A

Page 3

Worth mentioning that the techiques of [9] producing an exact answer could have been applied to polygon mosaics?
And that our work will include this but also...

Page 1-3

A polygon is a nonempty subset of tiles in a mosaic in which no tile is T_0 and all edges have 0 or 2 dotted lines drawn from their midpoint.

A polygon mosaic is a mosaic that has at least one polygon and in which every tile not in a polygon is T_0.

A messy polygon mosaic is a mosaic that has at least one polygon.

I'd like to dump "suitably connected" entirely because it focuses the reader on things we don't care about.

B

So much notation:

$\mathbb{S}^{(m,n)}$, $\mathbb{M}^{(m,n)}$, $\mathbb{F}^{(m,n)}$, $\mathbb{L}^{(m,n)}$

How much is <u>necessary</u>? How much is <u>helpful</u>?

(**) The first sentence of Section 2 should introduce framed binary lattice since $f: \mathbb{M} \to \mathbb{F}$ (not $f: \mathbb{M} \to \mathbb{L}$ except inasmuch as $\mathbb{F} \subseteq \mathbb{L}$)

(***) ~~For a framed binary lattice ℓ containing no cells $\begin{smallmatrix}01\\10\end{smallmatrix}$ or $\begin{smallmatrix}10\\01\end{smallmatrix}$,~~

Notice that $f^{-1}(\ell)$ does not make sense since f is not one-to-one.

"For a messy mosaic, applying $g \circ f$ gives the polygon mosaic formed by replacing each tile not in a polygon by T_0. For example, applying $g \circ f$ to the messy mosaic on the left of Fig 6 gives Fig 7, except that all tiles with dots are replaced by T_0."

$\boxed{C}$

Generally, there's more notation & definitions than helpful. For example: define v and not u. If u is needed, just talk about $|u(\ell)|$.

Not sure, but do we need <u>both</u> g and P? What if we defined a single function that would be, in the current notation, $P \circ g$?

(****) In proving Prop. 3.2, I'm unconvinced in Step b. You seem to be changing <u>all</u> 0s to 1s at once (note the reference to ℓ_a and ℓ_b) while also moving one step at a time. Very confusing. Then just generally I'm unconvinced. If we're leaving things to the reader, that should be more explicit — what are we asking them to check by cases? It helps to mention there are six cases in step b.

(Also in proof: define "flip" better?)

(I think the overall framework can be tweaked?? but I'll wait until the next draft.)

$\boxed{D}$

I continue to strongly feel that the proof of Prop 3.3 is needlessly complicated. I think you're proving the inclusion-exclusion principle in an awkward way, whereas we can simply state the principle (with or without proof) and then apply it to get something that emphasizes mosaics and not counting principles.

In particular, I'm struggling why we care about $\Sigma\, u(\ell)$ at all. Why calculate something we don't need or use?

$\boxed{E}$

A nice proposition to tie things together:

Given a Mosaic $\mathfrak{M}$,
$\mathrm{vof}(\mathfrak{M}) = \ldots$

Regarding the discussion at the bottom of page 5: I already had discomfort with this text because I don't think the discussion of u in this way is helpful or informative. By the time I reached Prop. 4.1 (page 11) I realized another problem — other than u(cell) or v(cell), I implicitly got from text & examples that $u(\ell)$, $v(\ell)$ is only for framed lattices. E.g. the discussion p.5 of $f^{-1}(\{\ell\})$ is not explicitly for a framed lattice ℓ, causing additional confusion to me (and probably more to an average reader).

$\boxed{F}$

Why are we calling these matrices "state matrices"? If we can't explain it to the reader, we shouldn't use the term. I've never really understood what "state" we're talking about.

Statement of Prop 4.1:

How about using the formatting used in the proof:

"The (i,j) entry of A_n [resp. B_n, C_n, D_n] equals

$v\begin{pmatrix} 0 & \beta_{n-1}(i) & 0 \\ 0 & \beta_{n-1}(j) & 0 \end{pmatrix}$ [resp. $v\begin{pmatrix} 1 & \beta_{n-1}(i) & 0 \\ 0 & \beta_{n-1}(j) & 0 \end{pmatrix}$,

$v\begin{pmatrix} 0 & \beta_{n-1}(i) & 0 \\ 1 & \beta_{n-1}(j) & 0 \end{pmatrix}$, $v\begin{pmatrix} 1 & \beta_{n-1}(i) & 0 \\ 1 & \beta_{n-1}(j) & 0 \end{pmatrix}$]."

The first paragraph of the proof can also then be much shorter — just observe that $\beta_0(0)$ is the empty string.

$\boxed{G}$

I'm struggling to understand the last two paragraphs of the proof of Prop 4.1, in part because ℓ is not clearly defined and changes! How about:

~~"Since A_n has size $2^{n-1} \times 2^{n-1}$ the (i,j) element of B_n gets multiplied by $v\begin{pmatrix}0&0\\0&1\end{pmatrix}$ to form the $(i, j+2^{n-1})$ element of A_{n+1}. That is~~

(After the existing equation giving A_{n+1})

"Since A_n has size $2^{n-1} \times 2^{n-1}$, the $(i, j+2^{n-1})$ element of A_{n+1} equals $v\begin{pmatrix}0&0\\0&1\end{pmatrix}$ times the (i,j) element of B_n. By our induction hypothesis, this equals

$$v\begin{pmatrix}0&0\\0&1\end{pmatrix} v\begin{pmatrix}0 & B_{n-1}(i) & 0\\ 1 & B_{n-1}(j) & 0\end{pmatrix}$$

$$= v\begin{pmatrix}0 & 0 & B_{n-1}(i) & 0\\ 0 & 1 & B_{n-1}(j) & 0\end{pmatrix} = v\begin{pmatrix}0 & B_n(i) & 0\\ 0 & B_n(j+2^{n-1}) & 0\end{pmatrix},$$

as desired. Similar arguments ~~(maybe show one more??)~~ show that the other blocks in A_{n+1} have the desired values, and then these arguments must be ~~repeated~~ adapted to prove the desired results for B_{n+1}, C_{n+1}, and D_{n+1}, but all the work is similar. (Maybe do one more e.g. the lower-left corner of D_{n+1}.)

$\boxed{H}$

The statement of Prop 4.2 is wrong. The fact that I read through the statement <u>and</u> the proof without realizing it is a clear sign that the proof needs work. The issue as I later <u>realized</u> is that the ~~a~~ left and right edges of each lattice in L must be all zeroes.