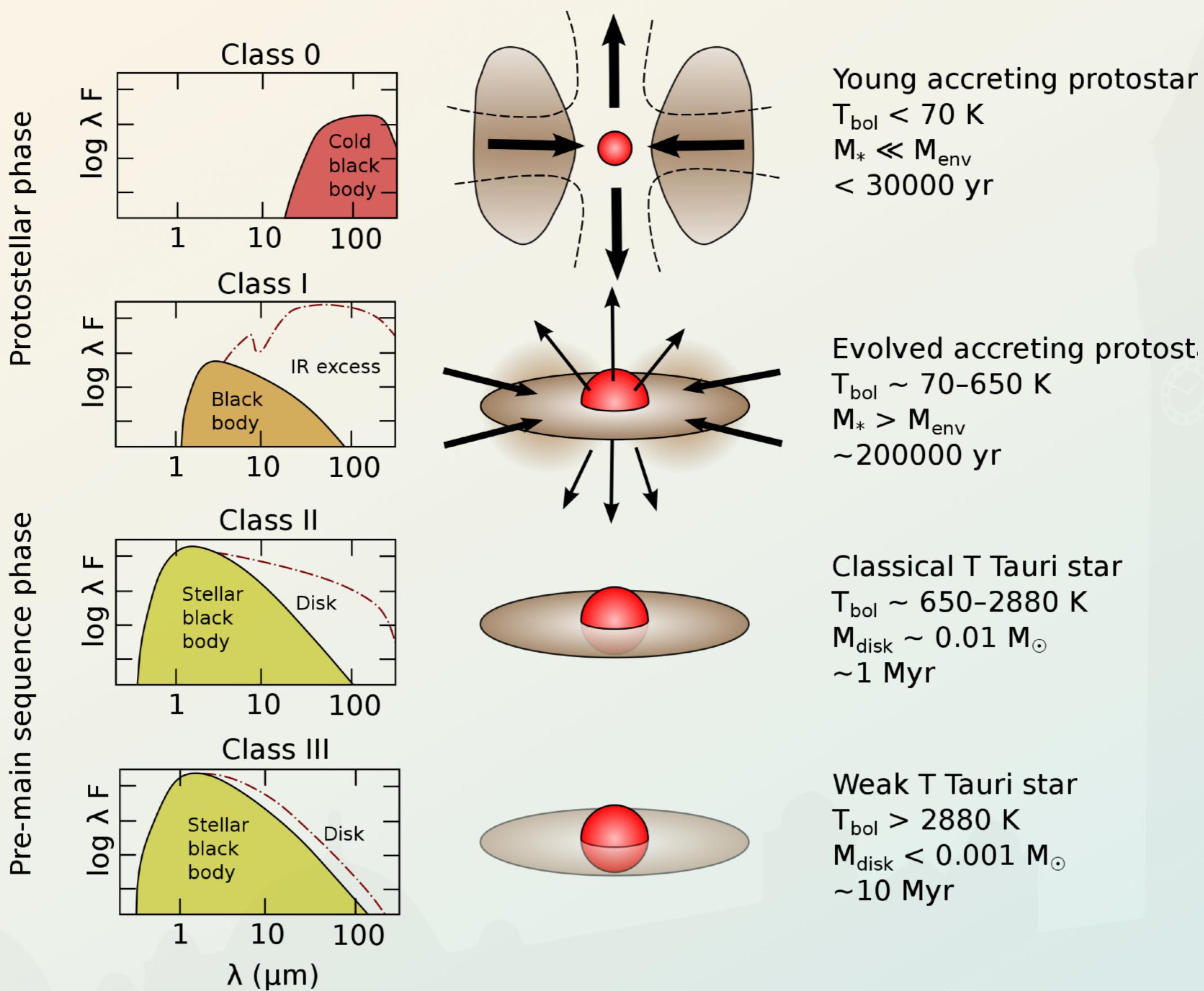
Because of residual angular momentum, the outer part of the nebula become a disc, a **protoplanetary disc**.

Planet formation happens within this disc

BRIEF HISTORY OF PLANET FORMATION



EVOLUTION OF YOUNG STELLAR OBJECTS



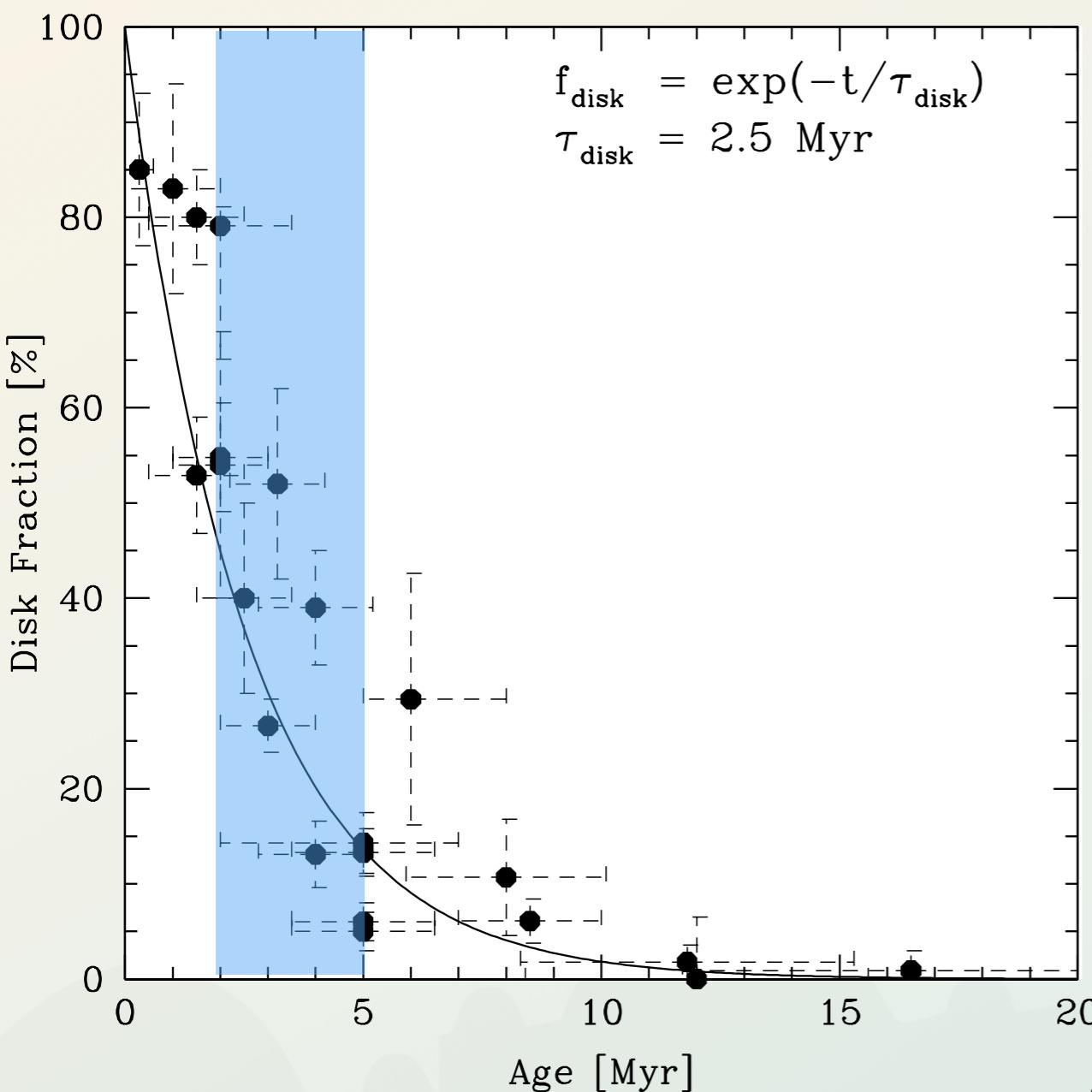
EVOLUTION OF YOUNG STELLAR OBJECTS

- Class 0** A giant molecular cloud $\sim 10^4 M_{\odot}$ collapses and fragments into smaller cores, who also collapse thus starting star formation ignites, but remain imbedded in an opaque cloud.
- Class I** Since the cloud had some non-zero angular momentum, it rotates. As it contracts, its rotation velocity increases and spreads the material into a disc. In addition powerful magnetic polar jets expel some of the mass. Both effects help reveal the protostar.
- Class II** Also called a classical **T-Tauri star**. A pre Main-Sequence star is visible and rapidly contracts. There is an InfraRed excess due to the colder, but large disc.
- Class III** Because of photo-evaporation and planet formation, a big inner gap forms, and most of the material in the disc is now gone. The overall spectrum is barely distinguishable from a stellar blackbody.
Most of the disc's material is accreted by the star.
A typical disc is $0.5 - 1 \times 10^{-1} M_{\star}$

WHEN DO PLANETS FORM?

During Class I and II planet formation is most active.

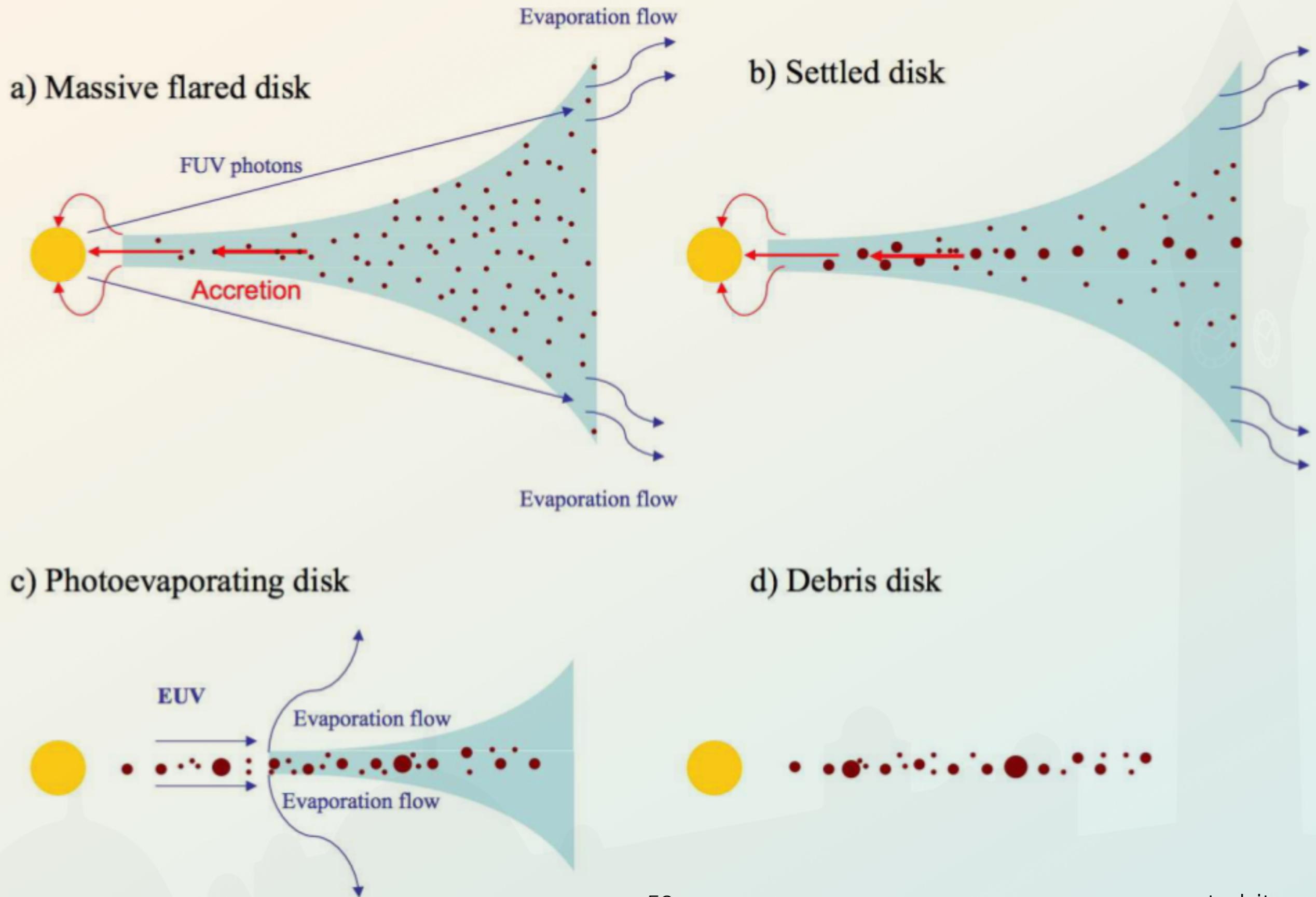
Most stars have no disc after 10 Myr. This picture is compatible with the clues in our solar system (Castillo-Rogez 2007, in blue)



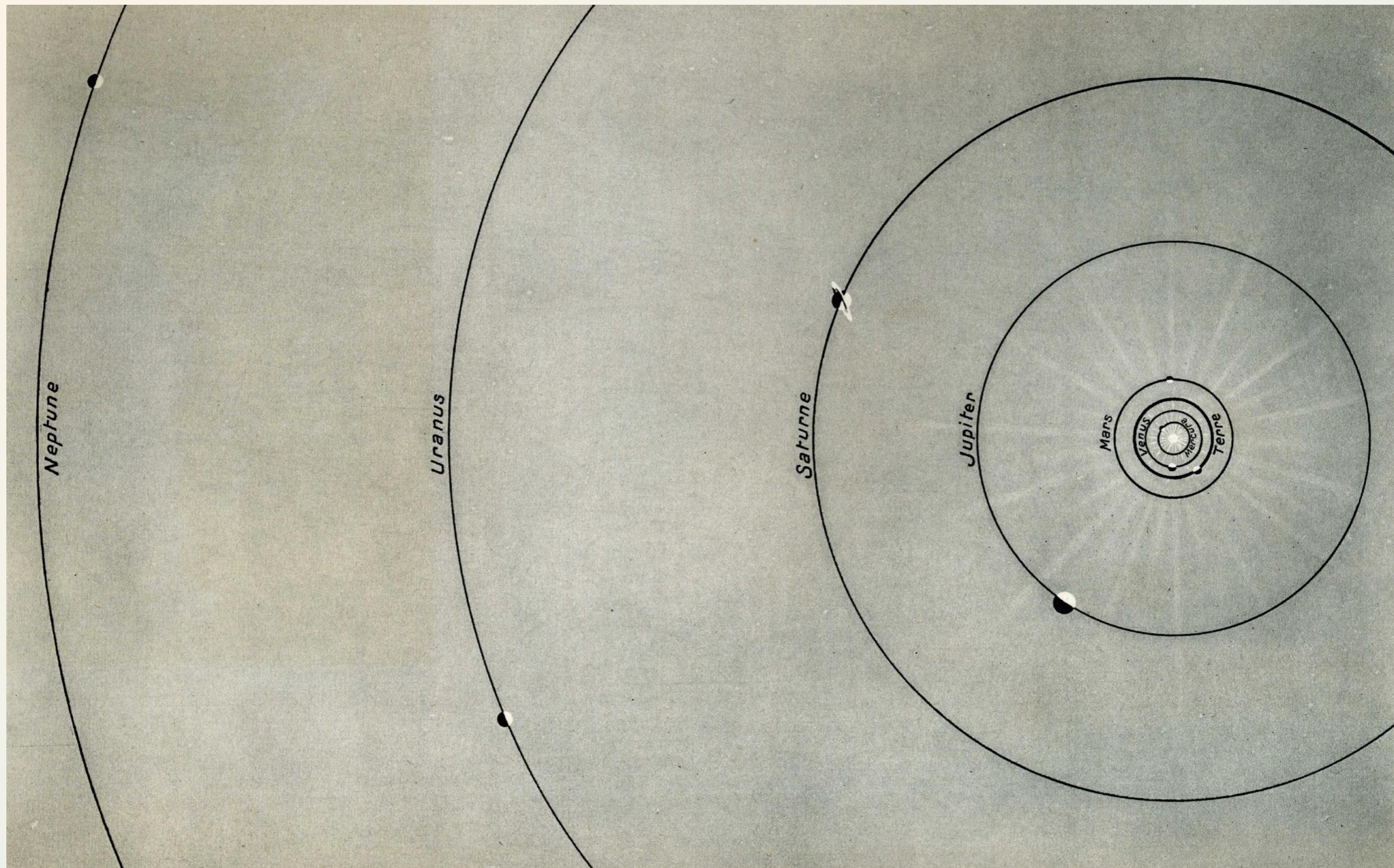
Gas-giant planet formation only has \sim 10-15 Myr to make the planet and bring it to where we detect them.

