Fluid Substitution Manual and Workflow for Matlab

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Theory. Arguably, the most important contribution to rock physics is Gassmann's (1951) fluid substitution theory. This theory allows us to compute the bulk modulus of porous rock filled with Fluid A if this modulus is known (measured) in the same rock but filled with Fluid B. These derivations were conducted under the assumption that the wave-induced pore pressure oscillations equilibrate within the sample over the wave period, meaning that Gassmann's is a low-frequency theory. Hence, it is applicable at the wireline and seismic frequency ranges, but not applicable to ultrasonic laboratory data obtained on *liquid-saturated* rock. It helps predict the seismic response of rock filled with any hypothetical fluid if it is measured in-situ where the pore fluid is known. For example, if the elastic properties of rock are measured in-situ in rock 100% filled with water, we can predict these properties in the same rock but filled with oil or gas.

Gassmann's theory provides the bulk modulus of fluid-saturated rock (K_{Sat}) as a function of the dry-rock bulk modulus (K_{Dry}), the bulk modulus of the solid phase (K_s), that of the pore fluid (K_f), and the total porosity (ϕ). It is assumed that the total porosity is the connected porosity, that is the hydraulically isolated pores that are not in communication with the rest of the pore space are not included. This theory also assumes that the shear modulus is fluid-independent:

$$K_{Sat} = K_s \frac{\phi K_{Dry} - (1+\phi)K_f K_{Dry} / K_s + K_f}{(1-\phi)K_f + \phi K_s - K_f K_{Dry} / K_s}, \quad G_{Sat} = G_{Dry}. \tag{1}$$

The latter equation can be rearranged as follows:

$$K_{Dry} = K_s \frac{1 - (1-\phi)K_{Sat}/K_s - \phi K_{Sat}/K_f}{1 + \phi - \phi K_s/K_f - K_{Sat}/K_s}, \quad G_{Dry} = G_{Sat}. \tag{2}$$

Equations 1 and 2 provide us with a fluid substitution recipe as follows. Assume that we know the bulk modulus K_{SatA} of rock saturated with Fluid A whose bulk modulus is K_{fA} .and its density is ρ_{fA}. Then from Equation 2 we obtain:

$$K_{Dry} = K_s \frac{1 - (1-\phi)K_{SatA}/K_s - \phi K_{SatA}/K_{fA}}{1 + \phi - \phi K_s/K_{fA} - K_{SatA}/K_s}. \tag{3}$$

The bulk modulus K_{SatB} of the same rock saturated with Fluid B is (Equation 1):

$$K_{SatB} = K_s \frac{\phi K_{Dry} - (1+\phi) K_{fB} K_{Dry} / K_s + K_{fB}}{(1-\phi) K_{fB} + \phi K_s - K_{fB} K_{Dry} / K_s}, \tag{4}$$

where K_{fB} is the bulk modulus of Fluid B.

The shear modulus of the rock remains the same, no matter what fluid it is saturated with (Figure 1).

Gassmann's theory is always correct, no matter what rock or fluid we are dealing with, as long as the assumptions of this theory are honored:

(1) This is a static theory, meaning that the **frequency of the elastic wave is low**.

(2) The rock frame's mineral is characterized by a **single bulk modulus**. This means that the rock is mono-mineralic, which appears to be a strong impediment for using this theory in a real situation. Yet, there is a simple recipe of dealing with this challenge (see later).

(3) The pore fluid is characterized by a single bulk modulus, which is, once again, an apparent impediment for using this theory at partial saturation. Yet, there is a simple recipe of dealing with this challenge (see later).

