COE352 Project #2

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$$u_t - u_{xx} = f(x,t)$$

$$\int_0^1 \phi(x)(u_t - u_{xx})dx = \int_0^1 \phi(x) f(x,t)dx$$

$$\int_0^1 \phi(x)u_t dx - \int_0^1 \phi(x)u_{xx}dx = \int_0^1 \phi(x) f(x,t)dx$$

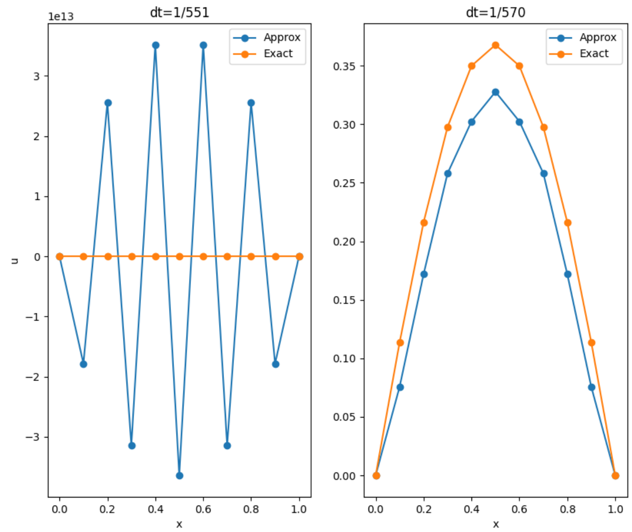
$$\int_0^1 \phi(x)u_{xx} = [u_x\phi(x)]_0^1 - \int_0^1 u_x\phi'(x)dx = (0) - \int_0^1 u_x\phi'(x)dx$$

$$\int_0^1 \phi(x)u_t dx + \int_0^1 u_x\phi'(x)dx = \int_0^1 \phi(x) f(x,t)dx$$

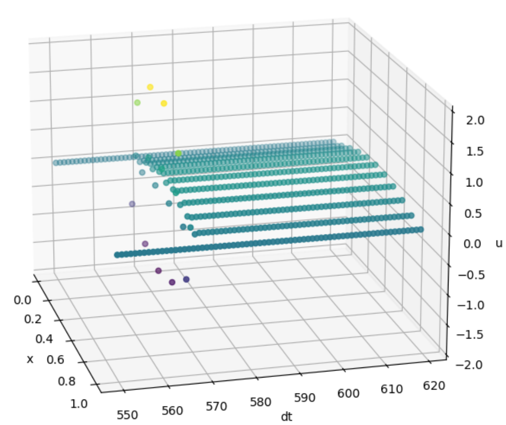
2

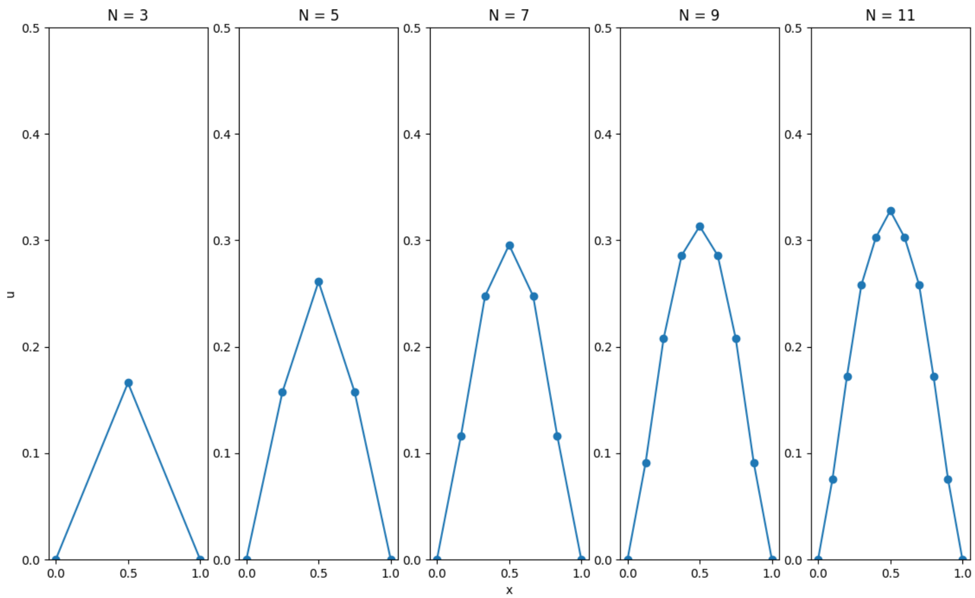
The timestep instability occurs at roughly $\Delta t = \frac{1}{570}$ (shown in the graphs below). The solution becomes more jagged, ”smaller”, and overall less accurate as N decreases.

Approx vs. Exact Solutions for F.E.



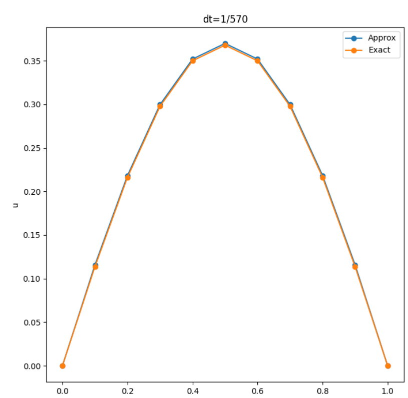
Final u vector vs. x and dt



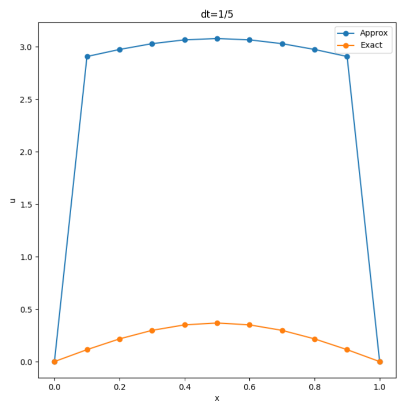


3

The following graph shows the Backward Euler approximation.



If the timestep is equal to or greater than the spatial step size, the approximate solution remains stable, however it can fail to adequately capture the energy flow of the system. This is shown in the graph below, where there appears to be a large addition of energy in the final vector.



This happens because large timesteps can result in large jumps in temperature each step (via numerical approximation inaccuracies). These potentially small errors combine over timesteps to net a significant error in the final timestep.

This is opposed to smaller timesteps which would more gradually model temperature changes and have much less numerical inaccuracies.