Evolution of Cosmic Structure

Lecture 6: Cosmology from Galaxy Clustering and Weak Lensing

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Gravitational instability in expanding medium

Matter-dominated

$$\delta(t) = At^{2/3} \propto a(t) \propto \frac{1}{1+z}$$

Linear growth

Radiation-dominated

$$\delta(t) = B_1 + B_2 \ln(t)$$

Logarithmic growth

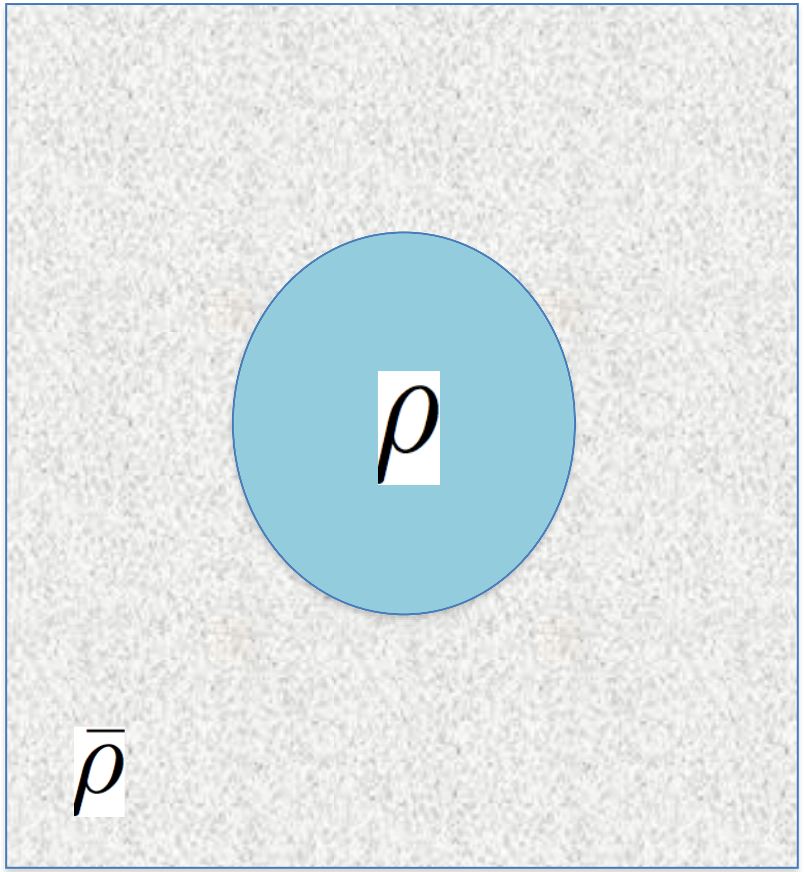
Cosmological-constant dominated

$$\delta(t) = C_1 + C_2 e^{-2H_\Lambda t}$$

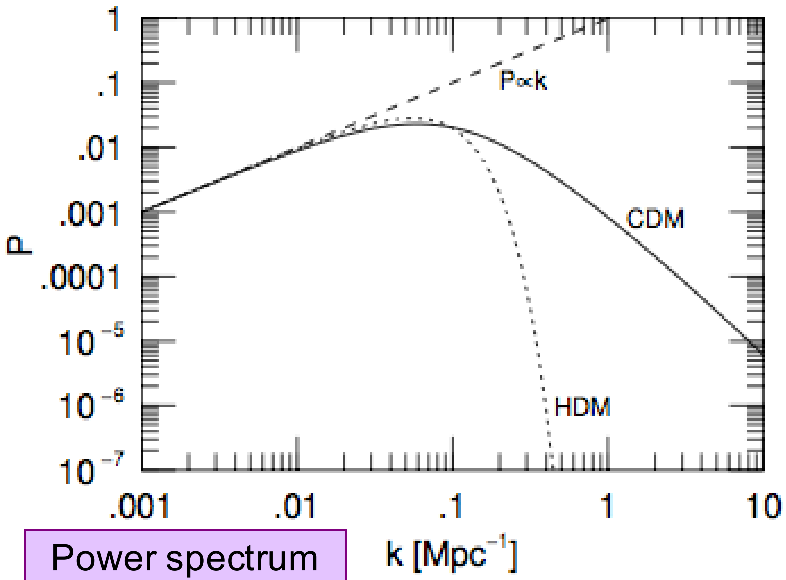
$$H_\Lambda = \text{constant}$$

No growth!

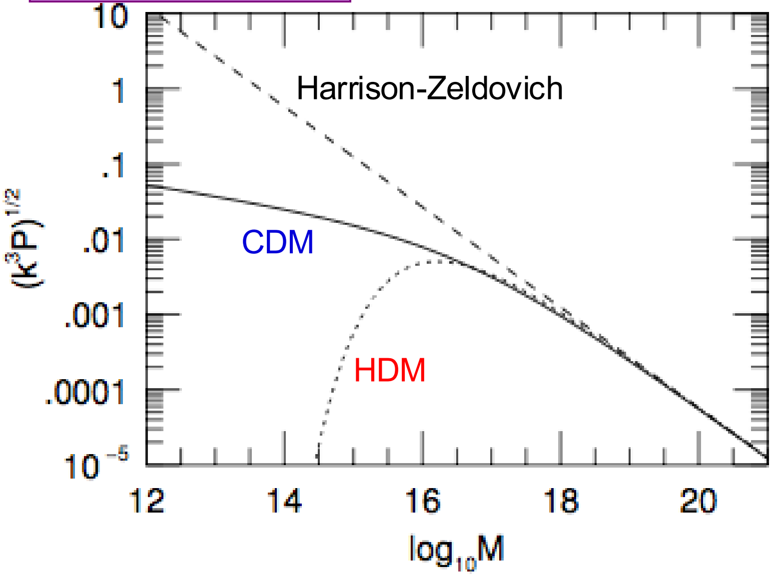
Matter phase dominates the growth of structure



Evolution of the power spectrum



Power spectrum and $\Delta M/M$ at t_{rm}



Recalling that the rms fractional mass fluctuations scale as $\frac{\Delta M}{M} \propto \sqrt{k^3 P(k)}$ we see in the plot below that the difference between CDM and HDM is fundamental. In CDM fluctuations on smaller mass scales have progressively larger fractional amplitudes, whilst for HDM there is a maximum in $\Delta M/M$ at a mass ~10^{16} $M_{\odot}$.

