Asteroids Progress Report

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Jack Dinsmore

Asteroid Dynamics

- **Ingredients**
  - Linear equation of motion

$$\ddot{r} = -\frac{\mu}{r^2}$$

  - Angular equation of motion

$$\dot{\omega} = I^{-1}(\tau - \omega \times (I\omega))$$

  - Torque
  - Moment of inertia

- **Initial conditions**
  - Asteroid starts on hyperbolic trajectory
  - Initially spinning around lowest energy principal axis

Asteroid Dynamics

- The torque is given by

$$\tau = -\frac{1}{2}\sum_{lm}(-1)^l \mathcal{J}_{lm}\sum_{l'm'} S_{l+l',m+m'}(D)$$
$$\Big[(i\hat{x}+\hat{y})(l'-m'+1)\mathcal{K}^*_{l',m'-1}$$
$$+(i\hat{x}-\hat{y})(l'+m'+1)\mathcal{K}^*_{l',m'+1}+2im'\hat{z}\mathcal{K}^*_{l'm'}\Big].$$

- MOI is diagonal:

$$I_{xx} = \frac{2}{3}\mathcal{K}_{20} - 2\mathcal{K}_{2,-2} - 2\mathcal{K}_{22} + \frac{2}{5}\mathcal{K}_{00}$$
$$I_{yy} = \frac{2}{3}\mathcal{K}_{20} + 2\mathcal{K}_{2,-2} + 2\mathcal{K}_{22} + \frac{2}{5}\mathcal{K}_{00}$$
$$I_{zz} = -\frac{4}{3}\mathcal{K}_{20} + \frac{2}{5}\mathcal{K}_{00}$$

Asteroid Dynamics

- To force a diagonal MOI, only the real part of K_{22} and K_{20} are nonzero.

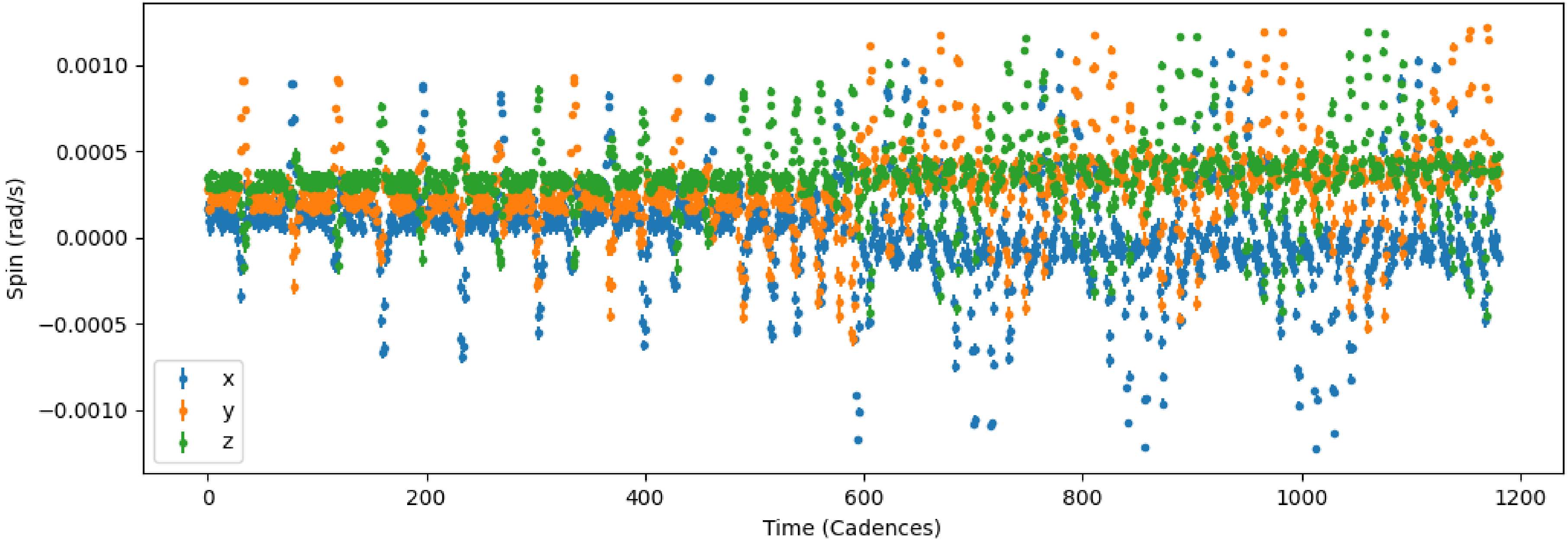
- We may choose $K_{1m} = 0$

- K_{00} is the mass

Data

- We collect spin data separated in time by "cadences."
  - E.g., 2 min to increase data, 1 hr to increase speed, etc.

