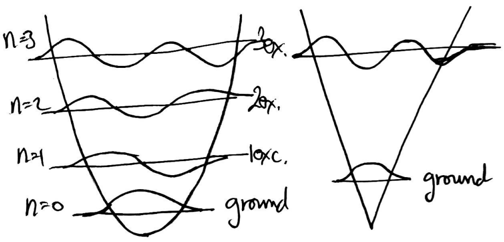
General Physics tutorial problems 1: Quantum mechanics - solutions

1. For both the square and parabolic well, the number of lobes starts with one in the ground state, and increases by 1 for each higher state. Thus we are looking for a wavefunction with 4 lobes, some spread "outside" the well (where the total energy is less than the potential), and a magnitude that depends on proximity to the sides of the well (more like the parabolic than the square well).

   The increased probability near the edges is hard to see for a low level, but we can at least get the number of nodes right, and try to get the change in curvature to correspond to where the kinetic energy goes negative.



2. Eliminating C,D, and E because they are not localised near the potential minimum, and B because it has the wrong symmetry with respect to x, we are left with the correct Gaussian form, A.

3. In the first case, 2 half-wavelengths fit in the well; 3 in the second case. The ratio of wavelengths is 3/2, hence the ratio of momenta, $p = \frac{h}{\lambda}$ is 2/3. Since $E = \frac{p^2}{2m}$, the ratio of energies is 4/9. The state with more curvature in ψ (more half-wavelengths) has higher enregy. Alternatively, you could remember the energy levels of the infinite square well, $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ and note that we are dealing with $n = 2$ and $n = 3$.

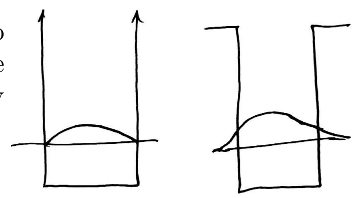
4. Kinetic energy $K = E - U = E$ before the drop, and $3E$ after. Wavevector $k = \frac{\sqrt{2mE}}{\hbar}$ changes to $\frac{\sqrt{6mE}}{\hbar}$. Thus the wavelength $\lambda = \frac{2\pi}{k}$ decreases by $\sqrt{3}$.

   Reflection coefficient $\left|\frac{\psi_R}{\psi_I}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2 = \left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)^2$.

5. Using non-relativistic formulae $E = \frac{p^2}{2m}; p = \hbar k$, we get $E = \frac{\hbar^2 k^2}{2m}$ or $\lambda = \sqrt{\frac{h^2}{2mE}}$. Thus for constant λ, the product mE has to remain constant, so $\frac{E_e}{E_n} = \frac{m_n}{m_e}$.

6. $|\psi|^2$ has a strong peak of value ~ 4 on the left hand side, so the most probable position is at this peak. Roughly integrating by eye, on the positive x side we have $|\psi|^2$ of height 1 and width 1, for an integrated probability of 1, while on the negative we have about 4 for a width of 0.5, so the particle is more likely to be found on the negative x side.

7. The wavefunction penetrates into the walls of the finite well and so spreads out slightly more in this case. The lower curvature of the wavefunction, or equivalently its longer wavelength, give a slightly lower energy in the finite well case.



   For the infinite well, applying $\Delta p \Delta x \sim \hbar$ by setting $\Delta x = a; \Delta p = p$, where a is the well width, gives $E = \frac{p^2}{2m} = \frac{\hbar^2}{2ma^2}$. This compares reasonably with $\frac{\hbar^2\pi^2}{2ma^2}$, the lowest level of the infinite square well.

   Outside the finite square well, $\psi \sim e^{-kx}$, with $k = \sqrt{\frac{2m(V-E)}{\hbar^2}}$, so the characteristic penetration distance $\frac{1}{k} = \frac{\hbar}{\sqrt{2m(V-E)}} \sim \frac{\hbar}{\sqrt{2mV}}$ for large well depth V.

8. $\hat{p}\psi = -i\hbar\frac{\partial\psi}{\partial x}$ with $\psi = Ae^{ikx}$ yields $\hat{p}\psi = \hbar k\psi$, an eigenstate of momentum $\hat{p}$ with eigenvalue $\hbar k$.

   Similarly, $\hat{H}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m}\psi$.
   The conclusion is that the plane wave state $\psi = Ae^{ikx}$ has both an energy and momentum that can be measured simultaneously and with arbitrary precision.