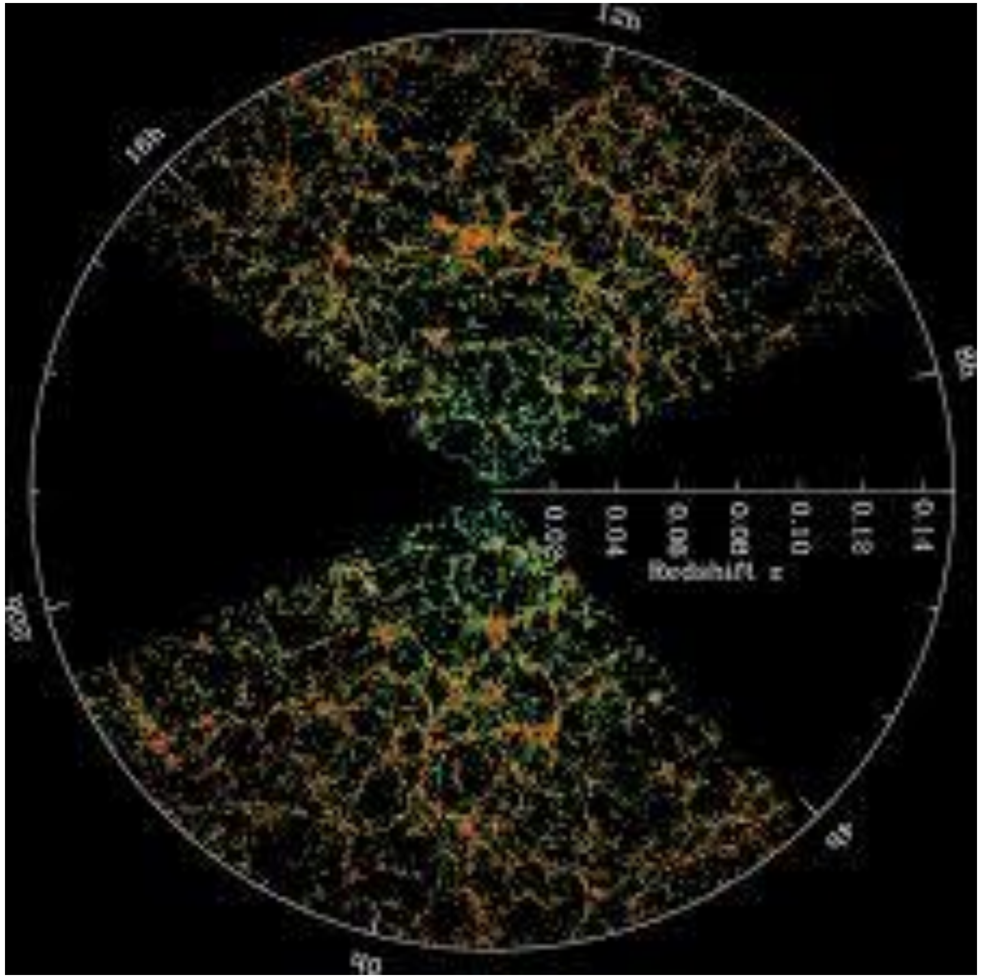
Note that in the matter-dominated era, which starts at t_{rm}, fluctuations both smaller and larger than the horizon grow with $\delta \propto a$, so that the power simply rises across the whole spectrum, and there is no change in shape. This means that the first structures to reach δ~1, after which they start to turn around and collapse, will be small for CDM, but ~10^{16} M for HDM. Small galaxies seen at high z therefore show that most of the DM must be cold.

