# Map Complexity: Comparison and Measurement

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# Map Complexity: Comparison and Measurement

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ABSTRACT. Visual complexity is defined in this paper as the degree to which the combination of map elements results in a pattern that appears to be intricate or involved. A test involving judgments of choropleth and isopleth map complexity yields three conclusions: (1) choropleth maps are consistently perceived as more complex than isopleth maps made from the same data; (2) the relationship between choropleth and isopleth maps can be closely described with a power function; and (3) the number of class intervals has a greater effect on complexity than does the pattern of the distribution mapped.

A measure of complexity applicable to the isopleth map as well as choropleth can be derived, then, using the power function. Three graph theoretic measures are considered as objective measures of choropleth map complexity, with correlations of 0.92 to 0.95 obtained between these measures and subjective complexity of choropleth maps. The graph theoretic measures for choropleth maps are then adjusted according to the power relationship observed between choropleth and isopleth subjective map complexity to yield a complexity measure for the latter map type. Similar procedures might be used to derive complexity measures for other map types.

The subject of thematic map complexity is one that is of wide interest in cartography. One reason for this interest is the assumption that map complexity may have an adverse effect on map effectiveness. While it is likely that, at certain levels, complexity does impede map communication, it is unlikely that all increases in map complexity will result in a corresponding decrease in effectiveness. An examination of the complexity-effectiveness relationship requires an understanding of both map complexity in general and the ways in which it can be measured. Progress toward such an understanding is the goal of the present study.

Map complexity is related to both the nature of the distributions mapped and the symbolization used in representing these distributions. Because cartographers have greater control over symbolization than over the distribution, differences in complexity among methods of symbolization deserve careful attention. Determination of whether complexity differences among symbolization types exhibit consistent patterns is essential before drawing conclusions about the influence of these differences on the effectiveness of the symbolization. Comprehensive analysis of map complexity requires the development of well-defined and repeatable measures of complexity for each form of symbolization considered. The most useful measures would be those that can be applied to a variety of symbolization types. If consistent relationships exist between the subjective complexity of maps using different methods of symbolization, a physical measure of map complexity developed for maps using one method of symbolization could be adapted to scale the complexity of maps using other methods. The specific goals of this study are to examine the relationship between subjective complexity of choropleth maps and shaded isopleth maps (two common methods of thematic symbolization) and, based on this relationship, to develop a physical measure of complexity that can be applied to both forms of symbolization.[1]

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## DEFINITION OF MAP COMPLEXITY

The study of map complexity has been hampered by the lack of a consensus on the definition of the term itself. Although many measures of complexity exist, they do not all measure the same thing. Differing interpretations of complexity have contributed to the difficulty of formulating a conceptual definition of complexity as it applies to maps. There are differences in the concept itself and there are various aspects of the map to which a concept is applied. To understand the variations in the concept of complexity as applied to maps, it is useful to refer to a standard dictionary definition. Because complexity is generally defined as the quality or state of being complex, it is the definition of complex that must be considered.

> Complex: 1. composed of interconnected parts; compound; composite. 2. characterized by a very complicated or involved combination of parts, units, etc. 3. so complicated as to be difficult to understand.[2]

These definitions represent two points of view. The first, presented in definitions 1 and 2, is concerned with the interconnectedness of parts. The second, represented by definition 3, emphasizes ease of understanding. A somewhat different dichotomy of meaning associated with complexity is pointed out by Brophy in his statement that

> . . . visual map complexity is that which is a direct consequence of the spatial differentiation of the graphic content of the map, while intellectual map complexity is that which is due to the meaning or significations contained in or ascribed to the symbolism.[3]

These two aspects of map complexity, visual and intellectual complexity, are likely to influence different stages of the cartographic communication process. Visual complexity should be most important in "map reading," defined by Morrison as those processes involved with perception and/or cognition of the information on the map (detection, discrimination, recognition, and estimation). Intellectual complexity, on the other hand, should have the greatest influence on the subsequent stages of interpretation and analysis (interaction of information received from the map with previously held information and adjustment of a person's cognition of reality as a result of this interpretation).[4]

It is the visual aspect of complexity over which the cartographer has the greatest control and, therefore, to which attention must first be directed. The visual aspect of map complexity is related primarily to pattern geometry within the map itself. Muehrcke probably comes the closest to expressing the essence of this concept in defining map complexity as ". . . the spatial variance in map pattern. . ." measured as ". . . the internal organization or dependence in map pattern."[5]

A third set of distinctions concerning the concept of map complexity is pointed out by Olson.[6] She makes a distinction between complexity as an inherent property of the distribution (dependent upon its pattern geometry but not on visual judgments) and complexity as a visual characteristic of the map (if we judge the map to be complex, it is). Characteristics of the underlying distribution were used by both Muehrcke and Morrison in the creation of a series of maps that varied in complexity while Olson used the map itself in her study of the relationship between measured and perceived complexity.[7] Although there should be a relationship between the complexity of the underlying distribution and the visual complexity of the map, the consistency of the relationship may depend on map type and other factors.

A definition of complexity that focuses on the visual characteristics of the map is adopted for purposes of this study. Map complexity is defined simply as the degree to which the combination of map elements results in a pattern that appears to be intricate or involved. In this sense, map complexity is a subjective impression formed by the map reader. Before attempting to develop a physical measure that approximates visual complexity and that can be adapted to both choropleth and shaded isopleth maps the relationship between subjective complexity for these two kinds of maps will be examined. This relationship will then be used to create an index of complexity for isopleth maps that is based on a measure that is directly applicable only to choropleth maps.

## COMPARISON OF CHOROPLETH AND ISOPLETH COMPLEXITY

Visual patterns of both choropleth and isopleth maps depend on the distribution of the data, the size, shape, and location of enumeration units, the method of data classification, and the boundaries between classes. Differences in pattern between choropleth and isopleth maps results from the assumption of continuity inherent in isopleth maps as well as the nature of the boundaries between symbols. On choropleth maps these boundaries are usually arbitrarily defined political boundaries whose specific location is not necessarily related to the distribution mapped. In contrast, isopleth map boundaries are a more direct function of the underlying distribution. The boundaries on choropleth maps tend to be angular or irregular (which is most noticeable when individual units are large) while boundaries on isopleth maps are smooth. In addition, adjacent areas on choropleth maps can differ in value by several classes (presumably resulting in greater visual contrast across the map) while adjacent areas on isopleth maps are also adjacent in value level.

