Evolution of Cosmic Structure

Lecture 3 – Inflation and the CMB

Suhail Dhawan

UNIVERSITY OF BIRMINGHAM

s.dhawan@bham.ac.uk

Learning Outcomes

What is known about the initial fluctuations from which cosmic structures have developed?

- Review of essential background cosmology
- Inflation and the horizon
- Seed fluctuations in the Universe at $z \sim 1000$: evidence from the CMB
- Recombination and the origin of the CMB
- The cause of the CMB temperature fluctuations

Three problems of the Universe

1. The Flatness problem – The universe must have been very close to the critical density at early times.

$$|1-\Omega|_r \propto a^2 \propto t$$

$|\rho - \rho_{crit}| < 10^{-55}$ for the Universe to last 10^{10} years

2. The horizon problem – The universe seems more uniform than it naively should be.

As calculated, the horizon in the radiation era is $d_{hor} = 2ct$. At t = 0.35 Myr, that is $d_{hor} = 0.2$ Mpc, or 1 degree on the sky.

3. The monopole problem – Some particle physics theories predict that magnetic monopoles should exist but none have been found.

How can we fix these problems?

Inflation and the horizon

Recall the acceleration equation: $\frac{\ddot{a}}{a} = -\ \frac{4\,\pi\,G}{3}\ \rho\ (1 + 3w)$

So, if $\mathrm{w} < -\frac{1}{3}$, then the $\rho(1 + 3w)$is negative, and the acceleration $\ddot{a}$ is accelerating.
Recall that the vacuum energy density has $\mathrm{w} = -1$, so, in that case,

$$\frac{\ddot{a}}{a} = -\ \frac{4\,\pi\,G}{3}\ \rho\ (1 + 3w) = \ \frac{8\,\pi\,G}{3}\ \rho$$

This second-order ODE has the solution;

$$a(t) = a_0 e^{\sqrt{\Lambda} t} = \mathrm{a_0 e^{Ht}}$$

Where $\Lambda \equiv H = \ \frac{8\,\pi G}{3}\rho$.

This solves the flatness problem by driving Ω towards 1. It also solves the horizon problem because the parts of the universe we see were once in contact.

Inflation and the horizon

The inflation theory envisages a massive exponential expansion of the Universe, at t~10^{-36}-10^{-34}s, involving ~100 e-foldings. This enlarges the horizon size at this very early time from ~10^{-27}m to ~1 pc (see Ryden § 11.4 for details), which is ample to encompass our whole sky. Hence the whole of our visible universe was in causal contact at that time. There is no consensus on the cause of inflation.

But why don't the photons all go away? Inflation should drop the temperature from T = 10^{28} K before inflation to 10^{-15} K after.

At the end of the inflationary epoch, the vacuum energy is released by the decay of the "false vacuum", reheating the universe back to the high temperature at which inflation was initiated. With the decay of the high vacuum energy, the subsequent evolution will initially be radiation-dominated, and the proper radius of the horizon will continue to expand at speed $d(d_{\rm hor})/dt$ ~2c.

Inflation and the horizon

But why are there still density fluctuations?

Quantum fluctuations of the vacuum state will be blown up to macroscopic sizes by the inflationary episode, and these can form the seeds from which subsequent growth of structure develops. Topological defects (cosmic strings, domain walls etc) have also been proposed as originators of primordial fluctuations.

The structure we see in the Universe today was seeded on quantum scales by virtual particles popping in and out of existence.

Inflation and the horizon

In the inflationary scenario, the inflaton field ϕ which drives inflation has quantum fluctuations $\delta\phi$ around its classical trajectory. These fluctuations can be characterized by their variance given as:

$$\langle \delta\phi^2 \rangle \sim \left(\frac{H}{2\pi}\right)^2$$

Where H is the Hubble parameter.

These fluctuations in the inflaton field cause fluctuations in the spacetime metric. These perturbations ζ, called curvature perturbation, generates the density fluctuations. For scalar perturbation,

$$\zeta \approx \frac{H}{\dot{\phi}} \delta\phi$$

These perturbations grow as the Universe through inflation, where $a(t) \propto \mathrm{e}^{\mathrm{Ht}}$ leads to the the wavelength of the fluctuation growing