Rocky planet formation can take longer. Earth formed around 100 Myr after the Sun.

WHEN DO PLANETS FORM?



SOLAR SYSTEM

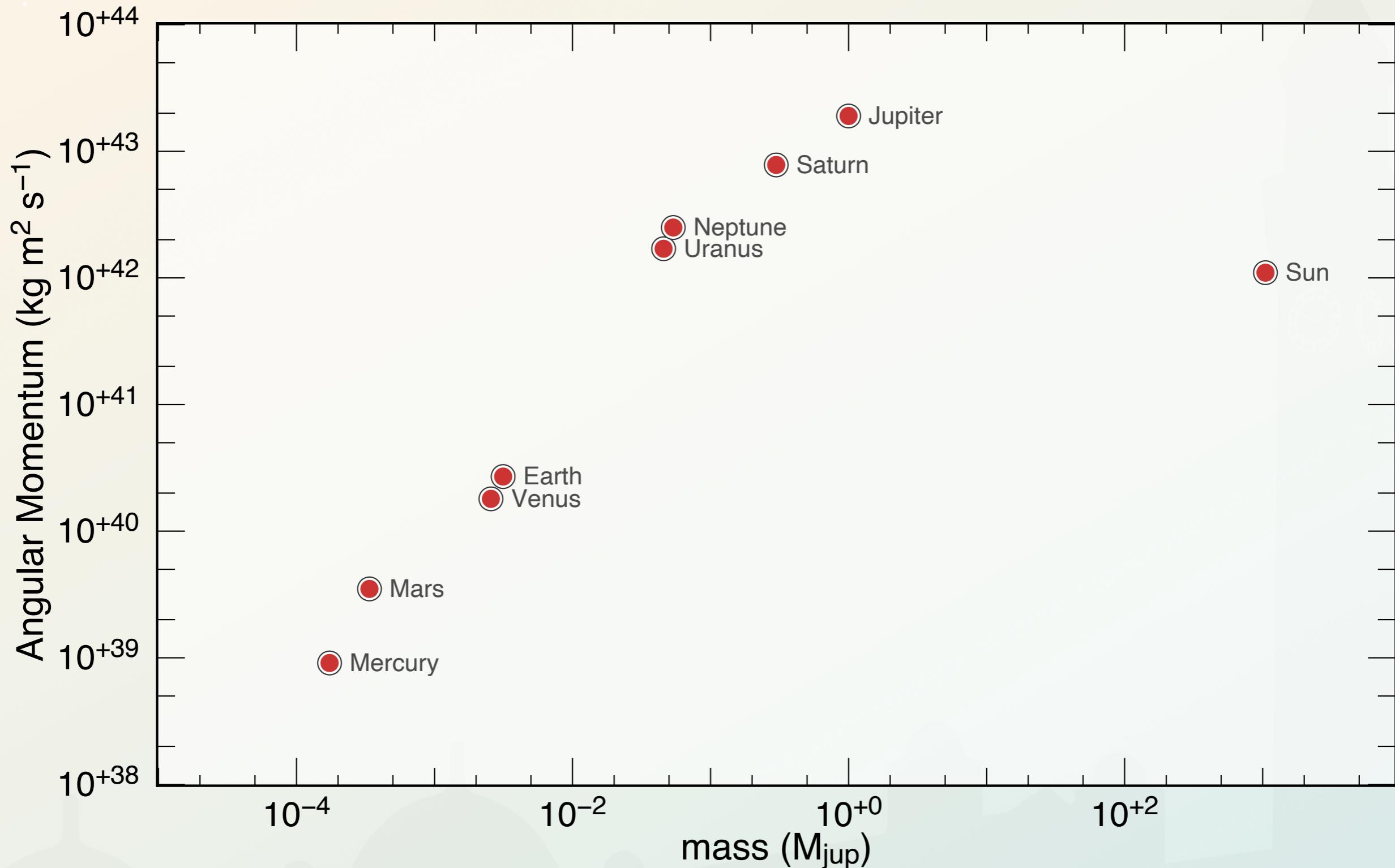


INVENTORY

- The Sun: 99.8% of the mass, <2% of the angular momentum
Removing the Sun: Jupiter is 70% of the mass, 60% of angular momentum
- Gas giants: Jupiter & Saturn
Ice giants: Uranus & Neptune
Rocky planets: Mercury, Venus, Earth & Mars
- A plethora of small bodies: **debris**
dwarf planets, satellites, asteroids, comets, ring systems
Main Belt, Kuiper Belt, Oort Cloud

without the solar system's example, we could not invent all that

ANGULAR MOMENTUM



ANGULAR MOMENTUM - EXERCISE 1

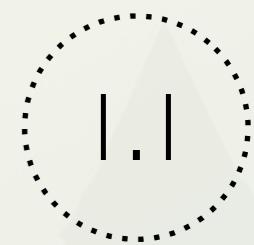
Problem 1 *What maximal value could the Sun's angular momentum take? Could it exceed the orbital angular momentum of planets within the Solar system? Starting from angular momentum, derive an expression for orbital angular momentum. To answer the question assume the Sun rotates at break-up velocity.*

Solution on canvas

ORBITAL ANGULAR MOMENTUM

Angular momentum

$$L = mvr$$



Keplerian velocity

$$v = \sqrt{\frac{GM_\star}{r}}$$

Orbital angular momentum

$$L = m_p \sqrt{GM_\star r}$$

Specific angular momentum

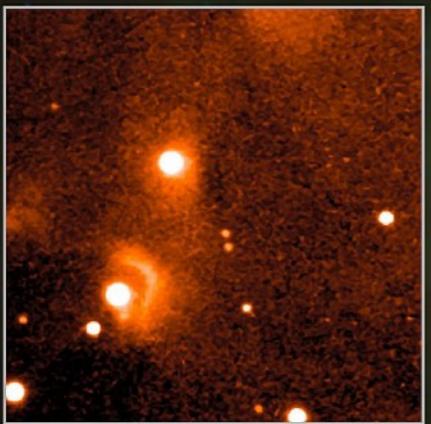
$$L = \sqrt{GM_\star r}$$

It is because of these equations that a disc forms around a protostar. Initially the size of the nebula r is large and v is small. L is constant and so is m . As it contract, it has to spin-up to conserve angular momentum, thus creating a disc.

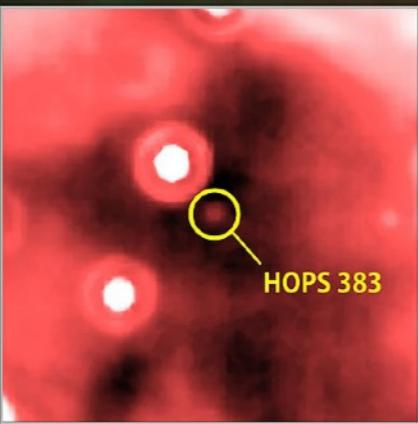
CLOUD COLLAPSE

HOPS 383: A deeply embedded protostar in outburst

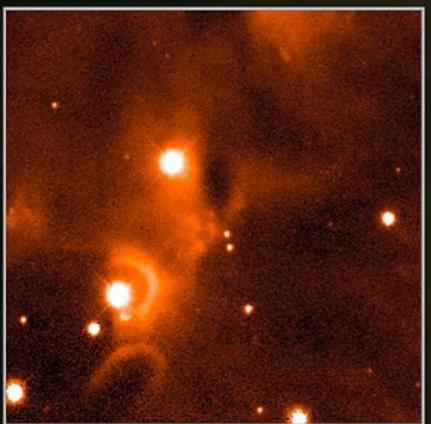
KPNO, 2000



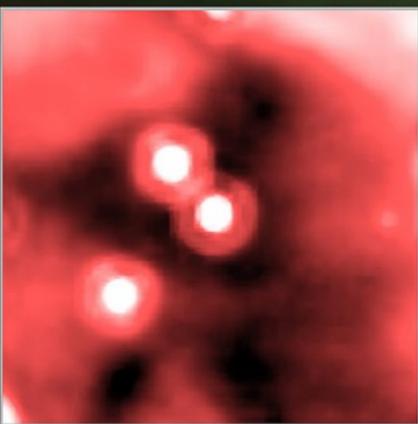
Spitzer, 2004



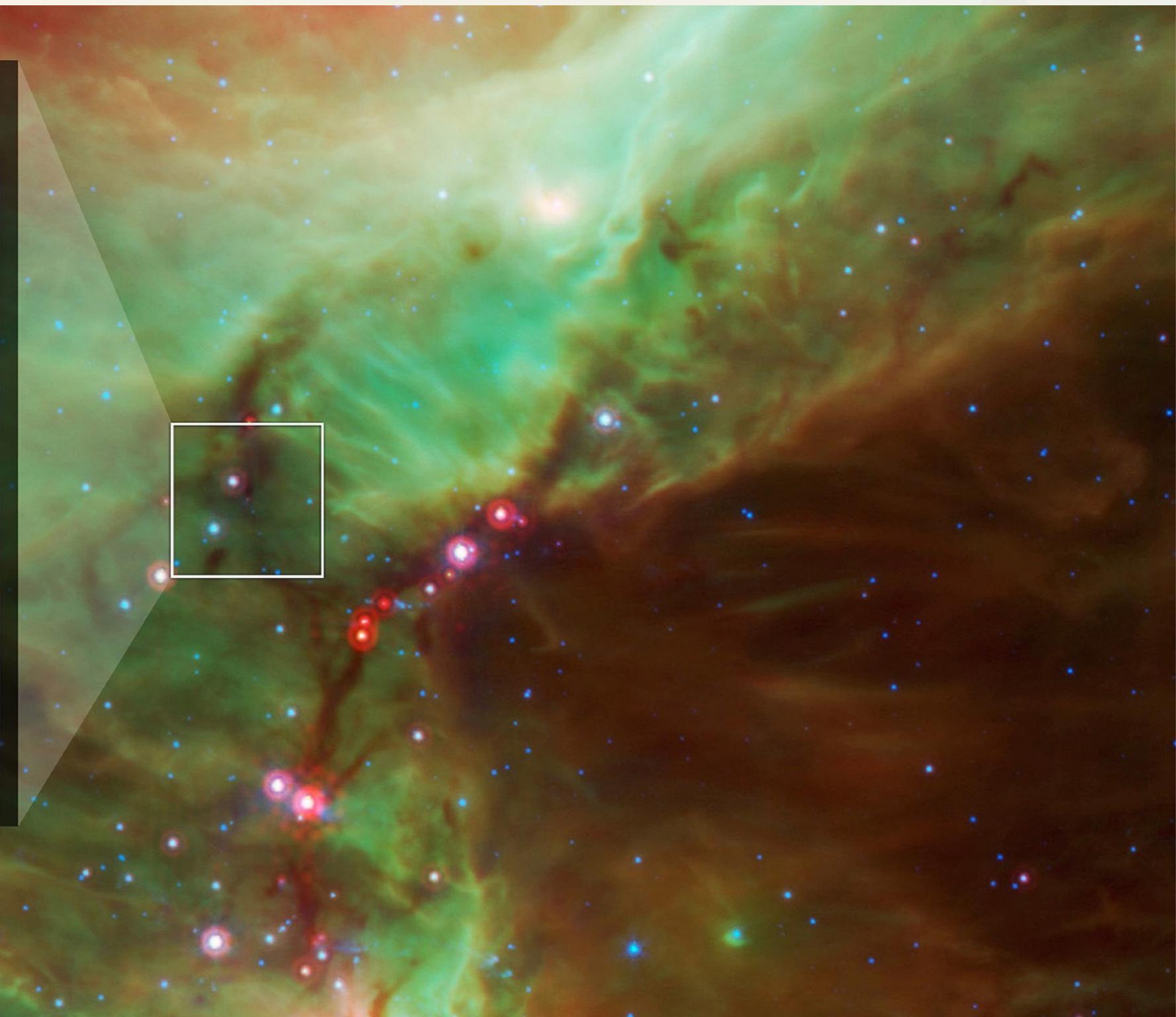
KPNO, 2009



Spitzer, 2008



1 arcminute



CLOUD COLLAPSE

To study how a cloud collapses and the mass of a typical fragment which will eventually become a star, we need to invoke the **Virial Theorem**.

$$2K + U = 0$$

1.4

This describes a gravitational system in equilibrium (e.g. the solar system). If the kinetic energy is too large, then the system expands, and should it be too low, the system contracts, in both cases until a new equilibrium is reached.

This theorem applies to galaxy clusters, stellar clusters, the thickness of the rings of Saturn etc. In the context of a nebular cloud, kinetic energy is given by the temperature of the gas. If temperature is too low, or density a little large the cloud will contract.

THE JEANS PARAMETERS

A uniform sphere has a gravitational energy written as:

$$U_{\text{grav}} = -\frac{3}{5} \frac{GM^2}{R^3}$$

1.5

The kinetic energy of a gas particle is:

$$K_{\text{gas}} = \frac{3}{2} k T$$

1.6

Combining both we get to:

Jeans length $R_{\text{jeans}} = \sqrt{\frac{15k}{4\pi G} \frac{T}{\rho \mu m_{\text{p}^+}}}$

1.7

Jeans mass $M_{\text{jeans}} = \left(\frac{3}{4\pi\rho} \right)^{1/2} \left(\frac{5kT}{G\mu m_{\text{p}^+}} \right)^{3/2}$

1.8

CLOUD COLLAPSE

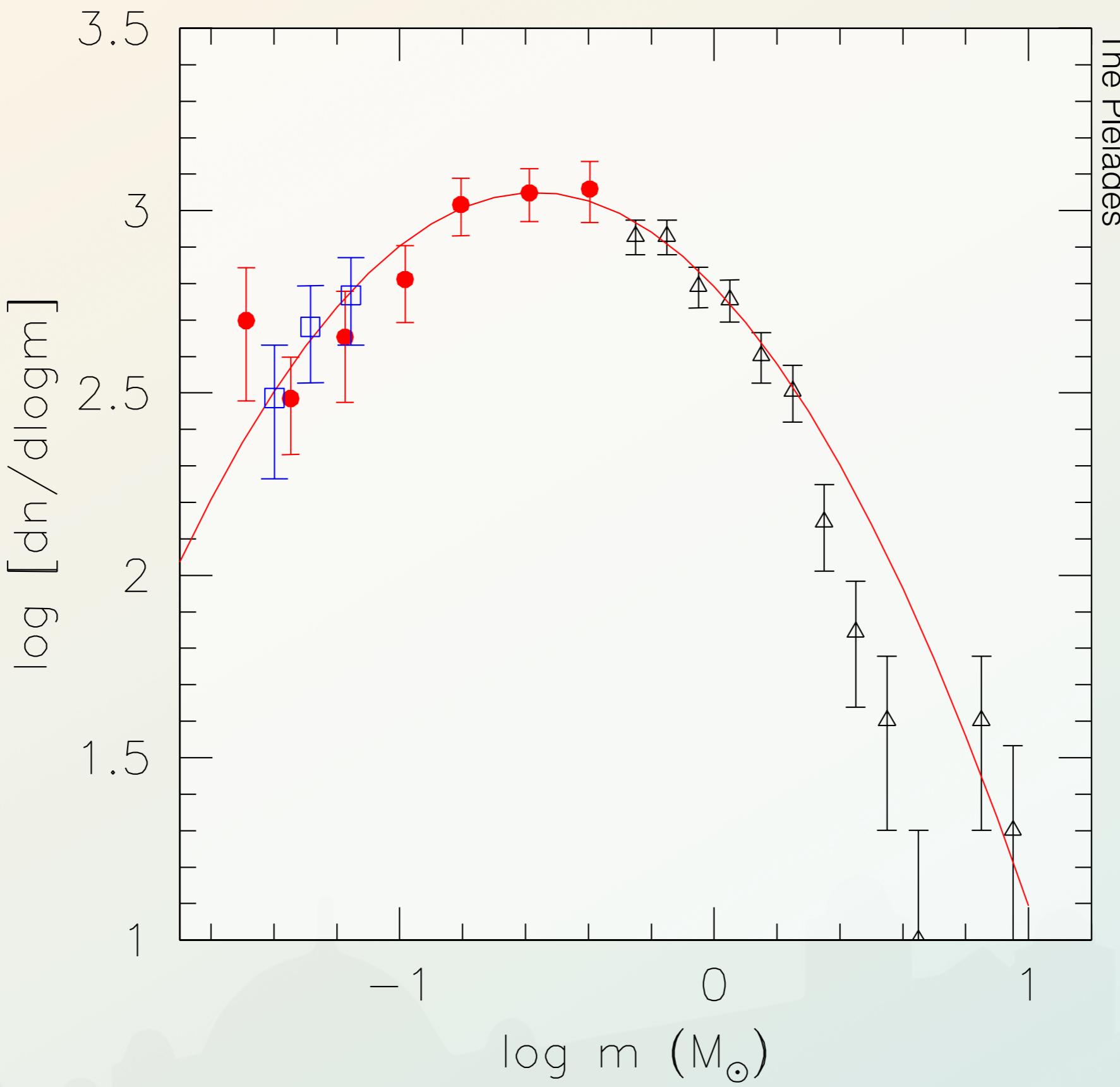
Using typical parameters, a giant molecular cloud is roughly $10^4 M_{\odot}$ but its Jeans mass is $\sim 400 M_{\odot}$.

