# Evolution of Cosmic Structure

# Lecture 9 - Halo mass function and the baryons

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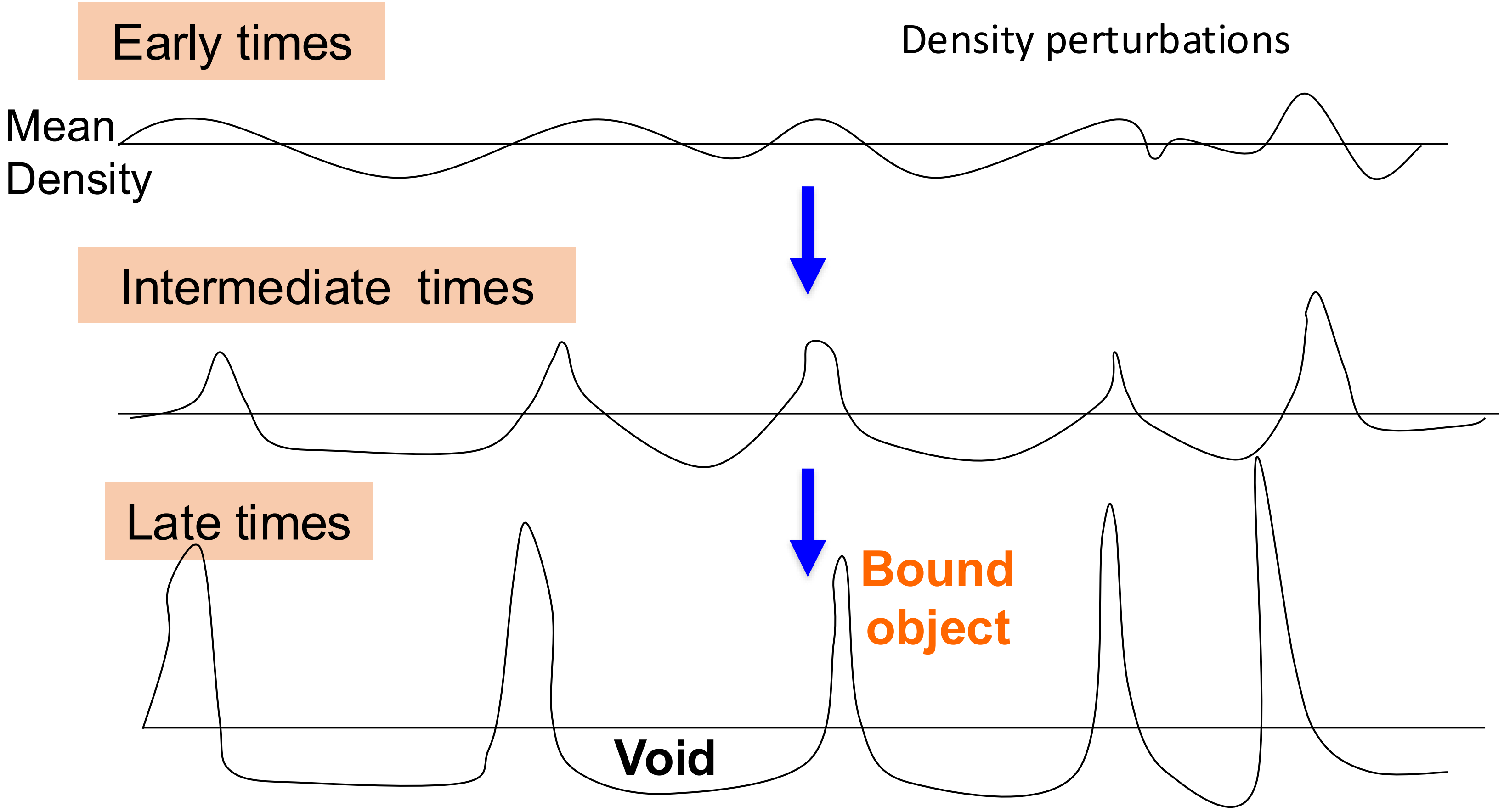
# Non-linear evolution and collapse

In the matter dominated era, if $\Omega_m \approx 1$, then perturbations on all wavelengths grow according to $\delta \propto a$, until $\delta \sim 1$. Once the local density in a perturbation exceeds the critical density of the Universe, its expansion will slow down and stall, and it can collapse.



# The spherical top-hat model

Substituting into $R = R_*(1 - \cos\eta)$ and $t = \alpha R_*(\eta - \sin\eta)$, we obtain, at early times

$$R(\eta) \approx R_* \frac{\eta^2}{2}(1 - \eta^2/12)\ , \qquad t(\eta) \approx \alpha R_* \frac{\eta^3}{6}(1 - \eta^2/20)\ .$$

It follows that $R$ can be expressed as a function of $t$ thus:

$$R(t) \approx \frac{R_*}{2}\left(\frac{6t}{\alpha R_*}\right)^{2/3}\left[1 - \frac{1}{20}\left(\frac{6t}{\alpha R_*}\right)^{2/3}\right]\ .$$

Hence $R \propto t^{2/3}$ at the outset – the well-known evolution for an $\Omega_m$=1 universe.