Based on the above criteria, it is hypothesized that choropleth maps are visually more complex than isopleth maps of the same distribution. At the lowest level of complexity (a one class map) each should have a value of zero. As complexity increases, there is likely to be some level beyond which differences in complexity are no longer apparent to an observer. A curvilinear relationship is therefore hypothesized, with choropleth and isopleth complexity equal at the low and high ends of the spectrum and the most noticeable differences found between these extremes (Fig. 1).

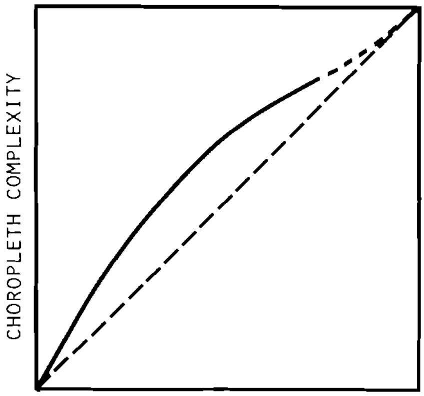


**Figure 1.** Hypothesized relationship between choropleth and isopleth map complexity. The range of each axis is from the minimum possible complexity to some maximum that would normally be found on maps.

### Test Maps

The objectives of the initial stage of this experiment were to test the hypothesis that choropleth maps are more complex than isopleth maps and to generate a psychological scaling of complexity to which physical measures can be compared. Methods for creating a psychological complexity continuum share one requirement: that a large sample of stimuli covering a broad range of values be presented to the subjects. To obtain a sufficient range of stimuli, eighty maps, forty choropleth and forty isopleth, were produced.

All maps used in the test have the same base (a section of contiguous U.S. counties chosen to be representative of areas with irregular boundaries). The base map selected has sixty enumeration units and is a section of a county map of central Kentucky (Fig. 2). The area represented was mapped at an unconventional orientation to minimize the possibility that impressions of complexity will be influenced by familiarity with the area. The mapped area was defined by a rectangular boundary.

The distributions to be mapped were chosen to represent a broad range of typical geographical distributions. Ten distributions representative of a range of

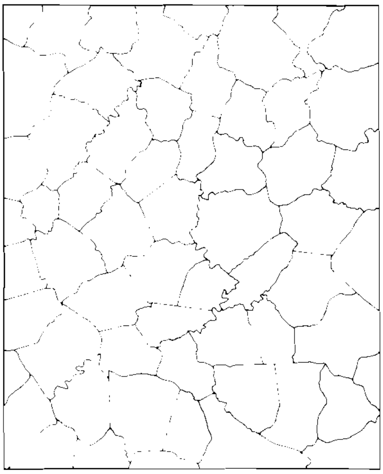

**Figure 2.** The base map.

socio-economic, agricultural, and climatic data were used. Maps with two, three, four, and five classes were produced. Although two-class maps are not commonly encountered, they are included in this study to extend the range of complexity and achieve more accurate scaling.

The maximum number of classes is restricted to five in light of the perceptual "rule" that the average person can easily distinguish only about five shades of gray.[8] Many classification techniques exist by which data may be grouped. In relation to visual map complexity a major difference in these classification systems is that they will result in different numbers of units in each class. To control for this variable, all data were classed in quantiles, thus resulting in an equal number of unit values in each class. Maps were produced without scales or legends.

For each of the ten distributions, two-, three-, four-, and five-class choropleth and isopleth maps were created yielding a total of forty of each kind of map. On choropleth maps, boundary lines between enumeration units with equal values were omitted. Using the visual centroids of the enumeration units as control points, isolines on the isopleth map were drawn using the Surface II Graphics System.[9] The two- through five-class choropleth-isopleth pairs for one of the ten distributions are presented in Figs. 3–6 respectively.

## Methodology

The initial procedure was to determine the subjective complexity of the test maps. This was accomplished by generating a psychological complexity scale to which physical complexity measures for each map could subsequently be compared.

Psychophysical scaling techniques can be categorized as direct or indirect. With indirect scaling, the step between the