This means the cloud will contract, but as it does so, several parts of the cloud contract separately (sound speed does not have time to cross the cloud before collapse happens). This means the cloud fragments.

As the cloud fragments, density increases in the fragments faster than temperature at first. The Jeans mass decreases too, thus the fragments also fragment into smaller clumps until roughly stellar mass fragments are created.

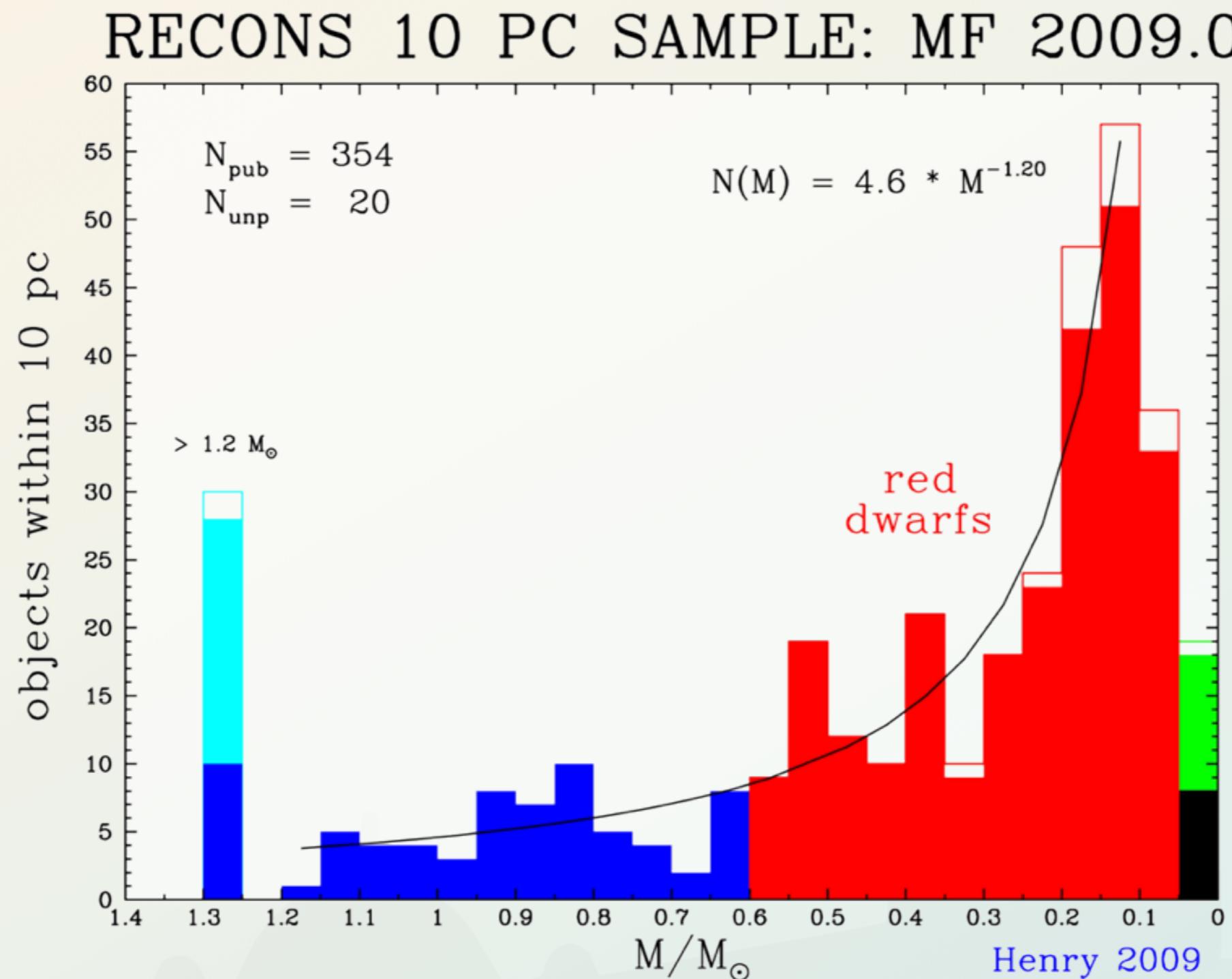
These progressive fragments lead to the Initial Mass Function in the form of a log-normal distribution. There are few small stars and few very massive stars. The peak is around $0.2 M_{\odot}$.

THE INITIAL MASS FUNCTION



THE SOLAR NEIGHBOURHOOD

The IMF can be seen in the mass distribution of stars around the Sun



PROTOPLANETARY DISC OBSERVATIONS

Kant & Laplace both saw the coplanarity of the Solar System as indication that everything started in a disc.

We now can image protoplanetary discs around young stars.

Using mm wavelengths who can traverse the obscuration caused by much of the dust and gas in a star forming region.

The images you will next show where the dust is. The wavelength depends on the size of the dust particle.

The images you will next have no depth, we only see the last surface of these objects, and as such is a 2D projection of a 3D object.

ALMA ARRAY



PLANET FORMATION

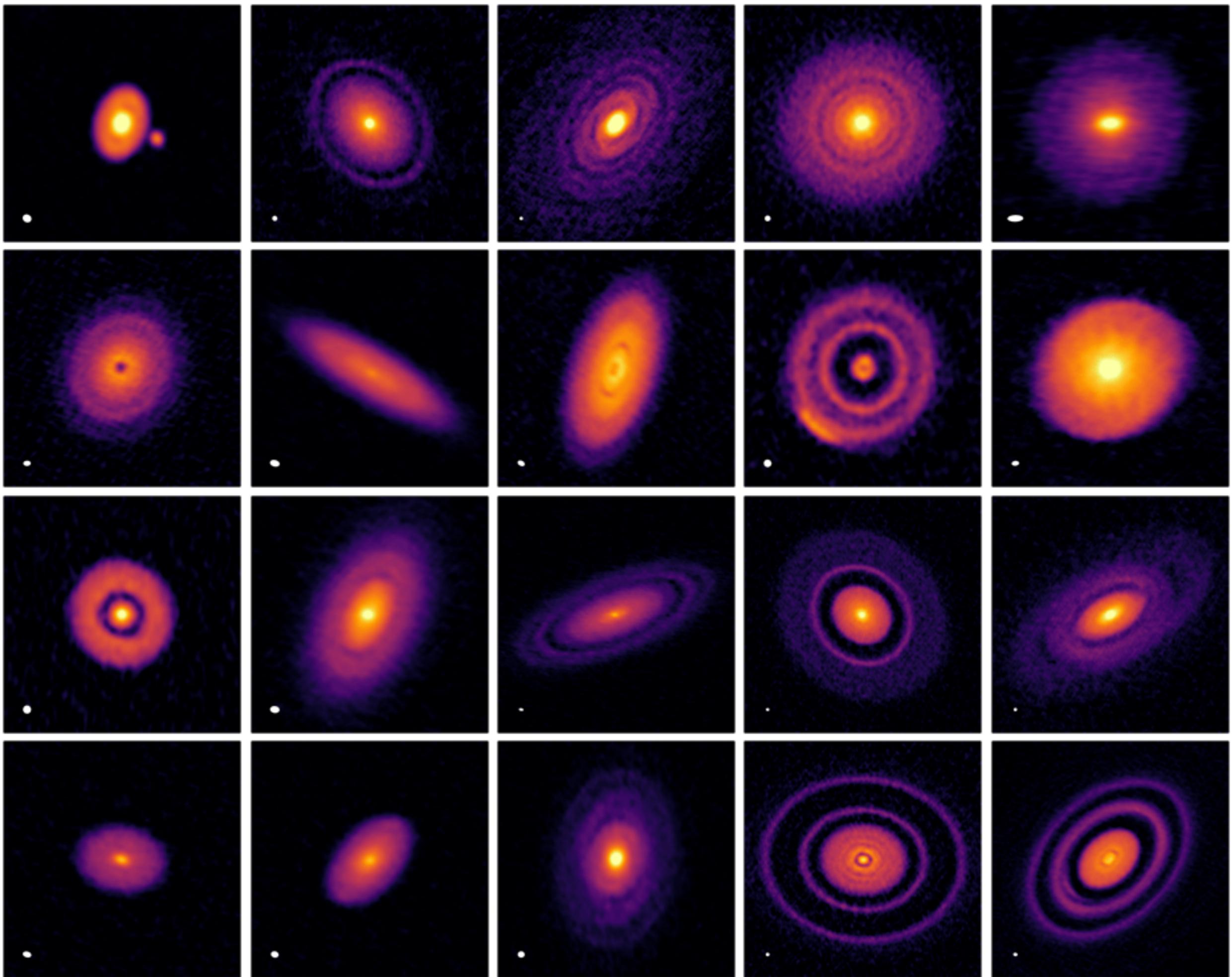
DRAFT VERSION DECEMBER 12, 2018
Typeset using L^AT_EX **twocolumn** style in AASTeX62

The Disk Substructures at High Angular Resolution Project (DSHARP): I. Motivation, Sample, Calibration, and Overview

SEAN M. ANDREWS,¹ JANE HUANG,¹ LAURA M. PÉREZ,² ANDREA ISELLA,³ CORNELIS P. DULLEMOND,⁴
NICOLÁS T. KURTOVIC,² VIVIANA V. GUZMÁN,^{5,6} JOHN M. CARPENTER,⁵ DAVID J. WILNER,¹ SHANGJIA ZHANG,⁷
ZHAOHUAN ZHU,⁷ TILMAN BIRNSTIEL,⁸ XUE-NING BAI,⁹ MYRIAM BENISTY,^{10,11} A. MEREDITH HUGHES,¹² KARIN I. ÖBERG,¹
AND LUCA RICCI¹³

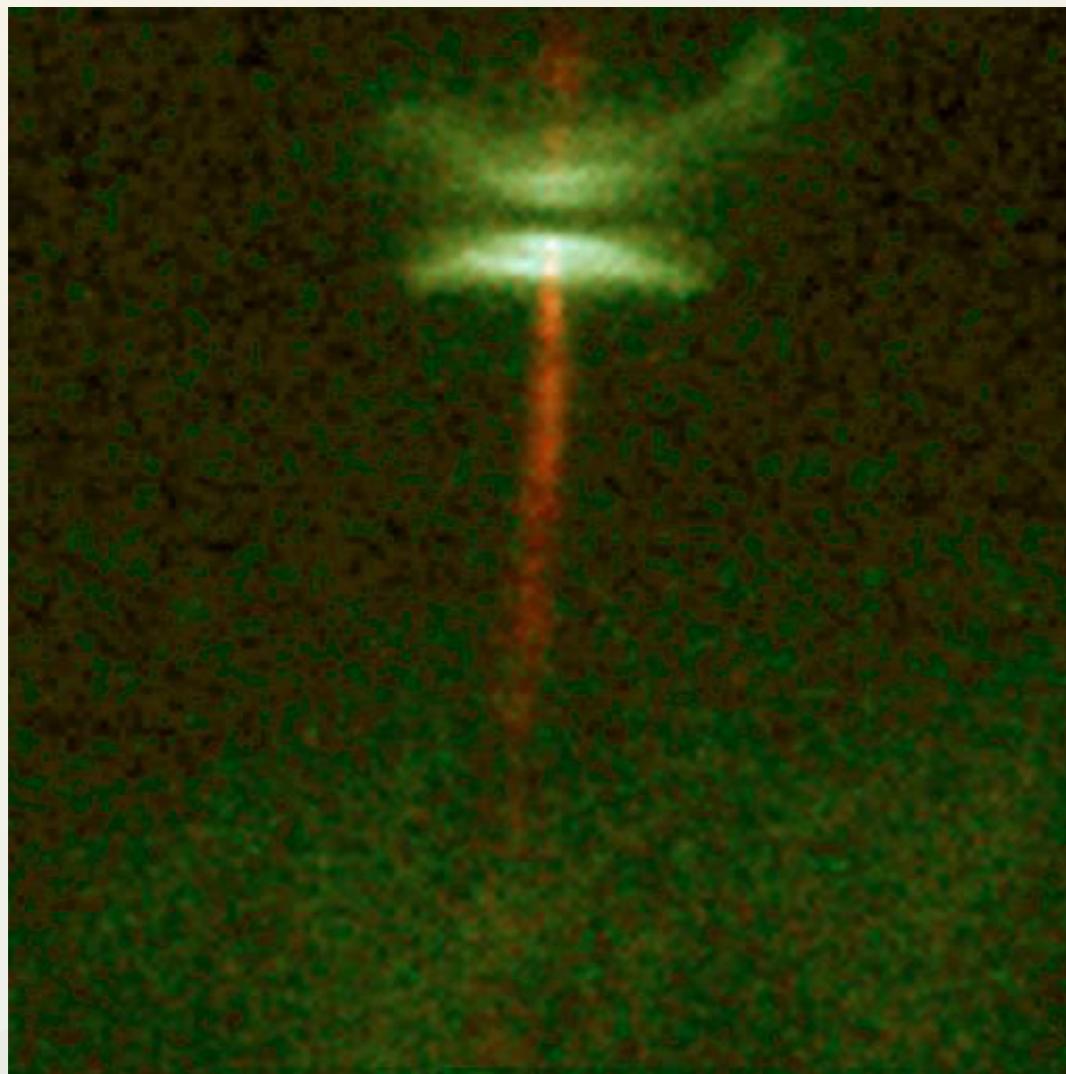
pdf of the paper is on canvas

<https://arxiv.org/abs/1812.04040>



GAP FORMATION

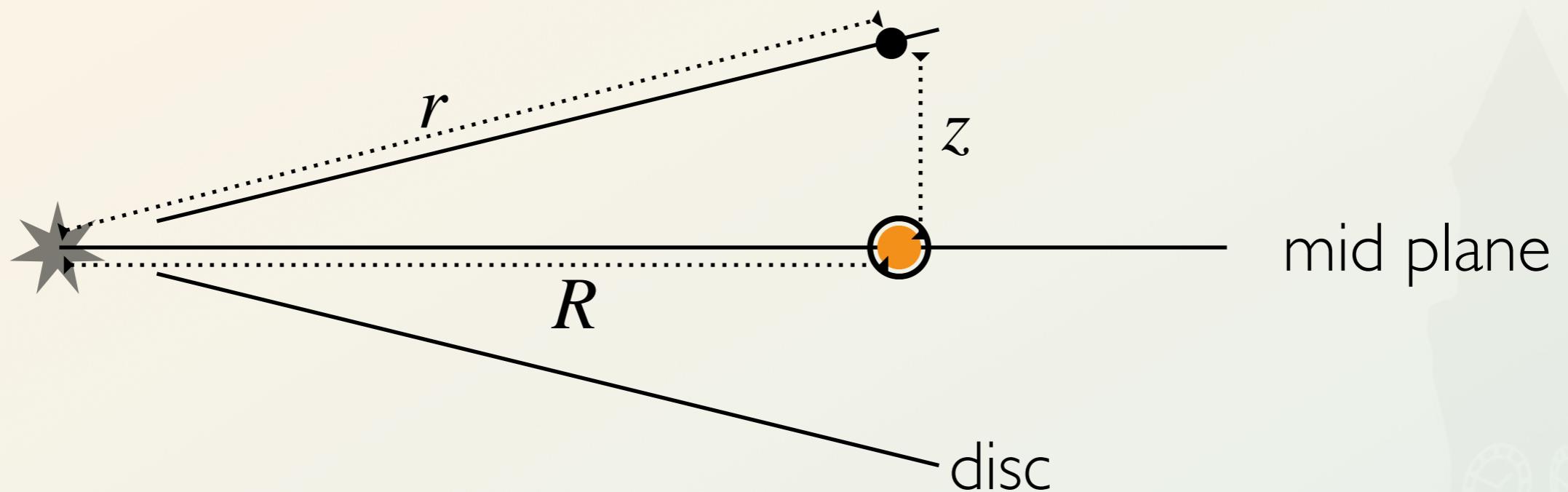
A planet opens a gap when it has accreted everything within its sphere of influence. So we need to calculate that.



