General Physics tutorial problems 1: Mechanics - solutions

1. At constant speed, forces on carriage must be balanced, hence the force pushing the carriage is equal and opposite to the frictional force on it from the track. Air resistance would also be part of the balance, but negligible at slow speeds.

2. In order to move along the circular path defined by the large sphere, a centripetal force must be applied to the bearing. This is provided by its weight, mg. With the bearing at an angle θ from the top of the sphere, resolving mg into a radial and tangential component, the radial component is $mg\cos\theta$. If this is larger than the necessary centripetal force, there will be a normal contact force that reduces the total centripetal force. At the point of departure, the radial component of mg just ceases to be sufficient (and the normal contact force vanishes, of course). Thus $mg\cos\theta = \frac{mv^2}{r}$ at departure. From the geometry, $\cos\theta = \frac{r-h}{r}$, so we can eliminate θ in favour of h. By conserving energy, $\frac{1}{2}mv^2 = mgh$ (this is easy because the bearing is sliding, so takes up no rotational energy). This allows us to substitute $2gh$ for v^2, and find $h = \frac{r}{3}$.

3. Friction provides the necessary centripetal force. The maximum frictional force is μmg. Equating to $\frac{mv^2}{r}$ gives a radius of $\frac{v^2}{\mu g}$.

4. As the mass reaches the natural length of the band, its kinetic energy is $\frac{1}{2}mv^2$. Stretching the band a distance x beyond this takes energy $\frac{1}{2}k_s x^2$ (integral of force times distance, where force $= k_s x$). At the point the mass stops, it has zero kinetic energy. Conserving energy, $x = \sqrt{\frac{mv^2}{k_s}}$.

5. Angular momentum, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. (i) 10 $\mathrm{kg\,m^2\,s^{-1}}$ upwards; (ii) 20 $\mathrm{kg\,m^2\,s^{-1}}$ downwards. It's easiest to calculate at the moment its velocity and displacement from the reference point are perpendicular, but the cross product ensures the angular momentum about any point is conserved, even though the mass is moving linearly.

6. Wave speed $c = \sqrt{\frac{T}{\rho}}$. It's reasonably easy to derive by equating the restoring force on a small segment Δx, which is proportional to both the tension and the second derivative of the displacement, to its mass times its acceleration: $\rho\,\Delta x\frac{\partial^2\psi}{\partial t^2} = T\frac{\partial^2\psi}{\partial x^2}\Delta x$, then comparing to the wave equation $\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}$. But it is probably worth remembering. The frequency of oscillation does not change at the join (both sides have to move the same way in order to stay joined) so since $c = \lambda f$, the wavelength tracks the speed. Doubling ρ reduces the speed, and hence the wavelength, by $\sqrt{2}$.

7. Phase velocity, $v_p = \frac{\omega}{k}$; group velocity, $v_g = \frac{d\omega}{dk}$. Using (de Broglie) $p = \frac{h}{\lambda} = \hbar k$ and $E = \hbar\omega$, we get $v_p = \frac{E}{p}$; $v_g = \frac{dE}{dp}$. From the given λ, we get $p = 6.6 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}$. Dividing by the neutron mass, we find its speed is $4 \times 10^3\,\mathrm{m\,s^{-1}}$, which justifies the use of non-relativistic expressions.

8. We conserve angular momentum, L. Initially, that of running child about centre of roundabout $= m_c v_c r$. In final state with roundabout rotating at angular velocity ω, L of roundabout $= I\omega$, and L of child $= m\omega r^2$. Moment of inertia of roundabout disc of density ρ and thickness t is $I = \frac{1}{2}Mr^2 = \frac{1}{2}\pi r^4 t\rho$. Hence $m_c v_c r = (\frac{1}{2}\pi r^4 t\rho + m_c r^2)\,\omega$. Substituting, I get $\omega = 0.03\,\mathrm{rad\,s^{-1}}$. Centripetal force $= \frac{mv^2}{r} = mr\omega^2 = 0.054\,\mathrm{N}$. Stopping torque $\tau = I\frac{d\omega}{dt} = I\frac{0.03}{5}$. Substituting total $I = \frac{1}{2}\pi r^4 t\rho + m_c r^2 = 10047$, we have $\tau = 60\,\mathrm{Nm}$.

9. Pressure on the right side liquid surface is atmospheric. Pressure at the same height on the left is atmospheric plus $2h\rho g$. The net force acting to move all the water towards equilibrium is thus $2h\rho gA$, where A is the cross-sectional area of the tube. Applying Newton's second law, $\frac{d^2h}{dt^2}\rho Al = -2h\rho gA$, where ρAl is the mass of water moved (having length l), and the minus sign indicates the force acts towards equilibrium.

   A restoring force proportional to the displacement from equilibrium causes simple harmonic motion. Comparing the standard expression $\frac{d^2y}{dt^2} = -\omega^2 y$, the frequency $\omega = \sqrt{\frac{2g}{l}}$, and $h = h_0\cos(\omega t)$. Without viscous damping, the oscillations maintain the same amplitude forever. With damping, they decay. The simplest model, of a frictional force proportional to velocity, produces an exponential decay.