**Figure 3.** Two-class choropleth-isopleth map pair.

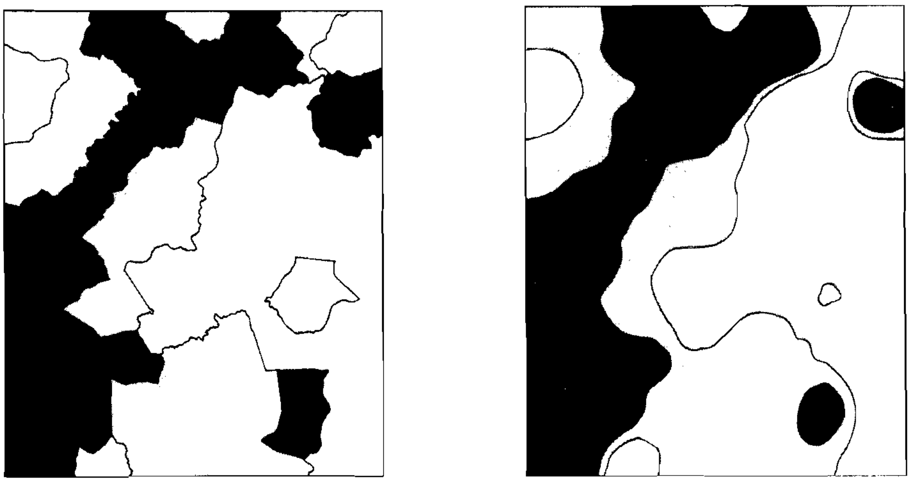

**Figure 4.** Three-class choropleth-isopleth map pair.

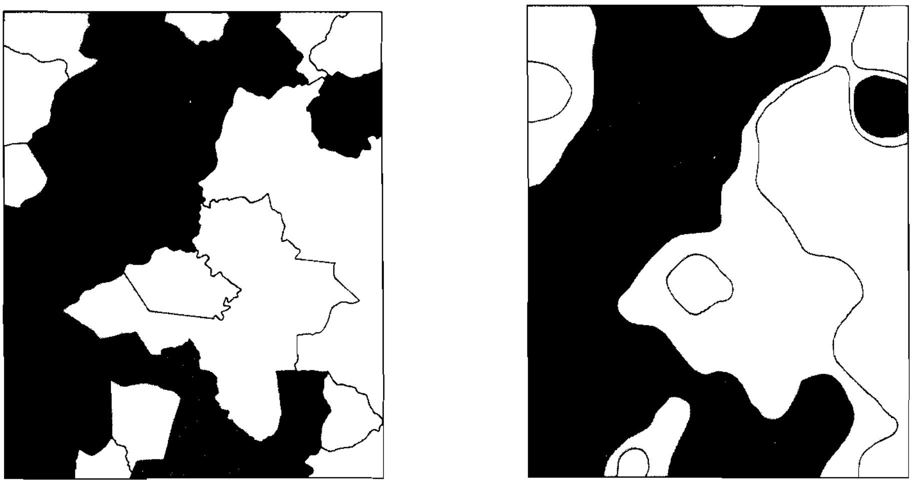

**Figure 5.** Four-class choropleth-isopleth map pair.

**Figure 6.** Five-class choropleth-isopleth map pair.

raw data and the final scale involves the application of a hypothetical psychological dimension to convert the ordinal data to interval scale data.[10] Alternatively, direct scaling minimizes the steps between the raw data and the final scaling by defining, in the instructions to the subjects, the quantitative property to be measured. Operationally, therefore, direct scaling can be more easily applied and should produce more accurate results when a particular characteristic such as complexity is being scaled. The one assumption that must be made is that the respondents are able to rate the complexity of maps quantitatively. Since direct scaling has successfully been applied to a wide variety of perceptual continua (including odors, visual brightness, loudness, gray tones, length of lines, and size of circles) it seems reasonable to accept this assumption here.[11]

Magnitude scaling is generally accepted as the most consistent form of direct scaling.[12] The most direct method of magnitude scaling is magnitude estimation. This method requires observers to match a number directly to the perceived magnitude of each stimulus. Two procedures can be followed in a magnitude estimation experiment. The first is to present the observer with a standard stimulus to which the magnitude of each other stimulus must be related. A drawback of this approach is that the slope of the function relating subjective to objective magnitudes can be influenced by the choice of the standard.[13]

To avoid the bias introduced by an arbitrary standard, an alternative method was developed by Stevens.[14] In this method no standard is presented or prescribed by the experimenter. Observers are simply directed to select the number that they find appropriate for the first and every subsequent stimulus. This procedure is used in the present study.

## Presentation of Test Maps

Sixty-seven introductory geography students were presented with a packet containing copies of the eighty test maps arranged in random order. Subjects were asked to assign values to represent the visual complexity of each map, defined as how intricate or involved the pattern appeared to be. Complete instructions were:

> a. You have each been presented with a set of maps arranged in random order. You will be asked to give me some information about how complex the maps appear to be. Before I explain more specifically what you are to do, I would like you to sort through the maps to get an idea of the range in complexity that exists in the set. By complexity I mean how intricate or involved the maps appear to be. (pause)
>
> b. What you are to do is to indicate how complex, or how complicated or involved, each map appears to be by assigning a number to it. Assign any number that seems appropriate to the first map. Then, one at a time, assign numbers to each additional map that reflect your impression of the map's complexity. There is no limit to the range of numbers that you may use. You may use whole numbers, decimals, or fractions. Try to make each number match the intensity as you perceive it. Keeping in mind that there are no limits to the range of numbers you may use, if a map appears more or less complex than any of those before it, assign a number that indicates this. Do not go back to look at maps you have already rated. Please write the numbers in the upper right-hand corner of each map.

Conversion of the raw data into the subjective complexity scale simply requires standardizing the data and calculating the central tendency for each group of observers. Since the method of value estimation used can result in extreme values, either the geometric mean or the median should be used as a measure of central tendency. According to both Engen and Stevens, distributions of subjective judgments are usually lognormal (i.e., the logarithms of the values form a normal distribution), with the geometric mean the most appropriate measure of central tendency,[15] and hence the measure chosen for use here.

A somewhat artificial source of variance is the variation in the range of numbers that may be used by different individuals. This source of variance is eliminated and the geometric means calculated through the application of a procedure outlined by Engen.[16] It consists of (1) converting each response value to its logarithm, (2) determining the mean value of each row (logarithm of the geometric mean of each observer's responses to all the stimuli), (3) determining the mean of all values obtained in step 2 (logarithm of the grand mean of all the responses for all observers to all stimuli in the original data matrix), (4) subtracting each of the individual mean log responses in step 2 from the grand mean log response determined in Step 3, and (5) adding the value obtained in step 4 to the row values obtained for each observer in Step 1.

### Results

The standardized geometric mean of complexity for each map is plotted in Fig. 7 with isopleth values on the top line and choropleth values on the bottom. Values range from a low of 1.5 for a two-class isopleth map to a high of 11.3 for a five-class choropleth map. Lines in the figure connect values for isopleth and choropleth maps that were created from the same data. Choropleth maps, as hypothesized, are viewed as more complex than isopleth maps. In fact, every choropleth map was judged to be more complex than its corresponding isopleth map. No statistical inference is necessary to demonstrate the significance of this relationship.

Visual complexity appears to be composed of at least two components, spatial variation of mapped data and the number of classes into which data are divided. Little overlap in complexity level judgment occurs among maps with different numbers of classes. This implies that the number of classes makes the greater contribution to complexity judgment. To evaluate the extent to which complexity level judgment is also influenced by the spatial variation in the map, the rank order of four-class maps was compared to that of two-, three- and five-class maps. The evaluation was based on a graphic comparison (Fig. 8) as well as on Kendall's tau rank correlations calculated between the set of four-class maps and the corresponding two-, three-, and five-class maps. The correlation analysis provides a measure of the probability that the similarity in order was random. This probability is

