Question 1)

      Using linear contrasts on the data from the Handicap Study (case0601), the **Bonferroni method** was applied to construct simultaneous confidence intervals for hypothesis tests comparing the following types of mobility handicaps:

1) $\mu_2 = \mu_3$ 2) $\mu_2 = \mu_5$ 3) $\mu_3 = \mu_5$

Where $\mu_2 = mean\ score\ for\ Group\text{:}Amputee$

      $\mu_3 = mean\ score\ for\ Group\text{:}Crutches$

      $\mu_5 = mean\ score\ for\ Group\text{:}Wheelchair$

Statistical software produced the following estimates, p-values, and **confidence intervals** for the group comparisons of interest, which suggest a **significant** difference **only in the first case**:

Parameter	Estimate	Standard Error	t Value	Pr > |t|	97.5% Confidence Limits	
Amputee vs. Crutches	-0.93571429	0.30859610	-3.03	0.0035	-1.64379159	-0.22763698

Parameter	Estimate	Standard Error	t Value	Pr > |t|	97.5% Confidence Limits	
Amputee vs. Wheelchair	-0.28928571	0.30859610	-0.94	0.3520	-0.99736302	0.41879159

Parameter	Estimate	Standard Error	t Value	Pr > |t|	97.5% Confidence Limits	
Crutches vs. Wheelchair	0.64642857	0.30859610	2.09	0.0401	-0.06164873	1.35450588

Question 2)

Multiple comparison procedures verified the following 95% confidence interval half-widths:

Procedure	Score	Procedure	Score	Procedure	Score
LSD	1.233	Dunnett	1.545	Scheffé	1.957
Tukey-Kramer	1.735	Bonferroni	1.794		

The code run to verify the procedural estimates and the output generated are reproduced below:

LSD

Proc Glm Data = Work.Import;

Class Handicap;

Model Score = Handicap;

Means Handicap / LSD CLDIFF;

Run; Quit;

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of t	1.99714
Least Significant Difference	1.2326

Comparisons significant at the 0.05 level are indicated by ***.				
Handicap Comparison	**Difference Between Means**	**95% Confidence Limits**		
Crutche - Wheelch	0.5786	-0.6540	1.8112	
Crutche - None	1.0214	-0.2112	2.2540	
Crutche - Amputee	1.4929	0.2602	2.7255	***
Crutche - Hearing	1.8714	0.6388	3.1040	***
Wheelch - Crutche	-0.5786	-1.8112	0.6540	
Wheelch - None	0.4429	-0.7898	1.6755	
Wheelch - Amputee	0.9143	-0.3183	2.1469	
Wheelch - Hearing	1.2929	0.0602	2.5255	***
None - Crutche	-1.0214	-2.2540	0.2112	
None - Wheelch	-0.4429	-1.6755	0.7898	
None - Amputee	0.4714	-0.7612	1.7040	
None - Hearing	0.8500	-0.3826	2.0826	
Amputee - Crutche	-1.4929	-2.7255	-0.2602	***
Amputee - Wheelch	-0.9143	-2.1469	0.3183	
Amputee - None	-0.4714	-1.7040	0.7612	
Amputee - Hearing	0.3786	-0.8540	1.6112	
Hearing - Crutche	-1.8714	-3.1040	-0.6388	***
Hearing - Wheelch	-1.2929	-2.5255	-0.0602	***
Hearing - None	-0.8500	-2.0826	0.3826	
Hearing - Amputee	-0.3786	-1.6112	0.8540	

Dunnett

Proc Glm Data = Work.Import;

Class Handicap;

Model Score = Handicap;

Means Handicap / Dunnet("None") CLDIFF;

Lsmeans Handicap / ADJUST = Dunnett PDIFF = Control("None");

Run; Quit;

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of Dunnett's t	2.50316
Minimum Significant Difference	1.5449

Comparisons significant at the 0.05 level are indicated by ***.				
Handicap Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
Crutche - None	1.0214	-0.5235	2.5664	
Wheelch - None	0.4429	-1.1021	1.9878	
Amputee - None	-0.4714	-2.0164	1.0735	
Hearing - None	-0.8500	-2.3949	0.6949	

Handicap	Score LSMEAN	H0:LSMean=Control Pr > |t|
Amputee	4.42857143	0.8597
Crutche	5.92142857	0.2918
Hearing	4.05000000	0.4516
None	4.90000000	
Wheelch	5.34285714	0.8836

Tukey-Kramer

Proc Glm Data = Work.Import;

Class Handicap;

Model Score = Handicap;

Means Handicap / Tukey CLDIFF;

Lsmeans Handicap / Adjust = Tukey;

Run; Quit

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of Studentized Range	3.96804
Minimum Significant Difference	1.7317