The higher order term shows us how the overdensity evolves as the radius lags the Hubble expansion, i.e.

$$\delta \equiv \frac{\delta\rho}{\rho} = -3\frac{\delta R}{R} \approx \frac{3}{20}\left(\frac{6t}{\alpha R_*}\right)^{2/3}\ .$$

This is the linear regime behaviour whereby δ increases in proportion to *a*.

# The spherical top-hat model

What happens at the final stage? The analytical top hat model gives system collapses to a point at $t_{coll} = 2\pi\alpha R_*$. In practice, however, we expect the dark matter particles to stream through the centre and *virialise*. The virial theorem requires a stable self-gravitating system to have a balance between gravitational potential energy (*V*) and kinetic energy (*T*), such that $V = -2T$.

If the energy in the collapse is conserved, then

$$E_{max} = E_{vir}$$

$$U_{max} = U_{vir} + T_{vir} = \frac{1}{2}U_{vir}$$

$$-\frac{GM^2}{r_{max}} = -\frac{GM^2}{2r_{vir}}$$

$$r_{vir} = \frac{r_{max}}{2}$$

A similar calculation to the previous one, with $r_{vir}$and $t_{coll}$ relative to the unperturbed radius give $\frac{\rho}{\bar{\rho}} = \frac{(6\,\pi\,)^2}{2} = 178$. This will be a key definition for a collapsed halo!

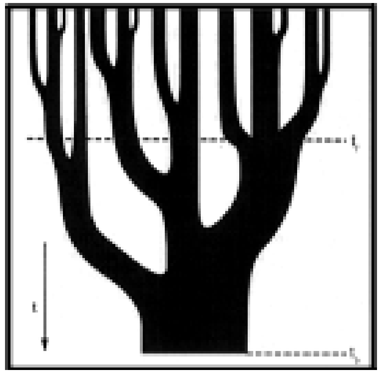
# Non-linear growth of structure

**How does the structure develop once the fractional density perturbations become large, and subject to gravitational collapse?**

**How do the baryons respond to the collapse of dark halos?**

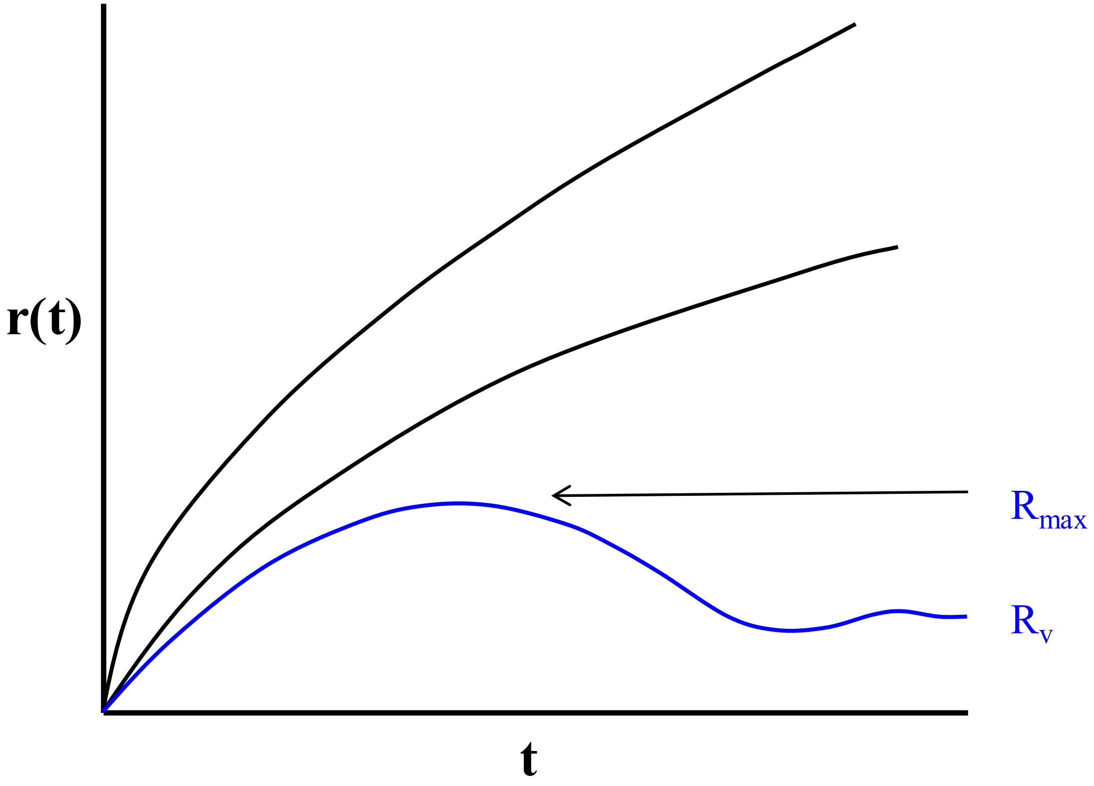
- Non-linear evolution and collapse
- The spherical collapse model
- Non-spherical collapse
- The halo mass function
- Hierarchical merging
- Baryon cooling and galaxy formation