$$\lambda(t) = \lambda_0 \mathrm{e}^{\mathrm{Ht}}$$

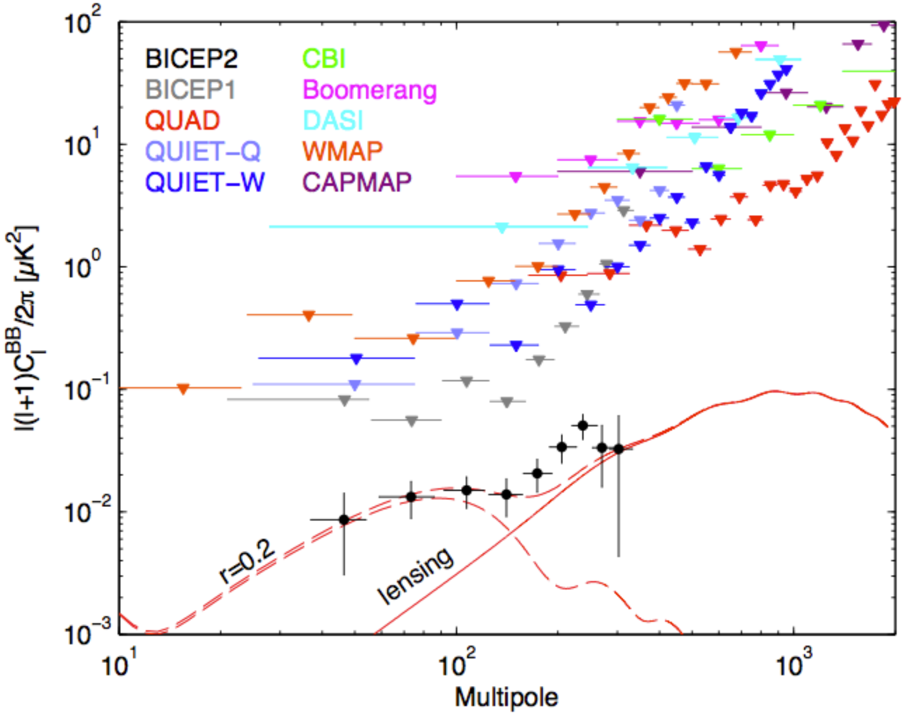
When the wavelength of the fluctuation reaches the horizon size, the fluctuation is said to freeze out.

Gravity waves and Inflation

Quantization of the gravitational field combined with the exponential expansion of inflation should produce a background of gravitational waves.

Though the waves themselves are much too weak to see directly, we may be able to find their imprint on the radiation CMB. The waves induce quadrupole anisotropies in the radiation field, which induce degree scale polarization. This can't be produced from density perturbations.

BICEP2 had claimed a detection of these gravitational wave imprints on the CMB.



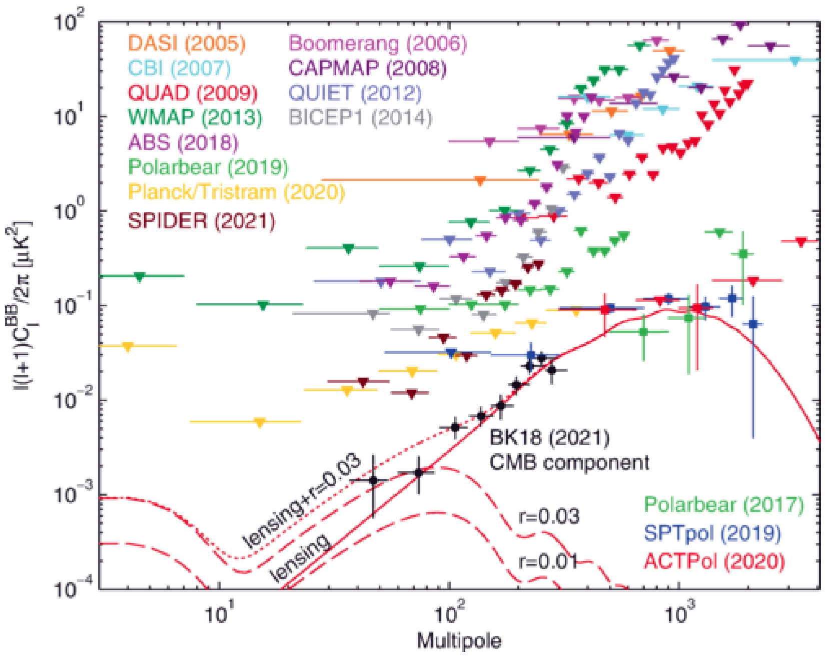
Dust!!

Gravity waves and Inflation

Quantization of the gravitational field combined with the exponential expansion of inflation should produce a background of gravitational waves.

Though the waves themselves are much too weak to see directly, we may be able to find their imprint on the radiation CMB. The waves induce quadrupole anisotropies in the radiation field, which induce degree scale polarization. This can't be produced from density perturbations.

BICEP2 had claimed a detection of these gravitational wave imprints on the CMB.



Dust!!

Recombination and the CMB

The origin of the CMB is discussed in some detail in Ryden § 9.