Handicap	Score LSMEAN	LSMEAN Number
Amputee	4.42857143	1
Crutche	5.92142857	2
Hearing	4.05000000	3
None	4.90000000	4
Wheelch	5.34285714	5

Least Squares Means for effect Handicap
Pr > |t| for H0: LSMean(i)=LSMean(j)

Dependent Variable: Score

i/j	1	2	3	4	5
1		0.1233	0.9725	0.9400	0.5781
2	0.1233		0.0278	0.4686	0.8812
3	0.9725	0.0278		0.6443	0.2348
4	0.9400	0.4686	0.6443		0.9517
5	0.5781	0.8812	0.2348	0.9517	

Comparisons significant at the 0.05 level are indicated by ***.				
Handicap Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
Crutche - Wheelch	0.5786	-1.1532	2.3103	
Crutche - None	1.0214	-0.7103	2.7532	
Crutche - Amputee	1.4929	-0.2389	3.2246	
Crutche - Hearing	1.8714	0.1397	3.6032	***
Wheelch - Crutche	-0.5786	-2.3103	1.1532	
Wheelch - None	0.4429	-1.2889	2.1746	
Wheelch - Amputee	0.9143	-0.8174	2.6460	
Wheelch - Hearing	1.2929	-0.4389	3.0246	
None - Crutche	-1.0214	-2.7532	0.7103	
None - Wheelch	-0.4429	-2.1746	1.2889	
None - Amputee	0.4714	-1.2603	2.2032	
None - Hearing	0.8500	-0.8817	2.5817	
Amputee - Crutche	-1.4929	-3.2246	0.2389	
Amputee - Wheelch	-0.9143	-2.6460	0.8174	
Amputee - None	-0.4714	-2.2032	1.2603	
Amputee - Hearing	0.3786	-1.3532	2.1103	
Hearing - Crutche	-1.8714	-3.6032	-0.1397	***
Hearing - Wheelch	-1.2929	-3.0246	0.4389	
Hearing - None	-0.8500	-2.5817	0.8817	
Hearing - Amputee	-0.3786	-2.1103	1.3532	

Bonferroni

Proc Glm Data = Work.Import;

Class Handicap;

Model Score = Handicap;

Means Handicap / Bon CLDIFF;

Lsmeans Handicap / Adjust = Bon;

Run; Quit;

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of t	2.90602
Minimum Significant Difference	1.7936

Handicap	Score LSMEAN	LSMEAN Number
Amputee	4.42857143	1
Crutche	5.92142857	2
Hearing	4.05000000	3
None	4.90000000	4
Wheelch	5.34285714	5

Least Squares Means for effect Handicap Pr > |t| for H0: LSMean(i)=LSMean(j) Dependent Variable: Score				
i/j	**1**	**2**	**3**	**4**	**5**
1		0.1838	1.0000	1.0000	1.0000
2	0.1838		0.0349	1.0000	1.0000
3	1.0000	0.0349		1.0000	0.4010
4	1.0000	1.0000	1.0000		1.0000
5	1.0000	1.0000	0.4010	1.0000	

Comparisons significant at the 0.05 level are indicated by ***.				
Handicap Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
Crutche - Wheelch	0.5786	-1.2150	2.3721	
Crutche - None	1.0214	-0.7721	2.8150	
Crutche - Amputee	1.4929	-0.3007	3.2864	
Crutche - Hearing	1.8714	0.0779	3.6650	***
Wheelch - Crutche	-0.5786	-2.3721	1.2150	
Wheelch - None	0.4429	-1.3507	2.2364	
Wheelch - Amputee	0.9143	-0.8793	2.7079	
Wheelch - Hearing	1.2929	-0.5007	3.0864	
None - Crutche	-1.0214	-2.8150	0.7721	
None - Wheelch	-0.4429	-2.2364	1.3507	
None - Amputee	0.4714	-1.3221	2.2650	
None - Hearing	0.8500	-0.9436	2.6436	
Amputee - Crutche	-1.4929	-3.2864	0.3007	
Amputee - Wheelch	-0.9143	-2.7079	0.8793	
Amputee - None	-0.4714	-2.2650	1.3221	
Amputee - Hearing	0.3786	-1.4150	2.1721	
Hearing - Crutche	-1.8714	-3.6650	-0.0779	***
Hearing - Wheelch	-1.2929	-3.0864	0.5007	
Hearing - None	-0.8500	-2.6436	0.9436	
Hearing - Amputee	-0.3786	-2.1721	1.4150	

Scheffé

Proc Glm Data = Work.Import;

Class Handicap;

Model Score = Handicap;

Means Handicap / Scheffe CLDIFF;

Lsmeans Handicap / Adjust = Scheffe;