However, the outer parts are thicker than the inner parts.

First, let's derive the shape of the disc on the board.

ESTIMATING A PROTOPLANETARY DISC PROFILE

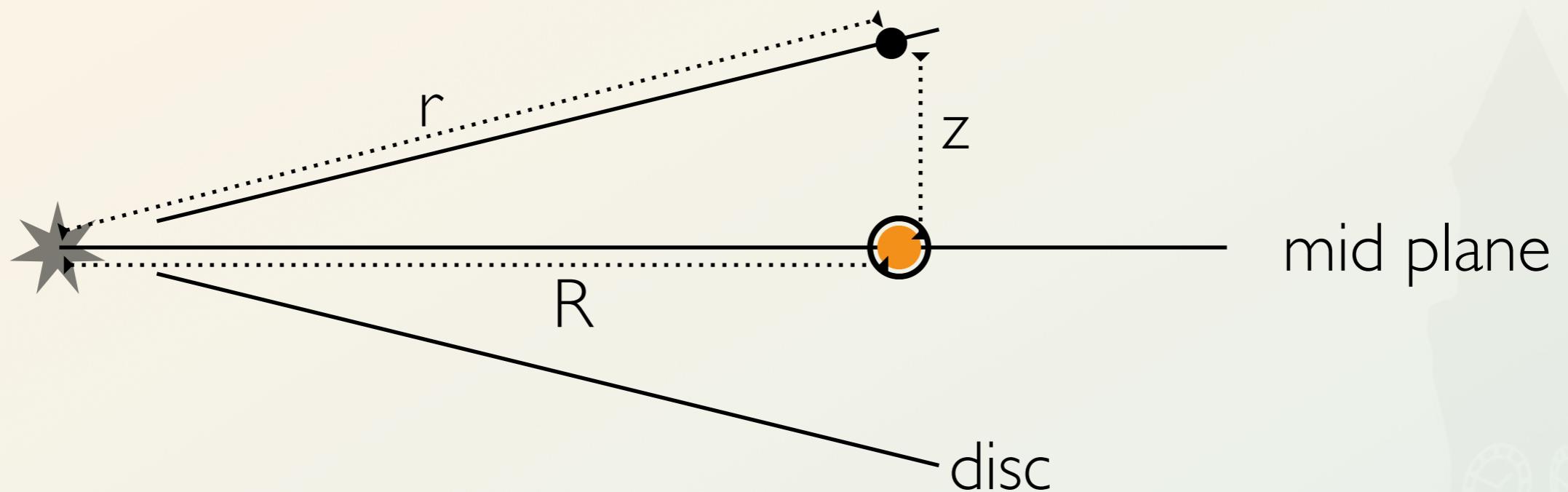


A disc has a thickness, z , on either side of the mid-plane. The thickness is expected to change (here to grow) as a function of the orbital separation (R).

Above the mid-plane, a particle is in suspension. There are gravitational forces keeping it in orbit. The disc below that particle has a mass so a force is acting trying to attract the particle down to the mid-plane. The reason it does not is due to temperature (kinetic energy).

Let's derive this thickness parameter (the scale height)

ESTIMATING A PROTOPLANETARY DISC PROFILE



mass density

$$\rho(z) = \mu m_{\text{p+}} n(0) e^{-E(z)/kT(R)}$$

1.9

we find that

$$E(z) = \frac{1}{2} \frac{G \mu m_{\text{p+}} M_\star}{R^3} z^2$$

1.10

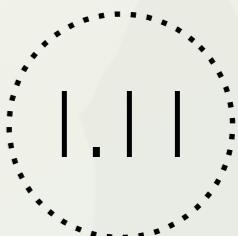
and get to

$$\rho(z) = \mu m_{\text{p+}} n(0) e^{-\frac{1}{2} \frac{z^2}{H^2}}$$

ESTIMATING A PROTOPLANETARY DISC PROFILE

with, the **Scale Height**

$$H = \sqrt{\frac{k T(R) R^3}{G M_\star \mu m_{\text{p}_+}}}$$

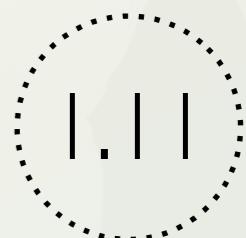


What does this mean?

- 68% of the mass is within 1 scale height (95%, within 2).
- H increases with R , disc gets thicker.
- H increases with $T \Rightarrow$ more random motion.
- for heavier μ , disc gets thinner (easier to puncture).

ESTIMATING A PROTOPLANETARY DISC PROFILE

$$H = \sqrt{\frac{k T(R) R^3}{G M_\star \mu m_{\text{p}_+}}}$$



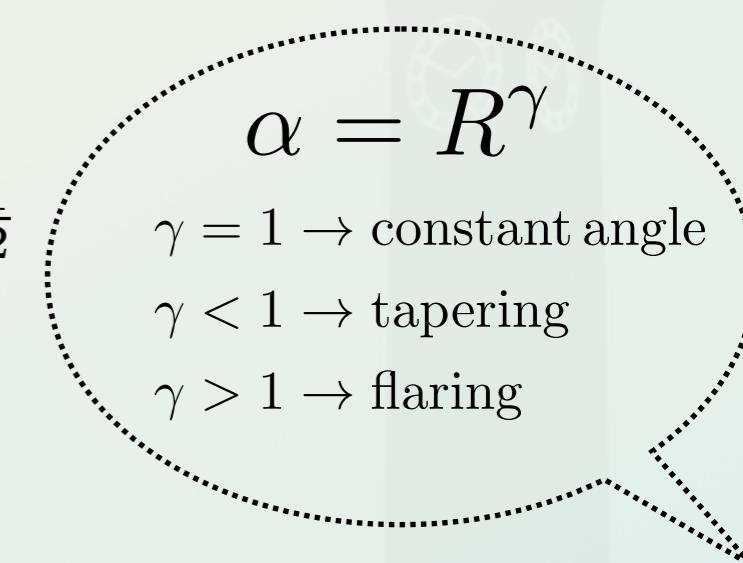
If T is constant, then

$$H(R) \propto R^{\frac{3}{2}}$$

and the opening angle

$$\alpha = H(R)/R = R^{\frac{1}{2}}$$

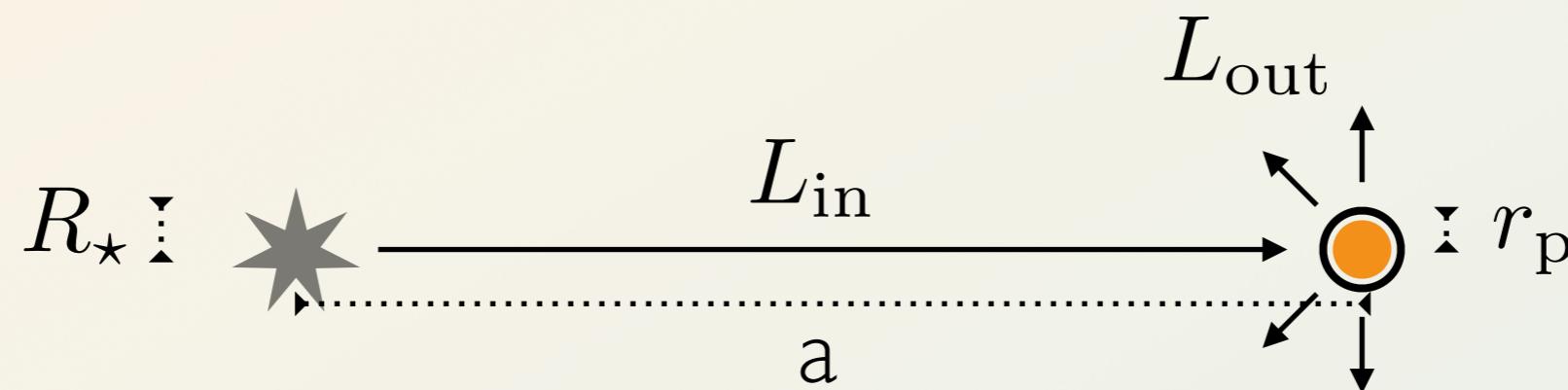
But T is not constant, it will decrease with R .



How does temperature vary with R ?

the equilibrium temperature

EQUILIBRIUM TEMPERATURE

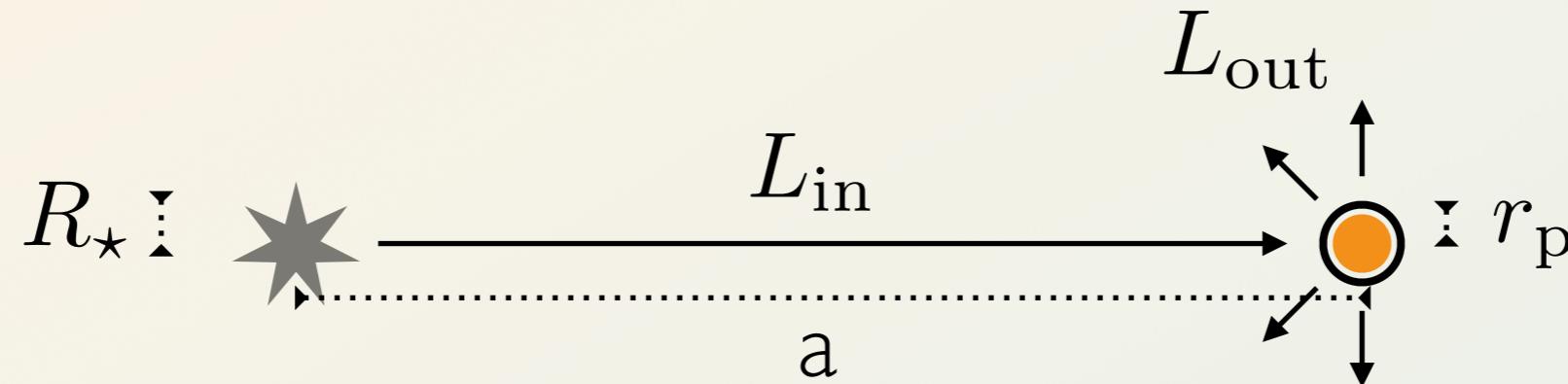


An irradiated object (a planet) will absorb that energy and heat up. This heat means the object emits (usually in the infrared). This creates a balance between incoming and emitted flux, which determines the equilibrium temperature.

To make this derivation we will assume that the planet re-radiates uniformly, but this needn't be the case.

If the planet is reflective, it will have albedo, A , defined the fraction of flux being reflected.

EQUILIBRIUM TEMPERATURE



The luminosity law

$$L = 4 \pi R^2 \sigma T^4$$

I.12

Equilibrium temperature $T_p = T_\star (1 - A)^{\frac{1}{4}} \sqrt{\frac{R_\star}{2a}}$

I.13

scaling relation $T_p = 255 \left(\frac{T_\star}{T_\odot} \right) \left(\frac{R_\star}{R_\odot} \right)^{1/2} \left(\frac{a_p}{1 \text{ AU}} \right)^{-1/2} K$
for $A = 0.3$
for $A = 0, T = 278 \text{ K}$

CORRECTING THE DISC PROFILE

$$H = \sqrt{\frac{k T(R) R^3}{G M_\star \mu m_{\text{p}_+}}}$$

1.11

equilibrium temperature

$$T_{\text{eq}} \propto 1/R^{1/2}$$

then

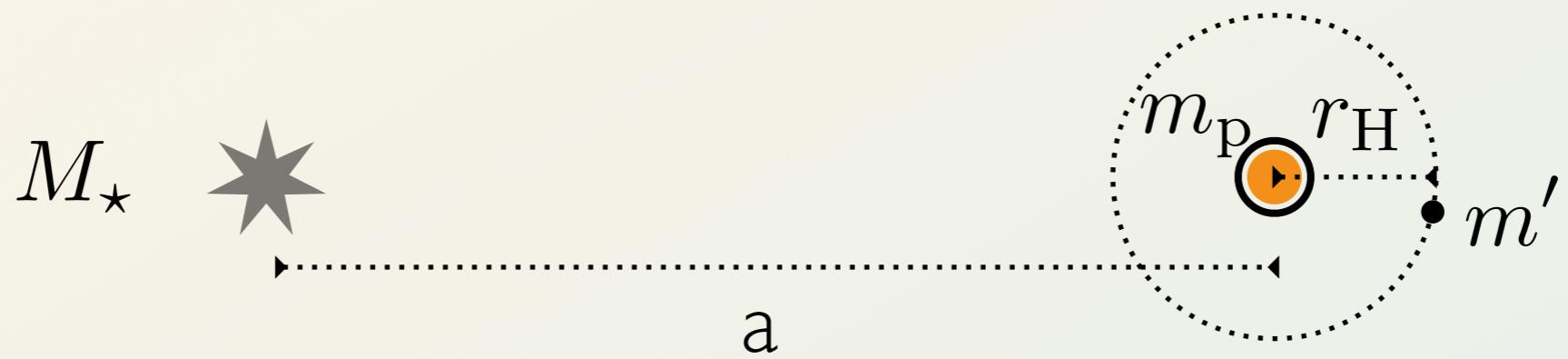
$$H(R) \propto R^{5/4} \rightarrow \alpha \propto R^{1/4}$$

Putting all numbers together we get

$$H(R) = 0.035 \text{ AU} \left(\frac{R}{1 \text{ AU}} \right)^{5/4}$$

THE HILL SPHERE

We now know the shape of a disc. Let's find the sphere of influence of a planet, its **Hill Sphere**



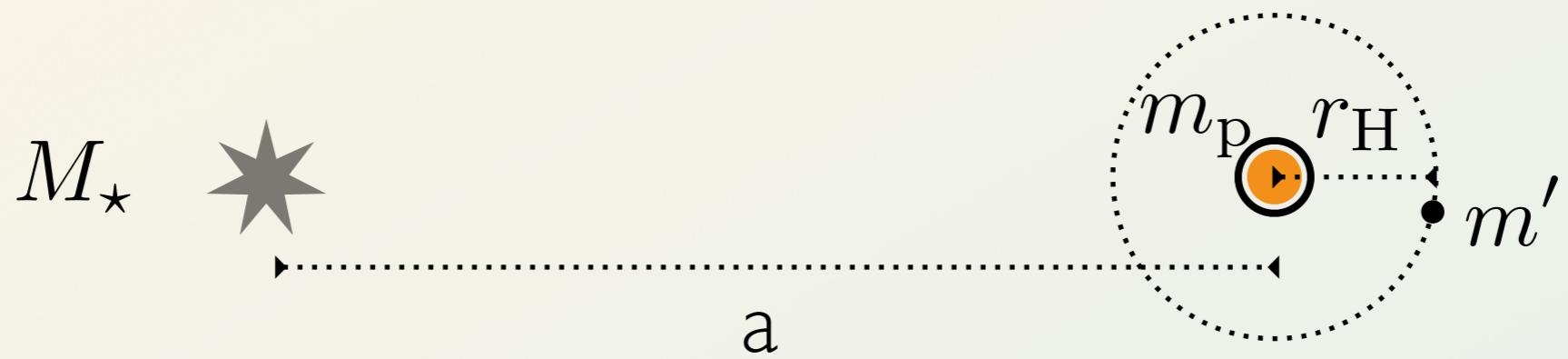
This is the region where the gravity of a planet dominates over the gravitational force of the star. The Moon has a Hill Sphere too, and the Sun as well, with respect to the rest of the Milky Way.

start with $F_{\text{tot}} = 0$

$$0 = -\frac{G M_\star m'}{(a + r_H)^2} + m' (a + r_H) \Omega^2 - \frac{G m_p m'}{r_H^2}$$

THE HILL SPHERE

We now know the shape of a disc. Let's find the sphere of influence of a planet, its **Hill Sphere**



Here we need to introduce a new concept:

Angular velocity

$$\Omega = \frac{2\pi}{P} = \sqrt{\frac{GM_\star}{a^3}}$$

and arrive at

$$r_H = a \sqrt[3]{\frac{m_p}{3M_\star}} = a \frac{r_p}{R_\star} \sqrt[3]{\frac{\rho_p}{3\rho_\star}}$$

I.14

I.15

GAP FORMATION

$$H(a) = 0.035 \text{ AU} \left(\frac{a}{1 \text{ AU}} \right)^{5/4}$$

$$r_H = a \sqrt[3]{\frac{m_p}{3 M_\star}} = a \frac{r_p}{R_\star} \sqrt[3]{\frac{\rho_p}{3 \rho_\star}}$$

Hill radius for Jupiter => ~ 0.35 AU
Disc scale height for Jupiter => ~ 0.275 AU

Jupiter will not fully open a gap.