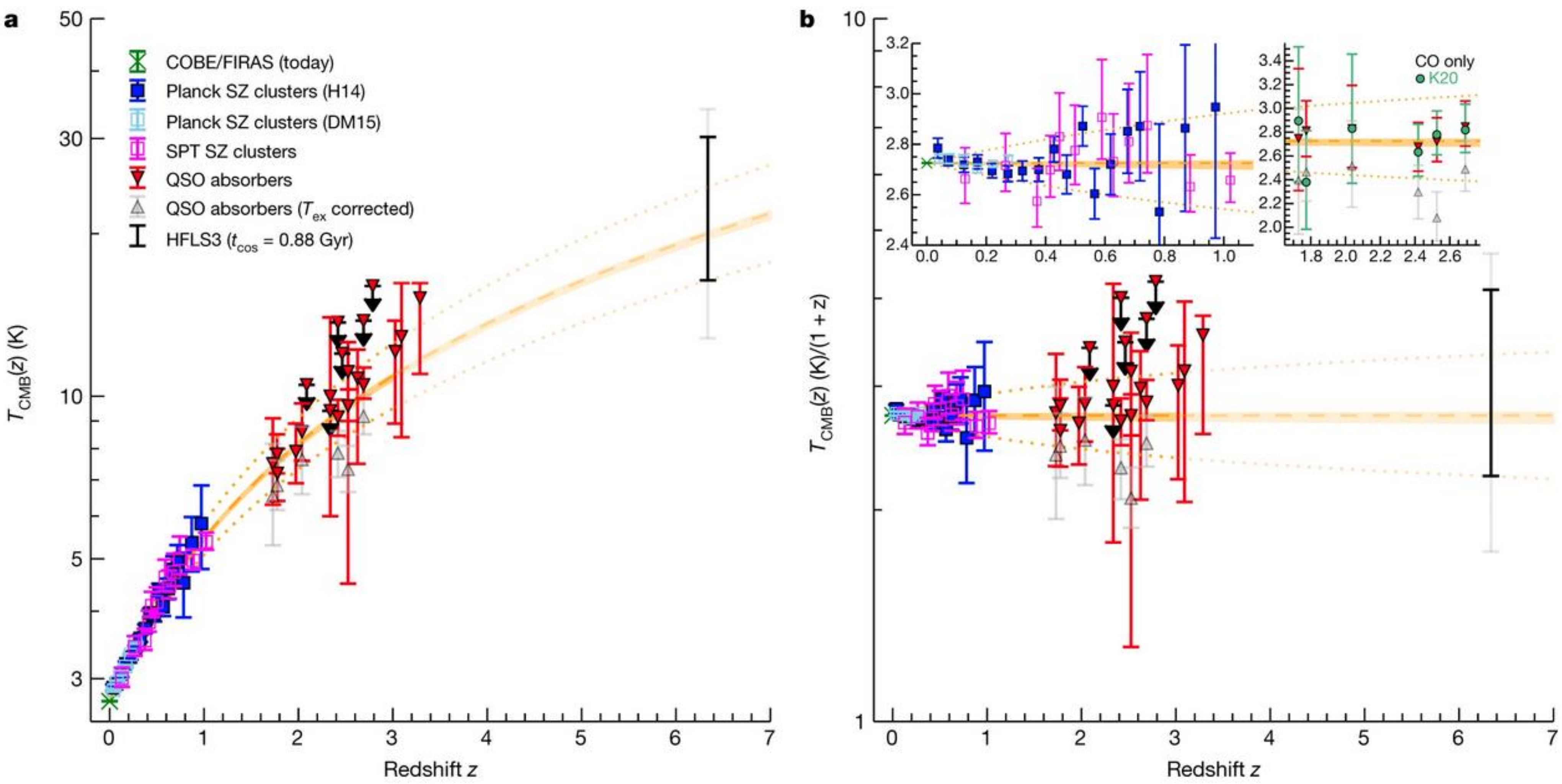
The early universe was ionized. And the high scattering cross-section of photons off free elections (σ_T=6.65 × 10^{-29}m^2) means they are tightly coupled.

As the Universe expands and cools, the radiation energy density drops as a^{-4}, whilst the energy density of matter drops only as a^{-3}. **Equality of the density of matter and radiation** is reached at T~9700 K (at t~50,000 yr), **recombination** follows at t~240,000 yr when T~3700 K. Thus, the baryons become neutral.