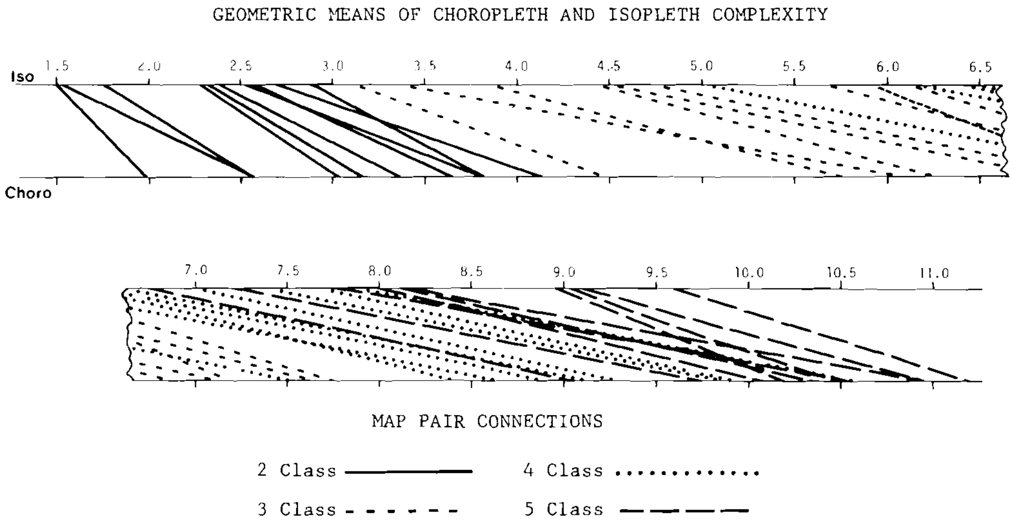


**Figure 7.** Comparison of choropleth and isopleth map complexity. The geometric mean of complexity for each map is plotted with values ranging from a low of 1.5 for a two-class isopleth map to a high of 11.3 for a five-class choropleth map. Lines connect values for choropleth and isopleth maps created from the same data. The character of the lines indicates the number of classes.

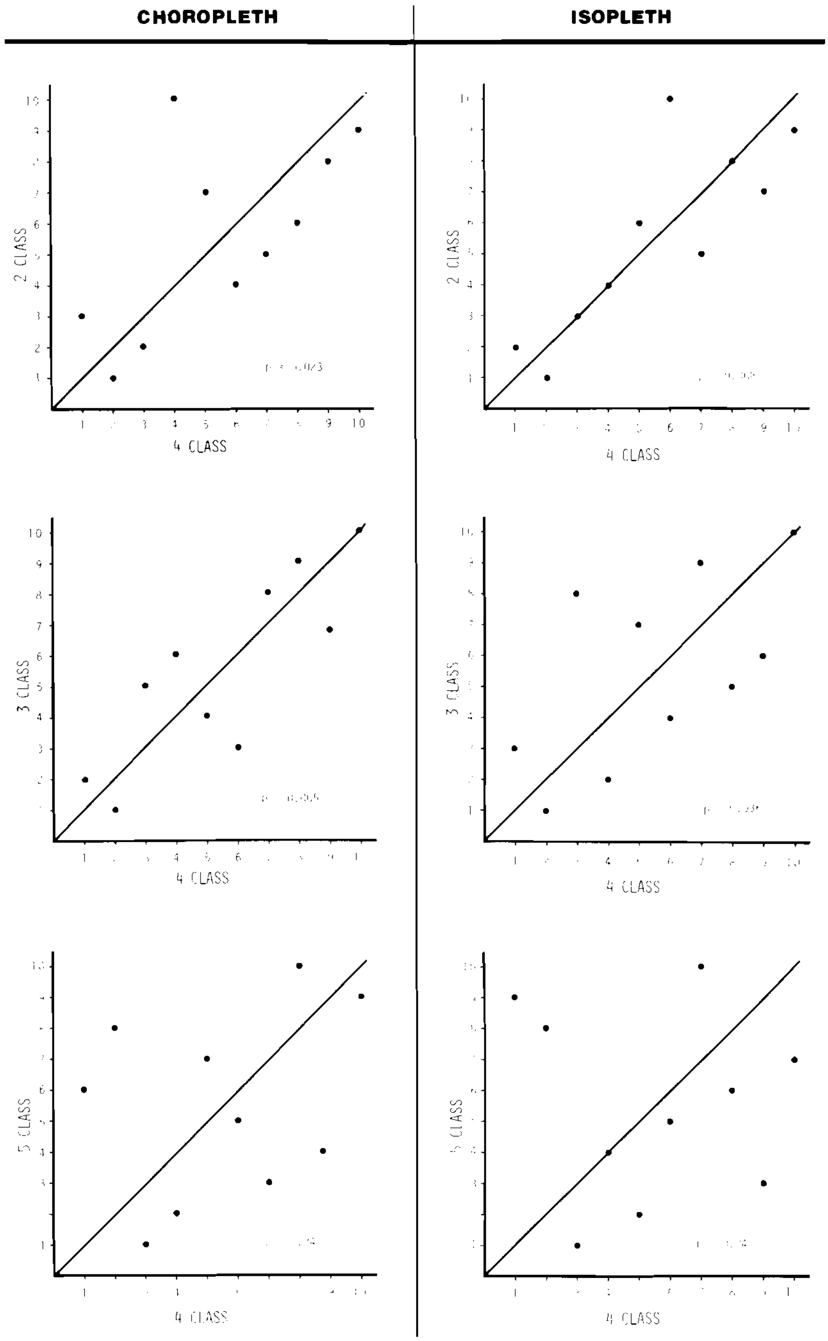


**Figure 8.** Plots of complexity rank order for four-class maps with two-, three- and five-class maps. Values presented with each graph represent the probability that the correspondence in order is random.

indicated on each graph. For both choropleth and isopleth maps, there seems to be a significant correspondence in the order of complexity judgments for distributions mapped with four classes compared to those with two or three classes. Much less correspondence exists between the order of four- and five-class maps. The distribution, therefore, seems to exert an influence on complexity but the influence is eliminated when data are divided into more than four classes.

The direction of the relationship between choropleth and isopleth maps was not the only item of interest; the character of the relationship was examined as well. As hypothesized, this relationship is curvilinear with the greater complexity of choropleth maps being most evident at the middle of the complexity range examined. These results support the contentions that for maps with very low complexity, both choropleth and isopleth maps are equally complex, and there is some level of complexity beyond which complexity differences are no longer apparent.

The visual appearance of the relationship of choropleth to isopleth map complexity suggests that a power function will provide a good fit. This function has the form $X = aY^b$, with $X$ and $Y$ in this case being complexity values for isopleth and choropleth maps respectively, $a$, a constant reflecting the unit of measurement, and $b$, the exponent of the function. This equation can be expressed logarithmically as $\log X = b (\log Y) + \log a$. The advantage of employing the latter formula is that it is expressed graphically as a straight line with $\log a$ as the $y$-intercept and $b$ as the slope of the line (Fig. 10), and the values of $\log a$ and $b$ can be found using ordinary regression. To determine the significance of the relationship, a correlation coefficient, $r$, was calculated for both the original values and the logarithmic values, with results of 0.964 and 0.987 respectively. (The line plotted on Fig. 10 represents the regression equation calculated for the logarithmic values and that in Fig. 9 represents the curve defined by this equation transformed back into a power function.)