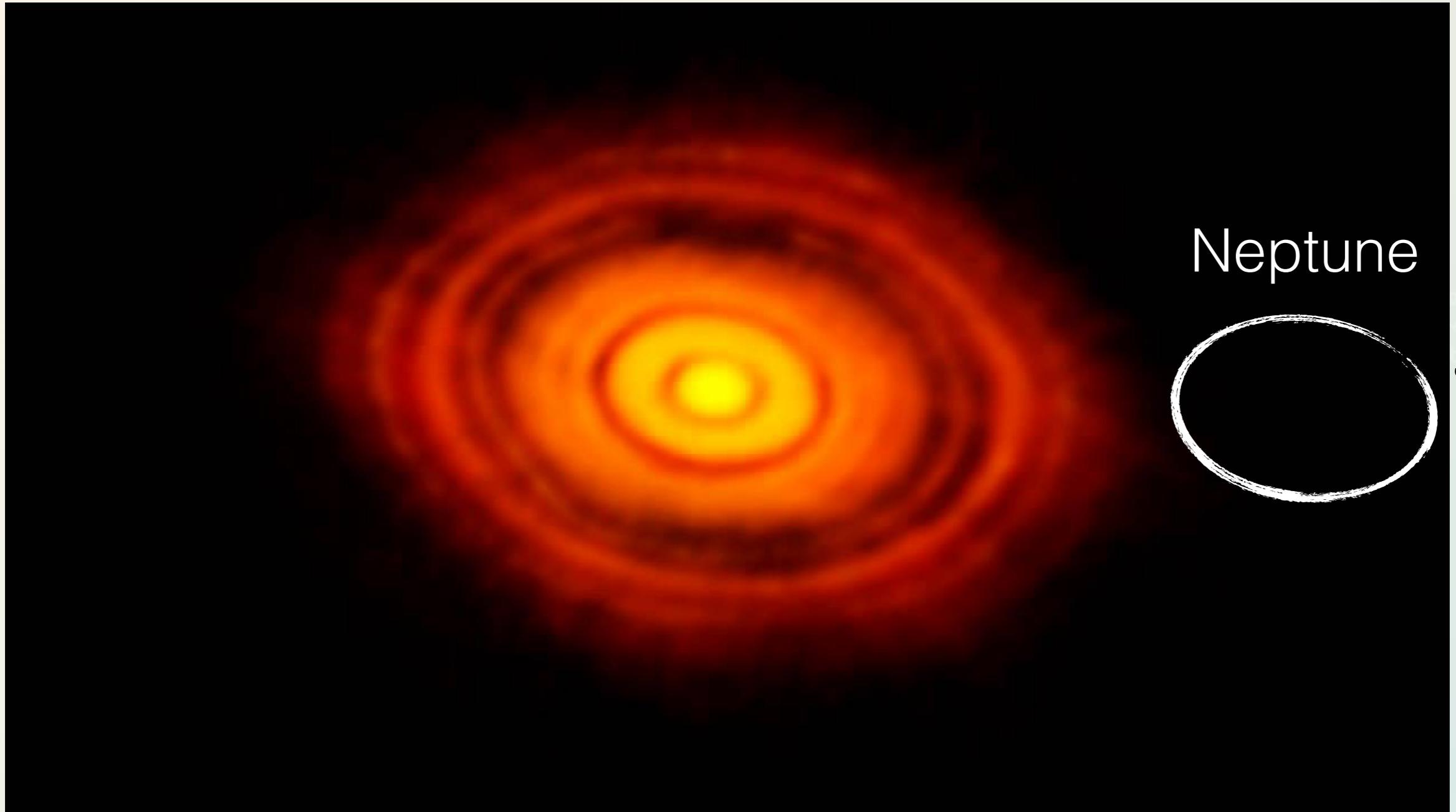
Hill radius for Earth => ~ 0.01 AU
Disc scale height for Earth => 0.035 AU

Earth wouldn't open a gap.

1.15

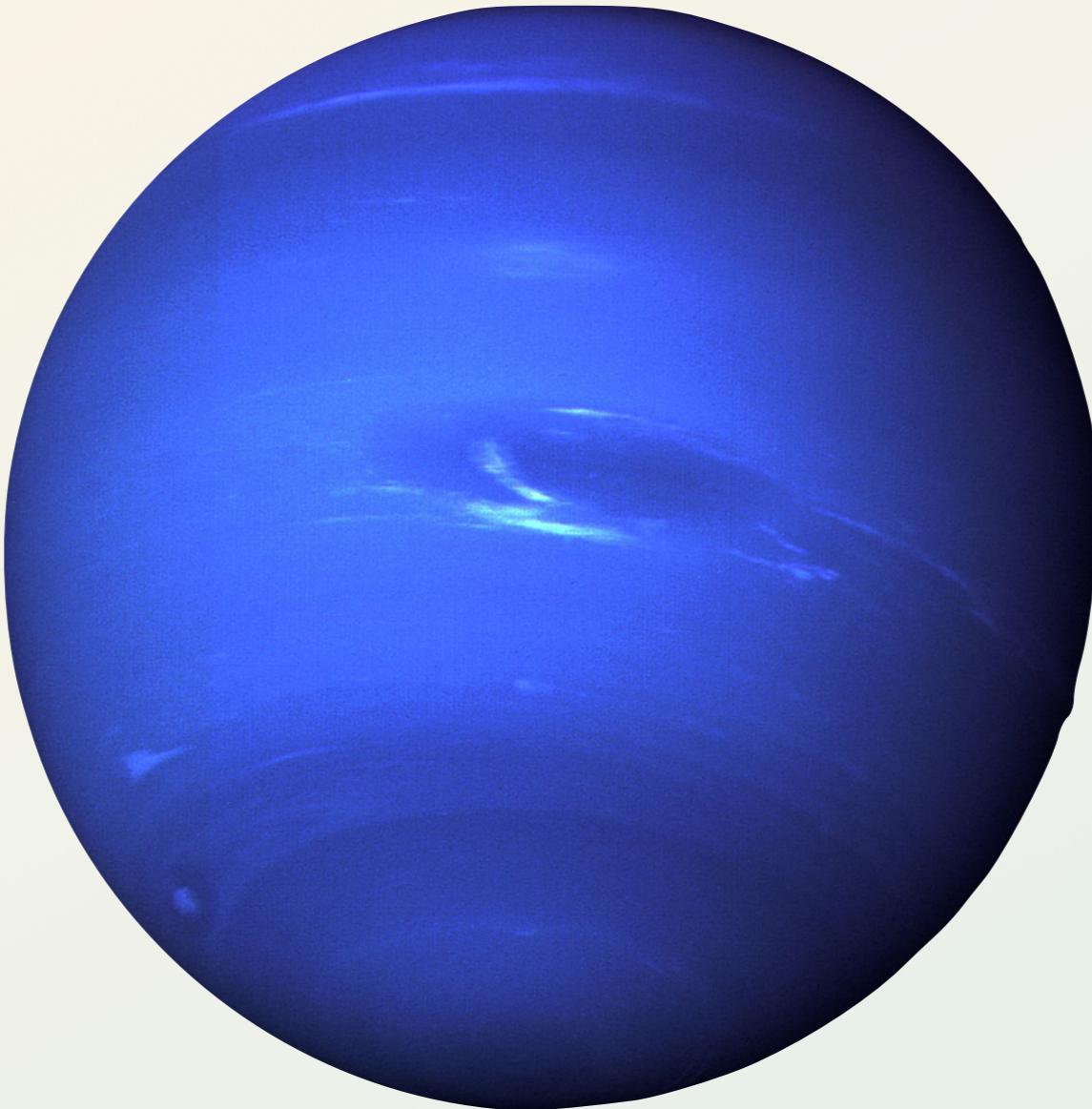
$\rho_\odot \sim \rho_{\text{Jup}}$
 $r_{\text{Jup}} \sim 0.1 R_\odot$

PROBLEM 3



What is the mass of the putative planets?

MATERIAL AVAILABLE TO GROW A PLANET



What object can retain gas?

=> need a certain mass => need some solid core first

MATERIAL AVAILABLE TO GROW A PLANET

The disc is mostly composed of gas, with a little dust, at a ratio roughly of 100 to 1.

To make big planets, gas needs to accrete, but can it be used?

One might think a planet grows from dust and gas available at the distance it is at. However this is not the case.

Usually there is not one location at which a planet forms, but building blocks form everywhere. We call these **planetesimals**.

Can planetesimals retain gas? Let's find out.

ESCAPE VELOCITY

Is the velocity needed, so that at infinity, you reach velocity = 0.

energy conservation

$$\frac{1}{2} m' v_{\text{esc}}^2 - \frac{G m_p m'}{r_p} = 0$$

escape velocity

$$v_{\text{esc}} = \sqrt{\frac{2 G m_p}{r_p}}$$

I.16

Poll

$$v_{\text{esc}} = 11.2 \left(\frac{m}{m_\oplus} \right)^{1/2} \left(\frac{r}{r_\oplus} \right)^{-1/2} \text{ km/s}$$

Surface gravity:

$$g = \frac{G m_p}{r_p^2}$$

I.17

To make planets with atmospheres, we need a solid core

Now, let's find if gas can escape.

GAS RETENTION

What does the velocity of gas depend on?

What is the kinetic energy of a gas molecule?

$$K_{\text{gas}} = \frac{3}{2} k T$$

1.6

Earth has $T = 286\text{K} \Rightarrow v \sim 0.5 \text{ km/s}$

Earth can keep its atmosphere

Eq. I.16 can be rewritten as

$$v_{\text{esc}} \propto r \sqrt{\rho}$$

Any body with $r \lesssim \frac{1}{5} r_{\oplus}$ is likely to lose its atmosphere.

Heavier molecules are easier to keep.

The only way for planetesimals to store volatiles is in solid form.

ICE LINES

The further from the star, the colder it is, and some volatile species can condense into solids, ready to be used to build a planet. These transitions are called **ice-lines**, or **snow lines**.

Remember the equilibrium temperature?

$$T_{\text{eq}} = T_{\star} \sqrt{\frac{R_{\star}}{2a}}$$

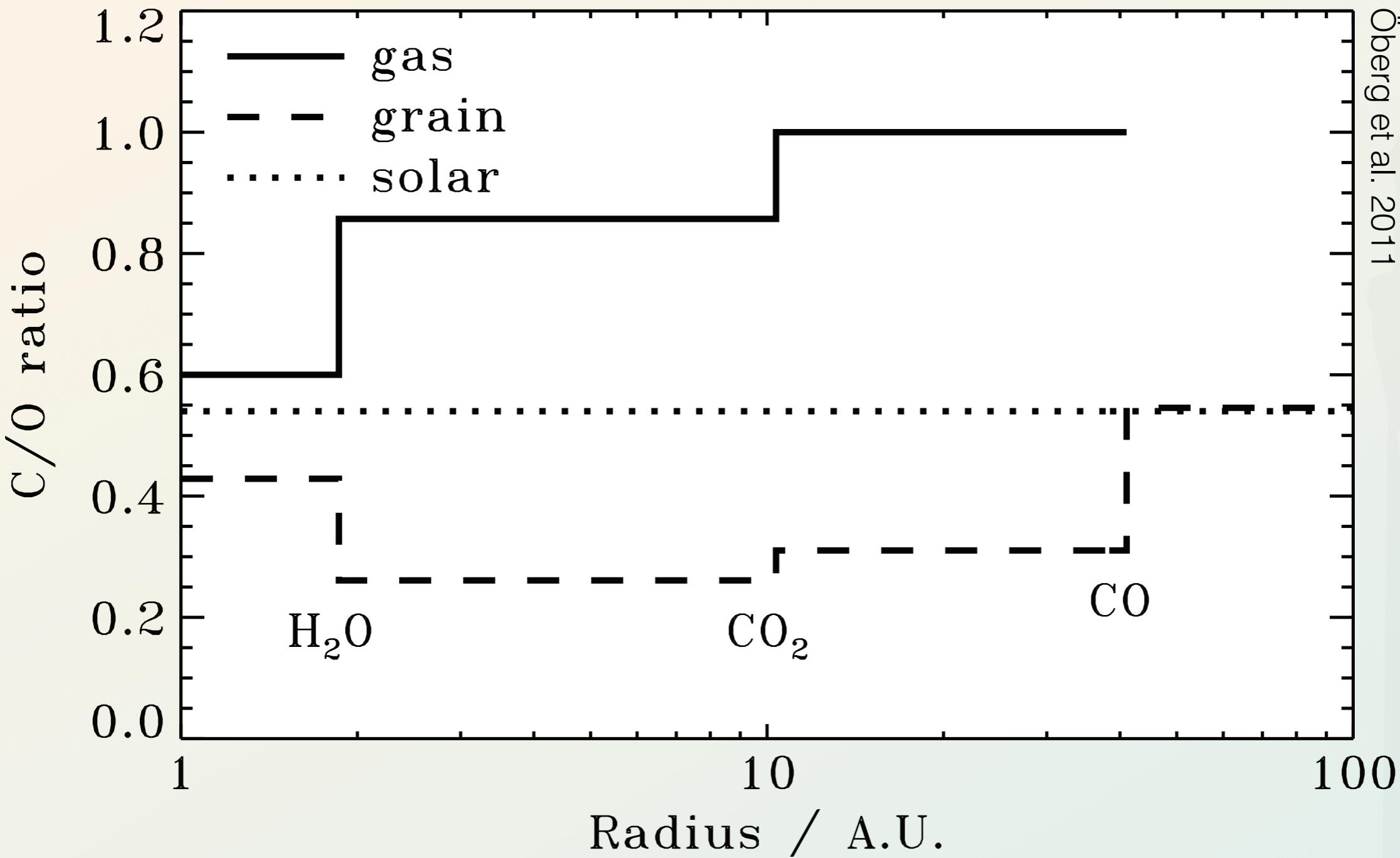
$$T_{\text{eq}} = 255 \left(\frac{T_{\star}}{T_{\odot}} \right) \left(\frac{R_{\star}}{R_{\odot}} \right)^{1/2} \left(\frac{a_p}{1 \text{ AU}} \right)^{-1/2} \text{ K}$$