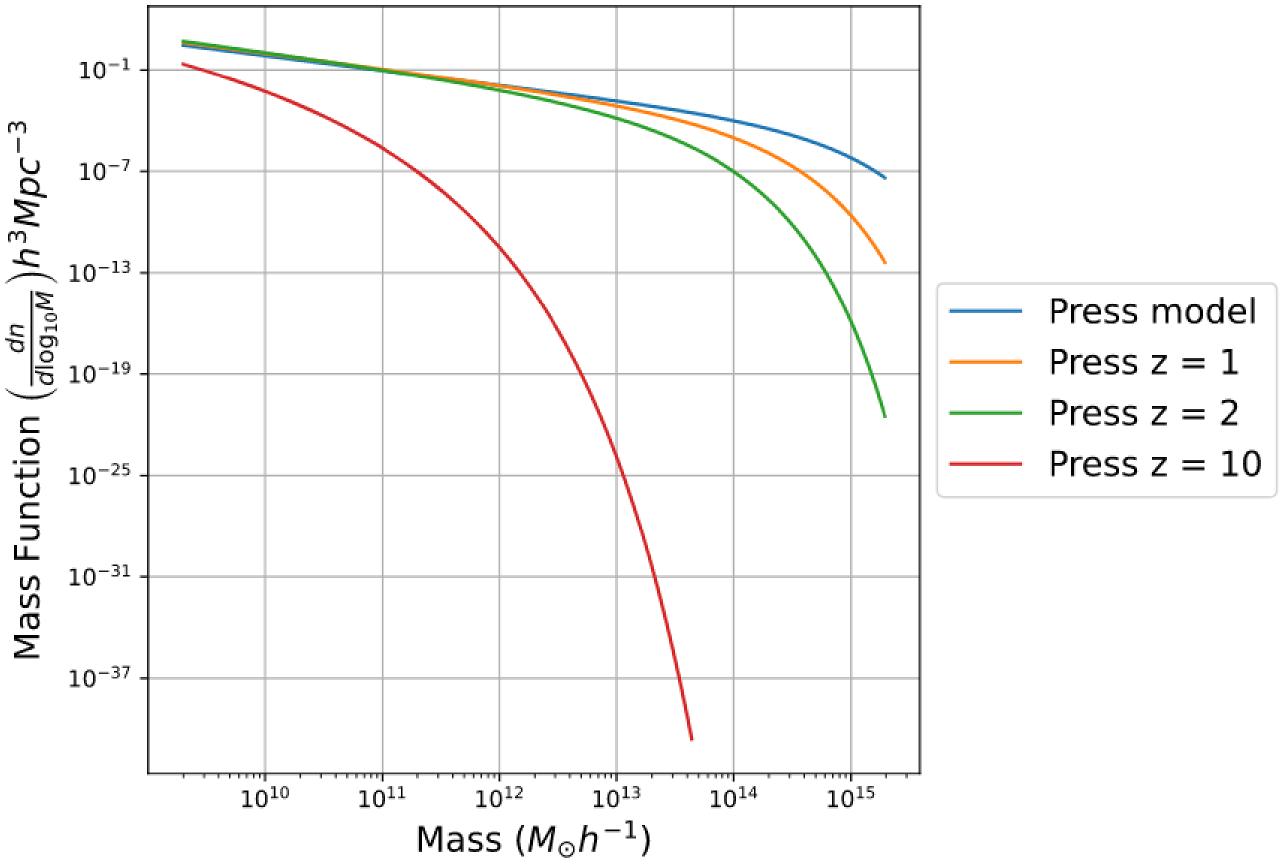
# The halo mass function



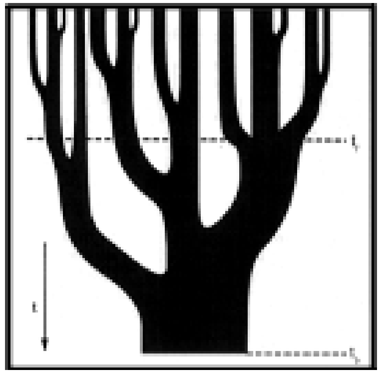
We would like to know the numbers of halos of different masses which are collapsing out of the Hubble expansion at any given time.

Evolution of radii of concentric shells near a spherical overdensity.