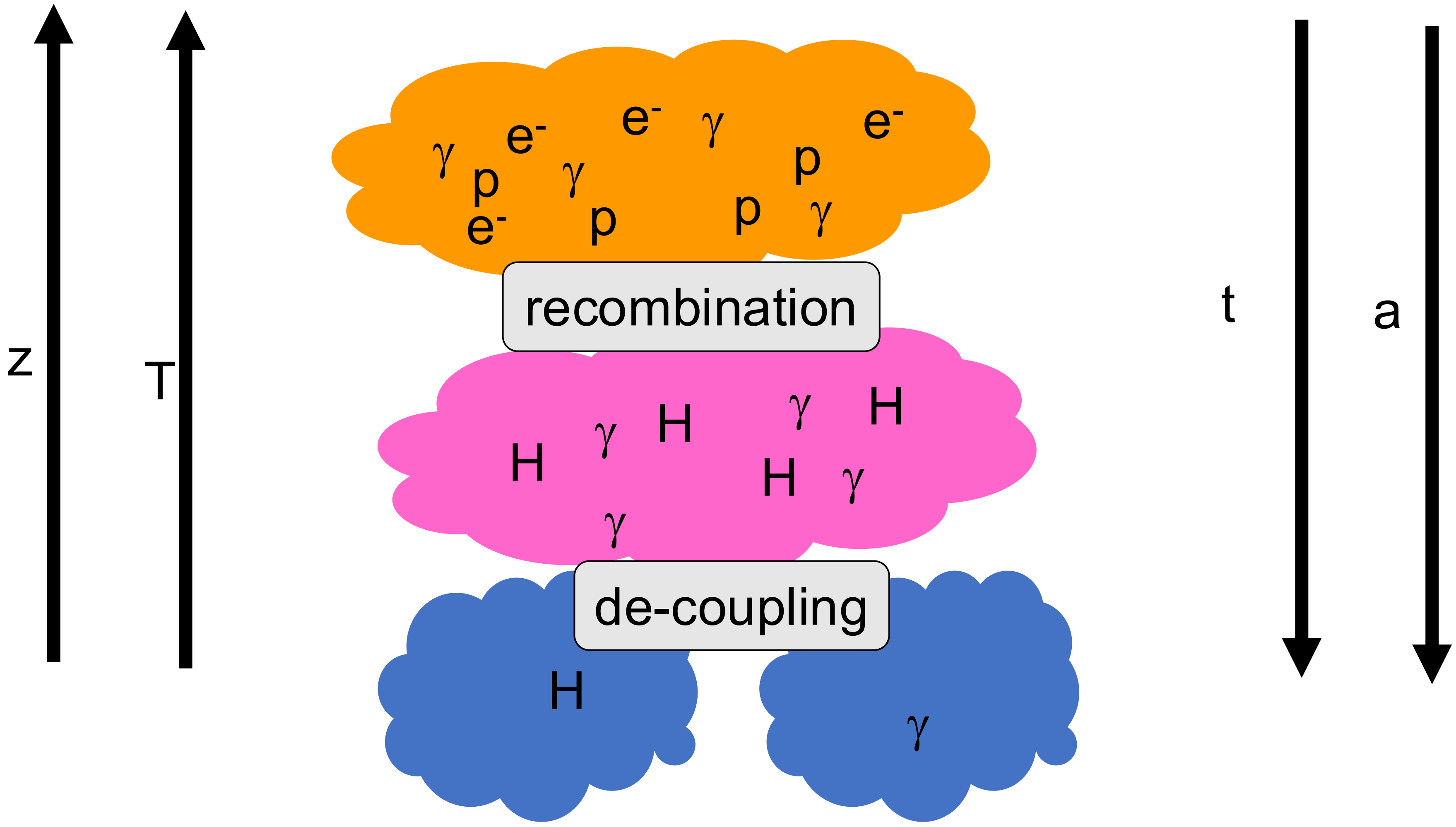
The scattering cross-section of photons off bound elections (away from line transitions) is much lower. So, following some further expansion (t~350,000 yr (z=1100) at T~3000 K) , the cross section has dropped enough for radiation to **decouple** from matter. (See Table 9.1 in Ryden.)

The CMB photons effectively travel freely from this **last-scattering surface**, with their blackbody spectrum cooling from 3000 K to 2.7 K as the universe expands by a factor of 1100.

