Asteroid Tidal Torque Progress Report

August 2, 2021

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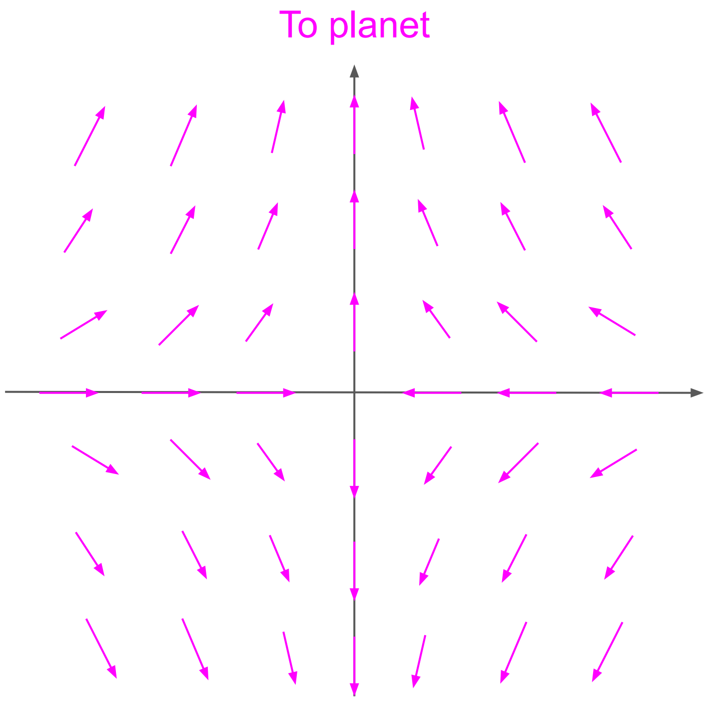
Introduction & equations of motion

Goal: extract asteroid shape and density from flyby



Tidal force map

- Gravity induces not just **tidal forces**, but **tidal torques**
- Tidal torques are applied to all parts of the asteroid, not just the surface
- Light curve data can tell us the **shape** of the outside, so tidal torque will tell us the **density** on the inside

Task list

- Derive the asteroid equations of motion
- Simulate the asteroid flyby
- Fit an asteroid model to flyby rotation data
- Extract a density distribution from light curve and rotation data
- Analyze the strengths and weaknesses of the model and fit method

- Done
- In progress
- Not yet done

The asteroid dynamics are defined by torque and MOI

Ingredients

- Linear equation of motion

$$\dot{\mathbf{D}} = \frac{\mu}{D^2}\hat{\mathbf{D}}$$

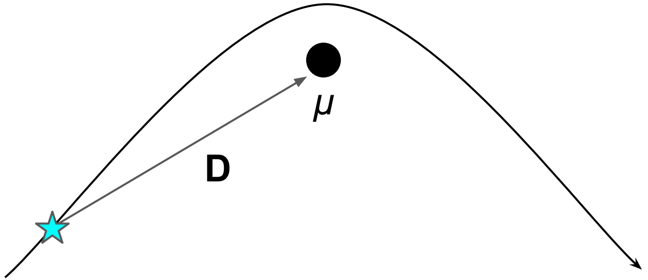
- Angular equation of motion

$$\dot{\omega} = I^{-1}(\tau - \omega \times (I\omega))$$

- Torque
- Moment of inertia

Initial Conditions

- Asteroid starts on hyperbolic trajectory
- Initially spinning around low energy principal axis



Torque and MOI can be expressed via K_{lm}

Torque is given by

Decays like $1/D^{l+l'}$

$$\tau = -\frac{1}{2}\sum_{lm}(-1)^l \mathcal{J}_{lm} \sum_{l'm'} S_{l+l',m+m'}(\mathbf{D})$$
$$\Big[(i\hat{\mathbf{x}}+\hat{\mathbf{y}})(l'-m'+1)\mathcal{K}^*_{l',m'-1}$$
$$+(i\hat{\mathbf{x}}-\hat{\mathbf{y}})(l'+m'+1)\mathcal{K}^*_{l',m'+1} + 2im'\hat{\mathbf{z}}\mathcal{K}^*_{l'm'}\Big].$$

MOI is diagonal

$$I_{xx} = \frac{2}{3}\mathcal{K}_{20} - 2\mathcal{K}_{2,-2} - 2\mathcal{K}_{22} + \frac{2}{5}\mathcal{K}_{00}$$
$$I_{yy} = \frac{2}{3}\mathcal{K}_{20} + 2\mathcal{K}_{2,-2} + 2\mathcal{K}_{22} + \frac{2}{5}\mathcal{K}_{00}$$
$$I_{zz} = -\frac{4}{3}\mathcal{K}_{20} + \frac{2}{5}\mathcal{K}_{00}$$

$$\mathcal{J}_{lm} \propto \int d^3r\,\rho_{\mathrm{Planet}}(\mathbf{r})R_{lm}(\mathbf{r})$$

$$\mathcal{K}_{lm} \propto \int d^3r\,\rho_{\mathrm{Asteroid}}(\mathbf{r})R_{lm}(\mathbf{r})$$