Run; Quit;

Alpha	0.05
Error Degrees of Freedom	65
Error Mean Square	2.666484
Critical Value of F	2.51304
Minimum Significant Difference	1.9568

Handicap	Score LSMEAN	LSMEAN Number
Amputee	4.42857143	1
Crutche	5.92142857	2
Hearing	4.05000000	3
None	4.90000000	4
Wheelch	5.34285714	5

**Least Squares Means for effect Handicap
Pr > |t| for H0: LSMean(i)=LSMean(j)**

Dependent Variable: Score

i/j	1	2	3	4	5
1		0.2238	0.9840	0.9642	0.7007
2	0.2238		0.0682	0.6051	0.9265
3	0.9840	0.0682		0.7545	0.3656
4	0.9642	0.6051	0.7545		0.9715
5	0.7007	0.9265	0.3656	0.9715	

Comparisons significant at the 0.05 level are indicated by ***.				
Handicap Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
Crutche - Wheelch	0.5786	-1.3782	2.5354	
Crutche - None	1.0214	-0.9354	2.9782	
Crutche - Amputee	1.4929	-0.4640	3.4497	
Crutche - Hearing	1.8714	-0.0854	3.8282	
Wheelch - Crutche	-0.5786	-2.5354	1.3782	
Wheelch - None	0.4429	-1.5140	2.3997	
Wheelch - Amputee	0.9143	-1.0425	2.8711	
Wheelch - Hearing	1.2929	-0.6640	3.2497	
None - Crutche	-1.0214	-2.9782	0.9354	
None - Wheelch	-0.4429	-2.3997	1.5140	
None - Amputee	0.4714	-1.4854	2.4282	
None - Hearing	0.8500	-1.1068	2.8068	
Amputee - Crutche	-1.4929	-3.4497	0.4640	
Amputee - Wheelch	-0.9143	-2.8711	1.0425	
Amputee - None	-0.4714	-2.4282	1.4854	
Amputee - Hearing	0.3786	-1.5782	2.3354	
Hearing - Crutche	-1.8714	-3.8282	0.0854	
Hearing - Wheelch	-1.2929	-3.2497	0.6640	
Hearing - None	-0.8500	-2.8068	1.1068	
Hearing - Amputee	-0.3786	-2.3354	1.5782	

Question 3)

    1) Problem

After an initial ANOVA test (HW #5) verified that, for at least two of the groups, there is a statistically significant difference in mean income for those with <12, 12, 13-15, 16, and >16 years of education, a series of multiple comparisons procedures were run to compare every group to every other group (Tukey-Kramer), as well as every other group to the control group of 12 years of education (Dunnett), to test for differences in the respective group means.

    2) Assumptions

As in HW #5, an initial assessment of this study design from the NLSY (which uses random probability sampling to estimate population means) and of the available data indicated that the assumption of **independence** should hold. To assess **normality** and **variance**, statistical software was used to produce distribution and probability histograms and QQ-plots of untransformed and logarithmic transformed data before determining that the total sample size (n = 2584) was sufficiently large and the relative variability sufficiently small to satisfy these assumptions, allowing for some general right-skewedness, and forgoing the logarithmic transformation.

    3) Test Selection and Execution

Firstly, when comparing every group to every other group, the Tukey procedure is appropriate, and in this case, because sample sizes are substantially different for each group (n = 136; 1020; 648; 406; 136; 374), the **Tukey-Kramer** modification is especially suitable for setting the familywise confidence level (95%) and providing the correct intervals. To produce the required output (Figure 1), the following code was run using statistical software:

Tukey-Kramer

Figure 1:

```
Proc Glm Data = Work.Import1;

Class Educ;

Model Income2005 = Educ;

Means Educ / Tukey CLDIFF;

Lsmeans Educ / Adjust = Tukey;

Run; Quit;
```

Alpha	0.05
Error Degrees of Freedom	2579
Error Mean Square	1.92E9
Critical Value of Studentized Range	3.86039

Figure 1 (cont.):

Educ	Income2005 LSMEAN	LSMEAN Number
12	36864.8961	1
13-15	44875.9568	2
16	69996.9729	3
<12	28301.4485	4
>16	76855.4626	5

Least Squares Means for effect Educ Pr > |t| for H0: LSMean(i)=LSMean(j) Dependent Variable: Income2005					
i/j	1	2	3	4	5
1		0.0026	<.0001	0.2031	<.0001
2	0.0026		<.0001	0.0006	<.0001
3	<.0001	<.0001		<.0001	0.1861
4	0.2031	0.0006	<.0001		<.0001
5	<.0001	<.0001	0.1861	<.0001	

Statistically significant differences in pairs of means and their respective confidence intervals and p-values are highlighted in the tables.