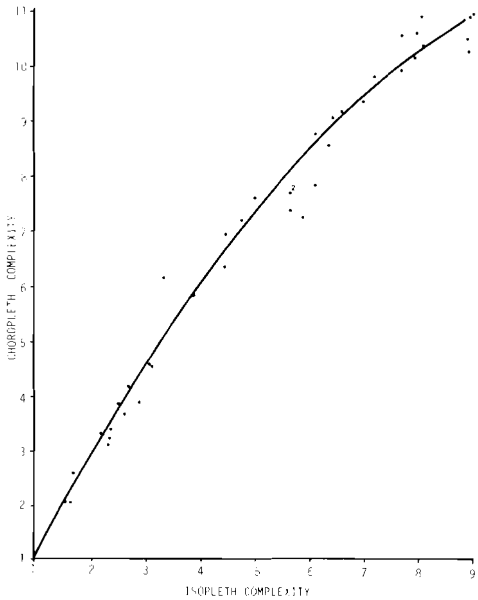


**Figure 9.** Plot of empirical choropleth versus isopleth map complexity.

## OBJECTIVE MEASUREMENT OF COMPLEXITY

The analysis of subjective complexity of choropleth and isopleth maps has demonstrated a very consistent relationship between the complexity of the maps using these two kinds of symbolization. This consistency can now be used in developing a physical measure of complexity that will be adaptable to both kinds of maps. If a physical measure of complexity can be shown to be an adequate approximation of the subjective complexity of either choropleth or isopleth maps, it can be transformed to scale the complexity of the other kind of map as well.

### Review of Existing Measures

A brief review of previously suggested physical measures of map complexity is necessary to provide a background for an

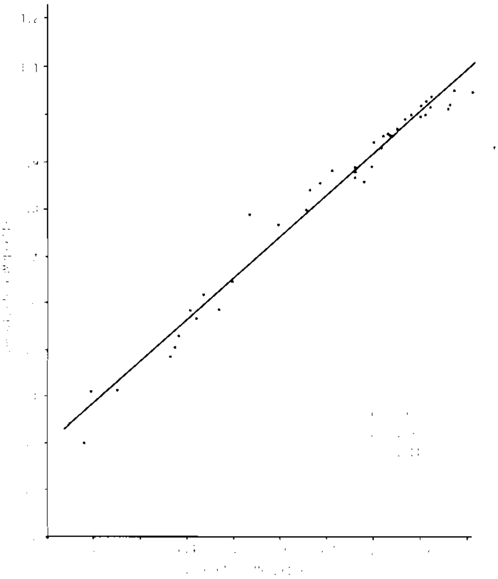

**Figure 10.** Plot of logarithmic values of choropleth and isopleth complexity.

explanation of the measures tested here. Several methods have been suggested for measuring the complexity of either choropleth or isopleth maps. These measures can be placed in two basic categories, *denumerable,* those dealing with individual features of the map (e.g., the number or length of edges), and *structural,* those focusing on the relationship between parts and reflected in some aggregate calculation (e.g., spatial autocorrelation).[17]

McCarty and Salisbury, defining complexity as "analogous to surface roughness as it is encountered in terrain analysis based on readings of topographic maps," outlined a structural measure of isopleth map complexity.[18] Their measure was a composite of: (1) the total number of intersections between a set of diagonal lines and the isopleth lines (diagonal crossings); (2) the total number of highs and lows; (3) the ratio of diagonal crossings to the number of isopleth intervals; (4) the ratio of the sum of highs and lows to the number of intervals; (5) the absolute product of over and under values obtained in the diagonal crossings measure.

Somewhat more attention has been directed to choropleth map complexity. Spatial autocorrelation, a structural measure, was introduced as a measure of choropleth map complexity by Olson.[19] Autocorrelation indicates the degree to which an arrangement of values departs from a random pattern. As applied to choropleth maps, it measures the degree of association between cells (enumeration units) and their neighbors, lagged over distance. Although there are several dimensions to spatial autocorrelation, Olson was able to demonstrate a high degree of correspondence (tau = 0.933) between autocorrelation as represented by an individual measure, Kendall's tau, and rank ordering of map complexity by respondents.

Based on Olson's research, spatial autocorrelation as measured by Kendall's tau appears to be an adequate measure of visual complexity for choropleth maps. There are, however, two limitations to its application. Olson's work is based on square grid cell choropleth maps and the correspondence of the measure to subjective complexity of more realistic maps with irregular boundaries is as yet undetermined. In addition, the Kendall's tau measure is not comparable across varying classification techniques or numbers of classes.[20]

A set of denumerable measures suggested by Muller that are based on graph theory represent an alternative to spatial autocorrelation for measuring choropleth map complexity.[21] After demonstrating that the boundaries on a choropleth map can be decomposed into faces, edges, and vertices, three measures (ratios in each case) were proposed: 1) the number of faces over the total possible faces, 2) the number of edges over the total possible edges, and 3) the number of vertices over the total possible vertices. Muller also suggested that a modification of the edge measure in which the length of edges was substituted for the number of edges would be a more accurate measure of complexity. A related measure, variation in region size, was proposed by Monmonier.[22]

Lavin has compared measures of the various aspects of spatial autocorrelation (including the Kendall's tau measure) with Muller's graph theoretic measures.[23] The comparison demonstrated a high level of redundancy among all measures tested. He concluded that any of these measures would be equally effective in measuring complexity. Although graph theory measures have not been compared to subjective complexity, their apparent redundancy with the Kendall's tau measure of autocorrelation suggests that graph theory measures will also prove to be highly related to the subjective complexity of choropleth maps. Graph theory measures also should not suffer from the major drawback of the Kendall's tau autocorrelation measure: the lack of comparability across classification techniques and number of classes.

Due to their greater flexibility, then, and their ease of computation, the graph theoretic measures were selected for examination as physical measures of map complexity. Graph theoretic measures are suitable only to the measurement of choropleth map complexity. The approach taken, therefore, is to evaluate the applicability of the measures to choropleth maps and then, based on the relationship demonstrated between subjective complexity of choropleth and isopleth maps, to assess the transformation of the measures to the scaling of isopleth complexity.