Simulation

Physical conditions restrict K_{lm}

- MOI must be diagonal in the principal axis frame

$$\Im\mathcal{K}_{22} = \mathcal{K}_{21} = 0$$

- The center of mass must be zero in the principal axis frame

$$\mathcal{K}_{1m} = 0$$

- There are only **3** low-order parameters:

$$\mathcal{K}_{00}\,, \quad \Re\mathcal{K}_{22}\,, \quad \mathcal{K}_{20}$$

(Mass)

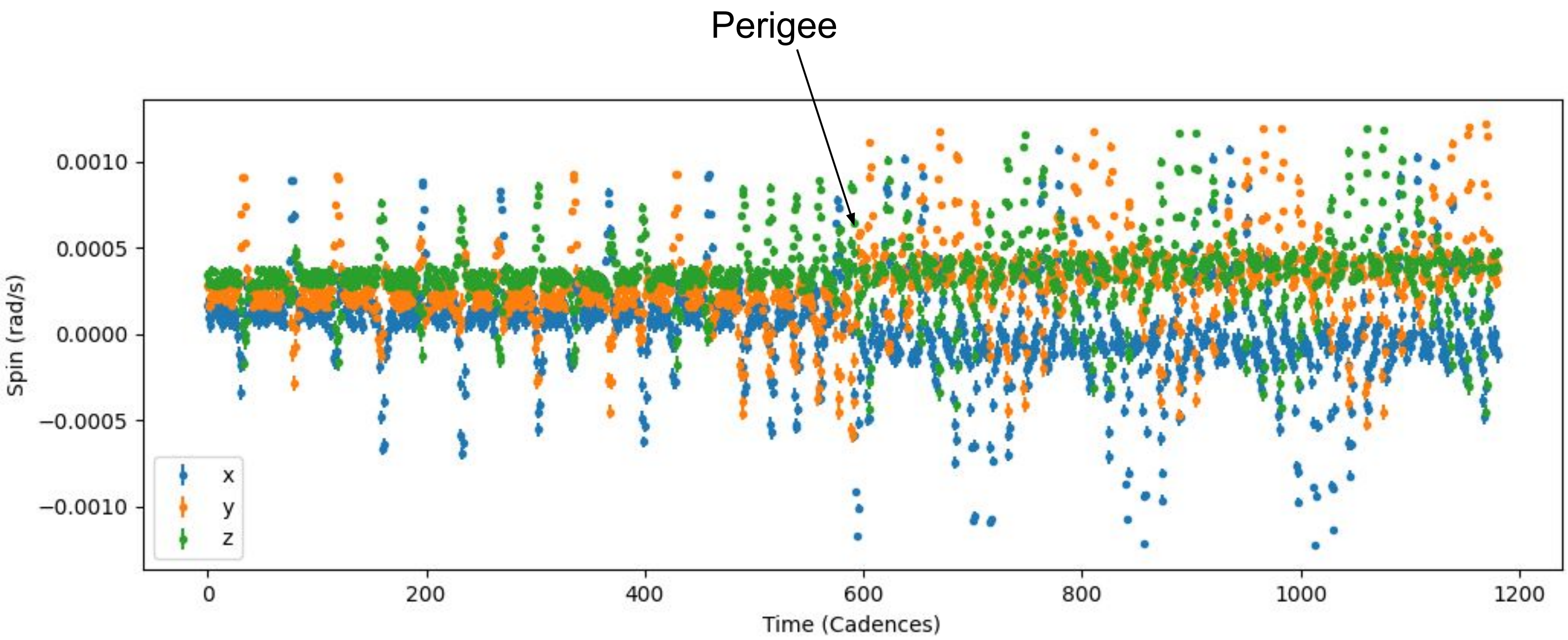
Data collection is done by simulation

We integrate the previous equations of motion numerically (SuperCloud)

Adjustable parameters:

- K_{lm}
- J_{lm}
- Orbit perigee and eccentricity
- Initial asteroid rotation speed and direction
- Initial orientation of the asteroid (one number, not three)
- Cadence of observation (e.g., 1 hour, 2 minutes)

We are able to simulate the flyby for any parameters



Fit method & results

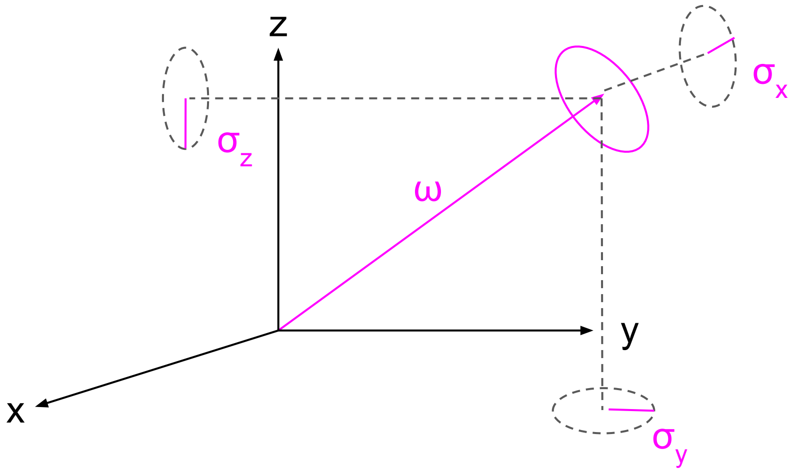
Our fit method uses a Markov Chain Monte Carlo (MCMC)

