Year 3 Assessed Problems

Semester 2

Assessed Problems 1

SOLUTIONS TO BE SUBMITTED ON CANVAS BY

Tuesday 24th February

at 09:00

Y3 Particle Physics 2025–2026
Assessed Problem 1

As always in assessed work, explain your arguments and working *fully* in all parts of your solutions in order to obtain full marks.

You will need to look up some particle properties to answer this question, for example using https://pdg.lbl.gov/. You should state any information you use.

1. A beam of K^- mesons strikes a stationary liquid-deuterium (LD_2) bubble chamber. The total flux (intensity) of the beam is 4×10^4 K^- per second, the target is 1.5 m deep, and all of the beam strikes the target.

   (a) Determine, and draw a Feynman diagram for, the K^-p interaction process requiring the lowest K^- beam energy which produces a final-state containing a Ξ^0, and which involves only the strong interaction. This reaction is referred to as $K^- \, p \to \Xi^0 + X$ in the following. **[3]**

   (b) Calculate the minimum energy of the K^- beam for this reaction to proceed. **[3]**

   (c) If the beam energy is set instead to 1.8 GeV and the cross-section for this process is 9 μb, calculate the rate of this $K^- \, p \to \Xi^0 + X$ interaction. Take the density of LD_2 to be 170 kg/m^3. **[4]**

Nuclear Physics Y3 Assessed Problems

Feb 2026

Nuclear Force

a) Why is the Woods-Saxon a good potential to use for the shell model? Explain why the edges of the potential are smeared. **[2 marks]**

Cross Sections

The general form for the differential cross section is given as:

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin(\delta_l)P_l(\theta)\right|^2 \tag{1}$$

$$k = \sqrt{\frac{2\mu E}{\hbar^2}} \text{ with } \mu \text{ as the reduced mass.} \tag{2}$$

A measurement is performed with $E = 25$ MeV neutrons incident on a proton target. Only the l=0 and l=1 waves contribute, and the measured phase shifts are $\delta_0 = \frac{\pi}{3}$ and $\delta_1 = \frac{\pi}{6}$.

b) Calculate the differential cross section as a function of the scattering angle, θ. Using $P_0(\theta) = 1, P_1 = \cos(\theta)$ **[3 marks]**

c) Calculate the total cross section in <u>barns</u> using:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega. \tag{3}$$

[5 marks]

Assessed problem set #1 for LH Condensed Matter Physics

1. The density of silver is 10.49 g/cm^3, and the effective mass of the valence electrons in silver is $1.0m_e$. Using a free-electron model for the electronic structure of silver, what is the electronic heat capacity of silver at 300 K as a fraction of the total heat capacity? The Debye temperature of silver is 215 K. In calculating the lattice heat capacity, you may assume that we are in the $T \gg T_{\mathrm{D}}$ limit. **[3]**

2. What is the Fermi temperature ($\varepsilon_{\mathrm{F}}/k_{\mathrm{B}}$) of silver? Based on that result, why do you expect the electronic heat capacity to be so much smaller than the lattice heat capacity at 300 K? **[1]**

3. What is the Seebeck coefficient of a two-dimensional metal, using the free-electron model? **[2]**

4. Show that a two-dimensional free-electron metal obeys the Wiedemann-Franz law. You may take as given that the resistivity of a 2D free-electron metal is given by $\rho = m^*/n_{2\mathrm{D}}e^2\tau$, and that $C_{\mathrm{el}} = (\pi^2/3)k_{\mathrm{B}}^2 T g(\varepsilon_{\mathrm{F}})$. **[3]**

5. The data below show the heat capacity of silver at low temperatures. Based on these data, what speed of sound do you derive from the Debye model of lattice heat capacity? **[1]**

T (K)	C (J/mol K)
3.00	0.0063
4.00	0.0134
5.00	0.0243
6.00	0.0402
7.00	0.0623
8.00	0.0925

Assessed Problems 1

Answer the following multiple choice questions with one of (a), (b), (c), (d), (e). All symbols have their standard meanings unless otherwise noted. Marks are available for a correct brief justification, even if the letter is incorrect. Only the first page of any work submitted will be marked.

