11-3-1

<u>New:</u> Binary "grid" in order not to assume first or last entry is zero.

Def

For a nonnegative integer $m < 2^t$, let $\beta_t(m)$ be the length t binary string representing m. (Thus if $m < 2^{t-1}$ then there is at least one leading zero.)

Notice that for $m < 2^t$, $\beta_{k+2}(2m)$ is the same string as $\beta_k(m)$ except with one extra zero at the beginning and one at the end, whereas $\beta_{k+2}(2^{k+1+2m})$ is the same string as $\beta_k(m)$ except with an extra 1 at the beginning and an extra 0 at the end.

Note: the indices of a matrix begin with 0, not 1, so an $m \times n$ A has entries $A_{i,j}$ where $0 \le i < m$ and $0 \le j < n$.

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~~Defn~~

For nonnegative integers k, define $A(k)$, $B(k)$, $C(k)$, and $D(k)$ to be the $2^k \times 2^k$ matrices ~~~~ whose entries are defined to be the valuation of the $2 \times (k+2)$ binary grid (with $k+1$ cells) that are ~~~~ given as follows

	Top Row	Bottom Row
$A(k)_{i,j}$	$\beta_{k+2}(2i)$	$\beta_{k+2}(2j)$
$B(k)_{i,j}$	$\beta_{k+2}(2i)$	$\beta_{k+2}(2^{k+1}+2j)$
$C(k)_{i,j}$	$\beta_{k+2}(2^{k+1}+2i)$	$\beta_{k+2}(2j)$
$D(k)_{i,j}$	$\beta_{k+2}(2^{k+1}+2i)$	$\beta_{k+2}(2^{k+1}+2j)$

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Proposition

1) $A(0) = (7)$, $B(0) = (1)$, $C(0) = (-1)$, and $D(0) = (1)$.

~~2) (For all nonnegative integers k~~
~~3) In the following we use 0 for the $2^k \times 2^k$ matrix of all zeros: (for all nonnega~~

2) For all nonnegative integers k,

$$A(k+1) = \begin{pmatrix} 7A(k) & B(k) \\ C(k) & D(k) \end{pmatrix}$$

$$B(k+1) = \begin{pmatrix} -A(k) & B(k) \\ 0 & D(k) \end{pmatrix}$$

$$C(k+1) = \begin{pmatrix} A(k) & 0 \\ C(k) & D(k) \end{pmatrix}$$

$$D(k+1) = \begin{pmatrix} A(k) & -B(k) \\ C(k) & 7D(k) \end{pmatrix}$$

where in these equations 0 is the $2^k \times 2^k$ matrix of all zeros.

proof
Needed

$\Rightarrow$ I think that for $k \geq 1$, my $A(k)$ is the transpose of what you write as $A^{(1,k+1)}$.

11-3-4

<u>Prop</u>

For all nonnegative integers k and pos integers m, the i,j entry of $[A(k)]^m$ equals the sum of the valuations of all $(m+1) \times (k+2)$ binary grids with top row $\beta_{k+2}(2i)$ bottom row $\beta_{k+2}(2j)$ and all zeros in the first and last column.

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$\boxed{\text{Not really helpful, so we can ignore this....}}$ ☆

Prop

For all nonnegative integers k and positive integers m, the ij entry of $\begin{pmatrix} A(k) & B(k) \\ C(k) & D(k) \end{pmatrix}^m$ is equal to the sum of the valuations of all $(m+1) \times (k+2)$ binary grids with top row $\beta_{k+2}(2i)$ bottom row $\beta_{k+2}(2j)$, and the final entry of every row equal to zero.