- Want to extract original parameters from sample rotation-vector data.
- The fit is complicated and highly sensitive to parameters, so simple fit methods won't work
- We use an MCMC (Markov Chain Monte Carlo) to extract full posterior probability distributions from flat priors and a **likelihood function**

$$\ln \mathcal{L} \propto \sum_{\text{data}} \frac{(y_i - y_i^*)^2}{\sigma_i^2}$$

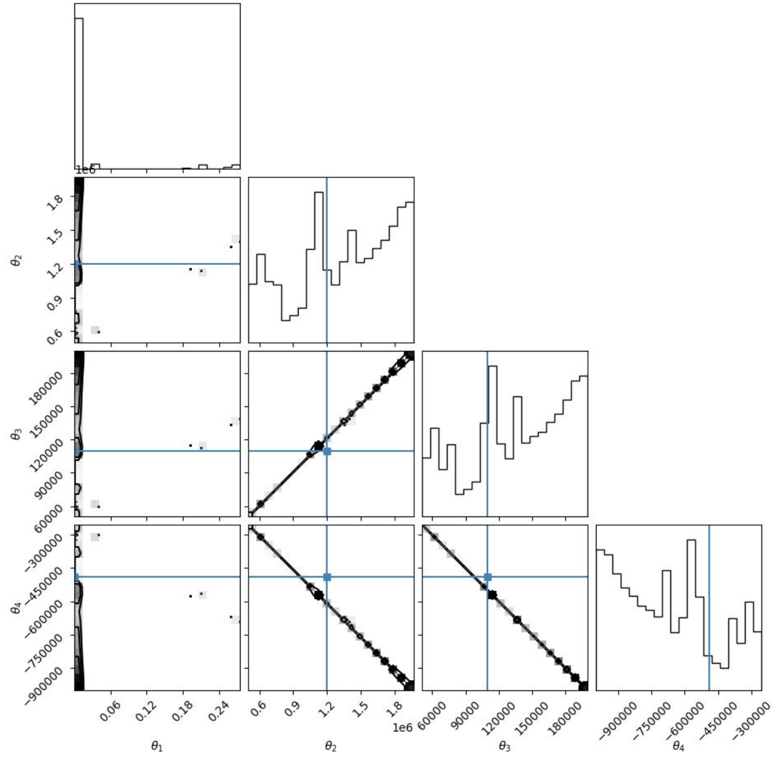
We use a semi-realistic error model

- Each rotational velocity component is shifted by a draw from a normal distribution
    - Its error is the standard deviation of that distribution
- The distribution comes from a small change in direction to the spin vector.



Fits are executed, but do not always recover initial parameters

- 17 fits run for initial roll (θ_1), $K_{00}(\theta_2)$, K_{22} (θ_3), K_{20} (θ_4)
- Converged after 200 cycles or so
- Results can be accurate



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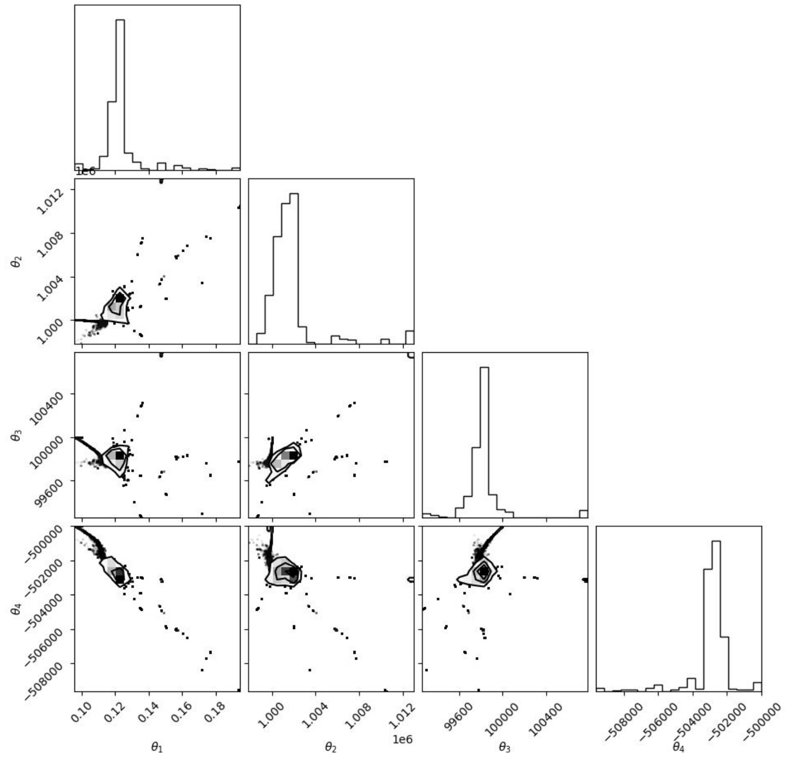