# The halo mass function



We would like to know the numbers of halos of different masses which are collapsing out of the Hubble expansion at any given time.

A clever approximate way of calculating this was derived by Press & Schechter by applying the top hat model to a growing set of Gaussian perturbations.

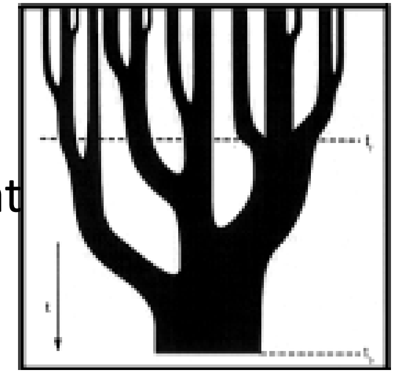
We saw in part 1 that if perturbation are a Gaussian random field, then the fluctuations on a mass scale $M$ (e.g., if one throws down spheres of radius $r = \left(\frac{3\,M}{4\,\pi\bar{\rho}}\right)^{1/3}$ which contain mass $M$ on average) are Gaussian distributed.

This is equivalent to saying that the probability of seeing a fractional mass fluctuation $\delta_M$, is

$$P\,(\delta_M) = \frac{1}{\sqrt{2\,\pi}\,\sigma_M}\,e^{-\frac{\delta_M^2}{2\sigma_m^2}}$$

where the variance on the mass scale M (known as $\sigma_M$) is related to the power spectrum on that scale.

# The halo mass function



These fluctuations grow linearly with the scale factor $a$ at first, and we have already seen that the linear growth law is

$$\delta = \frac{\delta\rho}{\rho} = \frac{3}{20}\left(\frac{6t}{\alpha R_*}\right)^{\frac{2}{3}}$$

In the non-linear top hat model, we saw that the turnaround occurs at time $t_{max} = \pi\alpha R_*$ and that final virialisation happens at $t_{coll} = 2\pi\alpha R_*$. At those two times, we calculated the density contrast to be 5.5 and 178 respectively.

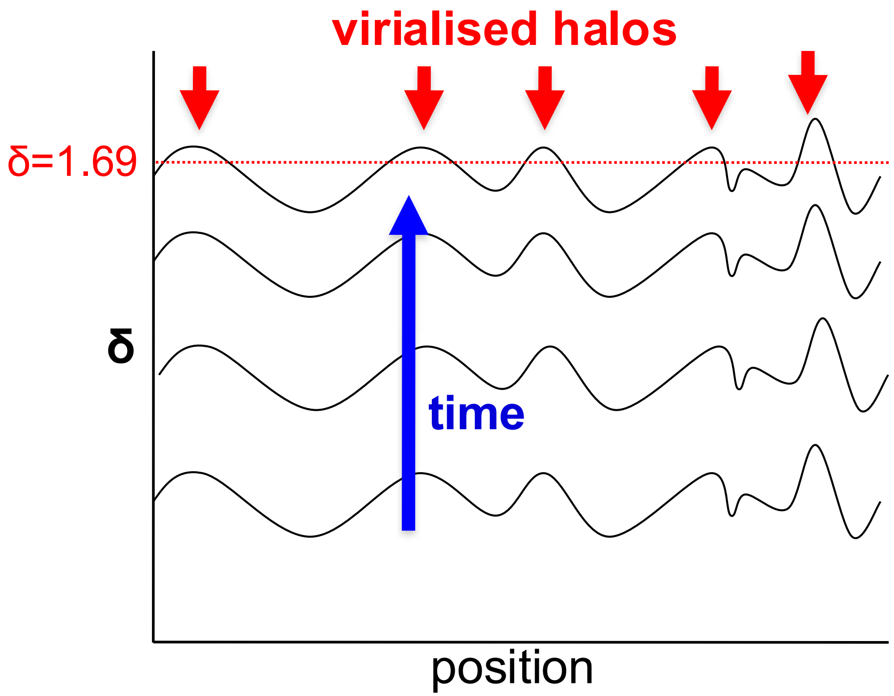
Let's just calculate what the linear model density contrast is at the collapse/virialisation time.

$$\delta_{coll} = \frac{3}{20}\left(\frac{6t_{coll}}{\alpha R_*}\right)^{\frac{2}{3}} = \frac{3}{20}\left(\frac{12\pi\alpha R_*}{\alpha R_*}\right)^{\frac{2}{3}} = \frac{3}{20}(12\,\pi)^{\frac{2}{3}} = 1.69$$

# The Press-Schechter model

The basic idea of the Press-Schechter approach is to evolve the perturbations linearly, but when $\delta$ reaches the value $\delta_c$ it is assumed to have formed a virialized halo.

Using this approach, it is possible to derive the mass function of halos of mass $M$ which are virialized at time $t$.