§ 2 – Cosmological inference and non-linear growth

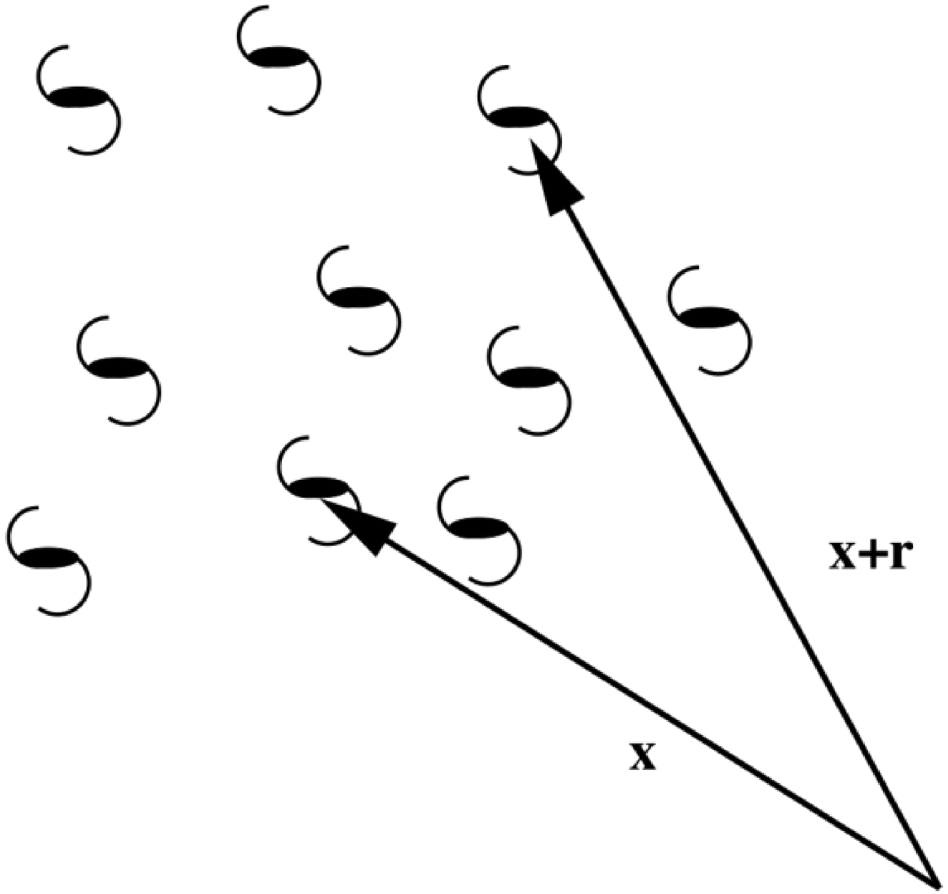
What is the linear growth function? How do we trace the matter power spectrum with observations? What is weak lensing?

- Linear growth function and the power spectrum
- Galaxy clustering: biased tracers of matter power spectrum
- Weak lensing
  - Measurement of S8

Galaxy Clustering as a cosmological probe

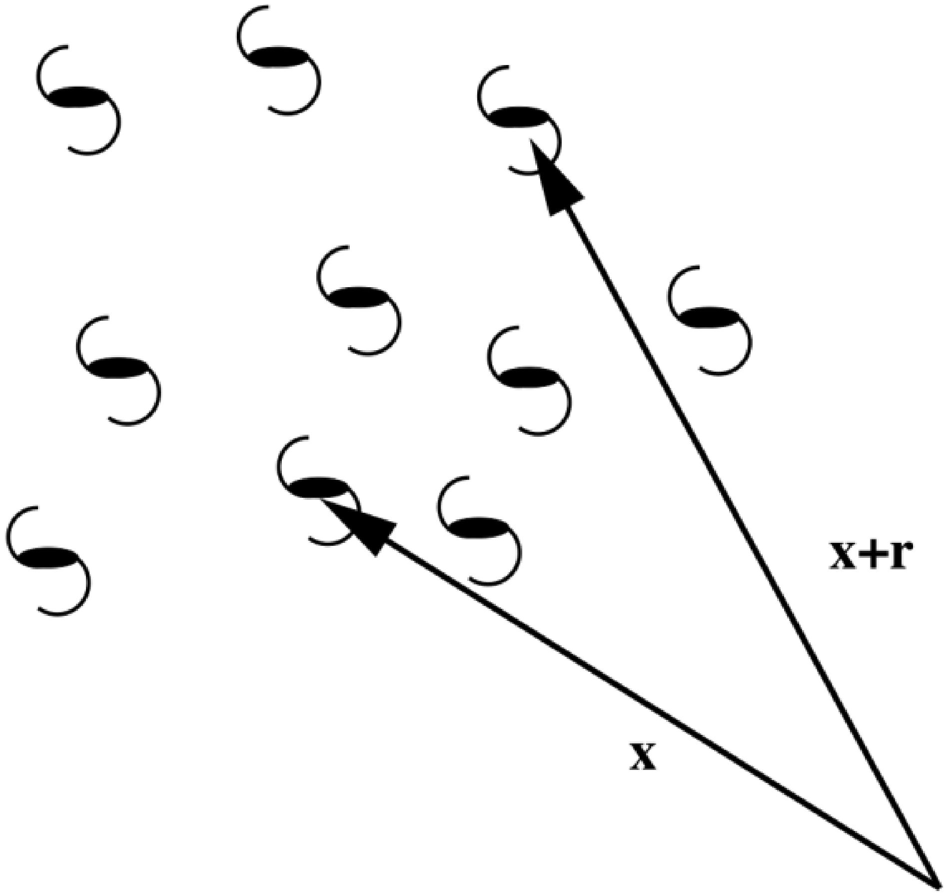


Correlation Function



A commonly used statistic for quantifying clustering is the second order of moment of the field, known as the auto-correlation function or correlation function

Correlation Function



$$\xi(\mathbf{r}) \equiv \langle \delta(\mathbf{x}) \cdot \delta(\mathbf{x}+\mathbf{r}) \rangle$$

Compared to dP = ndV

$$dP = n[1 + \xi(\mathbf{r})]dV$$

The correlation function is the Fourier transform of the matter power spectrum.