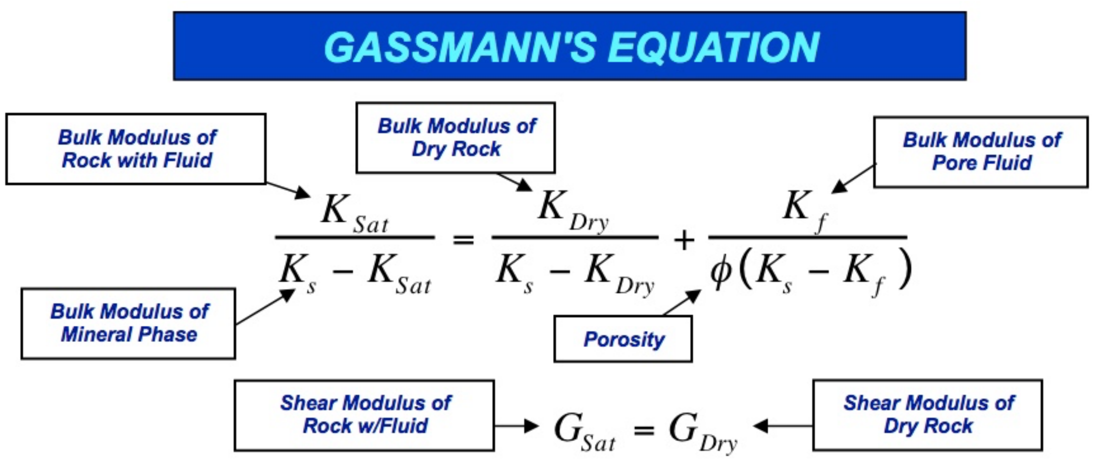

Figure 1. Illustration of Gassmann's theory.

It is important to remember that the bulk density ρ_b of the rock is also a function of the pore fluid. It depends on the porosity and density of the fluid (ρ_{fA} or ρ_{fB}):

$$\rho_{bB} = \rho_{bA} - \phi \rho_{fA} + \phi \rho_{fB}, \tag{5}$$

where ρ_{bA} and ρ_{bB} are the bulk densities of the rock with the two pore fluids, respectively.

Finally, we can compute the elastic-wave velocities, as well as other seismic attributes, once we know the elastic moduli:

$$V_{pB} = \sqrt{\frac{K_{SatB} + \frac{4}{3} G_{Dry}}{\rho_{bB}}}; \quad V_{sB} = \sqrt{\frac{G_{Dry}}{\rho_{bB}}}, \tag{6}$$

and

$$I_{pB} = \rho_{bB} V_{pB}; \quad \nu_B = \frac{1}{2} \frac{(V_{pB} / V_{sB})^2 - 2}{(V_{pB} / V_{sB})^2 - 1}, \tag{7}$$

where I_{pB} and ν_B are the P-wave impedance and Poisson's ratio of the rock filled with Fluid B, respectively. Although the shear modulus G is pore-fluid independent, V_s is, since the bulk density varies with varying fluid.

Let us finally describe the details required in practical fluid substitution, specifically the computation of K_s, ρ_s, K_f, and ρ_f.

The elastic moduli of the multi-mineral rock matrix K_s and G_s can be obtained using Hill's average as:

$$K_s = \frac{K_V + K_R}{2}, \quad G_s = \frac{G_V + G_R}{2}, \tag{8}$$

where

$$\begin{gathered} K_V = \sum_{i=1}^{N} f_i K_i, \quad G_V = \sum_{i=1}^{N} f_i G_i, \\ K_R^{-1} = \sum_{i=1}^{N} f_i K_i^{-1}, \quad G_R^{-1} = \sum_{i=1}^{N} f_i G_i^{-1}, \end{gathered} \tag{9}$$

where N is the number of the mineral components, f_i is the volume fraction of i^{th} mineral, and K_i and G_i are the bulk and shear moduli of the i^{th} component. The density is the arithmetic average of the densities of the components:

$$\rho_s = \sum_{i=1}^{N} f_i \rho_i, \tag{10}$$

The bulk modulus of the pore fluid K_f is

$$\frac{1}{K_f} = \frac{S_w}{K_w} + \frac{S_o}{K_o} + \frac{S_g}{K_g}, \tag{11}$$

where K_w, K_o, and K_g are the bulk moduli of water, oil, and gas, respectively. The density of the pore fluid ρ_f is the arithmetic average of those of the fluid components, water, oil, and gas:

$$\rho_f = S_w \rho_w + S_o \rho_o + S_g \rho_g, \tag{12}$$

Example 1: From Dry to Wet

Figure 2 shows the results of experimental measurements of the P- and S-wave velocity on an unconsolidated sand sample versus hydrostatic confining stress. The sample is dry. The bulk density of the sample and its porosity also vary as the stress increases. The sample's mineralogy is quartz, calcite, feldspar, and clay. The volumetric proportions of these components are shown in Table 1. These data are listed in MatLab space **SandB.mat**.

