Double Descent Demystified: Identifying, Interpreting & Ablating the Sources of a Deep Learning Puzzle

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A review by Jack Hanke

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Likely indirectly due to their work, we have all been in a similar situation. You solved a problem with a neural network and now have a large collection of inscrutable weights θ.

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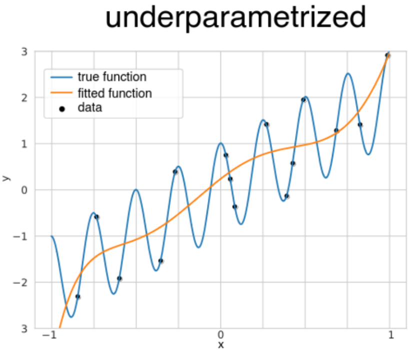
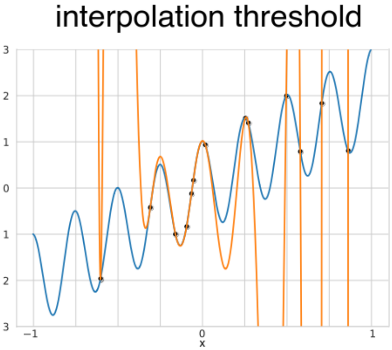
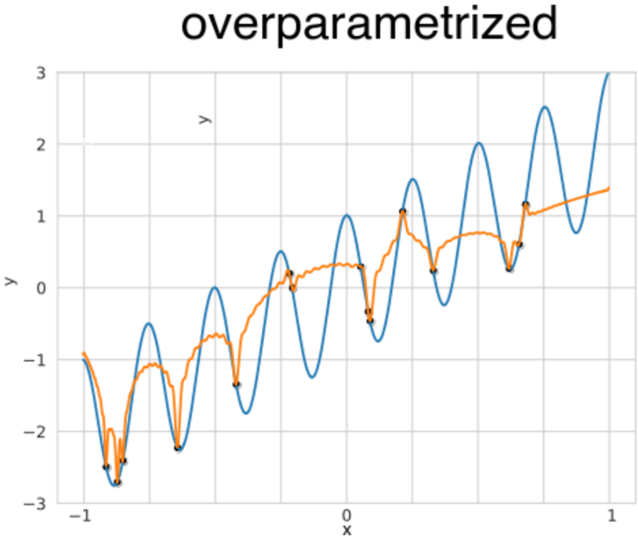
- Why does a model with so many parameters not just memorize the data? This is the *double descent problem*.
- The answer to this is (in part) this paper.

The Traditional View






overparametrized
y
3
2
1
0
−1
−2
−3
−1
0
1
x
y

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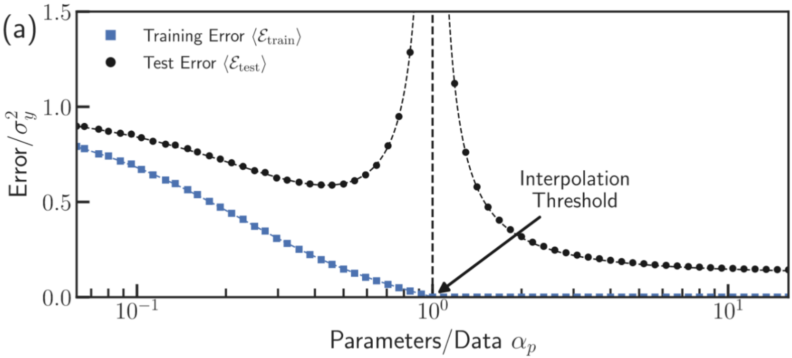
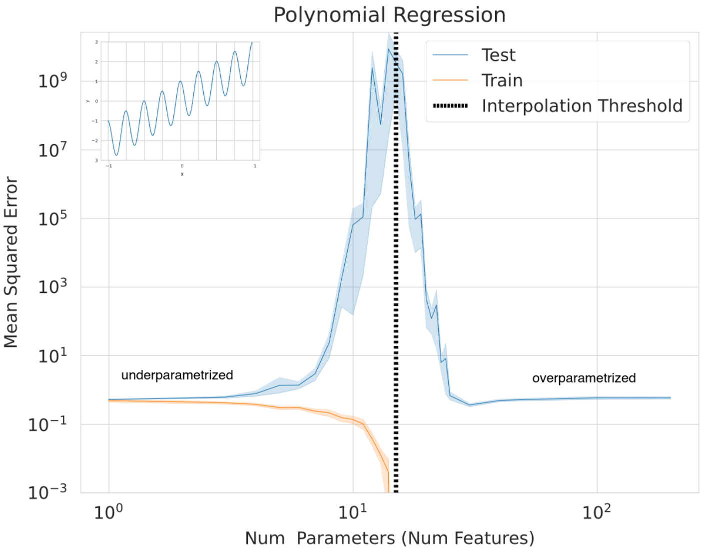


Double descent in polynomial regression - Empirical



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We will next study linear models, which have a fixed value of $P = D + 1$. Therefore, double descent occurs in the direction of increasing N.

Double descent in linear regression - Mathematical

The underparametrized regime is the classic least-squares minimization problem:

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For the overparameterized regime, the above optimization problem has infinite solutions. Therefore, we need to choose a different optimization problem:

$$\hat{\vec{\beta}}_{over} = \text{argmin}_{\vec{\beta}} ||\vec{\beta}||_2^2 \text{ s.t. } \forall n \in (1, \dots, N) \ \vec{x}_n \vec{\beta} = y_n$$

which is solved by

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Why this choice?

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We choose this optimization problem because *it is the optimization problem that gradient decent implicity minimizes*!

Double descent in nonlinear models - Intuition

TODO

Summary

TODO

Thank you for listening!