Robotic Arm Project Report

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1 Problem Description

The problem concerns programming a robotic arm to unscrew a bolt from its perch and drop the bolt onto a nail.The problem can be broken into approximately four stages:

1. Move the claw from the upright pose to the screwed pose.
2. Grip the bolt with the claw.
3. Move the claw from the screwed pose to the unscrewed pose, thereby unscrewing the bolt.
4. Move the claw from the unscrewed pose to the drop pose while gripping the bolt.
5. Release the bolt from the claw, dropping the bolt onto the nail.
6. Move the claw from the drop pose to the upright pose.

Note that gripping the bolt has relies on the action of a single joint. Thus, we essentially consider the arm to consist of 4 joints and consider the gripping or releasing action of the claw a separate action.

A CoppeliaSim model (to scale) of the arm, bolt, and nail is provided.

2 Problem Approach

We represent in a space frame such that $\hat{z}_s$ points upward from the ground, $\hat{y}_s$ points straight forward on the ground, and $\hat{x}_s$ such that $\hat{x}_s \times \hat{y}_s = \hat{z}_s$.

We then define the body frame such that $\hat{y}_b = \hat{z}_s$, $\hat{x}_b = \hat{x}_s$, and $\hat{z}_b = -\hat{y}_s$. We define the distance between the length between the shoulder and the elbow as ℓ_1, the length of the forearm as ℓ_2, and the length of the claw as ℓ_5.

We define the joint vectors as $\vec{\omega}_0 = \hat{z}_s$, $\vec{\omega}_1 = \hat{x}_s$, $\vec{\omega}_2 = \hat{x}_s$, and $\vec{\omega}_3 = \hat{z}_s$.

For convenience, we define the wrist joint's range such that $\theta_5 = 0°$ denotes the wrist's fully open position and $\theta_5 = 180°$ defines the wrist's fully closed position. Thus, the default upright position can be represented as $\vec{\Theta} = \vec{0}$.

2.1 Desired End Effector Frames

We can now represent the four end effector frames as homogeneous transformation matrices relative to the space frame.

2.1.1 Upright pose

Note that $X_{upright}$ is also the transformation matrix M from the space frame to the body frame. M can be evaluated by simple inspection: since $\hat{x}_b \parallel \hat{x}_s$, $\hat{y}_b \parallel \hat{z}_s$, and $\hat{z}_b \parallel -\hat{y}_s$. Thus,

$$R_{upright} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Since the claw lies $\ell_{all} = \ell_1 + \ell_2 + \ell_5$ units above the origin,

$$p_{upright} = \begin{bmatrix} 0 \\ 0 \\ \ell_{all} \end{bmatrix}.$$

2.1.2 Screwed pose

Note that, relative to the space frame, the desired end effector orientation involves rotating -ϕ_{bolt} about the negative z-axis, so

$$R_{screwed} = \mathrm{Rot}\left(-\hat{z}_s, \phi_{bolt}\right).$$

The position vector can be specified by cylindrical coordinates as

$$p_{screwed} = \begin{bmatrix} r_{screwed} \sin \phi_{bolt} \\ r_{screwed} \cos \phi_{bolt} \\ z_{bolt} - \ell_0 \end{bmatrix}.$$

2.1.3 Unscrewed pose

The unscrewed pose is very similar to the screwed pose; the only difference is a different radial distance to the bolt. Thus,

$$R_{unscrewed} = R_{screwed} = \mathrm{Rot}\left(-\hat{z}_s, \phi_{bolt}\right)$$

and

$$p_{unscrewed} = \begin{bmatrix} r_{unscrewed} \sin \phi_{bolt} \\ r_{unscrewed} \cos \phi_{bolt} \\ z_{bolt} - \ell_0 \end{bmatrix}.$$

2.1.4 Drop pose

The drop pose requires . The orientation of the drop pose can be thought of as rotating about the negative z-axis by ϕ_{nail}, similar to the bolt, and then rotating about the negative x-axis by $\pi/2 - \phi_{drop}$. Thus,

$$R_{drop} = \text{Rot}\left(-\hat{z}_s, \phi_{nail}\right) \text{Rot}\left(-\hat{x}_s, \pi/2 - \phi_{offset}\right).$$

The position vector of the nail can be specified similar to that of the bolt:

$$p_{drop} = \begin{bmatrix} r_{drop} \sin \phi_{drop} \\ r_{drop} \cos \phi_{drop} \\ z_{drop} - \ell_0 \end{bmatrix}.$$

3 Solution Design

We focus first on the desired end effector pose while ignoring the claw joint, deriving first a set of "partial" configurations for the first four joints before deriving the full set of configurations for all five joints.

3.1 Kinematics Formula

From inspection, we can see that

$$\mathcal{S}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathcal{S}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathcal{S}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \ell_1 \\ 0 \end{bmatrix}, \mathcal{S}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

We can then use the product of exponentials formula as

$$X = e^{\mathcal{S}_0 \theta_0} e^{\mathcal{S}_1 \theta_1} e^{\mathcal{S}_2 \theta_2} e^{\mathcal{S}_3 \theta_3} M.$$

We let $\vec{f}\left(\vec{\theta}\right)$ denote the end effector frame flattened into a 16-vector produced by the configuration 4-vector $\vec{\theta}$.