The number of halos per unit comoving volume in the mass range $M$ to $M + \Delta M$ is $\Delta M \frac{\mathrm{d}n}{\mathrm{d}M}$ where

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,t) = 2\frac{\rho\delta_c}{M^2}\frac{1}{\sqrt{2\pi}\,\sigma_m}\left|\frac{d\ln\sigma_m}{d\ln M}\right|\exp\left(-\frac{\delta_c}{2\,\sigma_M^2}\right)$$

Mass only appears through $\sigma_m$ and its derivative.

This rises in a power law fashion at low masses, but cuts off exponentially above the mass at which $\sigma_M \sim \delta_c$.

# The Press-Schechter model

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,t) = 2\frac{\rho\delta_c}{M^2}\frac{1}{\sqrt{2\pi}\,\sigma_m}\left|\frac{d\ln\sigma_m}{d\ln M}\right|\exp\left(-\frac{\delta_c}{2\,\sigma_M^2}\right)$$

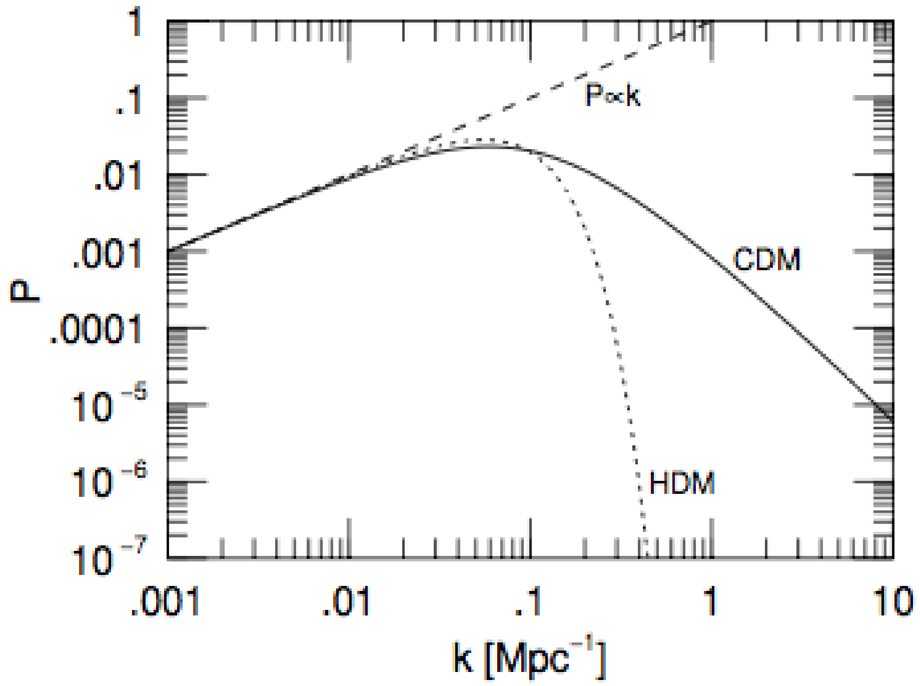
We can connect this directly to the slope in the power spectrum by recalling the earlier findings

$$\frac{\Delta M}{M} \propto \sqrt{k^3 P(k)} \propto L^{-(3+n)/2} \propto M^{-(3+n)/6}$$

We can get a reduced form by defining the variable $\nu = \frac{\delta_c}{\sigma_m}$

so it becomes

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,t) = \sqrt{\frac{2}{\pi}}\frac{\rho_m}{M}\left|\frac{n_e+3}{6}\right|\nu e^{-\nu^2/2}$$



# The halo mass function

Now, we can calculate the number density of collapsed halos per logarithmic mass. Let's try $5 \text{ X } 10^{14} \mathrm{M}_{solar}$ and let's assume $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_{M,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, $\sigma_8 = 0.8$. and $n_{\text{eff}} = -1.5$.

First calculate the comoving matter density:

$$\rho_m = \Omega_m \rho_{crit} = \Omega_m \frac{3H^2}{8\pi G} = 4.07 \times 10^{10} M_\odot \mathrm{Mpc}^{-3}$$

Find the mass inside an 8 Mpc radius sphere:

$$M_{R=8\mathrm{Mpc}} = \rho_m \frac{4}{3}\pi R^3 = 8.72 \times 10^{13} M_\odot$$

Now we can relate fluctuations on 8Mpc scales to those on the scale of the required mass (5E14 Msun)

$$\sigma_m = \sigma_8 \left[\frac{M_{M=5\times10^{14}M_\odot}}{M_{R=8\mathrm{Mpc}}}\right]^{-(n_{eff}+3)/6} = \sigma_8 \left[\frac{5\times10^{14}M_\odot}{8.72\times10^{13}M_\odot}\right]^{-(3-1.5)/6} = 0.52$$