Example Data



Asteroid Dynamics

- **Parameters:** K_{lm} control both shape and density distribution
  - Chosen so that they relate easily to torque
  - Moment of inertia derived from K_{lm} as well

$$\mathcal{K}_{lm} = \text{constants} \int d^3r\rho(r)Y_{lm}(\hat{r})r^l$$

Fit Method

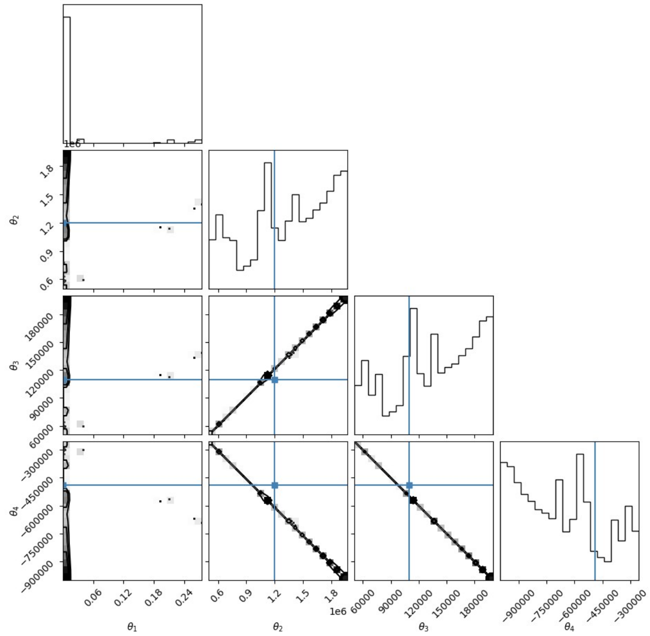
- With an MCMC, find the best K_{lm}s to maximize Gaussian likelihood:

$$\ln \mathcal{L} = -\sum_{\mathrm{data}} \frac{(y_i - y_i^*)^2}{\sigma_i^2}$$

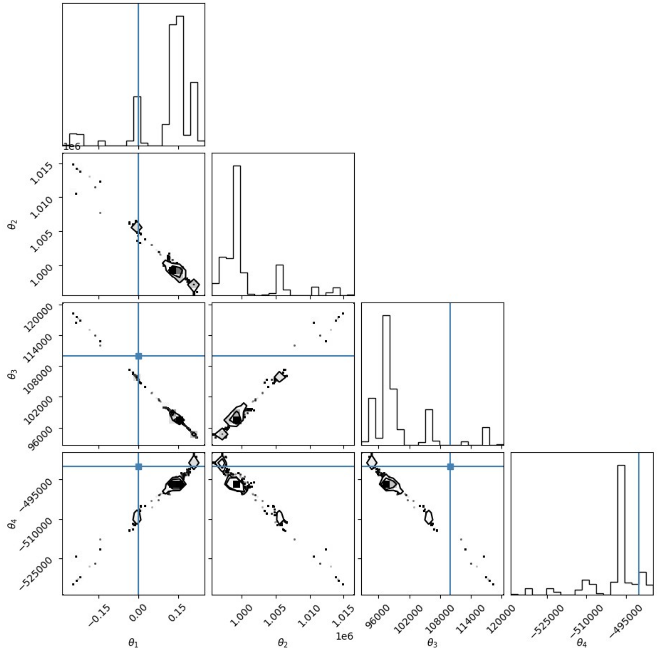
where error is simulated by rotating each spin vector by a small amount in a random direction.

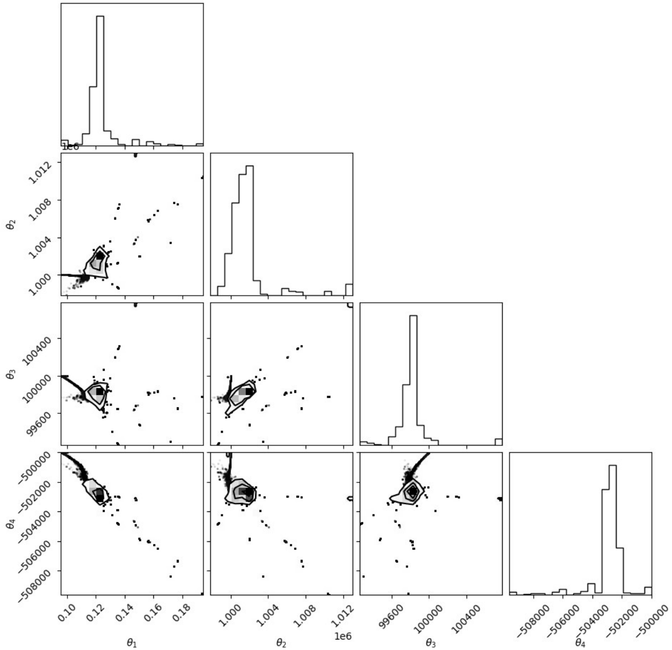
Current status

- Fits are running, but I'm still generating some to better understand them.
- Degeneracy sometimes appears with certain choices of parameters



More fit examples





Density distribution model

- Suppose we can extract K_{lm} from spin data.
- Suppose we can extract a shape parameterization from light curve data
- What is the density distribution?