We then use Mathematica to compute the 16-by-4 Jacobian matrix $J = \frac{d\vec{f}(\vec{\theta})}{d\vec{\theta}}$ and its pseudoinverse $J^{\dagger}$.

Beginning with configuration $\vec{\theta}_0 = \vec{0}$, we then can numerically approach the configuration needed for a desired end effector pose represented by 16-vector $\vec{x}_d$ using

$$\vec{\epsilon}_i = \vec{x}_d - \vec{f}\left(\vec{\theta}_i\right)$$
$$\Delta\theta_i = J^{\dagger}\left(\theta_i\right) \vec{\epsilon}_i$$
$$\theta_{i+1} = \theta_i + \Delta\theta_i.$$

We iterate at least until $\|\vec{\epsilon}_i\| < \varepsilon$ where $\varepsilon = 10^{-2}$ is our error threshold.

Pose	θ_0	θ_1	θ_2	θ_3	$\|\vec{\epsilon}_n\|$
Upright	$0°$	$0°$	$0°$	$0°$	0
Screwed	$-32.50°$	$15.95°$	$-105.93°$	$0.00°$	4.18×10^{-3}
Unscrewed	$-32.50°$	$20.87°$	$-110.84°$	$0.00°$	7.32×10^{-3}
Drop	$60.00°$	$-24.33°$	$-125.69°$	0.00	7.79×10^{-3}

3.2 Numerical solutions

We compute the partial configurations for each of four end effector poses after $n = 10$ steps:

We denote these partial configurations as $\vec{\theta}_{upright}, \vec{\theta}_{screwed}, \vec{\theta}_{unscrewed}$, and $\vec{\theta}_{drop}$ respectively.

3.3 Transitions

Steps 1, 4, and 6 can be accomplished without changing θ_3 or θ_5; however, step 3 requires changing θ_3 and steps 2 and 5 require changing θ_5. We find it most convenient to represent such changes as the following vectors:

$$\Delta\vec{\Theta}_{rotateCW} = \begin{bmatrix} 0° \\ 0° \\ 0° \\ -180° \\ 0° \end{bmatrix}, \Delta\vec{\Theta}_{rotateACW} = \begin{bmatrix} 0° \\ 0° \\ 0° \\ +180° \\ 0° \end{bmatrix}, \Delta\vec{\Theta}_{open} = \begin{bmatrix} 0° \\ 0° \\ 0° \\ 0° \\ -180° \end{bmatrix}, \Delta\vec{\Theta}_{close} = \begin{bmatrix} 0° \\ 0° \\ 0° \\ 0° \\ 180° \end{bmatrix}.$$

These configuration vectors can be added to the partial configuration vectors derived in the previous subsection to yield the full desired configurations for each step. We then derive our final set of full configurations:

1. The arm transitions from $\vec{\Theta}_{upright} = \begin{bmatrix} \vec{\theta}_{upright} \\ 0° \end{bmatrix}$ to $\vec{\Theta}_{screwed} = \begin{bmatrix} \vec{\theta}_{screwed} \\ 0° \end{bmatrix}$.
2. The arm transitions from $\vec{\Theta}_{screwed}$ to $\vec{\Theta}_{screwed} + \Delta\vec{\Theta}_{close}$.
3. The arm transitions from $\vec{\Theta}_{screwed} + \Delta\vec{\Theta}_{close}$ to
   (a) $\vec{\Theta}_{screwed} + \Delta\vec{\Theta}_{close} + \Delta\vec{\Theta}_{rotateACW}$ (rotate)
   (b) $\vec{\Theta}_{screwed} + \Delta\vec{\Theta}_{rotateACW}$ (open)
   (c) $\vec{\Theta}_{screwed}$ (rotate)
   (d) $\vec{\Theta}_{screwed} + \vec{\Theta}_{close}$ (close)
   (e) $\vec{\Theta}_{unscrewed} = \begin{bmatrix} \vec{\theta}_{unscrewed} \\ 0° \end{bmatrix} + \vec{\Theta}_{close}$.

4. The arm transitions from $\vec{\Theta}_{unscrewed}$ to $\vec{\Theta}_{drop} = \begin{bmatrix} \vec{\theta}_{drop} \\ 0° \end{bmatrix} + \vec{\Theta}_{close}$.
5. The arm transitions from $\vec{\Theta}_{drop}$ to $\vec{\Theta}_{drop} + \Delta\vec{\Theta}_{open}$.
6. The arm transitions from $\vec{\Theta}_{drop}$ back to $\vec{\Theta}_{upright}$.

4 Results and Analysis

Defining the space frame, body frame, and joint axes as such required transforming the programmed joint inputs to the actual measured values of the arm under the defined frames. The linear scaling employed worked extremely well, matching inputs almost exactly to the observed angles under designated axes.

Although the arm is simple enough that a trigonometric solution could have yielded an exact analytic solution, the numerical method employed here enjoys greater generalizability.

The mechanical limits of the elbow joint required that the upright position was changed to $-90°$. The jerkiness of the servo motion made the movement violent and the task more difficult. However, ultimately, the inverse kinematics approach was successful.