General Physics tutorial problems 2: Electromagnetism - solutions

1. Distance from centre to corner, r, given by $r\cos 30^\circ = a/2 \rightarrow r = a/\sqrt{3}$
   (i) Scalar potential from three charges adds: $3 \times \frac{\sqrt{3}q}{4\pi\epsilon_0 a}$.
   (ii) Three vectors sum to $E = 0$.
   (iii) Force on one charge from on neighbour $\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}$. Adding two such forces, each at 30° to the resultant, multiplies by $2\cos 30^\circ = 2\frac{\sqrt{3}}{2} = \sqrt{3}$, thus $F = \frac{1}{4\pi\epsilon_0}\frac{\sqrt{3}q^2}{a^2}$, directed away from the centre.
   (iv) Start with a single charge at the origin and the others infinitely far away. Bringing up a second charge to distance a increases the potential energy by $\frac{1}{4\pi\epsilon_0}\frac{q^2}{a}$, from the definition of potential. Bringing up a third so that it is a away from each of the first two increases the energy by two similar terms. Total potential energy is $\frac{3}{4\pi\epsilon_0}\frac{q^2}{a}$.

2. $\int_{\text{surface}} \mathbf{E}\cdot d\mathbf{s} = \frac{\Sigma q}{\epsilon_0}$. Evaluate over a cylindrical surface of radius 1 cm coaxial with wire. By symmetry, field is radial, normal to the surface. For length l, $2\pi r l E = \frac{\lambda l}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi r\epsilon_0}$, where λ is the linear charge density. Any charge inside would produce a field inside, again radial, pushing charges outwards. Charge can move in a metal, so charges move to outside.

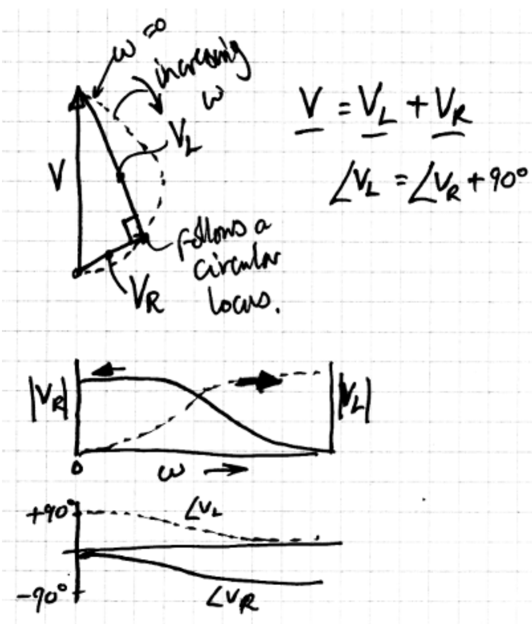
3. $q\mathbf{v}\times\mathbf{B} = \frac{mv^2}{r}$, hence $r = \frac{mv}{qB}$. Any component of $\mathbf{v}$ along $\mathbf{B}$ produces no force, hence motion in this perpendicular direction draws circle out into a helix.

4. Biot-Savart: $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{l}\times\hat{\mathbf{r}}}{r^2}$. Integrate around the circle so that $d\mathbf{l} = r\,d\theta$, everywhere perpendicular to $\mathbf{r}$. Hence $B = \frac{\mu_0 I}{2r} = \pi\times 10^{-5}$ T, axial.

5. Superposition: Set $V = 0$ (a short circuit), then I divides equally between the two equal vertical resistors, and its contribution to $V_{AB} = \frac{IR}{2}$. Next $I = 0$ (open circuit), and the applied V appears equally across the vertical resistors, $V_{AB} = -\frac{V}{2}$, negative because A is the negative end of the battery, and therefore more negative than B. Adding the two terms, which is allowed in a linear system (Ohm's law is linear), $V_{AB} = \frac{IR-V}{2}$.

6. Let the angle between the plane of the coil and B be $\theta = \omega t$. The instantaneous flux linking the coil of area πa^2 is $\phi = B\pi a^2\sin\omega t$. The EMF, $\epsilon = \frac{d\phi}{dt} = B\pi a^2\omega\cos\omega t$. The electical power dissipated is $\epsilon^2/R = B^2\pi^2a^4\omega^2\cos^2(\omega t)/R$. Averaging over a whole rotation, the average value of the $\cos^2$ term is $\frac{1}{2}$, hence the mean power is $B^2\pi^2a^4\omega^2/2R$.

7. Draw phasors for the voltages across the resistor and inductor. They have to always be perpendicular to each other, as they share the same current but their impedances are 90° out of phase. The vector sum of the two must equal the constant phasor of the applied voltage. See figure, from which the magnitudes and phases of the component voltages can be read off.



At $\omega = 0$, the inductor is negligible, so $V_R = V$; when $\omega L \gg R$, $V_L = V$. In between, their magnitudes become equal at $V/\sqrt{2}$.