ES 3011 C-2018

Lab #2: Modeling Dynamical Systems

INTRODUCTION

In this lab, you and a partner will develop mathematical models representing a mass-spring-damper, a RLC circuit, a car in motion, and a DC motor. These models will then be put into state-space representation which can later be used for simulation and analysis.

Please have the following outline for your report:

1. A few sentences of introduction of the topic of the lab.
2. Answers to each problem with concise explanations on your process in solving and outcome.
3. A paragraph concluding the report explaining the goals, what you learned, and any other conclusions.

MATLAB INTRO

Continuous linear time-invariant (LTI) systems have the following state-space representation:

$$(1) \qquad \dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$$

$$(2) \qquad \boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u}$$

'A' has size (nxn), 'B' has size (nxp), 'C' has size (qxn), and 'D' has size (qxp), however we won't use matrix D in this course. C, the output matrix, is typically an identity matrix which represents state variables to analyze (e.g. motor position, displacement, etc.). 'x' is representative of the state variables we choose such as position and velocity.

Here is an example of how to put system equations into state-space form:

Start with the following equation: $a\ddot{x} + b\dot{x} + cx = du$
Our desired output for control is to be the position, x.

1) Divide over the 'a' to get: $\ddot{x} + \frac{b}{a}\dot{x} + \frac{c}{a}x = \frac{d}{a}u$

2) Set $\boldsymbol{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$ which are the position and velocity state variables

3) Take derivative of $\boldsymbol{x}$**:** $\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$

4) Rearrange the equation in step 1 to put in form: $\ddot{x} = -\frac{b}{a}\dot{x} - \frac{c}{a}x + \frac{d}{a}u$

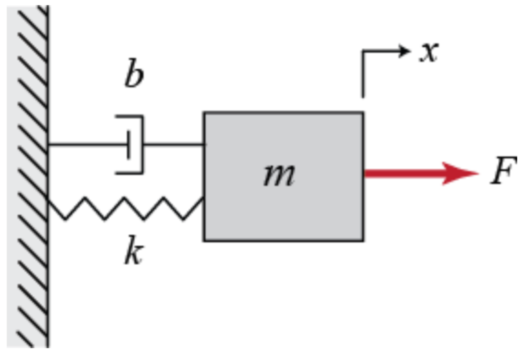
5) Put into state-space form: $\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{a} & -\frac{b}{a} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{d}{a} \end{bmatrix} u$

6) Since we want position, represent desired output as: $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

Note: The state-space equation is made in MATLAB by creating matrices A, B, C, and D and running the command "ss(A,B,C,D)".

MATLAB

I) Mass-Spring-Damper System

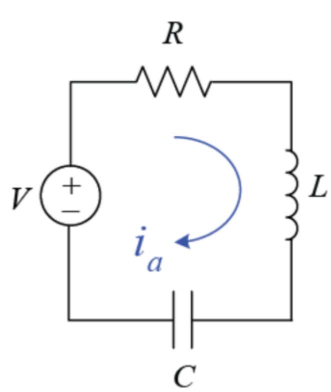


This system has the following properties:

m	Mass	5 kg
k	Spring Constant	1.0 N/m
b	Damping constant	0.5 N.s/m
F	Input Force	2 N

1. Draw the Free Body Diagram for this system.
2. Use Newton's 2nd Law to form the differential equation of this system.
3. Create the State-Space model in MATLAB using the properties above. Include the code and answer in your report.

II) RLC Circuit



This system has the following properties:

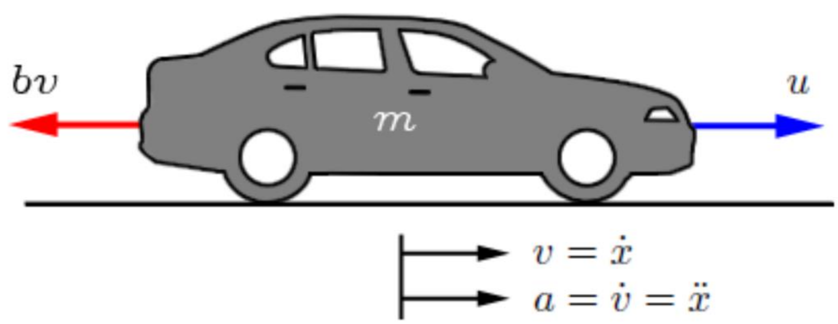
L	Inductance	5 H
C	Capacitance	300 F
R	Resistance	100K Ohm

1. Write the equation that results using Kirchhoff's voltage law.

   Note: $\boldsymbol{Charge}: \boldsymbol{q} = \int \boldsymbol{idt}$

2. Create the State-Space model in MATLAB using symbols with the MATLAB "syms" command. Choose charge on the capacitor and current through the inductor as the state variables. Set the current through the inductor as the output. Include the code and answer in your report.

Cruise-Control Car



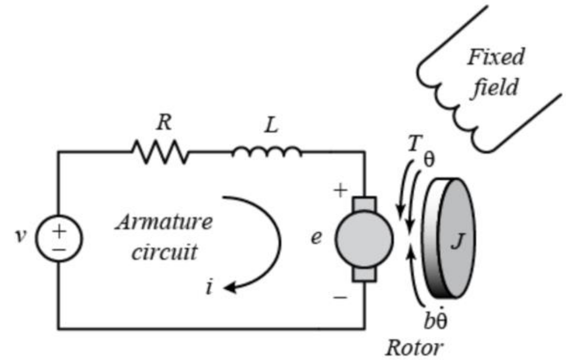
This system has the following properties:

m	Vehicle mass	1500 kg
b	Damping Coefficient	50 N.s/m

1. Write the system equation resulting from this free body diagram of a car.

2. Create the State-Space model in MATLAB using the properties above. Include the code and answer in your report.

MOTOR POSITION



Newton's 2nd Law and Kirchhoff's voltage law gives us these equations:

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

This system has the following properties:

J	Moment of inertia of the Rotor	3E-6 kg.m^2
b	Motor friction constant	3.5E-6 N.s/m
K	Electric Force and Motor Torque Constant	0.025 V/rad/sec
R	Electric Resistance	5 Ohm
L	Electric Inductance	3E-6 H

1. Write the State-Space form in MATLAB with the motor position, motor speed, and armature current as the state variables using the properties above. Let the armature voltage be the input and the rotational position be the output. Hint: Matrix A will be a 3x3, Matrix B will be a 3x1, Matrix C will be a 1x3, and Matrix D will be 0.