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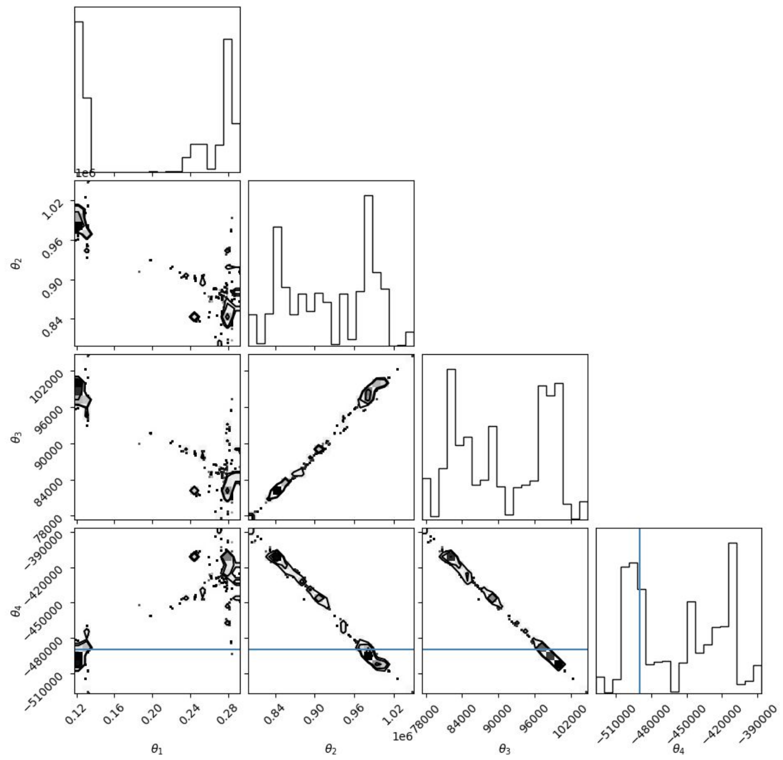
- Results can be inaccurate
- Degeneracy can be semi-broken



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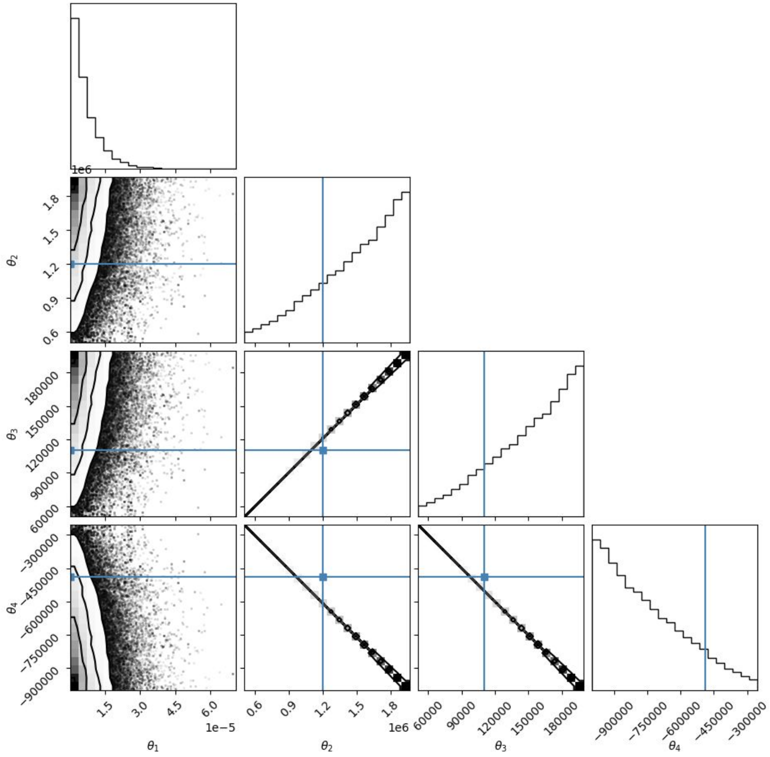
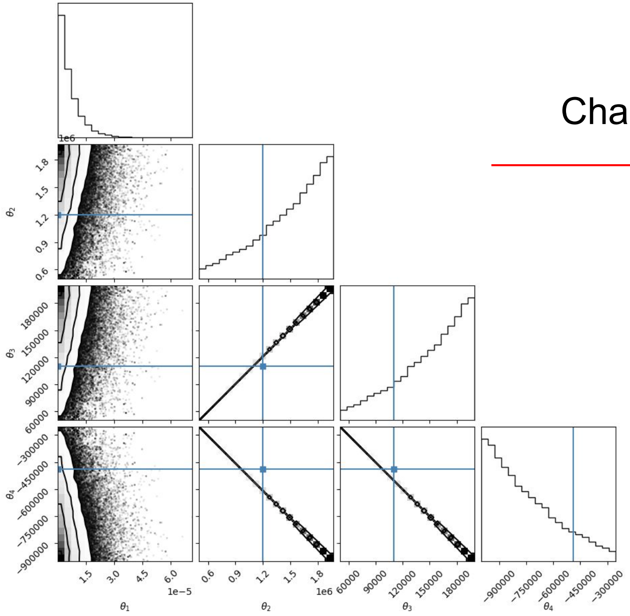
- Converged after 200 cycles or so
- Results can be accurate
- Results can be inaccurate
- Degeneracy can be semi-broken
- Degeneracy can obscure true values