Density distribution model

- Idea:
    1) Separate the shape model into N different sections
    2) Give each section its own density
    3) Find densities that give the fitted K_{lm} values
- Use N = number of K_{lm}s so that the math is as simple as N volume calculations and a matrix inversion!

Density distribution model

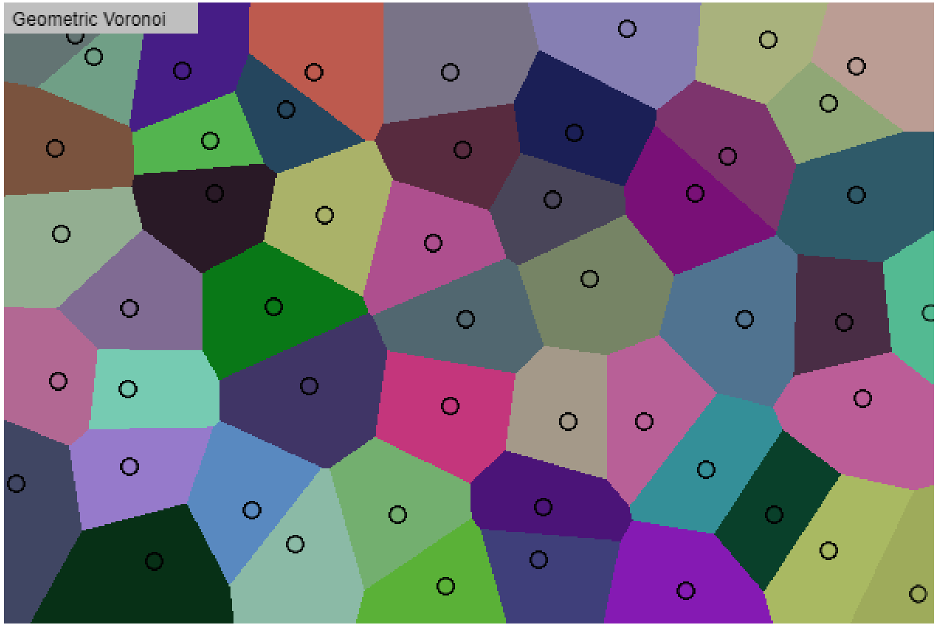
$$\mathcal{K}_{lmn} = \text{constants} \int_{A_n} d^3r r^l Y_{lm} \qquad \text{Asteroid} = \bigcup_{n=1}^{N} A_n$$

$$\begin{pmatrix} \mathcal{K}_{00} \\ \mathcal{K}_{11} \\ \mathcal{K}_{10} \\ \vdots \end{pmatrix} = \begin{pmatrix} \mathcal{K}_{001} & \mathcal{K}_{002} & \mathcal{K}_{003} & \cdots \\ \mathcal{K}_{11} & \mathcal{K}_{12} & \mathcal{K}_{113} & \\ \mathcal{K}_{10} & \mathcal{K}_{102} & \mathcal{K}_{103} & \\ \vdots & & & \ddots \end{pmatrix} \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \vdots \end{pmatrix}$$

$$\mathcal{K}_{lm} = \mathcal{K}_{lmn}\rho_n \implies \rho_n = \mathcal{K}_{lmn}^{-1}\mathcal{K}_{lm}$$

Density distribution model

- How do you choose the N sections?
  1) Find a sphere that encloses the asteroid
  2) Fill the sphere with N points
  3) Repopulate all points outside the asteroid model until they're all inside
  4) Form Voronoi cells based on the points



Density distribution model

- What if the cells are shaped weird?
- K_{lm} is linear in density!

$$\mathcal{K}_{lm} = \text{constants} \int d^3r \rho(r) Y_{lm}(\hat{r}) r^l$$

- So the average of two density distributions with equal K_{lm} gives the same K_{lm}
- Smooth the density distribution by averaging many density distributions

Density distribution model

- Pros:
  - Easy to execute
  - Smoothing allows for a more natural distribution
  - Non-degenerate
  - Not too complicated
- Cons
  - A shape model is needed
  - Cannot react to uncertainties in shape model
  - Distribution is difficult to control

Conclusion

- Done:
  - Simulation of asteroid has been made
  - Fits of asteroid parameters have been started
  - Way to extract asteroid density from parameters has been proposed
- To do:
  - Fit higher order parameters
  - Implement asteroid density extraction
  - Fit to light curve data?