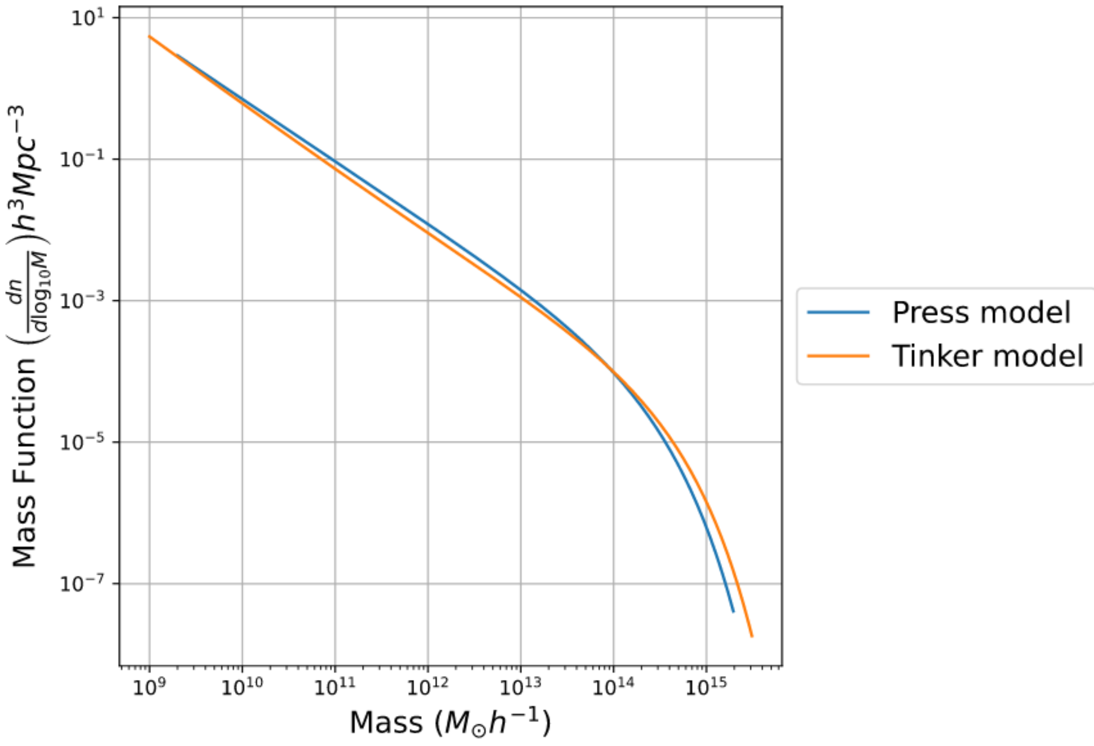
# The halo mass function

That means that,

$$\nu = 1.69/\sigma = 1.69/0.52 = 3.25$$

Then,

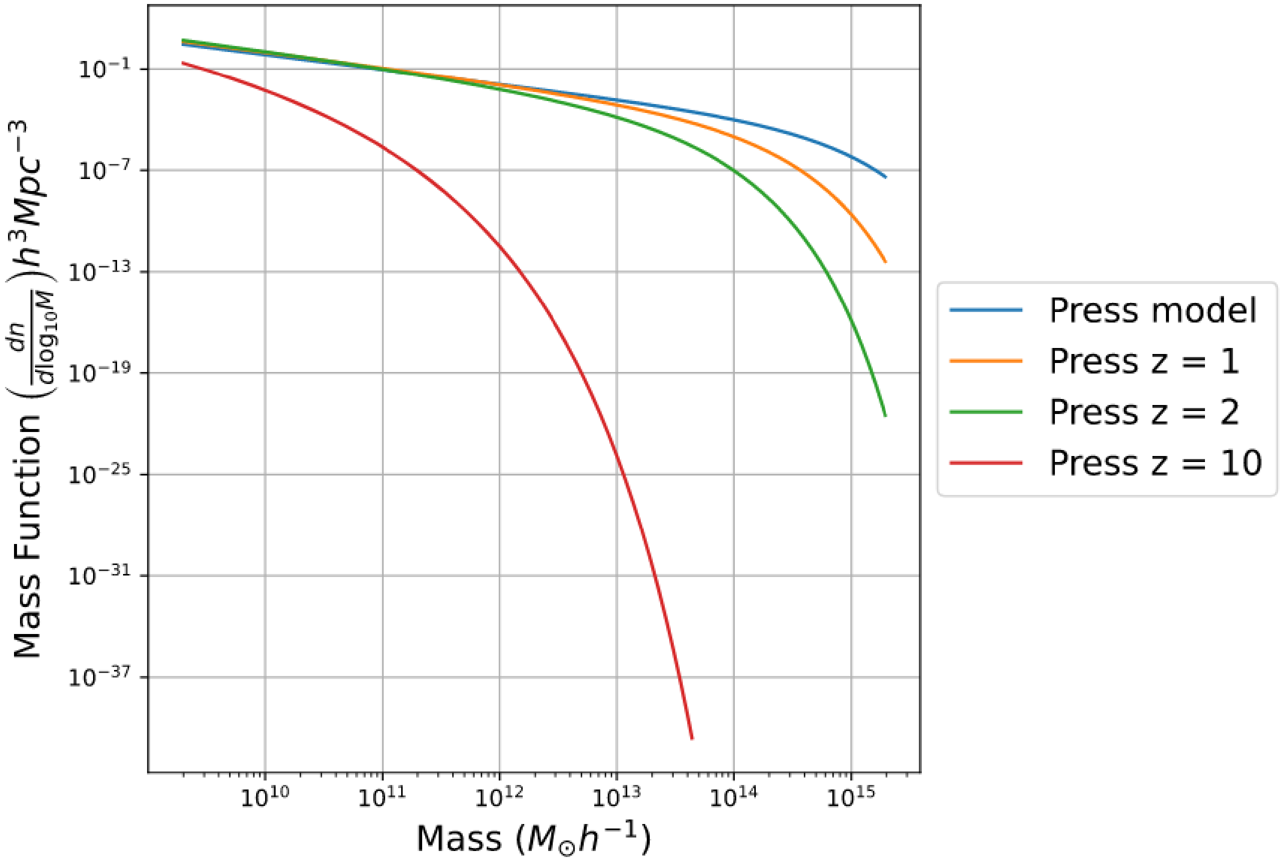
$$\frac{dn}{d\log M} = \sqrt{\frac{2}{\pi}}\frac{\rho_m}{M}\left|\frac{n_{eff}+3}{6}\right|\nu e^{-\nu^2/2} = 8.05\times 10^{-7}\mathrm{Mpc}^{-3}$$