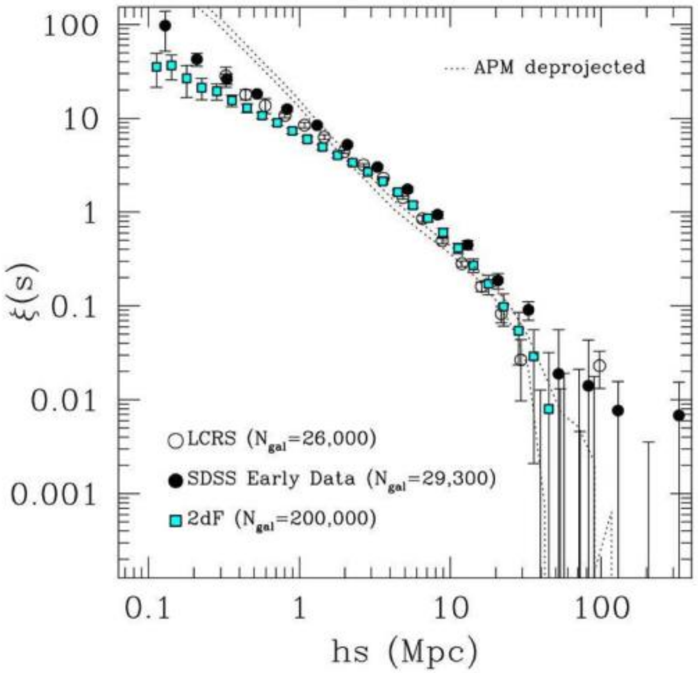
Cosmology with the Correlation Function

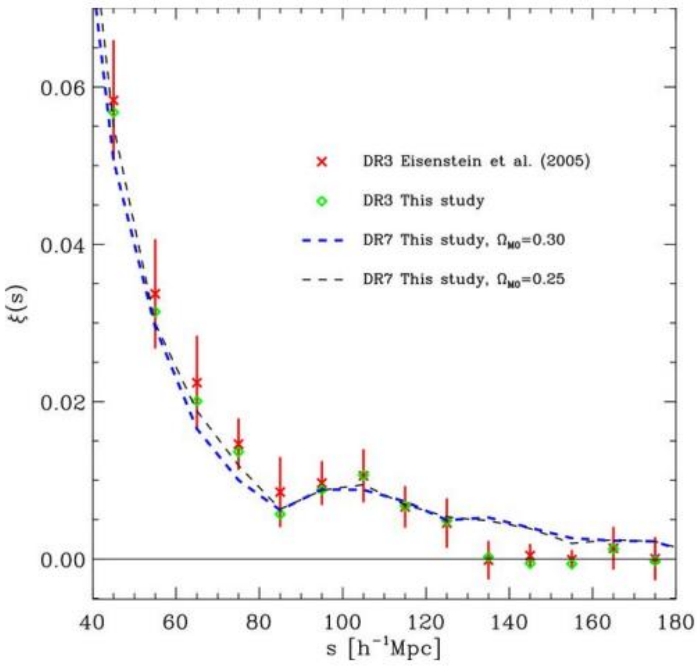
The time of equality depends on how much matter there is:
•More matter → equality happens earlier
•Less matter → equality happens later

Earlier equality:
•more time for growth
•less suppression of small-scale modes

Later equality:
•less time for growth
•stronger suppression

Thus the **shape** of clustering today tells us *when* equality happened → which tells us Ω_m.





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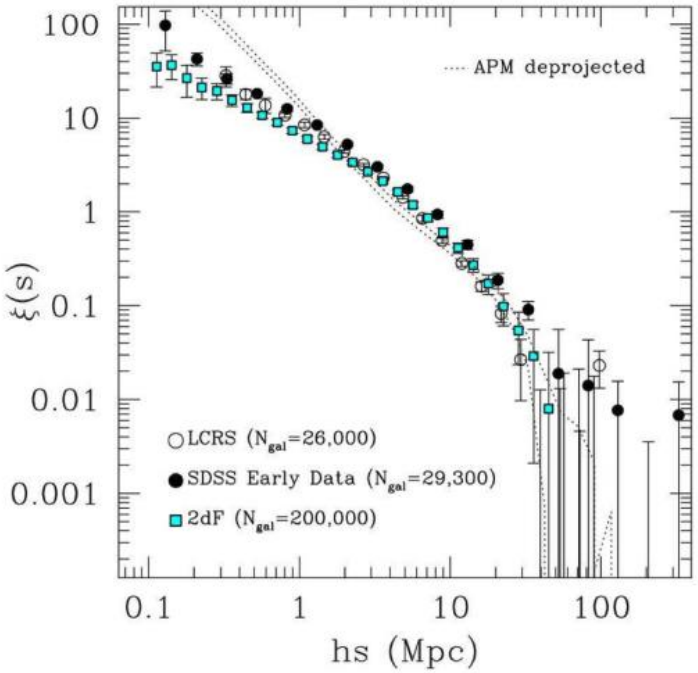

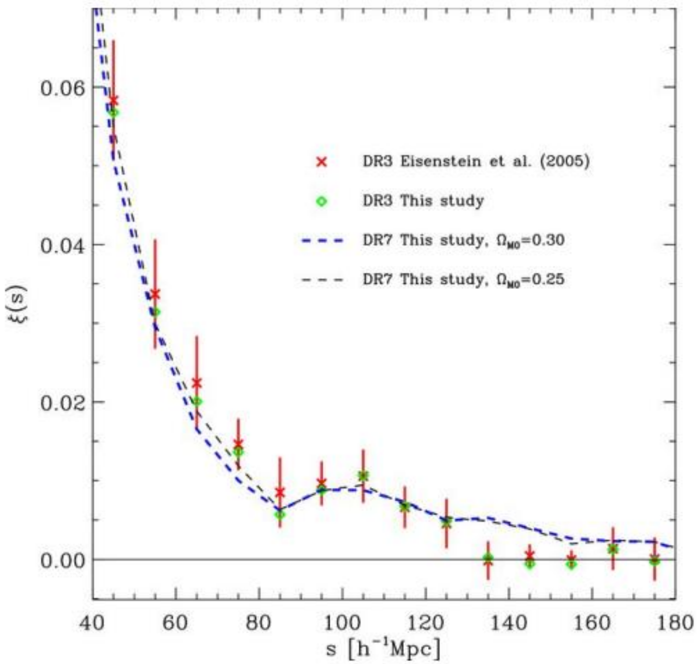

Galaxy Clustering as a cosmological probe

1.Traditionally, cosmological constraints from galaxy clustering have been quoted in terms of constraints on the rms amplitude of mass fluctuations σ(z, R)

$$\sigma^2(R) = \int \frac{d^3k}{(2\pi)^3} P_{\mathrm{lin}}(k) \left|W(kR)\right|^2$$

Where P_{lin} is the dimensionless power spectrum, W is the window function and k is the wavenumber.