1. [**2 marks**] A system has equation of motion

$$\ddot{q} = -\eta q^2 \dot{q} + \lambda q^3, \tag{1}$$

where η and λ are constants. With a suitable definition of quantity p, which of the following represents the system as a dynamical system for $\boldsymbol{X} = [q,\ p]^T$?

(a) $\dot{\boldsymbol{X}} = \begin{bmatrix} \frac{p}{m} \\ -\eta q^2 p + \lambda q^3 \end{bmatrix}$

(b) $\dot{\boldsymbol{X}} = \begin{bmatrix} p \\ -\eta q^2 p + \lambda q^3 \end{bmatrix}$

(c) $\dot{\boldsymbol{X}} = \begin{bmatrix} \frac{p}{m} \\ \eta q^2 p - \lambda m q^3 \end{bmatrix}$

(d) $\dot{\boldsymbol{X}} = \begin{bmatrix} p \\ \eta q^2 p - \lambda q^3 \end{bmatrix}$

(e) None of the above

2. [**2 marks**] Consider a Hamiltonian of the form

$$\mathcal{H}(q,p) = \frac{p^2}{2m} + V(q), \tag{2}$$

for some potential $V(q)$, $V(0) = 0$. Which of the following has to hold if we know that the system has a periodic orbit with energy E, period τ, and extreme values of q, p given as $\pm q_0$, $\pm p_0$? You can assume $V(0) = 0$ and

$$V'(0) = 0, \qquad V'(q)\big|_{q\neq 0}\,. \tag{3}$$

(a) The period τ is independent of the energy E.

(b) $p_0 \propto \sqrt{E}$

(c) $q_0 \propto \sqrt{E}$

(d) $V''(0) < 0$

(e) None of the above (i.e. none are necessarily true)

3. [**2 marks**] Which of the following is necessarily true for a general Hamiltonian system?
   (a) The Hamiltonian is conserved
   (b) Phase space density is conserved along all orbits
   (c) The phase portrait corresponds to contours $\mathcal{H} = E$
   (d) All of the above
   (e) None of the above (i.e. none are necessarily true)

4. [**2 marks**] Which of the following type of fixed point is *not possible* in a linear system?
   (a) Circulation
   (b) Saddle
   (c) Focus
   (d) Spiral
   (e) None of the above (i.e. all are possible)

5. [**2 marks**] A dynamical system is described using polar coordinates r, ϕ with $r \geq 0$, $0 \leq \phi < 2\pi$,
$$\dot{r} = r^3(a - br), \qquad \dot{\phi} = r - \phi. \tag{4}$$
   For which values of the parameters a and b does the system have a stable limit cycle?
   (a) $a > 0$, $b > 0$
   (b) $a > 0$, $b < 0$
   (c) $a < 0$, $b > 0$
   (d) $a < 0$, $b < 0$
   (e) None of the above

Assessed Problem Sheet 1: ECS

1) σ_8 is the rms amplitude of the mass fluctuations at cluster scales. From the scientific literature, find a measurement for σ_8. Cite the appropriate literature and state two limitations of the measurement. [4]

2) Consider two light pulses emitted at $t_e + \delta t_e$ and $t_o + \delta t_o$. Starting from the FRW metric derive the relation between the scale factor, $a(t)$ and redshift, z for a light pulse with emitted wavelength λ_e and observed wavelength λ_o. Assume the universe is flat and expansion is slower than the time difference between the emission of the pulses, i.e. $a(t_e) \sim a(t_e + \delta t_e)$. (Hint: $ds^2 = 0$ for light) [3]

3) Consider that the inflaton field ϕ has a density, ρ and pressure P given by

$$\rho_\phi = \dot{\phi}^2 + V(\phi) \tag{1}$$

$$P_\phi = \dot{\phi}^2 - V(\phi) \tag{2}$$

what condition will be necessary for the field to behave like the cosmological constant. [3]