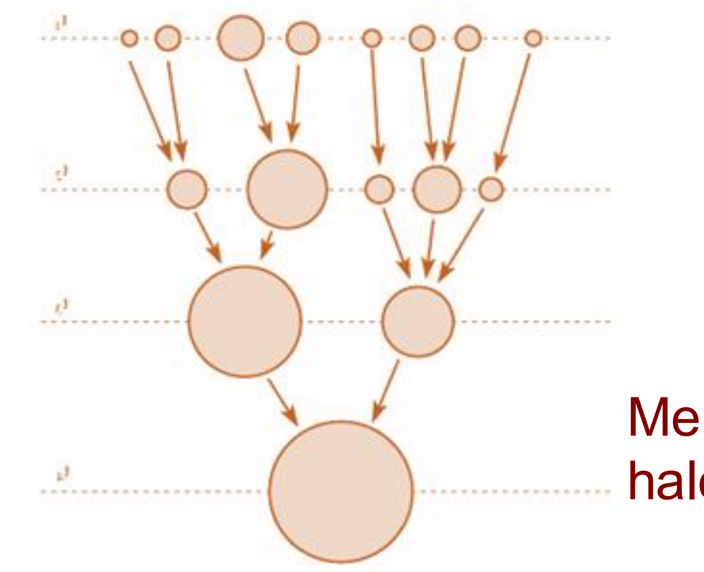


# Hierarchical merging

The P-S mass function therefore evolves in time like this:

We see that small objects collapse first, and then successively larger and larger ones. During this process, collapsed haloes may find themselves within larger collapsing structures. In this case they will grow through mergers and also accretion.



Merger tree, showing how larger halos are built up from smaller ones.

# Baryon cooling and galaxy formation

As the baryonic material is concentrated into collapsing halos, it will be compressed and heated. A rich variety of extra physics now comes into play.

If the halo contains gas, then new gas falling in will hit this supersonically and will shock, converting its kinetic energy into thermal energy. Its temperature will therefore be raised to the "virial temperature" $T_v$, derived from the equation

$$\frac{3}{2}kT_v = \frac{GM\mu m_{\mathrm{H}}}{r_v}$$

where $r_v$ is the "virial radius" out to which the system is virialised.

We have already seen that from the top-hat model the mean density of a newly virialised halo is 178 times the critical density of the Universe. It is therefore customary to round this up to 200, and estimate the radius out to which a halo is virialised as $r_{200}$, the radius within which the mean density is 200x the critical density – i.e. for an object of mass $M$, we have

$$M = \frac{4\pi}{3}r_{200}^3 \times 200\rho_c$$

Putting in some relevant numbers, the halo masses of typical large galaxies are ~$10^{12}M_\odot$ and we find $r_{200} \propto M^{1/3} = (M/10^{12}M_\odot)^{1/3} \times 206\ \mathrm{kpc}$

and $T_{200} \propto M^{2/3} = (M/10^{12}M_\odot)^{2/3} 1.0 \times 10^6\mathrm{K}$, where we have taken $\mu$=0.6 .