Conventionally, σ is measured at z = 0 and R = 8 h^{-1} Mpc, hence, the parameter of interest is σ_8. 8h^{-1} corresponds to the size of massive clusters / superclusters.

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$$\sigma^2(R) = \int \frac{d^3k}{(2\pi)^3} P_{\rm lin}(k)\, |W(kR)|^2$$

So the **final amplitude** today depends on:
- initial conditions (A_s),
- expansion history,
- gravity,
- neutrinos,
- dark energy.

It also governs the growth of structure, e.g.
Via cluster abundance, galaxy clustering, weak lensing

Caveats in galaxy clustering inference

- Galaxy clustering traces the galaxy overdensity and not directly the matter density. The two are related as δm = bg δ_g. The relationship between the galaxy power spectrum
and the matter power spectrum is consequently

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$$P_{gg}(k,z) = b^2(k,z)P(k,z),$$

Caveats in galaxy clustering inference

δm = bg δ_g.

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-

Why are galaxies biased tracers?

“Because they are only found in collapsed halos”

Galaxy bias

When an overdensity, δm, grows to be above a certain critical density, its collapses to form a gravitationally bound structure, known as a **halo**.

Details of the theory to follow

Dark matter exists everywhere.
Galaxies form only where matter collapses into **bound halos**, and only some halos host observable galaxies.
This *selection* immediately produces bias.

If you randomly sampled matter particles, you would get an unbiased tracer.
If you select **only collapsed objects**, you get a biased tracer.
Everything that follows is a consequence of this fact.

Galaxy bias

The change in galaxy bias can mimic the effect of growth of structure, making it a fundamental degeneracy in deriving cosmology from galaxy clustering data

Further, galaxy bias depends on the galaxy type and on the galaxy formation history (often termed “assembly bias”), and thus typically needs to be measured directly from the data. Even on linear scales, where galaxy bias is expected to be scale-independent (i.e., constant in wavenumber k), its time dependence is a priori unknown.

•Dense environments quench star formation → red galaxies are more clustered.

On halo scales (kpc–Mpc), baryonic processes modify the mapping:
gas cooling,
star formation efficiency,
AGN/SN feedback,
mergers.

Caveats in galaxy clustering inference

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$$P_{gg}(k,z) = b^2(k,z)P(k,z),$$

Another complication is that typical clustering measurements are made partly in the quasi-linear and non-linear regime, corresponding roughly to scales r<10 h-1 Mpc (or k>0.1 h-1 Mpc1) at z = 0. Non-linear corrections to the matter power spectrum also depend on scale, while additional scale dependence in galaxy clustering is brought about by baryonic effects, as well as the dependence of galaxy bias b(k, z) on scale. The resulting effects on the galaxy power spectrum are not theoretically tractable and must be calibrated with N-body simulations— ideally hydrodynamical simulations which contain both the baryon and the dark matter particles.

Ways to break the degeneracy

- Since Pm is directly proportionate to s8, increasing the bias can directly be compensated for with a decrease in s8, leaving the power spectrum unchanged

- There are a couple of routes taken to break this degeneracy between the galaxy bias and cosmology

- The first includes probing higher order statistiscs, e.g. the bispectrum, which is the Fourier transform of the three-point correlation function. The galaxy bispectrum has a different dependence on the bias parameter, the combination of (Pg, Bg) breaks the degeneracy between (b1, s8)

- The second involves combining galaxy clustering with a different physical probe, e.g. weak lensing or galaxy-shear cross correlation. The reason for this approach being powerful is that weak lensing probes the cosmic shear power spectrum which is not sensitive to the galaxy bias

Ways to break the degeneracy: Higher Order Statistics

Looking at correlations between > 2 points on the sky, e.g. 3-point correlation function. The Fourier transform gives us the "bi-spectrum".

This statistic has a different dependence on the bias compared to the power spectrum, hence, some of the degeneracy between the bias and cosmology can be broken

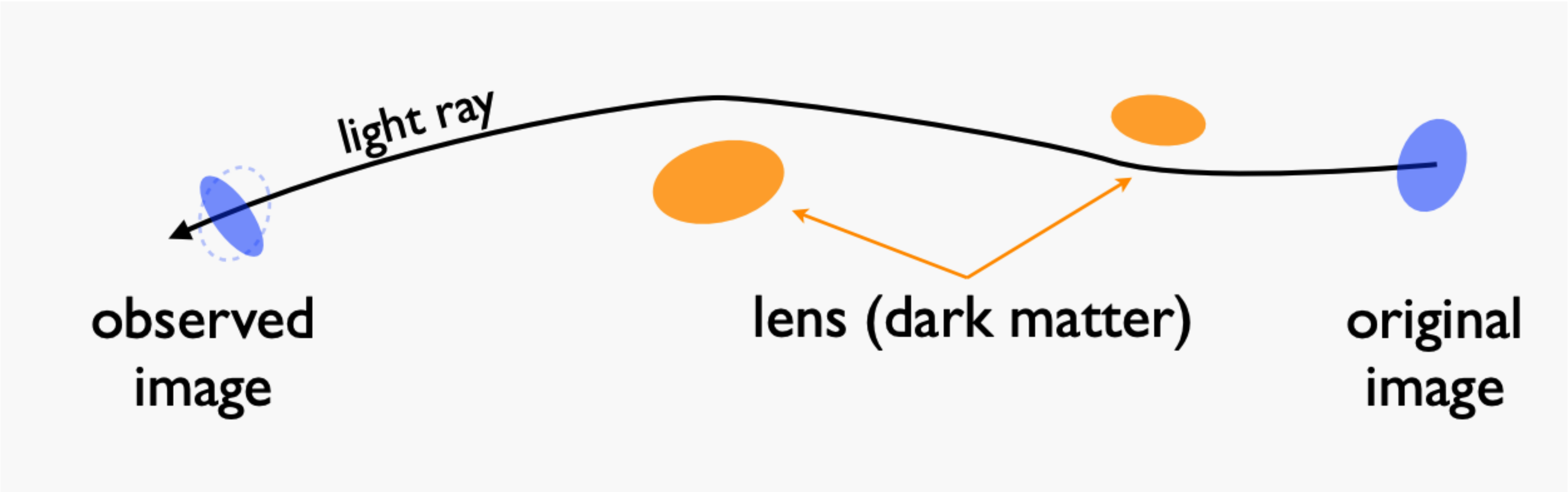
Bispectrum tells us "which triangular configurations are preferred"

