General Physics – solutions to thermal problems

1 (a). Isothermal:
$PV = nRT$ for ideal gas, where $n = 1$ mole and T is constant (isothermal). For $V \to V/2$,
(i) $P \to 2P$;
(ii) Internal energy of an ideal gas depends only on the temperature, so change is 0;
(iii) Work done on gas during compression $W = \int_V^{V/2} P\,dV = nRT \ln \frac{1}{2}$. But the internal energy U does not change (see above), so by the first law, the gas transfers heat $Q = W$ to the bath that is maintaining it at constant temperature. The change in entropy of the gas is therefore $\frac{\Delta Q}{T} = -R \ln \frac{1}{2}$.
(iv) Isothermal means the temperature does not change.

1 (b). Isentropic/Adiabatic:
(i) Ideal gas law is still true, but system is isolated so T can vary. $PV^\gamma =$ constant, where $\gamma = 5/3$ for a monatomic gas ($C_V = \frac{3}{2}R$; $C_P = C_V + R$; $\gamma = C_P/C_V$). $P_2 = 2^{5/3}P_1$
(ii) $\Delta U = C_V \Delta T$ is always true for an ideal gas. $C_V = \frac{3}{2}R$, and $\Delta T = T(2^{2/3} - 1)$, from part (iv). Thus $\Delta U = \frac{3}{2}R(2^{2/3} - 1)T$.
(iii) $\Delta S = 0$ by definition.
(iv) $TV^{(\gamma-1)}$ is constant. Thus $T_2 = 2^{\gamma-1}T_1$; $\Delta T = T(2^{2/3} - 1)$.

2. Just like an electrical problem:
(Current/heat) flow = (potential/temperature) difference / (electrical/thermal) resistance. Thermal resistance $R_{\text{th}} = \frac{1}{\sigma}\frac{L}{A}$ (again, just like the electrical case). So heat flow, $\dot{Q} = \Delta T/R_{\text{th}} = \pi$ watts.

3. Heat lost, $C(T_h - T_f) =$ heat gained, $C(T_f - T_c)$, so final temperature $T_f = \frac{T_h + T_c}{2}$.
$\Delta S_h = \int_{T_h}^{T_f} \frac{C}{T} dT = C \ln T_f/T_h$, likewise $\Delta S_c = C \ln T_f/T_c$, so $\Delta S = C \ln \frac{T_f^2}{T_c T_h} = C \ln[(T_c + T_h)^2/4T_cT_h]$, which is zero if $T_c = T_h$, and positive for different positive temperatures.

4. Number of microstates consistent with 3ϵ is ${}^{10}C_3 = \frac{10!}{7!\,3!} = 120$. $S = k_B \ln 120$. Maximum value is when half the atoms have ϵ, so 5.

5. Energy levels are $(n + \frac{1}{2})\hbar\omega$, where $n = 0$ is the ground state and $n = 3$ is third excited state, so $\Delta E = 3\hbar\omega$. For harmonic oscillator, $\omega = \sqrt{\frac{k_s}{m}}$. The probability ratio is given by the Boltzman factor, $\exp -\frac{3\hbar\sqrt{\frac{k_s}{m}}}{k_B T}$.

6. For a reversible engine, $Q_h/T_h = Q_c/T_c$. Temperatures are constant, so Q can represnt quantity of heat or quantity of heat per unit time (rate of heat flow). The work done on the engine, or electrical power, all exits as heat at the hot end, so $Q_h = W + Q_c = 100\,\text{kW}$. Solving these equations gives $W = 100\,\text{kW} \times (1 - \frac{283}{293}) = 3.4\,\text{kW}$.

7. $\epsilon\sigma(T_1^4 - T_2^4)$.

8. Moment of inertia I on a torsional support k_s is analogous to a mass on a spring. $\omega = \sqrt{\frac{k_s}{I}} = \sqrt{\frac{2k_s}{mr^2}}$. Equipartition applies to both the potential and kinetic energies of this oscillatory mode, so that in thermal equilibrium at T, considering only the kinetic energy term, $\langle \frac{1}{2} I \omega^2 \rangle = \frac{1}{2} k_B T$, so $\langle \omega \rangle = \sqrt{\frac{2k_B T}{mr^2}}$.