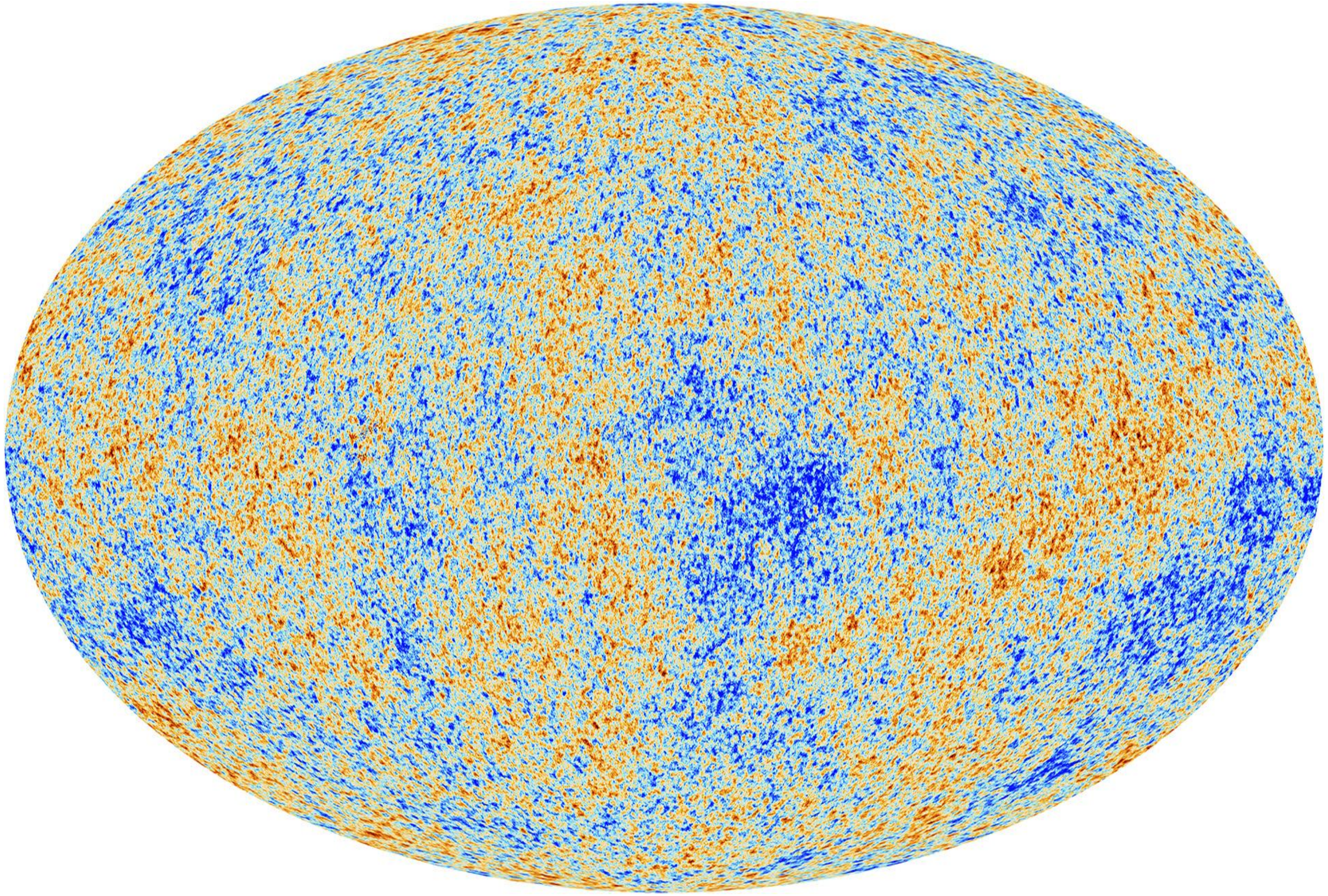
Time evolution of CMB



De-coupling and Recombination



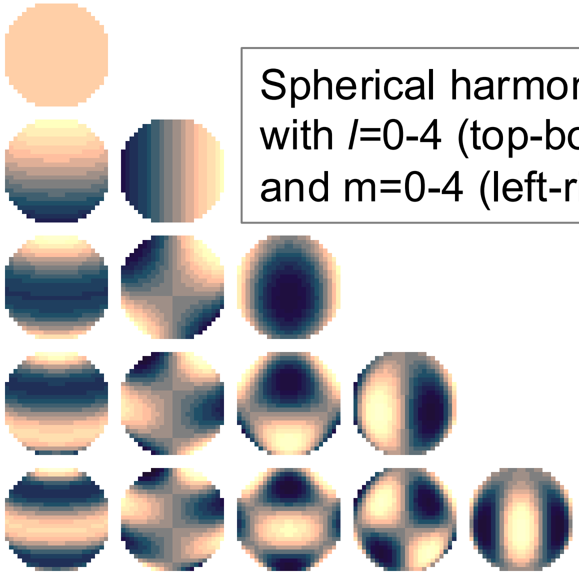
The CMB fluctuations



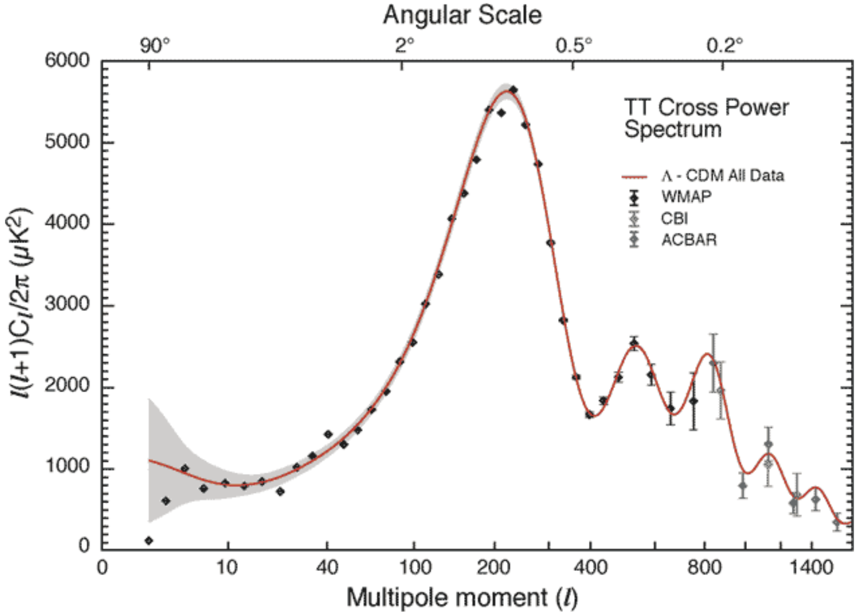
Planck
Year 1
data

CMB temperature fluctuations are present with a fractional amplitude of $\sim 10^{-5}$ as shown in these images from COBE and WMAP (1 year and 7 year data).