$$\delta_g = b_1 \delta + \tfrac{1}{2} b_2 \delta^2 + \cdots$$

$$B_g = b_1^3 \, B_m \; + \; b_1^2 b_2 \left[P_m(k_1) P_m(k_2) + \mathrm{cyc.} \right] \; + \; \cdots$$

Statistic	Scaling
Galaxy power P_g	$(b_1 \sigma_8)^2$
Galaxy bispectrum B_g	$b_1^3 \sigma_8^4$

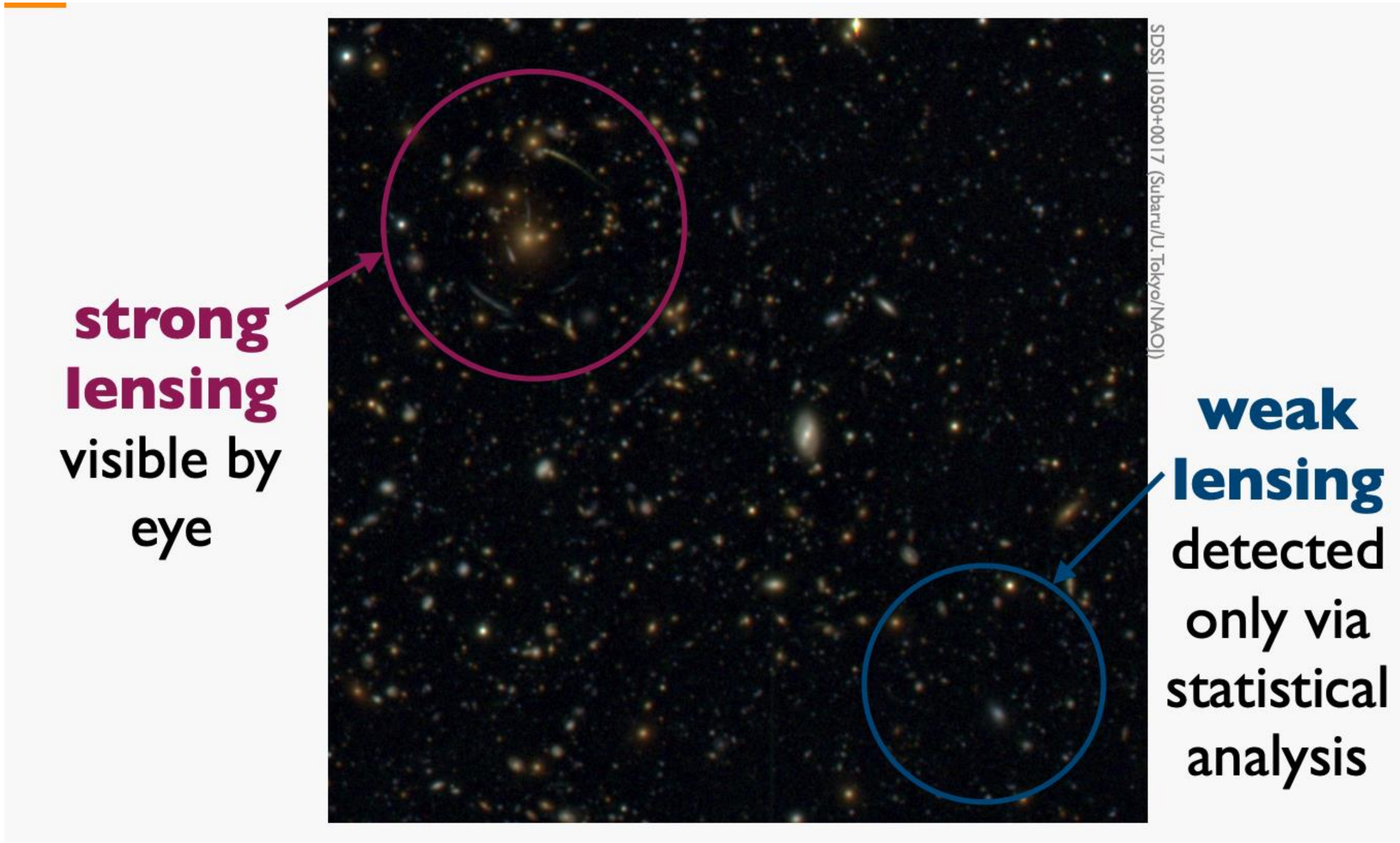
Gravitational Lensing and cosmology



Lensing contains a wealth of information about the matter content of the universe (amongst other important parameters)