I.13

At some certain distance, water vapour will become ice

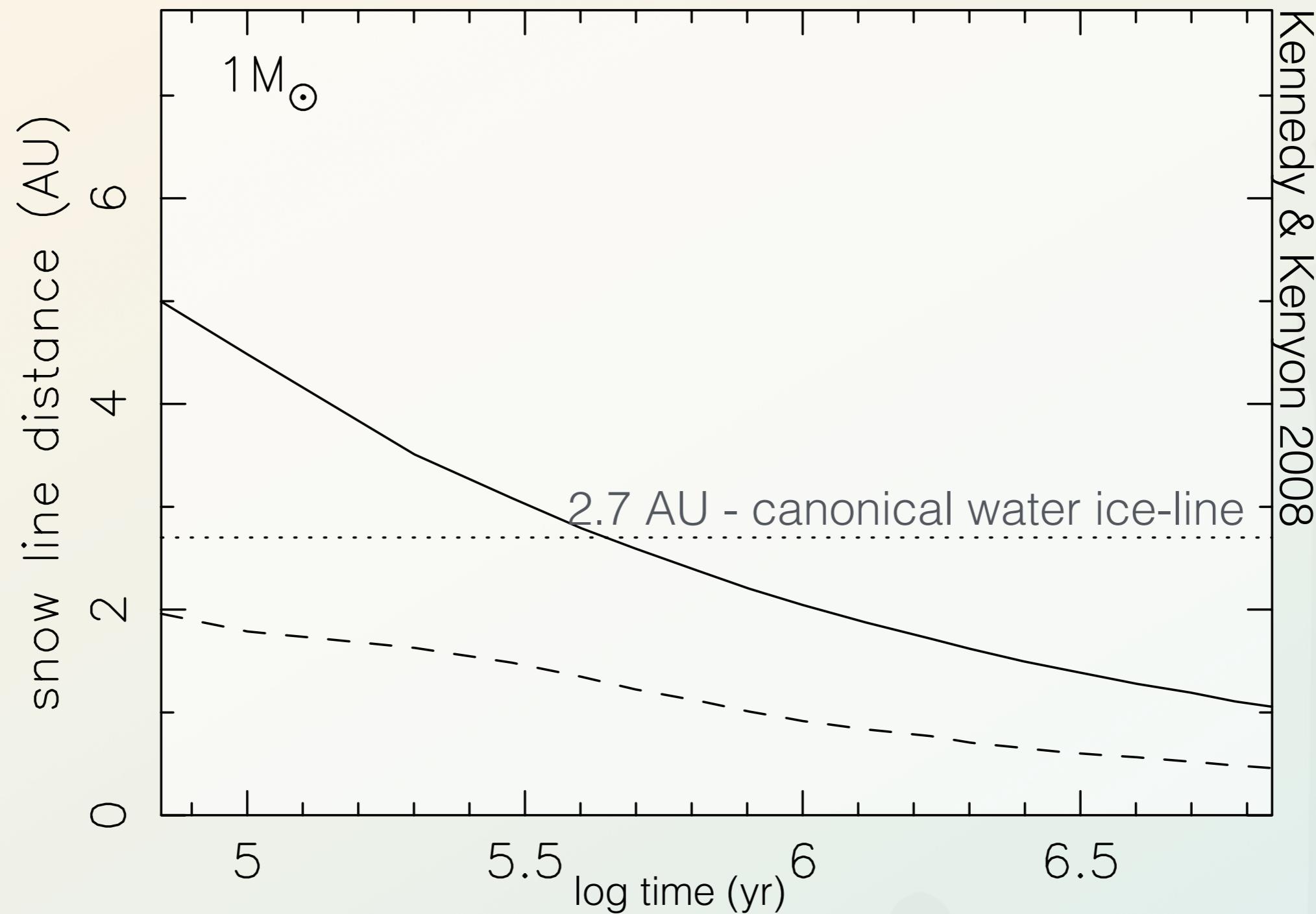
CO vapour will become CO ice
etc.

ICE LINES



Around 2.7 AU there is the water ice line. Water freezes into grains. Thus there are more materials to make planets. The gas loses oxygen to the grains, thus the C/O ratio increases. If a planet attracts an atmosphere its composition will reflect the gas disc's chemistry (and not the grains).

ICE LINE



Early, the disc itself is hot (the star is hotter at the beginning too), thus the ice line is farther out early on, but it gradually moves closer, and eventually reaches the distance for sublimation temperature in vacuum ($T_{\text{eq}} \sim 200$ K)

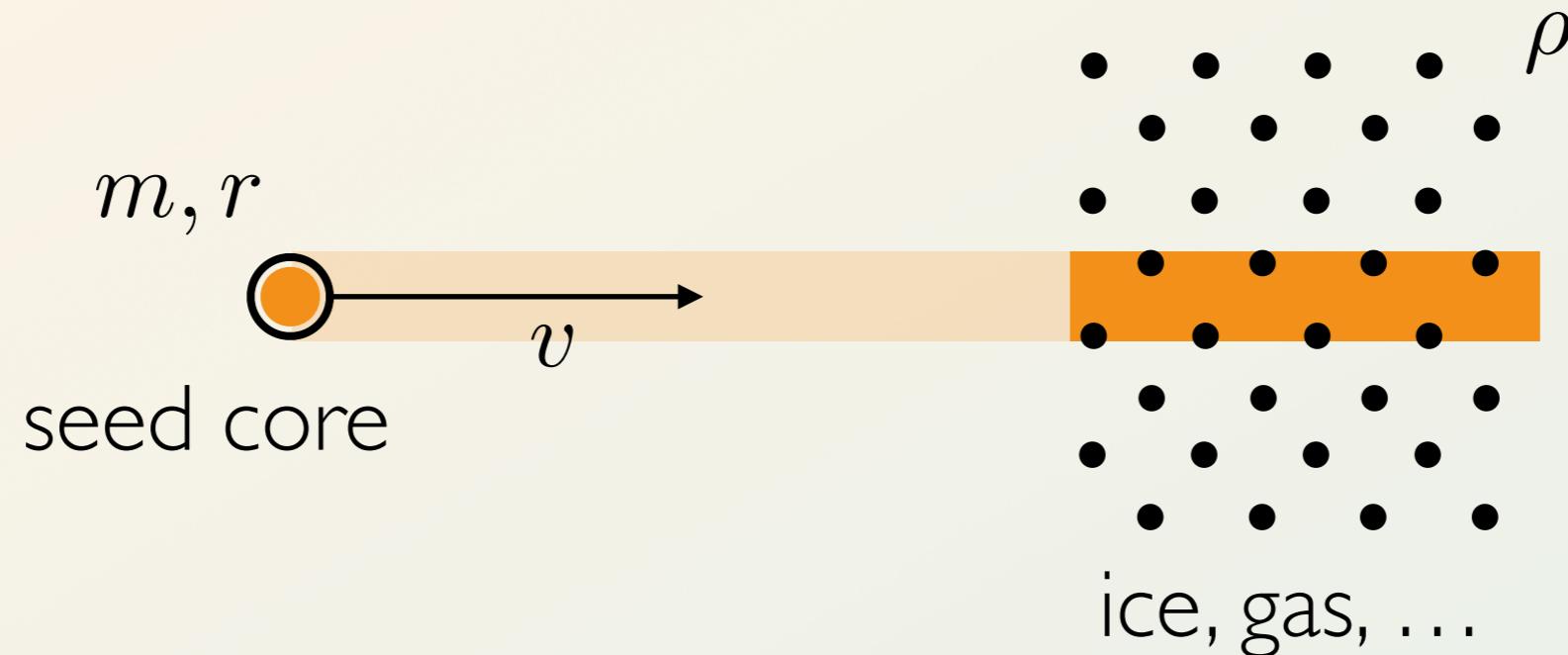
PLANET FORMATION: CORE-ACCRETION

Since planetesimals cannot have gas, planets need to form from solid blocks first. They combine together, and eventually form an object with enough mass to accrete the gas from the nebula (hopefully before the disc disperses).

This scenario is called the core-accretion scenario.

We will now look at how solid accretion works, and then how gas accretion happens.

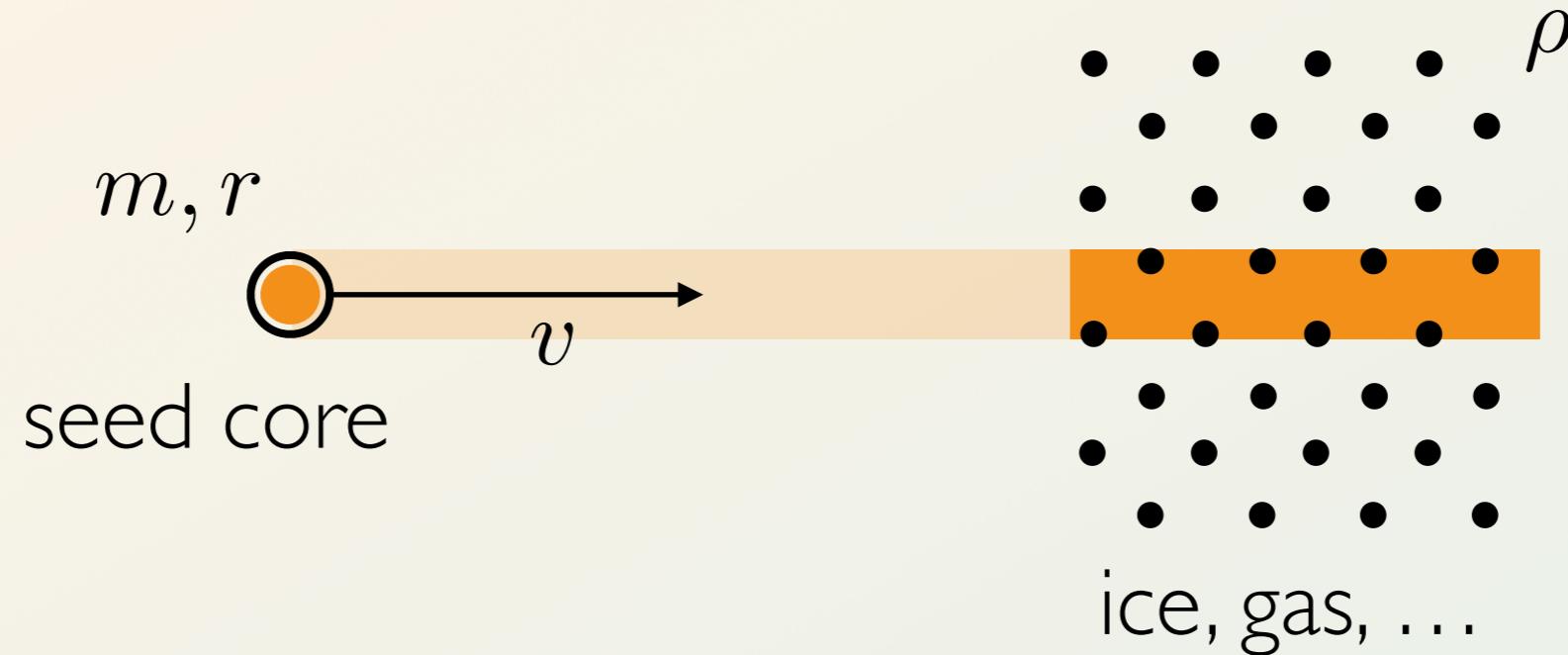
CORE ACCRETION



Let's take a seed planet, an object that managed to grow a little bit bigger than the others. The disc is viscous and orbits slower than Keplerian velocity (the disc also accretes, creating a flow towards the star). Because of its size, the seed orbits a little faster and thus seeds material coming towards it.

We will assume that anything along its path accretes on the seed, allowing it to grow in size and in mass.

CORE ACCRETION



let's assume all particles hitting the seed core are accreted

core accretion is to estimate

$$\frac{dm}{dt} = \pi r^2 v \rho = \sigma v \rho$$

1.18

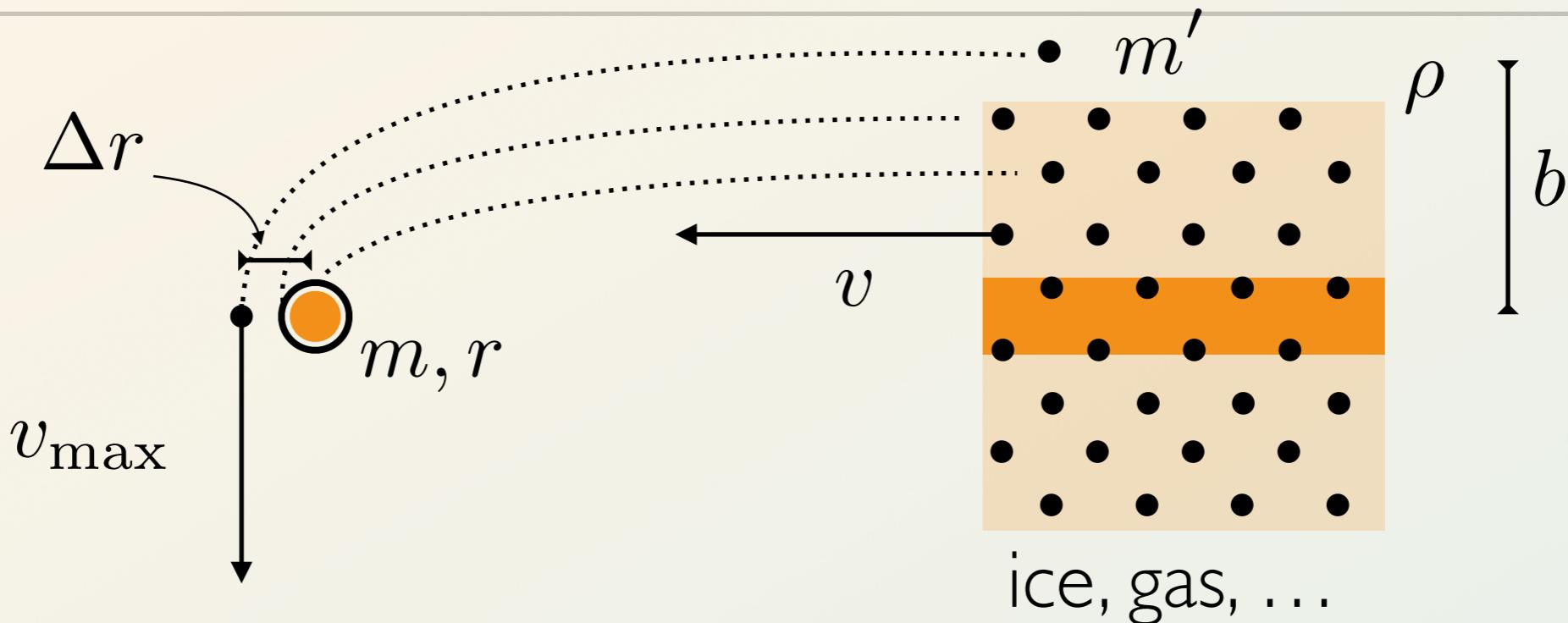
$$\frac{dm}{dt} \propto m^{2/3}$$

cross section

this cross section is correct if m is small.

at larger distances, density increases because of ice lines

GRAVITATIONAL FOCUSING



conservation
of energy

$$\frac{1}{2} m' v_{\max}^2 - \frac{G m m'}{\Delta r} = \frac{1}{2} m' v^2$$

1.19

conservation
of momentum

$$v_{\max}^2 = \frac{v^2 b^2}{\Delta r^2}$$

1.20

using 2.2 arrive at

$$b^2 = r^2 (1 + \Theta) \text{ with } \Theta = \frac{v_{\text{esc}}^2}{v^2}$$

1.21

RUNAWAY CORE ACCRETION

Rewrite I.18 as

$$\frac{dm}{dt} = \pi r^2 v \rho (1 + \Theta)$$
$$\propto r m \propto m^{4/3}$$

I.22

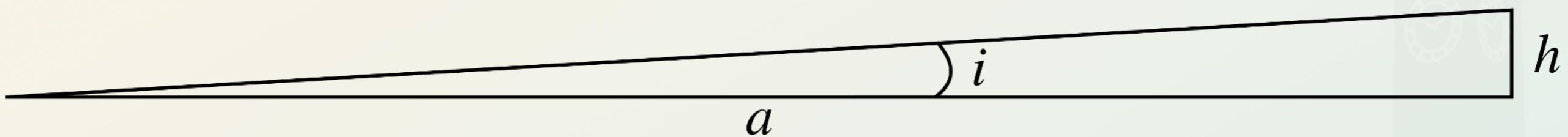
If you integrate, it diverges to infinity => runaway growth

Out of an entire population, if one gets a little more massive, then it will dominate and grow much faster than objects stuck at eq. I.18

SURFACE DENSITY

We observe in 2D rather than 3D, this means it is sometimes convenient to rewrite some of the equation in terms of what is being measured: an observable.