## Application of Graph Theory to Choropleth Map Complexity

In graph theory a graph is defined as a collection of faces, edges, and vertices. A choropleth base map can be considered a graph in which the faces, edges, and vertices are represented by map units, boundaries between units, and the joining points of these boundaries. The base map used in the present study appears in Fig. 11 as such a graph. In this map there are 60 faces, 175 edges and 116 vertices. Assuming that edges between faces that have equal values are

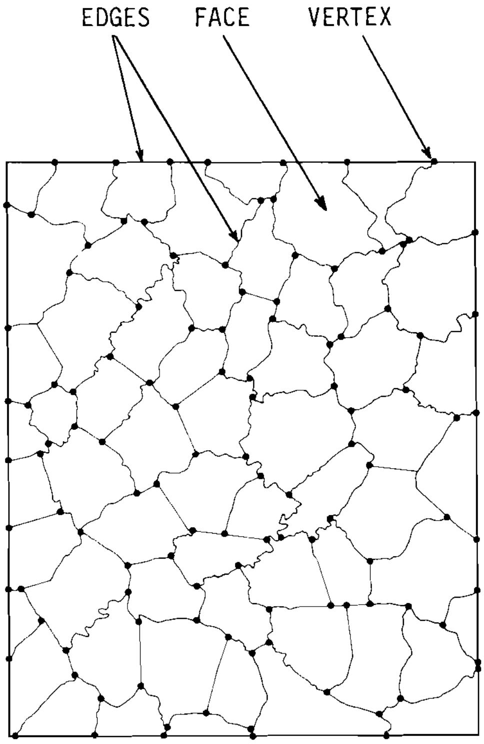


**Figure 11.** Graph transform of the base map.

omitted, the number of faces, edges, and vertices is a function of the size, shape, arrangement, and number of cells (Fig. 12). The more variation there is in the size of areas, the more edges and vertices will usually be required to make up a given number of units or faces. As the distribution of values mapped ranges from a homogeneous arrangement to a dispersed arrangement, the number of faces, edges, and vertices will all increase. Similarly, the less compact the units or the greater the number of units, the more faces, edges, and vertices there will be. The number of data classes also exhibits a pronounced effect on the number of faces, edges, and vertices; an increase in classes results in increased values of these variables (Fig. 13).

The number of faces, edges, and vertices seems to be related to the visual complexity of the map. Realizing this, Muller suggested that ratios of the number of faces, edges, and vertices on a map to the maximum possible numbers

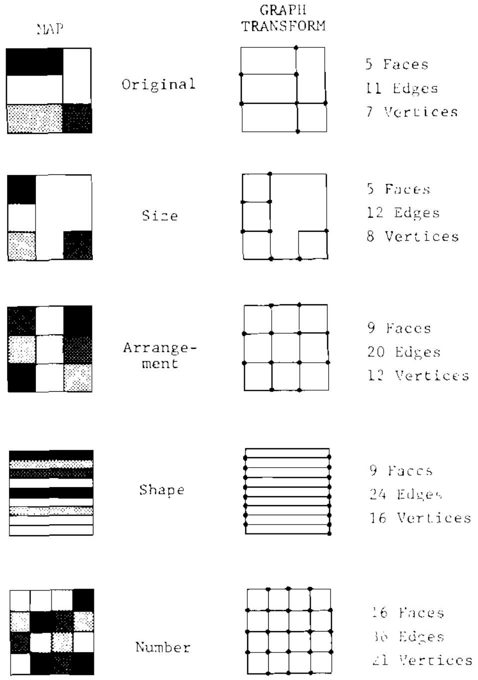


**Figure 12.** Influence of variation in map units on the number of faces, edges, and vertices.

be used as a measure of complexity.[24] In the case of maps with four or more classes, it can be shown that it is possible for every neighboring unit to belong

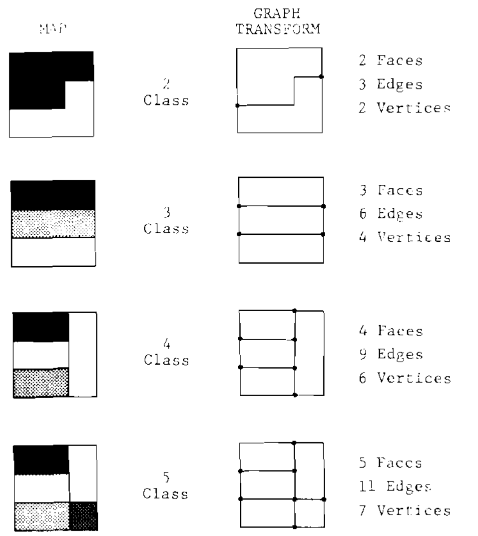


**Figure 13.** Influences of the number of class intervals on the number of faces, edges, and vertices.

to a different class and, therefore, the maximum values are equal to the number of faces, edges, and vertices on the original map. On two- or three-class maps, however, the maximum possible number of faces (therefore edges and vertices) will usually be less than the number of original units.

The deficiency of the graph theory measures as previously used is eliminated by simply using the total number of faces, edges, and vertices in the original base map, rather than the maximum possible number for any given choropleth map, as the denominator in calculating the complexity ratios. The complexity measures for faces and vertices become:

$$\text{Complexity}_F = \frac{\text{observed number of faces}}{\text{number of original faces}}$$

$$\text{Complexity}_V = \frac{\text{observed number of vertices}}{\text{number of original vertices}}$$

Neither the face nor the vertex measure takes into account the size of faces. Simply dividing the observed number of edges by the total of original edges would have the same drawback. To partially account for face-size variation, Muller proposed using the number of original edges that fall between categories on the map rather than the number of observed edges.[25] This measure results in a rough approximation of the length of edges, which tends to reflect face size. Although Muller expressed the opinion that a measure of actual edge length would result in an increase in accuracy over edge number, a comparison of his results for the two measures finds them redundant ($r = 0.96$). Edge length is probably a significant improvement over the number of edges only in cases where there is a large variation in length of the original edges. The complexity measure relating to edges used in the present investigation therefore is:

$$\text{Complexity}_E = \frac{\text{number of original edges left}}{\text{number of original edges}}$$

Each of the measures is calculated for the forty choropleth maps used in the complexity scaling procedure. If the