The CMB fluctuations



Spherical harmonics with l=0-4 (top-bottom) and m=0-4 (left-right).



The characteristics of the CMB fluctuations on the celestial sphere can be studied quantitatively by expanding them as a series of spherical harmonics

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} a_{lm} Y_l^m(\theta,\phi)$$

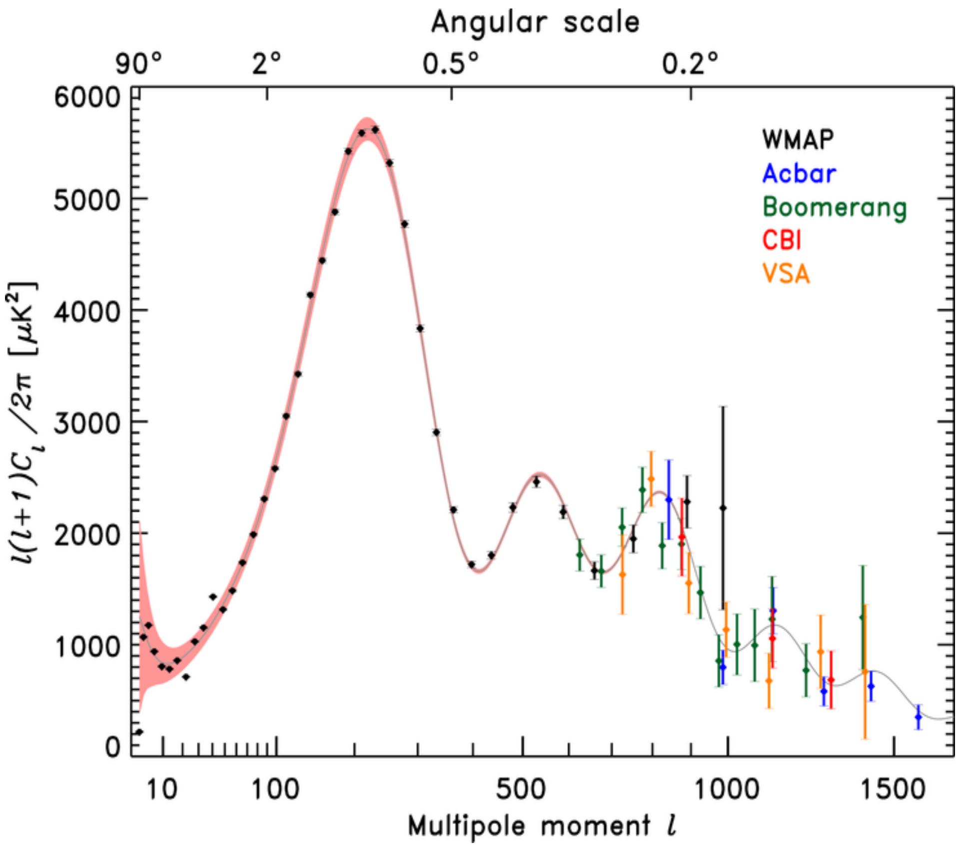
The spatial frequency of the fluctuations is then characterised by the value of l, and so the variation in fluctuation amplitude with spatial frequency is generally displayed as a sort of power spectrum.

Power spectrum of CMB fluctuations fitted with a ΛCDM model

The origin of the CMB fluctuations

The spatial frequency of the fluctuations shows complex structure, which can be understood in terms of oscillatory behaviour of the strongly coupled photon-baryon fluid as it falls into developing dark matter potential wells around the time of recombination.

Radiation escaping from a potential well of depth $\Delta\phi$ is redshifted, giving (from a GR treatment incorporating the effect of time dilation) a fractional temperature drop of $(\delta T/T)=(\Delta\phi/3c^2)$. This is known as the *Sachs-Wolfe* effect and dominates the fluctuations on scales greater than the horizon scale θ_H. This accounts for the trend in power at low (l<200) multipole moments.