Credit: M. Oguri

Gravitational Lensing and cosmology



Lensing occurs in different regimes. Here strong lensing is a rare phenomenon, seen as the arcs of the lensed background source. Weak Lensing is an imprint of the dark matter on the shapes of galaxies

Quantities of interest in Lensing

$$A \equiv \frac{\partial \vec{y}}{\partial \vec{x}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{x})}{\partial x_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \Psi(\vec{x})}{\partial x_i \partial x_j} \right)$$

$$\frac{\partial^2 \Psi(\vec{x})}{\partial x_i \partial x_j} \equiv \Psi_{ij}$$

$$\begin{aligned}
\left(A - \frac{1}{2}\mathrm{tr}A \cdot I \right)_{ij} &= \delta_{ij} - \Psi_{ij} - \frac{1}{2}(1 - \Psi_{11} + 1 - \Psi_{22})\delta_{ij} \\
&= -\Psi_{ij} + \frac{1}{2}(\Psi_{11} + \Psi_{22})\delta_{ij} \\
&= \begin{pmatrix} -\frac{1}{2}(\Psi_{11} - \Psi_{22}) & -\Psi_{12} \\ -\Psi_{12} & \frac{1}{2}(\Psi_{11} - \Psi_{22}) \end{pmatrix}
\end{aligned}$$

Quantities of interest in Lensing

$$
\begin{aligned}
\frac{1}{2}\mathrm{tr}A &= \left[1 - \frac{1}{2}(\Psi_{11} + \Psi_{22})\right]\delta_{ij} \\
&= \left(1 - \frac{1}{2}\triangle\Psi\right)\delta_{ij} = (1-\kappa)\delta_{ij}
\end{aligned}
$$

$$
\begin{aligned}
A &= \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \\
&= (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}
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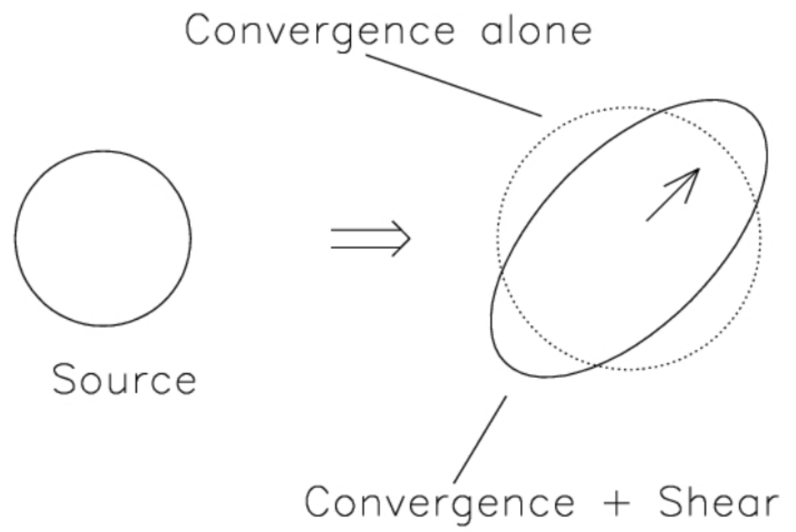
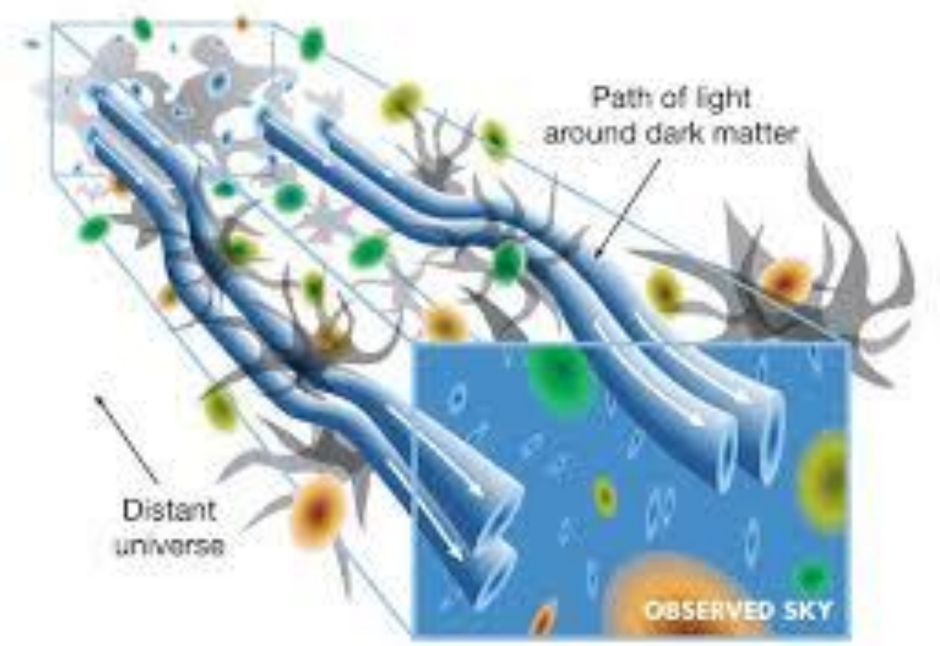


Ways to break the degeneracy: Weak Lensing

Weak gravitational lensing—subtle distortions in galaxy shapes due to the intervening large-scale structure—is a very powerful probe of the growth of cosmic structure. This method, proposed in the 1960 s and first detected in the year 2000, is now a standard-bearer for the probes of large-scale structure. Weak lensing also goes under the name cosmic shear as one statistically measures the amount of shearing of the observed shape of galaxies caused by photon deflections, which in turn informs about the projected mass along the line of sight

The principal feature of cosmic shear is the absence of galaxy bias. Even though measuring galaxy shapes (for cosmic shear) is more challenging than measuring galaxy positions (for galaxy clustering), this absence of possible degeneracies between the bias and the growth of structure makes cosmic shear a premier cosmological probe.