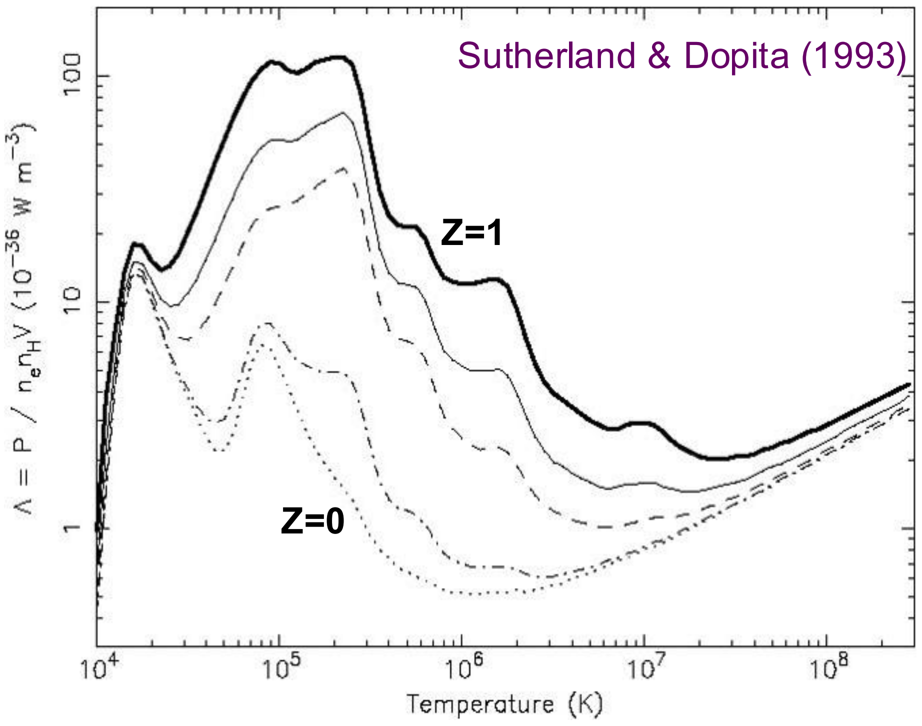
# Baryon cooling and galaxy formation

What happens next depends upon the ability of gas to cool. This is a strong function of temperature, since at $T$>>$10^5$ K atoms are mostly ionized, reducing their ability to radiate.

Though it may appear from the cooling function plot that gas at T>$10^6$K radiates more effectively, the cooling time $\tau = nkT/n^2\Lambda$, is actually *longer* at high $T$, since the thermal energy per particle ($3kT/2$) is higher.

The result is that gas in halos with masses greater than about $10^{12}$ $M_\odot$ is unable to cool effectively. This sets a natural upper limit to the mass of galaxies. In lower mass systems, gas is able to cool and form stars, whilst in more massive halos most of the baryons remain in the form of hot intergalactic gas.

Emissivity=$n_e n_H \Lambda$




Cooling function for a H+He plasma with Z=0 to 1 solar abundance

# Growth of structure - summary

- Seed fluctuations in density are believed to have been put in place very early, probably during the inflationary era, with fractional mass fluctuations which are progressively larger on smaller mass scales.
- Fluctuations on scales larger than the horizon grow linearly with $a$, but during the radiation-dominated era ($t$<50 kyr) dm fluctuations within the horizon can grow only slowly, so the power spectrum of fluctuations is reduced on smaller scales. However, smaller mass scales still have larger $\Delta M/M$.
- If the majority of the dark matter were hot (i.e. relativistic) then it would stream out the developing dm potential wells and obliterate power on small scales. The resulting suppression in the collapse of galaxy-scale masses is inconsistent with observation of galaxies at high $z$, so CDM must dominate.
- Once matter becomes dominant, $\delta$ grows linearly with $a$ on all scales, but baryons are unable to collapse into the developing dm fluctuations until after decoupling ($t$ ~350 kyr) since it is supported by radiation pressure.

# Growth of structure - summary

- As δ grows to become ~1 its growth accelerates and we enter the non-linear regime. The top-hat collapse model allows a simple analysis of the process of turnaround and recollapse.
- Combining the statistics of Gaussian fluctuations with the expected non-linear evolution, it is possible to estimate the developing mass function of virialised halos. Low mass halos tend to collapse first, followed by progressively larger structures.
- This leads to a process of hierarchical merging, as collapsed objects (as well as uncollapsed material) aggregate into larger and larger structures.
- Baryons are heated as they collapse into developing potential wells and their subsequent behaviour depends upon whether they can cool. For galaxy mass halos ($M<10^{12}M_{\odot}$) this is generally the case, and gas will cool out at the centre of halos and may form stars. For more massive systems, cooling is less effective. This sets a natural upper mass limit for galaxies.
- The detailed behaviour of baryons cannot be treated analytically, but can be investigated using numerical simulations.