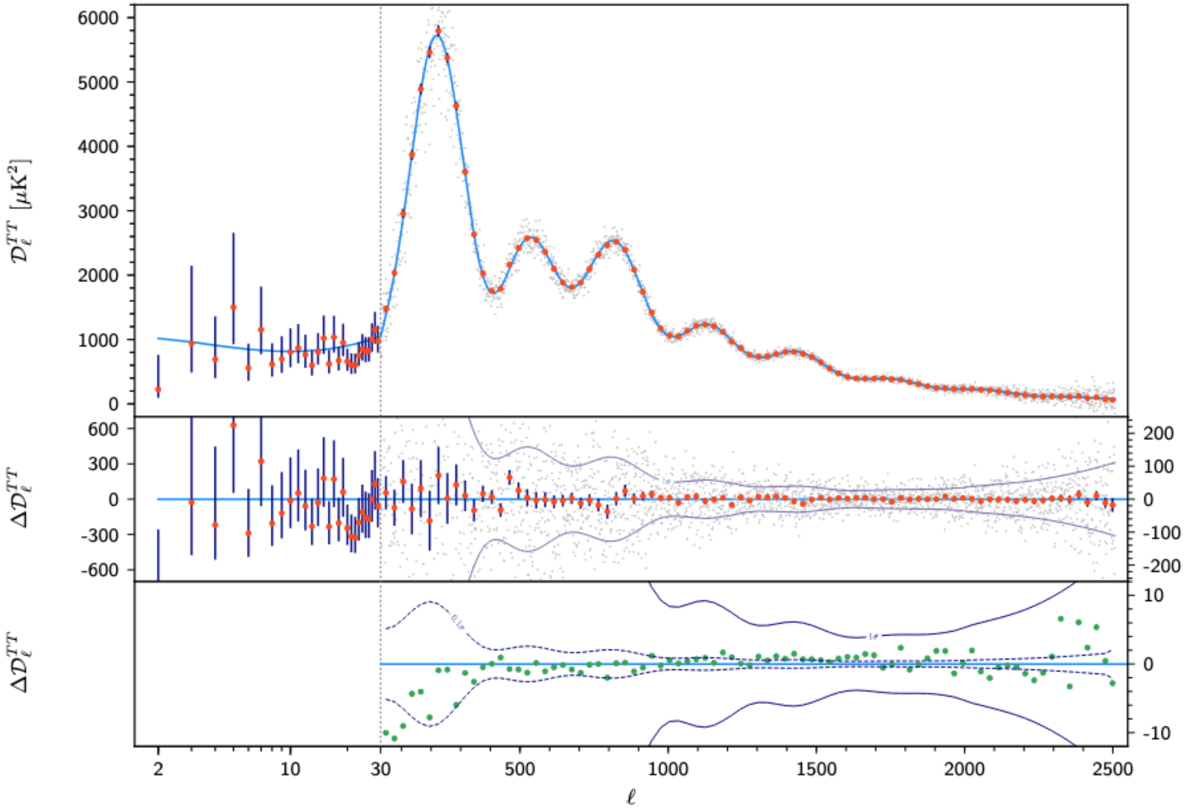


Power spectrum of CMB fluctuations from WMAP and a variety of ground-based microwave experiments – Hinshaw et al 2006

The origin of the CMB fluctuations

The spatial frequency of the fluctuations shows complex structure, which can be understood in terms of oscillatory behaviour of the strongly coupled photon-baryon fluid as it falls into developing dark matter potential wells around the time of recombination.

Radiation escaping from a potential well of depth $\Delta\phi$ is redshifted, giving (from a GR treatment incorporating the effect of time dilation) a fractional temperature drop of $(\delta T/T)=(\Delta\phi/3c^2)$. This is known as the *Sachs-Wolfe* effect and dominates the fluctuations on scales greater than the horizon scale θ_H. This accounts for the trend in power at low (l<200) multipole moments.