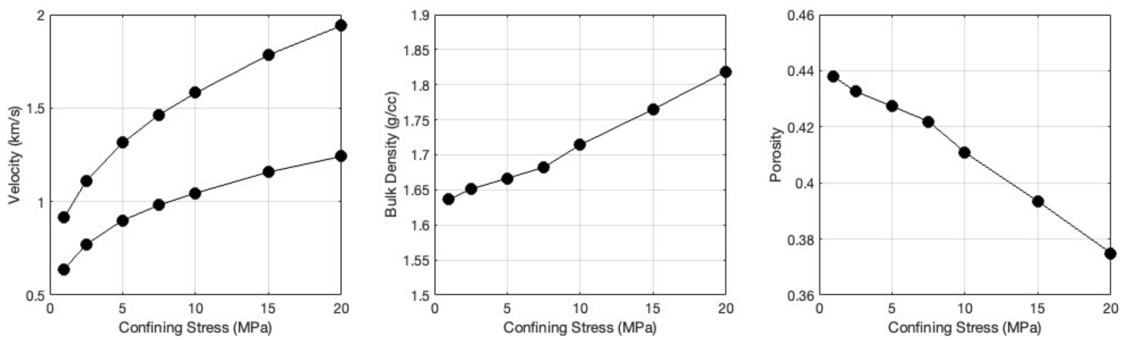

Figure 2. Left to right. Measured dry-sand P- and S-wave velocity; bulk density; and porosity versus stress.

Table 1. Mineral fractions and elastic moduli and density.

Mineral	Volume Fraction	Bulk Modulus (GPa)	Shear Modulus (GPa)	Bulk Density (g/cc)
Quartz	0.50	36.60	45.00	2.65
Calcite	0.10	76.80	32.00	2.71
Feldspar	0.20	75.60	25.00	2.63
Clay (Kaolinite)	0.20	21.00	7.00	2.65

Function

[Ks,Gs,Ms,RHOs]=MineralProperties(Quartz,Clay,Feldspar,Calcite,Dolomite,Anhydrite)

computes the elastic properties of the composite mineral frame with the component fractions given in Table 1. The component elastic moduli and density are given inside this function and can be changed by the user as needed for a concrete case study. The resulting bulk modulus of the mineral frame is K_s = 41.078 GPa.

Let us next compute the elastic properties of the sand under examination by assuming it is fully saturated with brine whose bulk modulus is K_w = 2.80 GPa and density ρ_w = 1.03 g/cc. In order to use Gassmann's theory as expressed by Equation (1), we also need the bulk modulus of the dry sand K_{Dry}. It can be computed from the measured velocities V_p and V_s, and density (ρ), together with other elastic properties, shear (G) and compressional (M) moduli, the P-wave (I_p) and S-wave (I_s) impedances, and Poisson's ratio (ν) as:

$$\begin{aligned}
&M = \rho_b V_p^2; \quad G = \rho_b V_s^2; \quad K = M - \tfrac{4}{3}G; \\
&I_p = \rho_b V_p; \quad I_s = \rho_b V_s; \quad \nu = \frac{1}{2}\frac{(V_p / V_s)^2 - 2}{(V_p / V_s)^2 - 1}.
\end{aligned} \tag{13}$$

This operation implemented by function

[M,G,K,Ip,Is,PR]=Velocity2Moduli(Vp,Vs,RHO)

computes the above-mentioned elastic properties. They are also stored in **SandB.mat**.