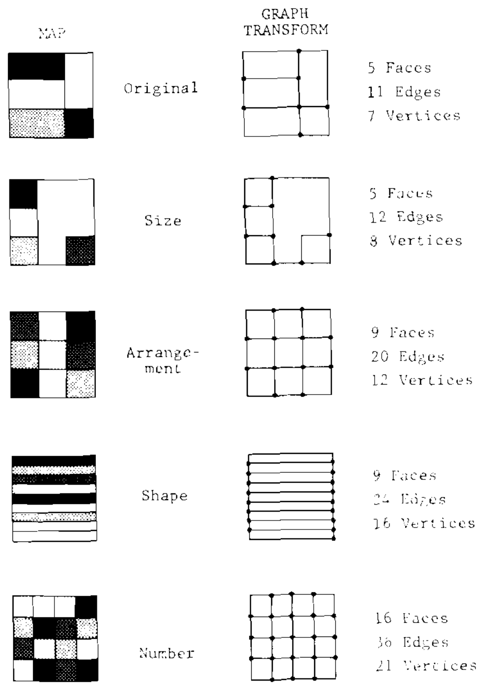


**Figure 12.** Influence of variation in map units on the number of faces, edges, and vertices.

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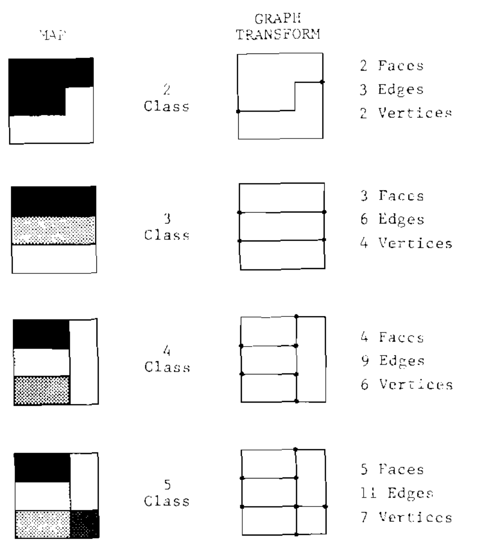


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$$\text{Complexity}_V = \frac{\text{observed number of vertices}}{\text{number of original vertices}}$$

Neither the face nor the vertex measure takes into account the size of faces. Simply dividing the observed number of edges by the total of original edges would have the same drawback. To partially account for face-size variation, Muller proposed using the number of original edges that fall between categories on the map rather than the number of observed edges.[25] This measure results in a rough approximation of the length of edges, which tends to reflect face size. Although Muller expressed the opinion that a measure of actual edge length would result in an increase in accuracy over edge number, a comparison of his results for the two measures finds them redundant ($r = 0.96$). Edge length is probably a significant improvement over the number of edges only in cases where there is a large variation in length of the original edges. The complexity measure relating to edges used in the present investigation therefore is:

$$\text{Complexity}_E = \frac{\text{number of original edges left}}{\text{number of original edges}}$$

Each of the measures is calculated for the forty choropleth maps used in the complexity scaling procedure. If the measures are accurate reflections of choropleth map complexity, a power function can be expected to provide the best description of the relationship between the measures and subjective complexity. Linear regression of the logarithms of subjective complexity with those of the objective complexity measures is used as a means of evaluating the correspondence of each measure with subjective complexity. The regressions result in correlation coefficients of 0.96, 0.97, and 0.98 for the face, vertex, and edge measures respectively. The logarithmic values and regression lines are plotted in Figs. 14-16. In contrast to Muller, but in agreement with Lavin, a high degree of redundancy is found among the three complexity measures ($r$ = 0.94 to 0.97).[26] The high correlation values suggest that no significant improvement in the explanation of variance in subjective complexity would be achieved by a multivariate analysis.

## Adaptation to Isopleth Map Complexity

Each of the three measures tested results in an adequate approximation of the subjective complexity of choropleth maps. What remains, then, is to determine the feasibility of adapting one of the measures to the scaling of isopleth map complexity. Considering the similarity of results, the choice of a measure must be an arbitrary one. The edge measure has the potential for accuracy over a larger range of maps due to its reflection of face-size variation, and, therefore, will be used here in attempting to relate a physical measure to the subjective complexity of isopleth maps.

It has been demonstrated that subjective complexity of choropleth maps is related to the edge ratio measure according to a power function. A power function has also been shown to provide an explanation of the variation in subjective complexity of choropleth and isopleth maps. Utilizing these functional relationships, it is possible to transform the edge number measure to apply to isopleth map complexity. The correlation between the values predicted by this transformation with the values for subjective complexity of isopleth maps is used to evaluate the adequacy of this measure. A correlation coefficient of 0.976 is obtained. The edge number ratio appears to be a reasonable measure of both choropleth and isopleth map complexity, with its use for the latter being indirect.

Although the calculation for isopleth maps depends on information for choropleth maps, this limitation is not as se-