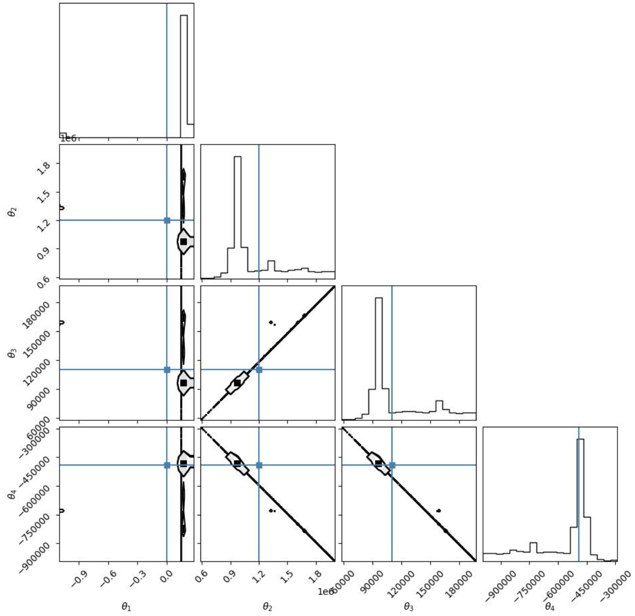


Tentative conclusions about fit behavior

- Degeneracy seems to be controlled by the choice of spin vector



Change spin



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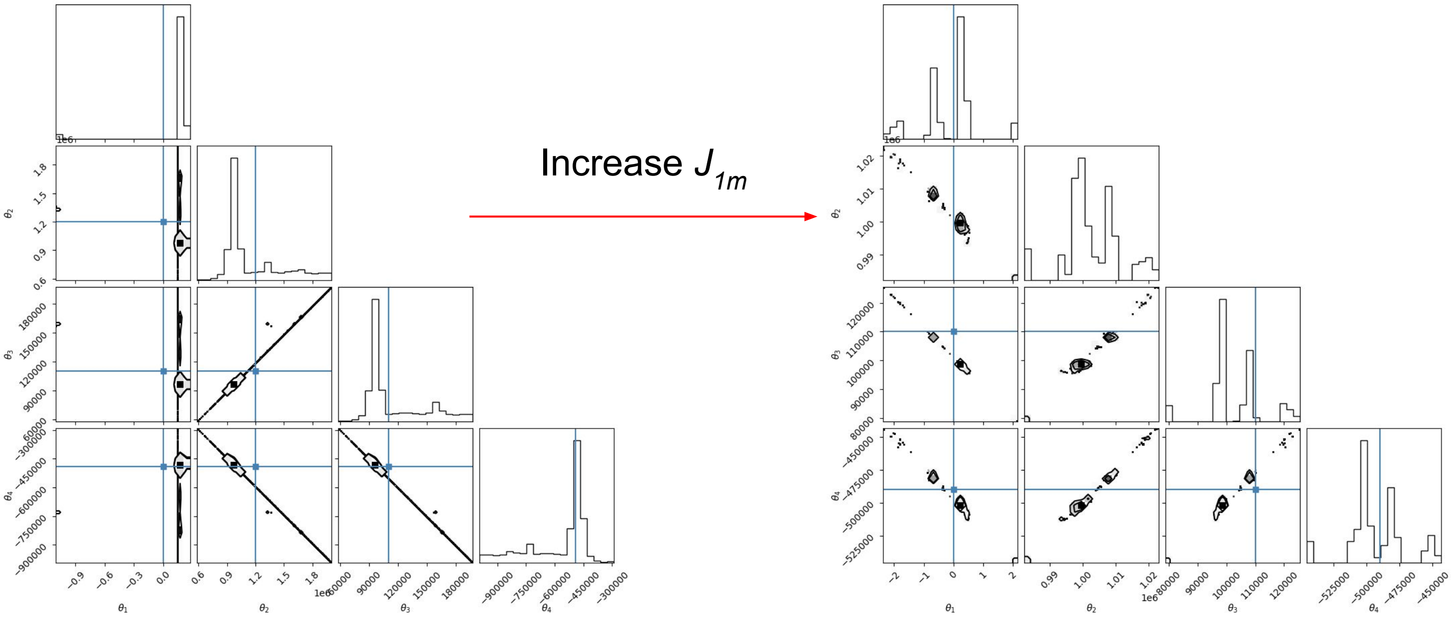
- Higher J_{1m} seems to increase accuracy