The elastic properties of the water-saturate (wet) sand are now computed by function

[KSat,RHOBSat,VpSat,VsSat]=FluidSubGassmMathWorks(KDry,G,RHOBDry,Ks,Phi,Kf,RHOf)

The case-specific call is

[KWet,RHOBWet,VpWet,VsWet]=FluidSubGassmMathWorks(K,G,RHOB,Ks,Phi,Kw,RHOw)

The results for the wet-sand velocity and density are shown in Figure 3. All these results are saved in **SandB.mat.**

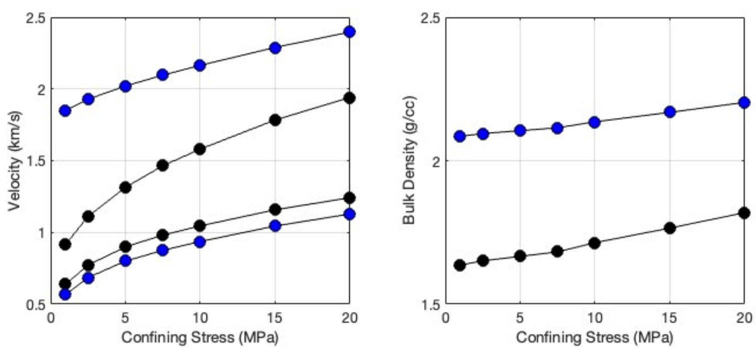

Figure 3. Left to right. P- and S-wave velocity in dry sand (black) and in wet sand (blue) versus stress; bulk density of dry sand (black) and wet sand (blue) versus stress.

Example 2: From Wet to Oil-Saturated Sand

Assume now that we know the velocities and bulk density of the water-saturated sand as computed in the first example. Let us now compute these properties in the sand that is 40% water saturated (S_w = 0.4) with the other fluid being light oil whose bulk modulus is 0.4177 GPa and density is 0.6111 g/cc. First, we need to compute the bulk modulus and density of the immiscible oil/water pore fluid system according to Equations 11 and 12. It is implemented by function

[Kf,RHOf]=FluidMixture(Kw,Ko,Kg,RHOw,RHOo,RHOg,Sw,So,Sg)

Since gas is not present in the system, we assign S_g = 0 and S_o = 1-S_w = 0.6. Also, because gas is not present, we can arbitrarily assign Kg = 0.03 GPa and ρ_g = 0.30 g/cc, all included in **SandB.mat**. The resulting computed pore fluid properties are K_f = 0.6333 GPa and ρ_f = 0.7767 g/cc.

Fluid substitution from FluidA to FluidB is according to Equations 3, 4, and 5. It is implemented by function

[VpSatB,VsSatB,RHOBSatB]=FluidA2FluidBGassmann(VpSatA,VsSatA,RHOBSatA,Ks,Phi,KfA,RHOfA,KfB,RHOfB)

In this concrete example, the call is

[VpSatOil,VsSatOil,RHOBSatOil]=FluidA2FluidBGassmann(VpWet,VsWet,RHOBWet,Ks,Phi,Kw,RHOw,Kf,RHOf)

The results are plotted in Figure 4 and are given in **SandB.mat**.

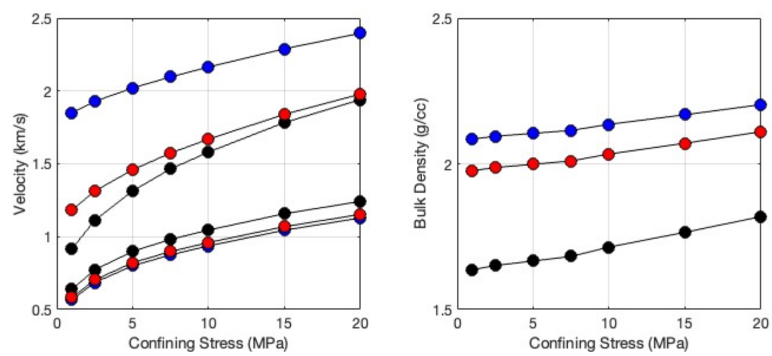

Figure 4. Left. P- and S-wave velocity in dry sand (black, wet sand (blue), and sand with oil (red) at S_w = 0.4 versus stress. Right. Same for the bulk density.

In fact, this last function can be directly used to conduct fluid substitution examples discussed earlier in this manual. Also, all the functions provided here can be applied to wireline data for fluid substitution along the entire depth of well log data.