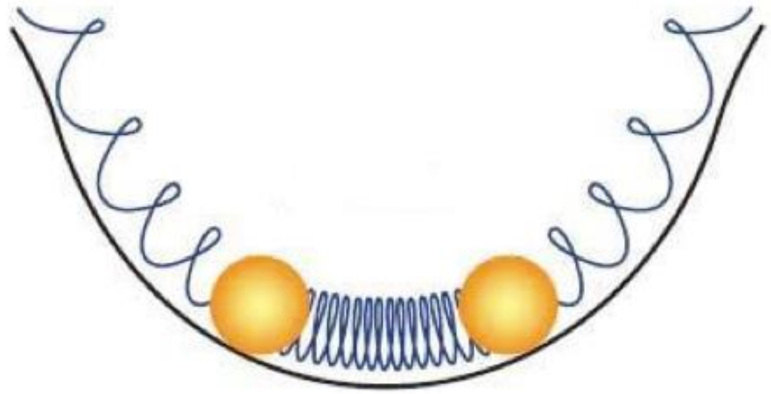


Power spectrum of CMB fluctuations from Planck 2018

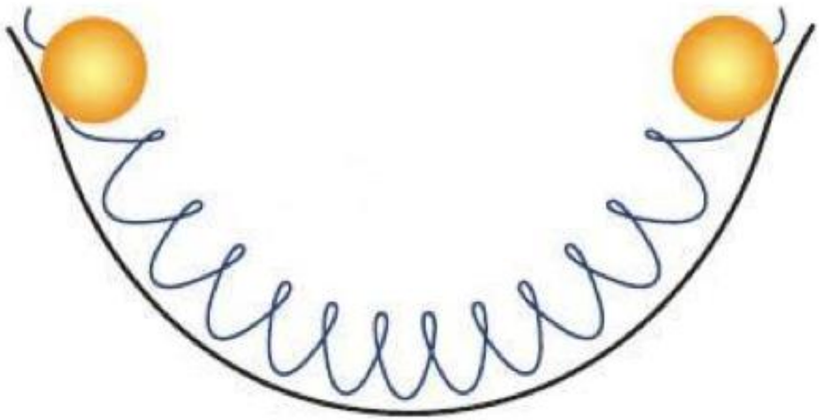
The origin of the CMB fluctuations

As was calculated in ObsCos, the physical radius of the horizon at decoupling is d_{hor}≈0.2 Mpc. Its angular size on the sky is therefore $\theta_H = d_{hor}/d_A$. The angular diameter distance to the surface of last scattering at z~1100 is only about 13 Mpc. Hence,

$$\theta_H \approx 0.2\text{Mpc}/13\text{Mpc} = 0.015\ \text{rad} \approx 1^\circ.$$

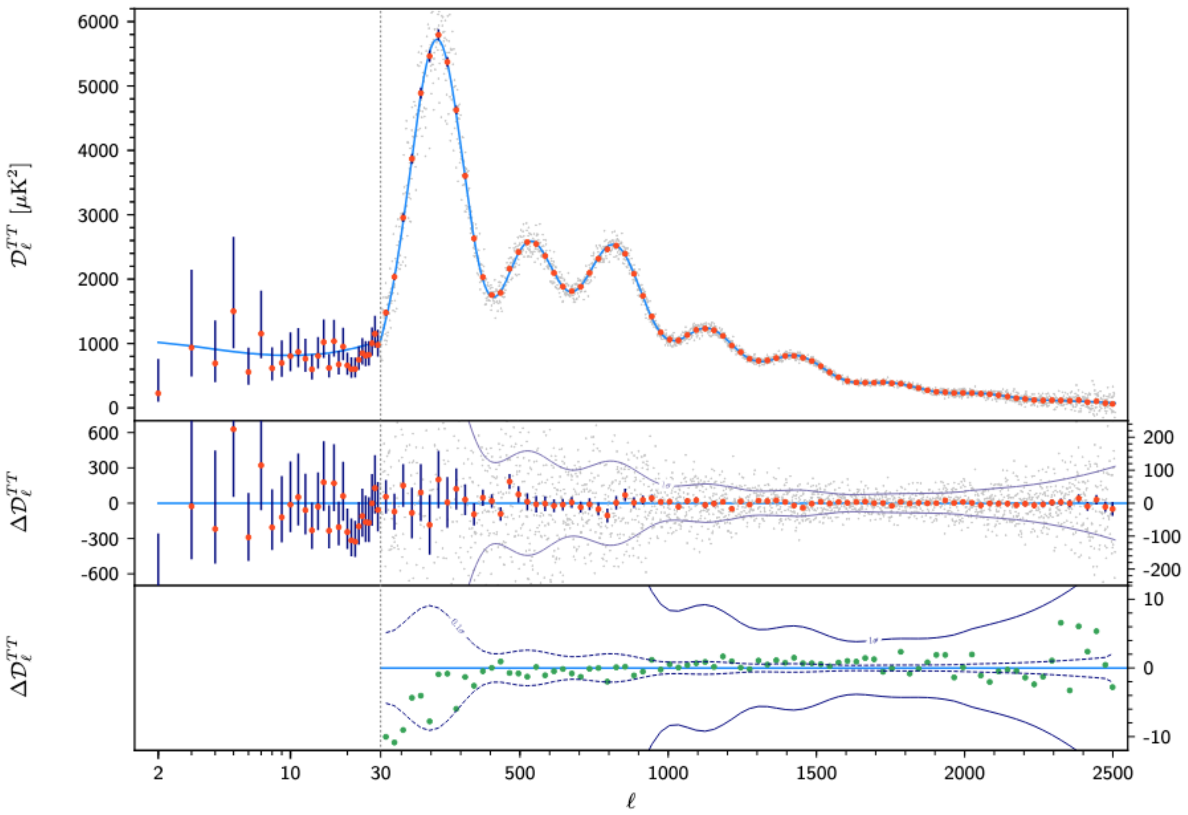


For fluctuations smaller than the horizon size (i.e. those on scales <1°) the pressure of the photon-baryon medium comes into play. Fluid falling into potential wells will be compressed, raising its density and temperature, and hence its pressure. It will then "bounce" and re-expand. As the horizon grows, longer wavelength modes are able to collapse.



The origin of the CMB fluctuations

The primary peak in the power spectrum is caused by modes which have just had time to collapse to maximum compression in the time between entering the horizon and decoupling. Perturbations on somewhat smaller scales will have had time to rebound, and so will not be so hot. The second peak corresponds to fluctuations on sizes small enough to allow a full in-out oscillation (generating a cool spot), the third peak to in-out-in, and so on.



Power spectrum of CMB fluctuations from WMAP and a variety of ground-based microwave experiments – Hinshaw et al 2006