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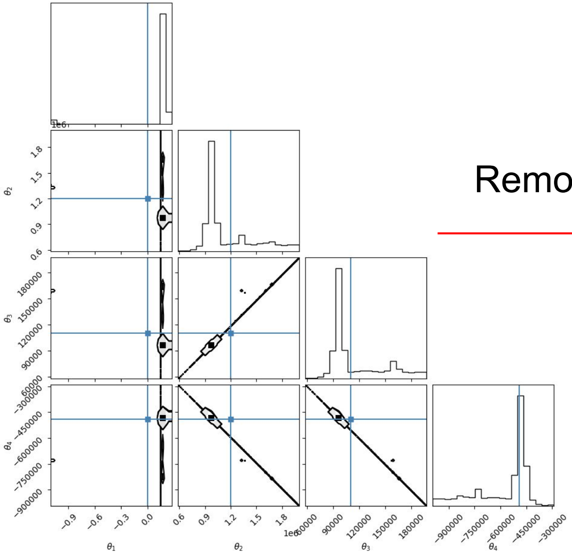
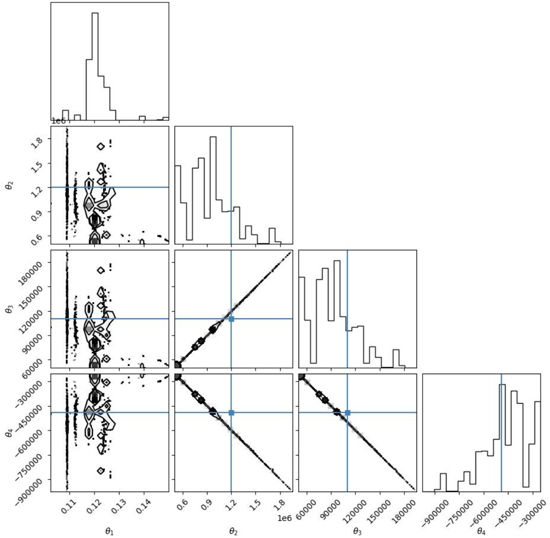
- Higher J_{1m} seems to increase accuracy
- Off diagonal J_{1m} may not help much



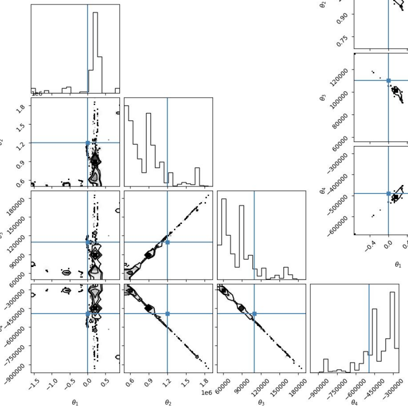
Remove J_{11}, $J_{1,-1}$

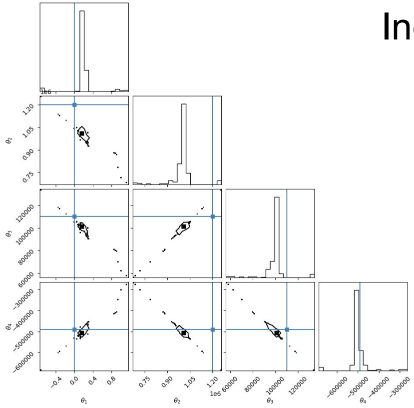


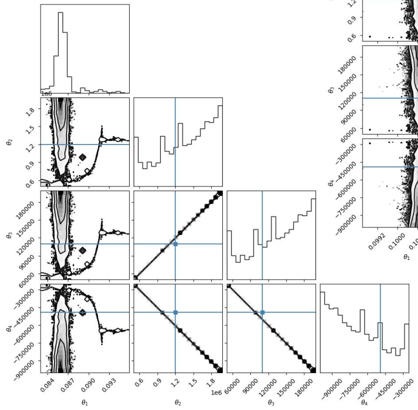
Tentative conclusions about fit behavior

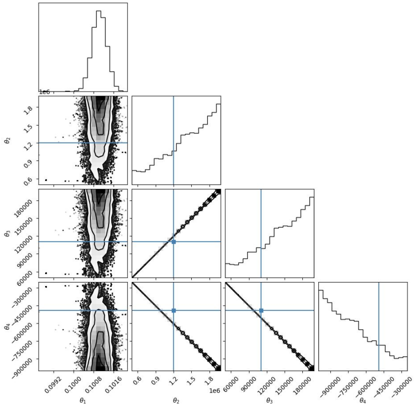
- Higher J_{1m} seems to increase accuracy
- Off diagonal J_{1m} may not help much
- Higher cadence doesn't necessarily help

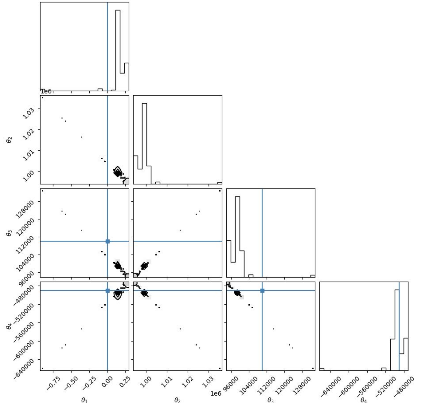
Increase candence











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- *b*

Density distribution

The density distribution model must yield physical results

Cannot

- predict negative density
- be spherically symmetric
- assume some radial density profile

Must

- Reproduce K_{lm}
- Reproduce light curve data

- Assume that light curve data gives us a **surface shape**, and flyby spin data gives us $\boldsymbol{K_{lm}}$.
- What is the density distribution?