A side on disc looks like that:



The height, h , is maintained due to velocity, which we refer to as the speed of sound. Thus we can write: $h = \frac{c_s}{\Omega}$

Therefore, the sound speed is

$$c_s = \frac{H}{a} v_{\text{Kep}}$$

1.23

H/a or H/R is called the **aspect ratio** of the disc

SURFACE DENSITY

We rewrite the 3D density ρ in 2D, which we call the surface density Σ , which is what can be measured with facilities like ALMA.

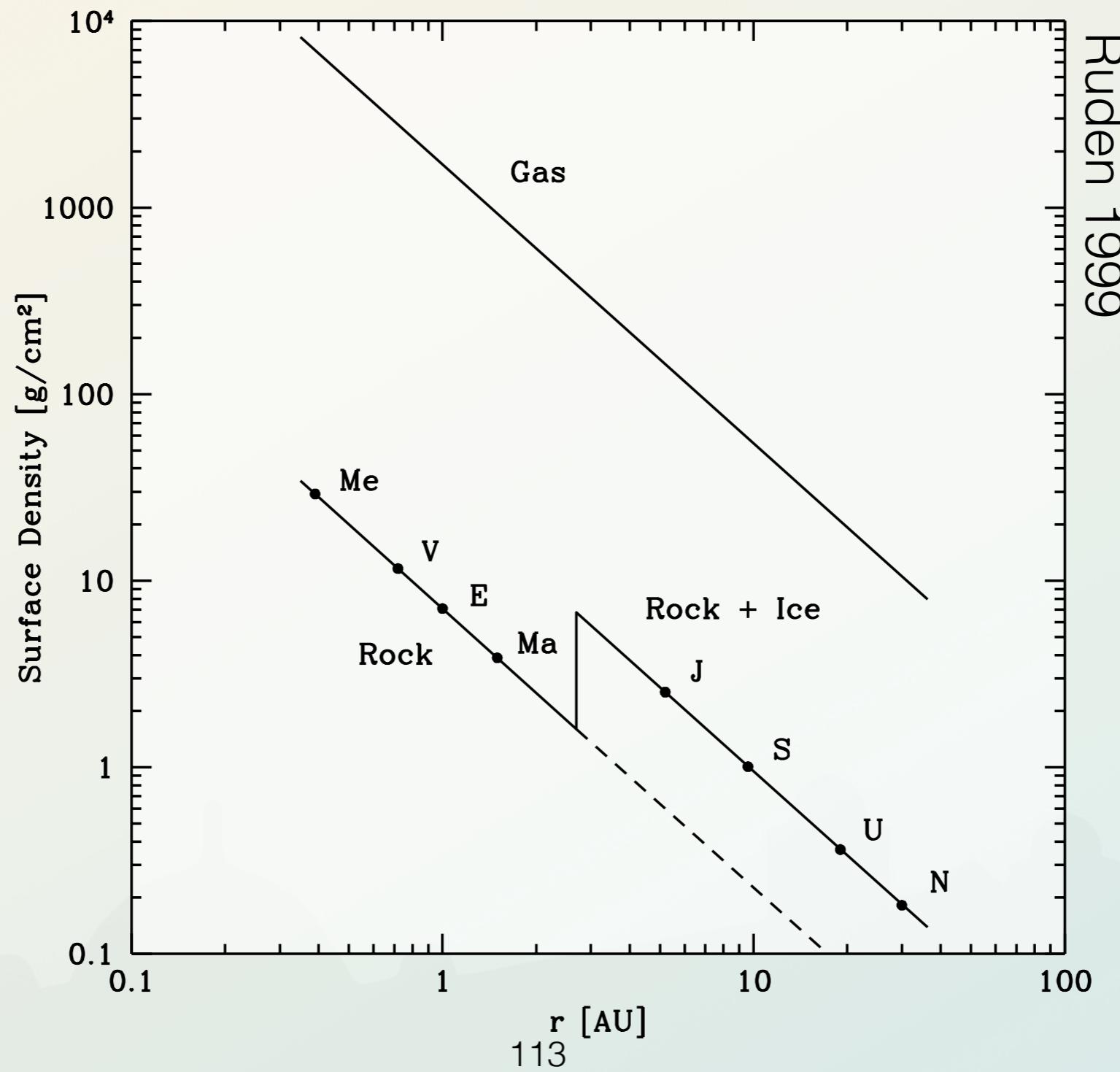
$$\frac{dm}{dt} = \frac{1}{2} \Sigma \Omega \pi r^2 (1 + \Theta)$$

1.24

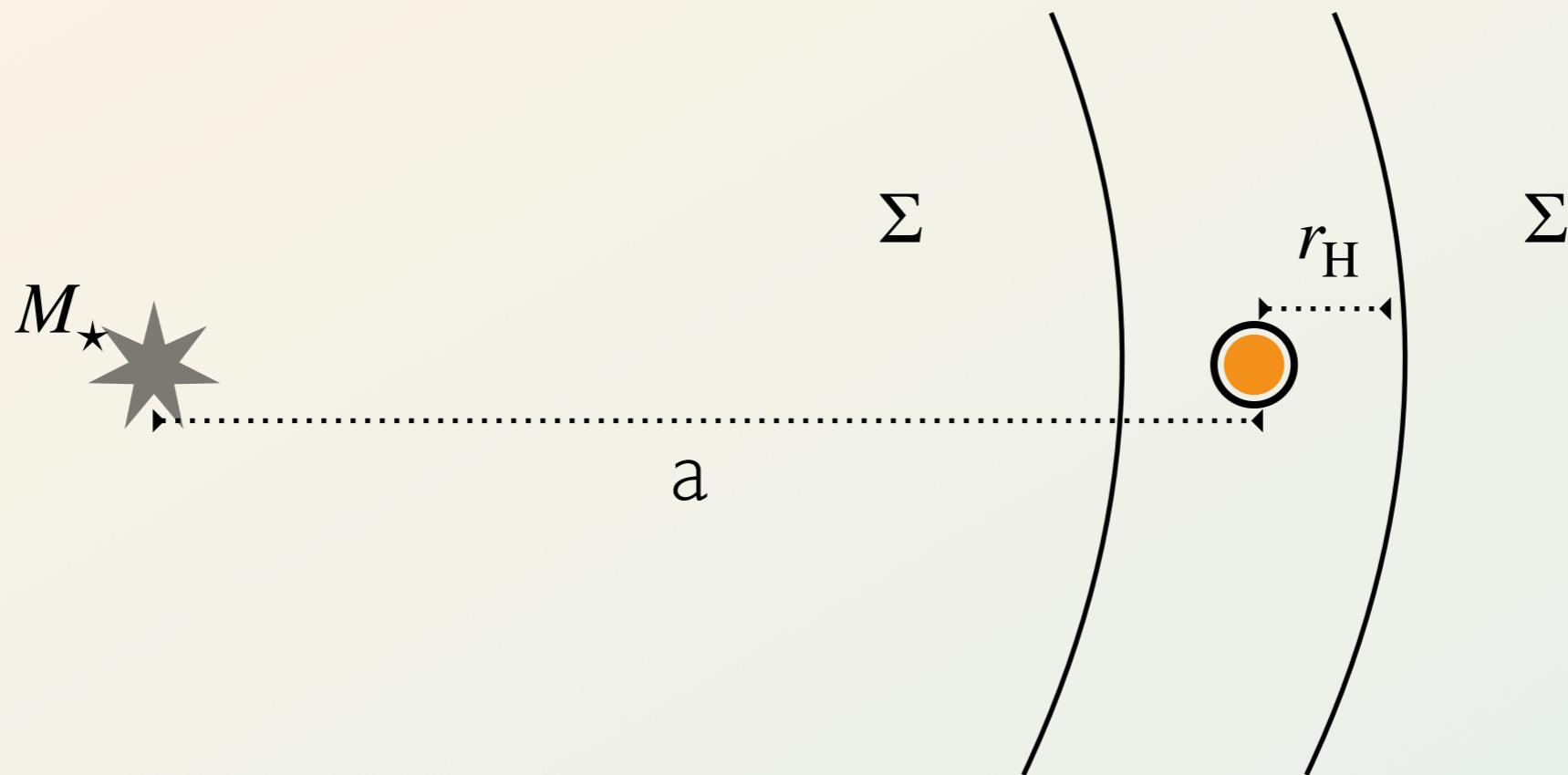
Bright areas in HL Tau are areas of higher surface densities.

MINIMUM SOLAR NEBULA

Take the solar system, break it apart and spread it into a disc. This is the minimum amount of matter needed to make us. If stellar/planet formation is 100% efficient, then all the Minimum Solar Nebula is used.



ISOLATION MASS



The **isolation mass** is the mass after the planet emptied its gap.

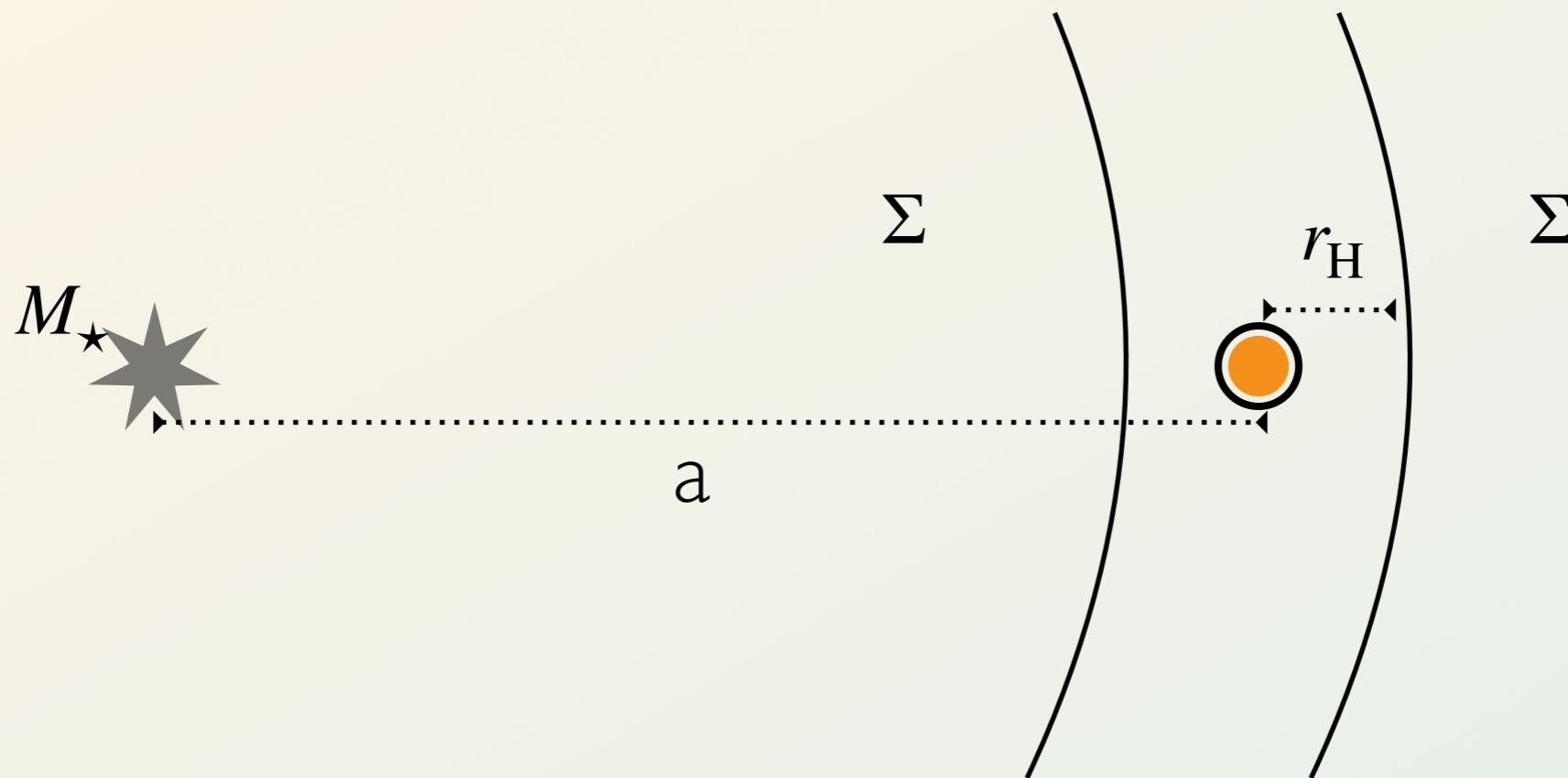
the minimum feeding zone of the planet is $2 \pi a (2 r_H) \Sigma$

we arrive at

$$m_{\text{iso}} = 8 \frac{\pi^{3/2}}{\sqrt{3}} a^3 M_\star^{-1/2} \Sigma^{3/2}$$

1.25

ISOLATION MASS



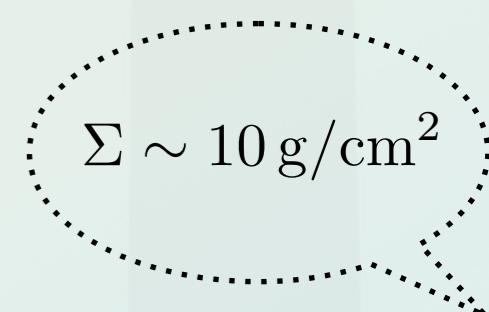
assuming the planet feeds inside of one Hill radii on either side

at 2 AU:

$$m_{\text{iso}} \sim 0.08 M_\oplus$$

at 10 AU:

$$m_{\text{iso}} \sim 10 M_\oplus$$



True feeding zone is likely inside of 2-3 Hill radii on either side

GAS ACCRETION

A core has formed, now it needs to accrete gas. To do that, the sound speed of the gas (which is determined by its temperature) needs to be lower than the escape velocity. A larger body can thus accrete gas that is hotter.

escape velocity

$$v_{\text{esc}} = \sqrt{\frac{2 G m}{r}}$$

1.17

sound speed

$$c_s = \frac{H}{a} v_{\text{Kep}}$$

1.25

For $v_{\text{esc}} > c_s$ we can solve for m :

$$m \geq \sqrt{\frac{3}{32 \pi} \frac{M_\star^3}{\rho a^3}} \left(\frac{H}{a} \right)^3$$

1.26

GAS ACCRETION

Because velocity has a distribution (a Maxwellian), high velocity particles in an atmosphere could escape. 50% of the gas particles in an atmosphere exceeds the mean if $v_{\text{esc}} = c_s$.