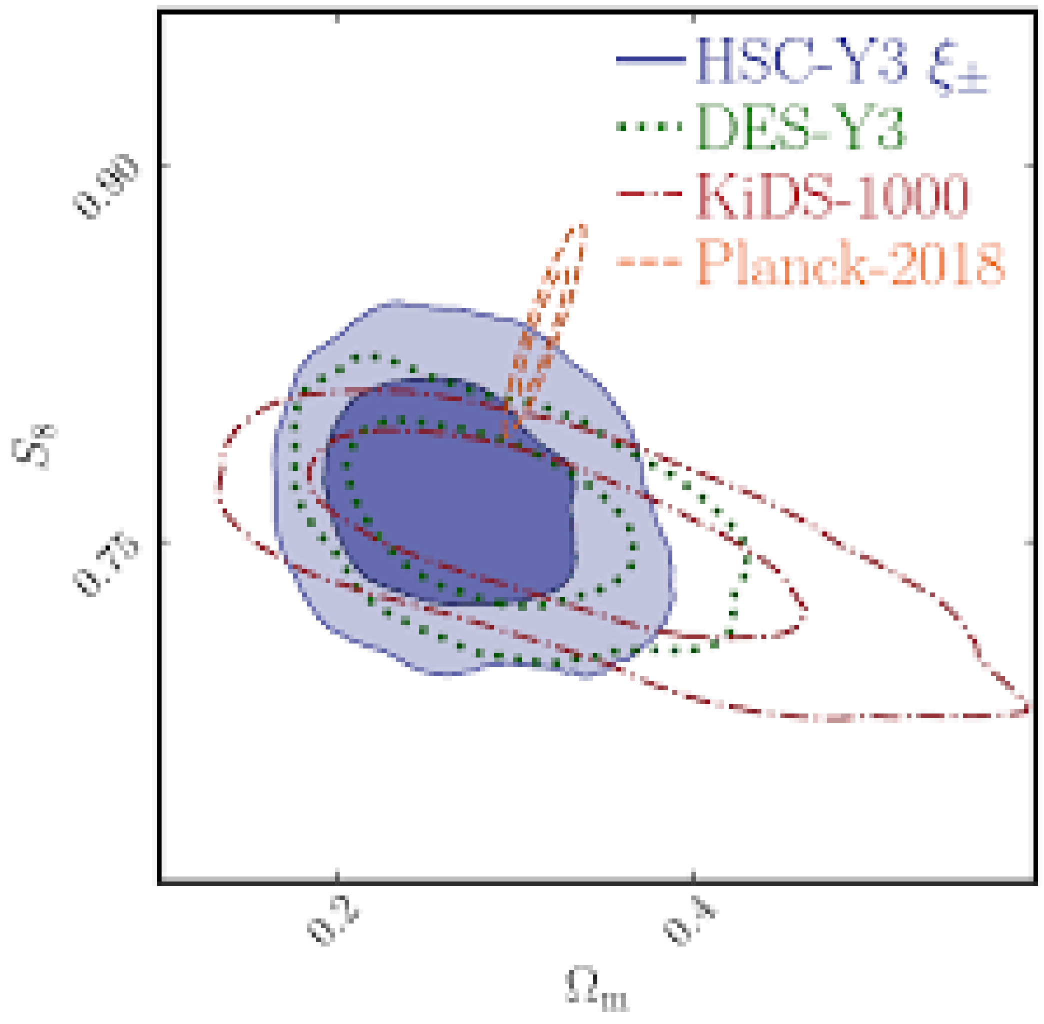
$$C_\ell^{\gamma\gamma} = \int_0^{\chi_H} d\chi\, \frac{W^2(\chi)}{\chi^2}\, P_\delta\left(k = \frac{\ell}{\chi}, z(\chi)\right)$$

Key quantities in weak lensing

$$\kappa = \frac{d_{\mathrm{L}} d_{\mathrm{LS}}}{d_{\mathrm{S}}} \int_0^{\chi_{\mathrm{S}}} \nabla^2 \Phi \, d\chi$$

where d_L, d_S, and dLS are, respectively, the distance from the observer to the lens and to the source, and the distance between the lens and the source. Here, d is the angular diameter distance which is related to the comoving distance r via dðzÞ 1⁄4 rðzÞ=ð1 þ zÞ, while v is the coordinate distance and is related to the comoving distance via standard relations involving a sine or a sinh; in a flat universe, r 1⁄4 v. Further, Φ is the three-dimensional gravitational potential that gets integrated along the line of sight in the above equation. A closely related quantity is shear γ which, along with the convergence, makes up elements of a 2 X 2 matrix whose components define the distortion of an image at any point on the sky.

Cosmology from 3 X 2pt



Galaxy-shear cross correlation and 3 X 2pt analysis

In addition to measuring shapes of distant galaxies across the sky and correlating them, there are other ways to leverage weak-lensing observations to learn about the growth of structure in the universe. One option is to measure the correlation of the shears of background galaxies with the positions of the foreground galaxies.

This correlation is known under the name galaxy–galaxy lensing, and effectively measures the lensing efficiency of foreground galaxies. The method should perhaps be called galaxy–galaxies lensing, as it correlates the position of one foreground galaxy with the shapes of a number of background galaxies that are near it on the sky. Another better name is galaxy–shear cross-correlation, as galaxy positions are correlated with (other galaxies') shear.