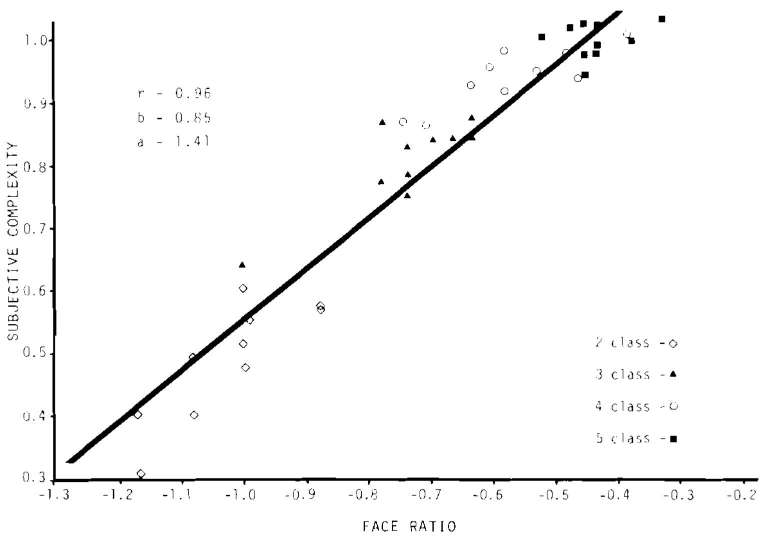


**Figure 14.** Relationship between complexity and the face ratio.

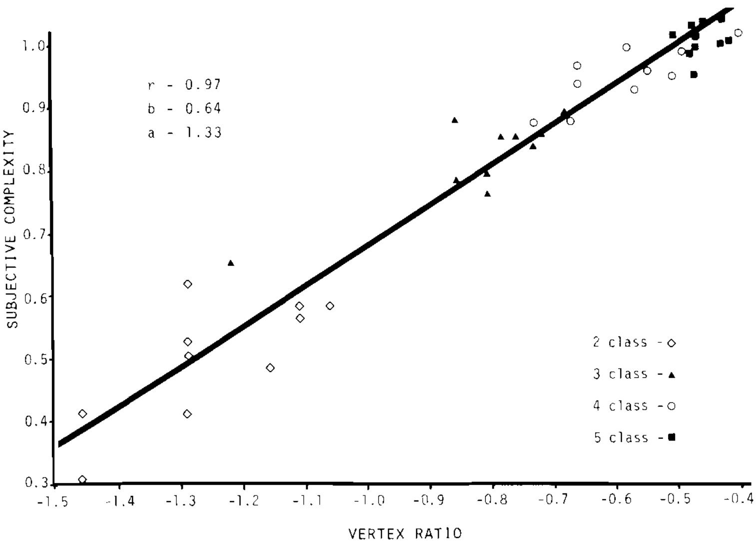


**Figure 15.** Relationship between complexity and the vertex ratio.

vere as it may first appear. If the graph structure of the enumeration units is digitally encoded, the measure is quite simple to calculate. A computer algorithm can be used to determine the

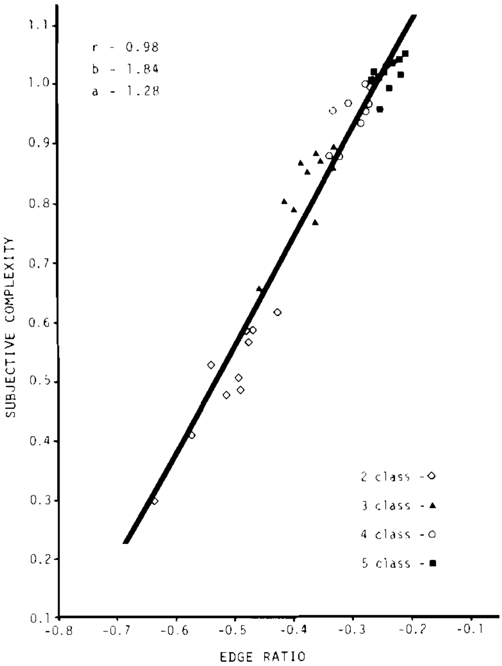


**Figure 16.** Relationship between complexity and the edge ratio.

edges of the original base map (graph) that separate units in different categories. From this number the edge number ratio can be calculated without actually having to produce a choropleth map. This feature would make it practical to use the measure not only in situations in which comparison between choropleth and isopleth complexity is the objective, but as a measure for the evaluation of individual isopleth maps as well.

Use of the measure for choropleth maps is dependent on knowledge of the parameters of the power function relating choropleth complexity to the edge number ratio. Application to isopleth maps is dependent on this knowledge as well as that of the parameters of the power function relating isopleth to choropleth complexity. The particular functions obtained here are likely to be related, at least in part, to the specific characteristics of the sample maps examined. The relationship of choropleth complexity to the edge number ratio, as well as that between choropleth and isopleth complexity, may vary with the number of enumeration units on the map or with different methods of classification that result in unequal numbers of units in each class. General application of the measure to all choropleth and isopleth maps will require examination of these questions.

As presented, the measure has two potential applications. One is to study the influence of map complexity on map effectiveness. For a specific set of test maps, the parameters relating the edge number ratio to choropleth and isopleth maps could be determined. It would then be possible to derive a physical measure (the edge number ratio) of the complexity of each test map. The advantage here is that such a measure is well-defined and repeatable, thus allowing for comparisons with other research findings.

An additional use is for situations in which a series of maps using the same base (e.g., a state) are to be produced. Once the parameters relating the edge number ratio to complexity of choropleth and isopleth maps were determined, complexity of any subsequently produced maps could be measured without recourse to psychological scaling.

## CONCLUSIONS

A functional relationship has been demonstrated between choropleth and isopleth map complexity with choropleth maps, as expected, being judged more complex. A graph theory measure (the edge number ratio) appears to provide an adequate measure of both choropleth and isopleth map complexity, although in the latter instance it is an indirect measure. These results provide the groundwork for examination of the relationship between communication effectiveness and complexity of these two kinds of maps, since the relationship of one variable to another cannot be determined if no measure of the variable exists.

Several aspects of the complexity-effectiveness relationship could be addressed in future research. Most important perhaps is testing the hypothesis that map effectiveness decreases with increases in complexity. While this is probably true in a general sense, the relationship is not likely to be a simple linear one. Before it is possible to evaluate whether or not a map is too complex for its intended purpose, this relationship must be understood. Because the complexity-effectiveness relationship may not be constant across symbolization, an examination of the relationship is necessary for each form of symbolization considered. Although isopleth maps have been found to be visually less complex than choropleth maps, it does not necessarily follow that they are more effective. Furthermore, we cannot ignore the fact that isopleth maps are based on the assumption of smoothness and contiguity of the distribution, and if that assumption is unreasonable an isopleth map simply cannot be used. In addition to differences due to symbolization, it may be found that the influence of complexity on map effectiveness varies with the situation in which the map is used or with different levels of training on the part of the map user.

The complexity measures tested here seem adequate for examining the impact of complexity on effectiveness. Many questions concerning map complexity, however, remain unanswered. One of the more obvious questions is how complexity of maps using symbolization other than choropleth or shaded isopleth compares with that of maps using these two methods. The relationship between complexity of choropleth maps and maps using other symbolization, such as dots or perspective, should be considered. If the relationship is consistent, it may be possible to use, as a surrogate measure, the edge number ratio for choropleth maps with the appropriate transformation. With no consistent relationship, an alternative measure designed specifically for each form of symbolization would be required.

An additional aspect of complexity that deserves further attention was brought to light with the creation of a complexity scale. Complexity appears to be the result of two somewhat independent factors: spatial variation of the mapped data and the number of classes into which data are divided. While the complexity measure developed here combines the two factors, they could prove to have an independent influence on map effectiveness. It may, therefore, be necessary to consider these factors separately.

Because cartographers have greater control over the number of classes, it may be particularly important to consider this aspect of complexity separately. As the number of classes decreases, map complexity decreases. At the same time, however, accuracy and perhaps visual and intellectual interest will drop. There may be a lower limit of complexity below which its decrease is of no consequence (or adverse consequence) to map effectiveness. One role of the cartographer must be to reach for the optimum in complexity rather than the least possible complexity.


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