This is why we will use $v_{\text{esc}} > 3 \times c_s$ as a criterion for young planets. Adapting eq. 1.26, we get for typical values:

$$H/a \sim 0.1 \quad a = 1 \text{ AU} \quad \rho \sim 5 \text{ g cm}^{-3}$$

$$m \geq 0.5 \text{ M}_\oplus$$

which fits with what is seen within the Solar system

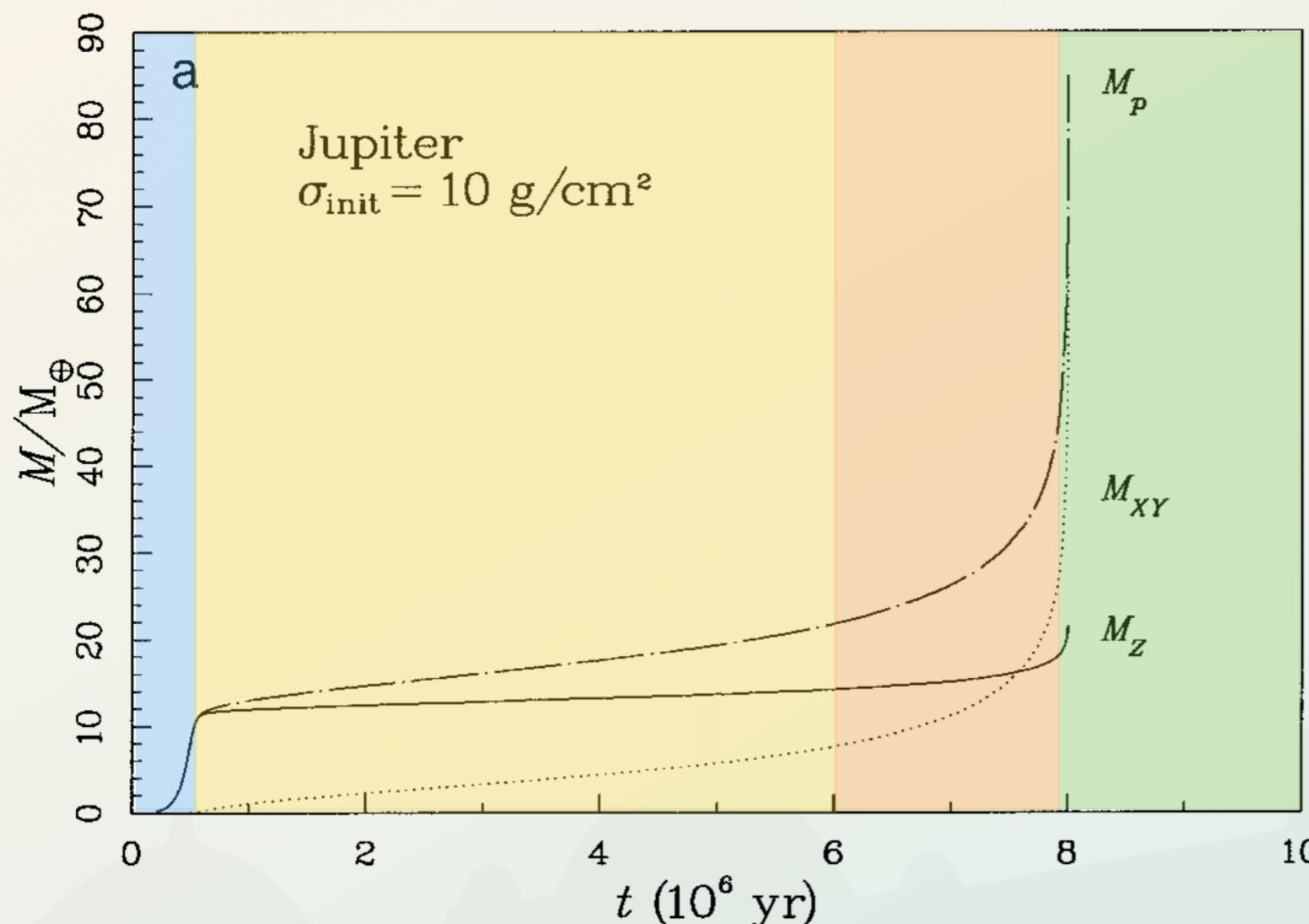
Retaining and acquiring an atmosphere is however quite different, with the acquisition a rather complicated problem to solve.

GAS ACCRETION

A critical mass to obtain much gas is

$$m_p \sim 10 M_\oplus$$

Once as much gas as solids are accreted, the planet goes through runaway gas accretion, until it empties its feeding zone.

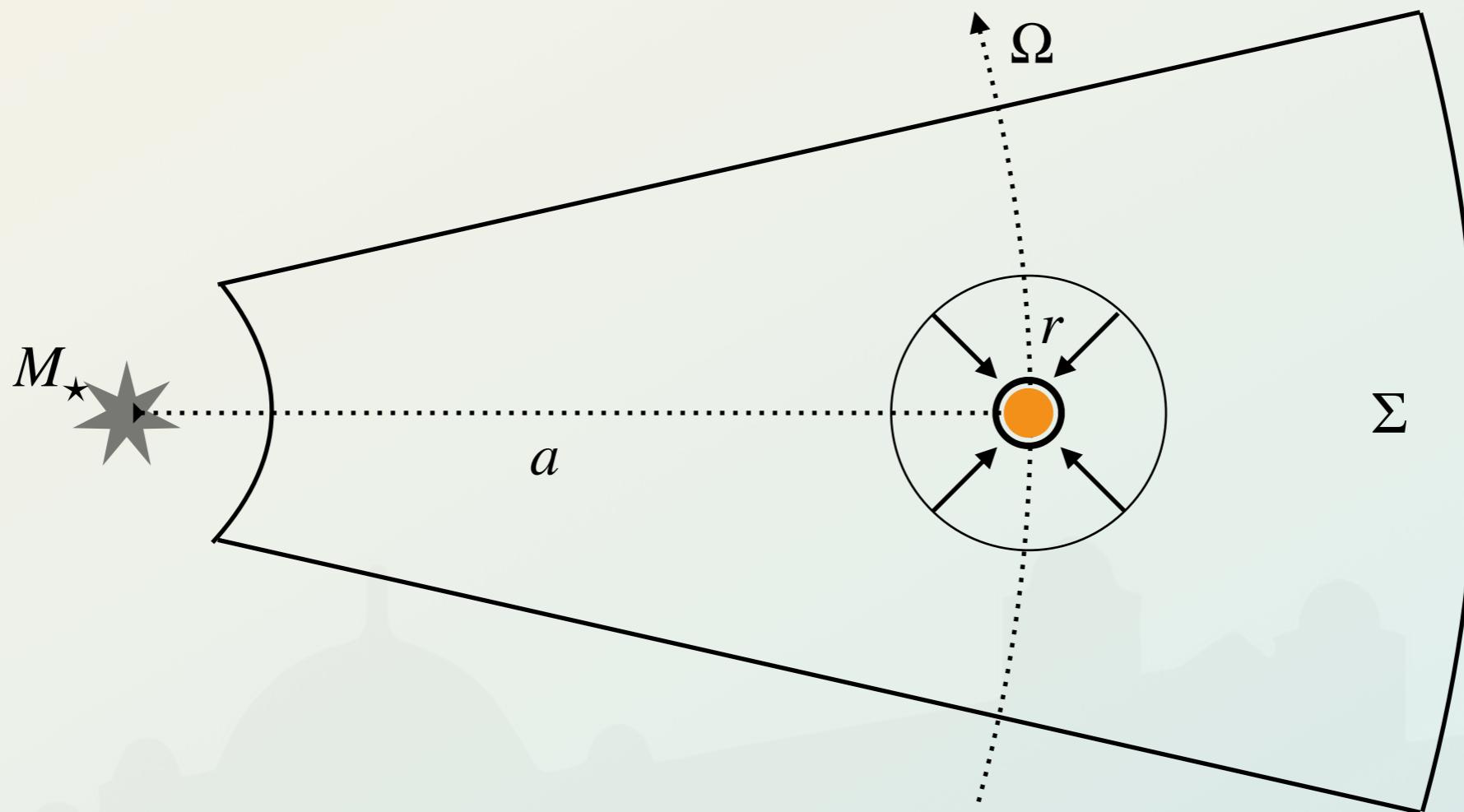


- Core accretion;
- Hydrostatic growth;
gas accretion requires the planet to contract before more gas can accrete. It is slow.
- Runaway accretion;
when gas dominates
- Termination;
Feeding zone is empty, the planet contracts

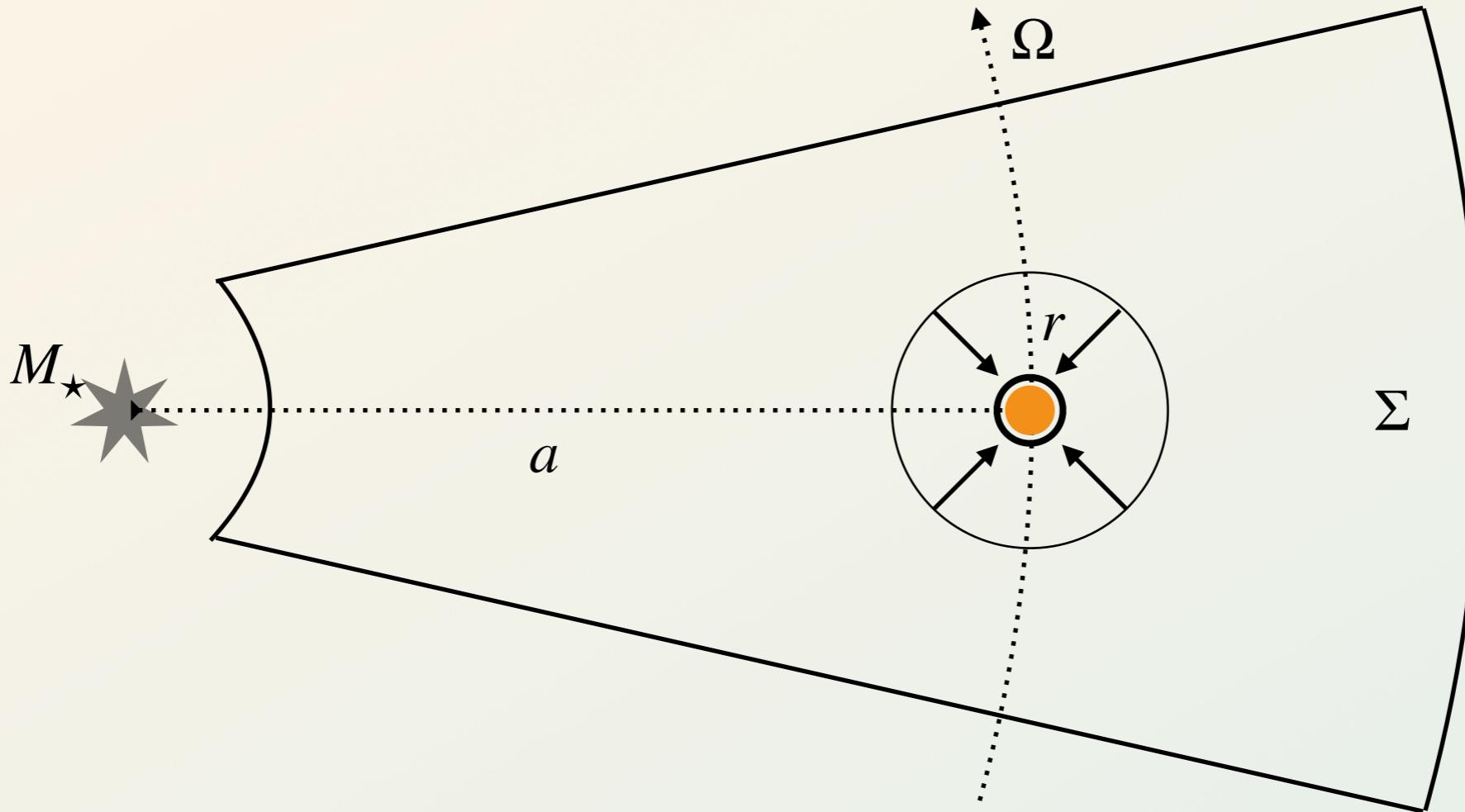
THE DIRECT COLLAPSE HYPOTHESIS

also called the gravitation instability scenario

This is an alternate planet formation scenario, working similarly to the stellar formation process. Instead of a nebula fragmenting on its own weight, the protoplanetary disc fragments due to rotational instabilities



THE DIRECT COLLAPSE HYPOTHESIS



Direct collapse happens in parts of the disc which pass a certain criterion, called the **Toomre criterion**.

First, let's consider the mass m within an area that is collapsing:

$$m = \Sigma r^2$$

THE DIRECT COLLAPSE HYPOTHESIS

Then we take the energy budget in the system, including the gravitational energy, the kinetic energy (as usual), but importantly the rotational energy as well.

gravitational energy

$$U = - \frac{G m^2}{r} = - G \Sigma^2 r^3$$

kinetic energy

$$K = \frac{1}{2} m c_s^2 = \frac{1}{2} \Sigma r^2 c_s^2$$

rotational energy

$$R = \frac{1}{2} m \Omega^2 r^2 = \frac{1}{2} \Sigma \Omega^2 r^4$$

1.27

For collapse,

$$U + K + R < 0$$

which leads to

THE DIRECT COLLAPSE HYPOTHESIS

For collapse,

$$U + K + R < 0$$

which leads to

the Toomre criterion

$$Q = \frac{c_s \Omega}{\pi G \Sigma} < 1$$

I.28

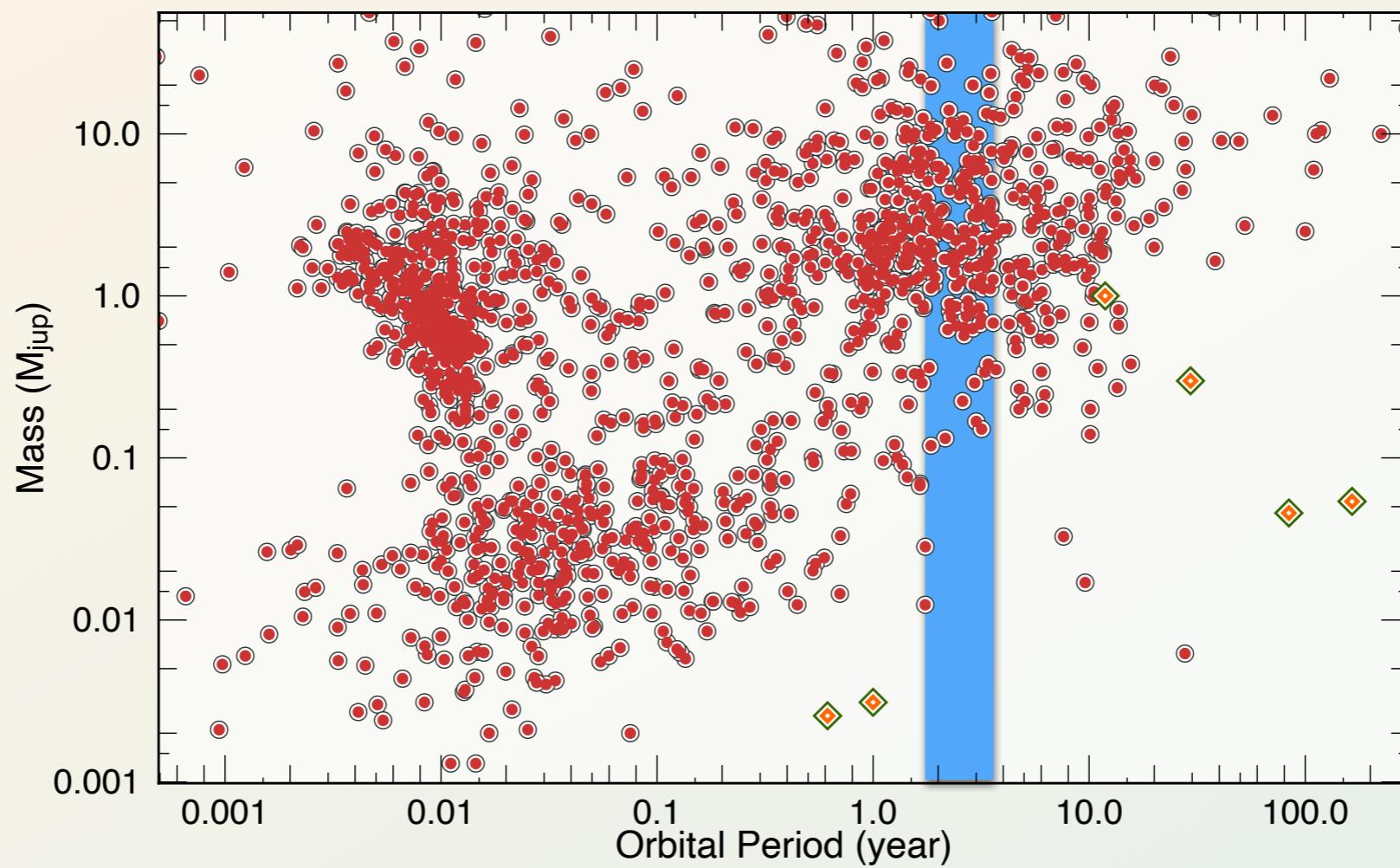
Parts of the disc with $Q < 1$, will collapse. Those are where the mass overcomes the two velocity terms. Introducing the local disc mass, we can get another insightful condition:

$$\frac{m_d}{M_\star} > \frac{h}{R}$$

I.29

Gravitational instability happens where the mass ratio exceed the aspect ratio which we know is about 0.1. This mass ratio is close to the maximum which is observed. Hence the direct collapse scenario is deemed unlikely, but plausible.

EXOPLANET POPULATION



Most planets with substantial amount of gas (i.e. likely formed beyond the water ice-line) are detected within the ice-line. This implies orbital migration of some sort.

Towards the end of the course we will look at disc-driven and disc-free migration.