Our density model consists of chunks of uniform density

1. Separate the asteroid into N chunks (N is the number of K_{lm}s known)
2. Calculate, K_{lmn}, the K_{lm} values for each chunk
3. Set up as matrix equation

known　　　known　　　unknown

$$\mathcal{K}_{lm} = [\mathcal{K}_{lmn}]\rho_n$$

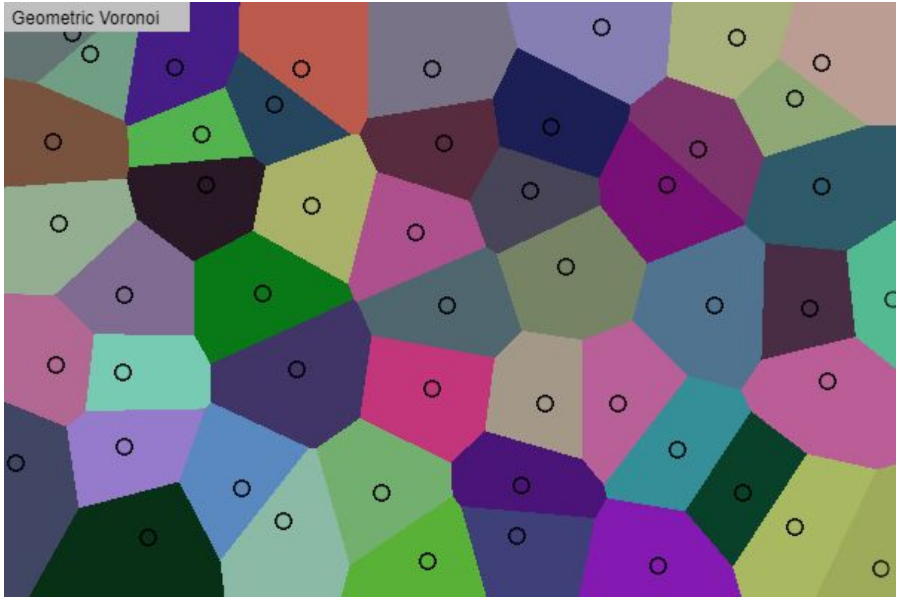
4. Solve matrix equation

$$\rho_n = [\mathcal{K}_{lmn}]^{-1}\mathcal{K}_{lm}$$

How do you make the chunks? Voronoi cells

How do you choose the N sections?

1. Find a sphere that encloses the asteroid
2. Fill the sphere with N points
3. Repopulate all points outside the asteroid model until they're all inside
4. Form Voronoi cells based on the points



Edges of Voronoi cells can be redrawn

Do we really want hard edges in random locations in our model?

- We can redo the density calculation with different Voronoi cells
- Then average together the density distributions because K_{lm} is linear in density

$$\mathcal{K}_{lm} \propto \int d^3 r \rho_{\text{Asteroid}}(\mathbf{r}) R_{lm}(\mathbf{r})$$

Pros and Cons of this density model

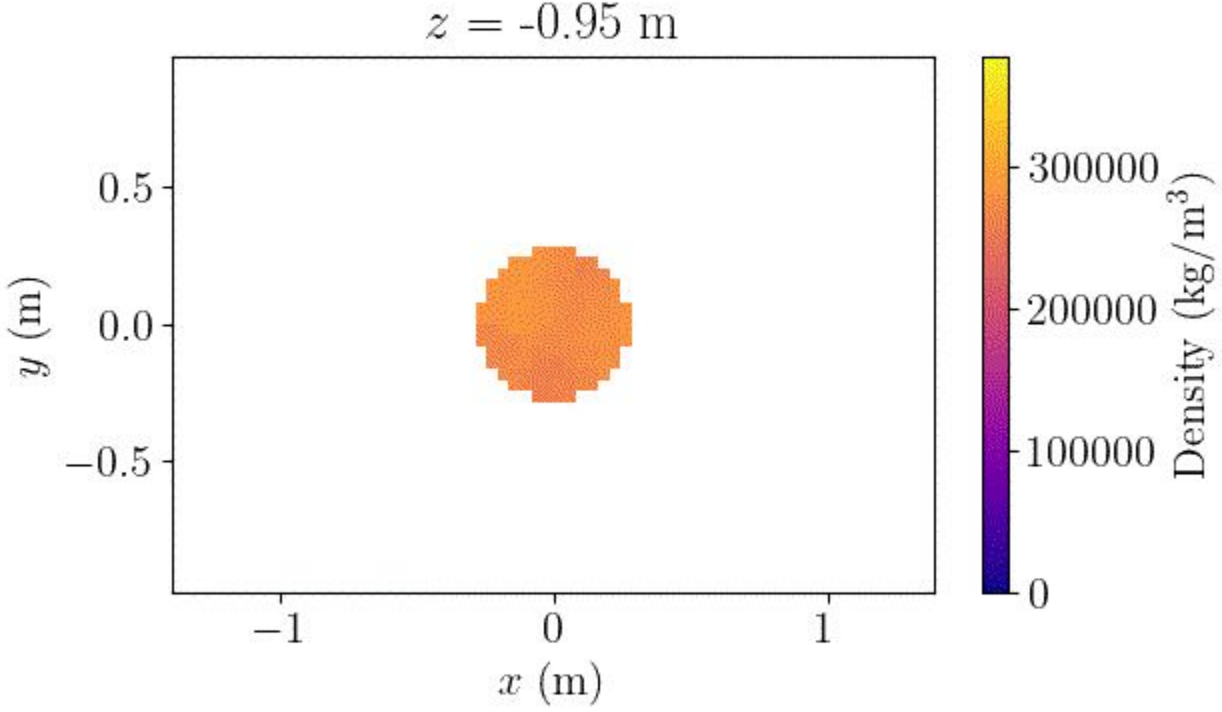
Pros:

- Massively parallelizable
- Works for any shape provided by light curve data
- Density calculation is nondegenerate
- Uncomplicated

Cons:

- A fully known shape model is needed
- Cannot yet account for uncertainties in K_{lm} or shape
- Voronoi cell calculation *is* degenerate
- Average does not necessarily converge

We can generate a density model given K_{lm} and a shape



Spherical shape

$I_{zz} > I_{xx}, I_{yy}$

Center of mass = origin

Conclusion

- We have derived the equations of motion for an asteroid flyby

$$\tau = -\frac{1}{2}\sum_{lm}(-1)^l \mathcal{J}_{lm} \sum_{l'm'} S_{l+l',m+m'}(\mathbf{D})$$
$$\Big[(i\hat{\mathbf{x}} + \hat{\mathbf{y}})(l' - m' + 1)\mathcal{K}^*_{l',m'-1}$$
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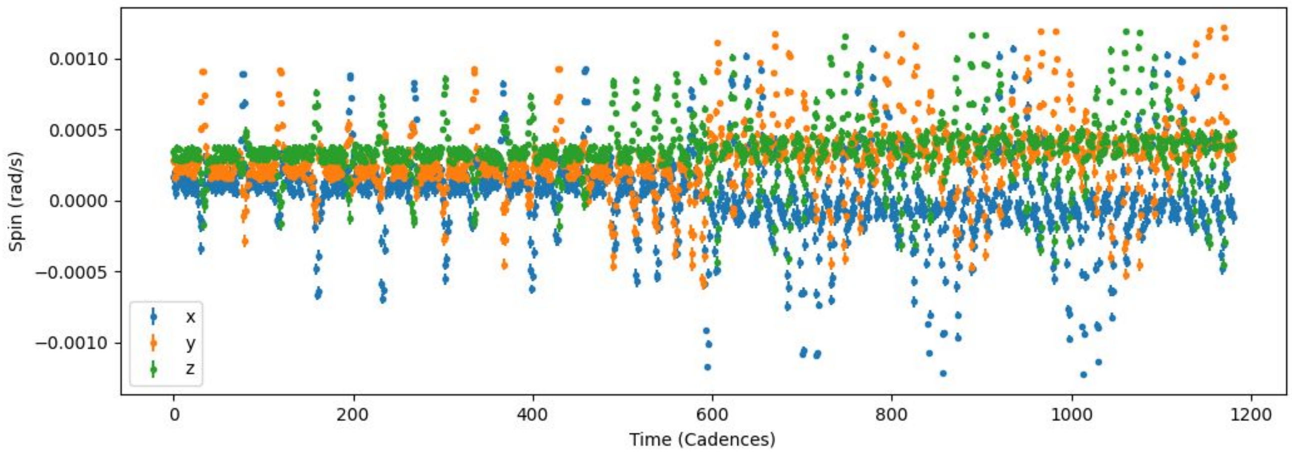
Conclusion

- We have derived the equations of motion for an asteroid flyby
- We have simulated the asteroid over the flyby



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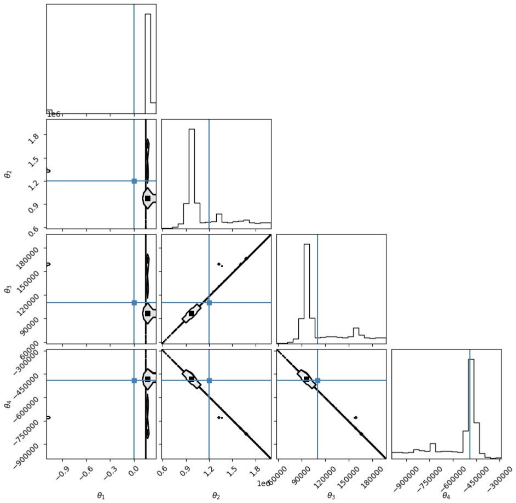
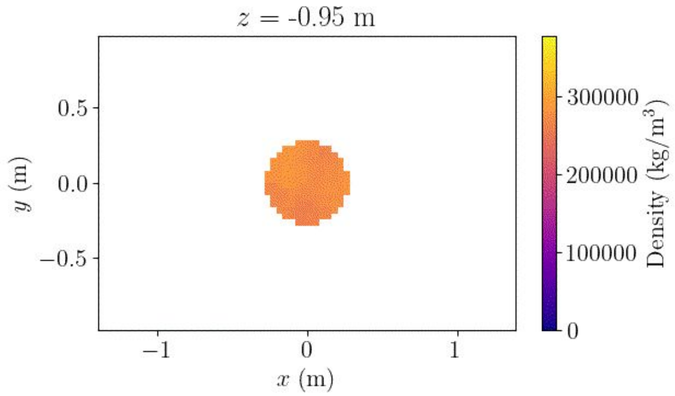


Conclusion

- We have derived the equations of motion for an asteroid flyby
- We have simulated the asteroid over the flyby
- We have fitted parameters to sample flyby data
- We have extracted a density distribution from asteroid parameters



To do

- Play with fit parameters to analyze
  - What causes fits to be degenerate?
  - What causes fits to find incorrect values?
- Assess the success of fits in different scenarios
  - Precision on higher orders of K_{lm}
  - Effect of increased cadence, different J_{lm}, higher perigee, etc.