Comparisons significant at the 0.05 level are indicated by ***.			
Educ Comparison	Difference Between Means	Simultaneous 95% Confidence Limits	
>16 - 16	6858	-1714	15431	
>16 - 13-15	31980	24212	39747	***
>16 - 12	39991	32760	47221	***
>16 - <12	48554	36577	60531	***
16 - >16	-6858	-15431	1714	
16 - 13-15	25121	17550	32692	***
16 - 12	33132	26113	40151	***
16 - <12	41696	29845	53546	***
13-15 - >16	-31980	-39747	-24212	***
13-15 - 16	-25121	-32692	-17550	***
13-15 - 12	8011	2002	14020	***
13-15 - <12	16575	5293	27856	***
12 - >16	-39991	-47221	-32760	***
12 - 16	-33132	-40151	-26113	***
12 - 13-15	-8011	-14020	-2002	***
12 - <12	8563	-2355	19482	
<12 - >16	-48554	-60531	-36577	***
<12 - 16	-41696	-53546	-29845	***
<12 - 13-15	-16575	-27856	-5293	***
<12 - 12	-8563	-19482	2355	

Secondly, when comparing every other group to the control group of 12 years of education, the **Dunnett** procedure is appropriate to account for the correlation due to the common appearance of the reference group average, setting the familywise confidence level (95%) and providing the correct intervals by using a multivariate t-distribution that accounts for the correlation. To produce the required output (Figure 2), the following code was run using statistical software:

Dunnet   Figure 2:

Proc Glm Data = Work.Import1;

Class Educ;

Model Income2005 = Educ;

Means Educ / Dunnett("12") CLDIFF;

Lsmeans Educ / Adjust = Dunnett PDIFF = Control("12");

Run; Quit;

Alpha	0.05
Error Degrees of Freedom	2579
Error Mean Square	1.92E9
Critical Value of Dunnett's t	2.48068

Comparisons significant at the 0.05 level are indicated by ***.			
Educ Comparison	Difference Between Means	Simultaneous 95% Confidence Limits	
>16 - 12	39991	33420	46561	***
16 - 12	33132	26754	39511	***
13-15 - 12	8011	2551	13472	***
<12 - 12	-8563	-18486	1359	

Educ	Income2005 LSMEAN	H0:LSMean=Control Pr > |t|
12	36864.8961	
13-15	44875.9568	0.0011
16	69996.9729	<.0001
<12	28301.4485	0.1180
>16	76855.4626	<.0001

Statistically significant differences in pairs of means and their respective confidence intervals and p-values are highlighted in the tables.

4)   Interpretation and Conclusion

On the basis of the results of the **Tukey-Kramer** procedure, one would identify a statistically significant difference in means at the ($\propto = .05$) level for all pairwise group comparisons *excluding* those between individuals with **(12 and <12)** and **(16 and >16)** years of education. This would suggest that although graduating (12) or not graduating high school (<12) has relatively little impact on differences in mean income for the populations of concern, there is a significant difference between those with some college study (13-15) and those without (12), and likewise, between those who have graduated college (16) and those who have not (13-15). The other end of the education-level spectrum produced similar results, indicating that there is relatively little difference in mean income between those with (>16) and without (16) advanced degrees.

On the basis of the **Dunnett** procedure, one would identify a statistically significant difference in means at the ($\propto = .05$) level for all pairwise group comparisons between the **control (12)** and every other group *excluding* those with **(<12)** years of education—a determination consistent with the results of the Tukey-Kramer procedure. This would suggest that although there is relatively little difference in group mean income at the bottom of the education-level spectrum, that difference becomes increasingly significant with higher levels of education, with an especially large increase as one moves from the non-graduate (13-15) to the graduate (16) level.

Bonus Question)

1)   Problem

Firstly, with the limited data on mean achievement test scores from the Equity in Group Learning Study (ex0522), a linear contrast was formed to see if the performance of low-ability students increases steadily with the ability of the best student in the group. Assuming equidistant levels of group ability, the contrast estimate and 95% confidence interval were identified using the following group coefficients:

(Low = -3; Low-Medium = -1; Medium-High = +1; High = +3)

Secondly, a **Dunnett** multiple comparisons procedure was run to determine which group composition differences are associated with different levels of test performance, using the (High-ability) group as the control.

2)   Assumptions

Assuming the results were derived from each student being assessed only once, the assumption of **independence** should hold. Though the individual observations were not provided in the data set, visual assessment of the table of group means and standard deviations suggested likewise that the assumptions of **normality** and **variance** should hold also. For verification, scatterplots, histograms, and residual plots were produced using statistical software (Figure 3), providing rough evidence of a **linear trend** amongst the groups **consistent with the contrast estimation**:
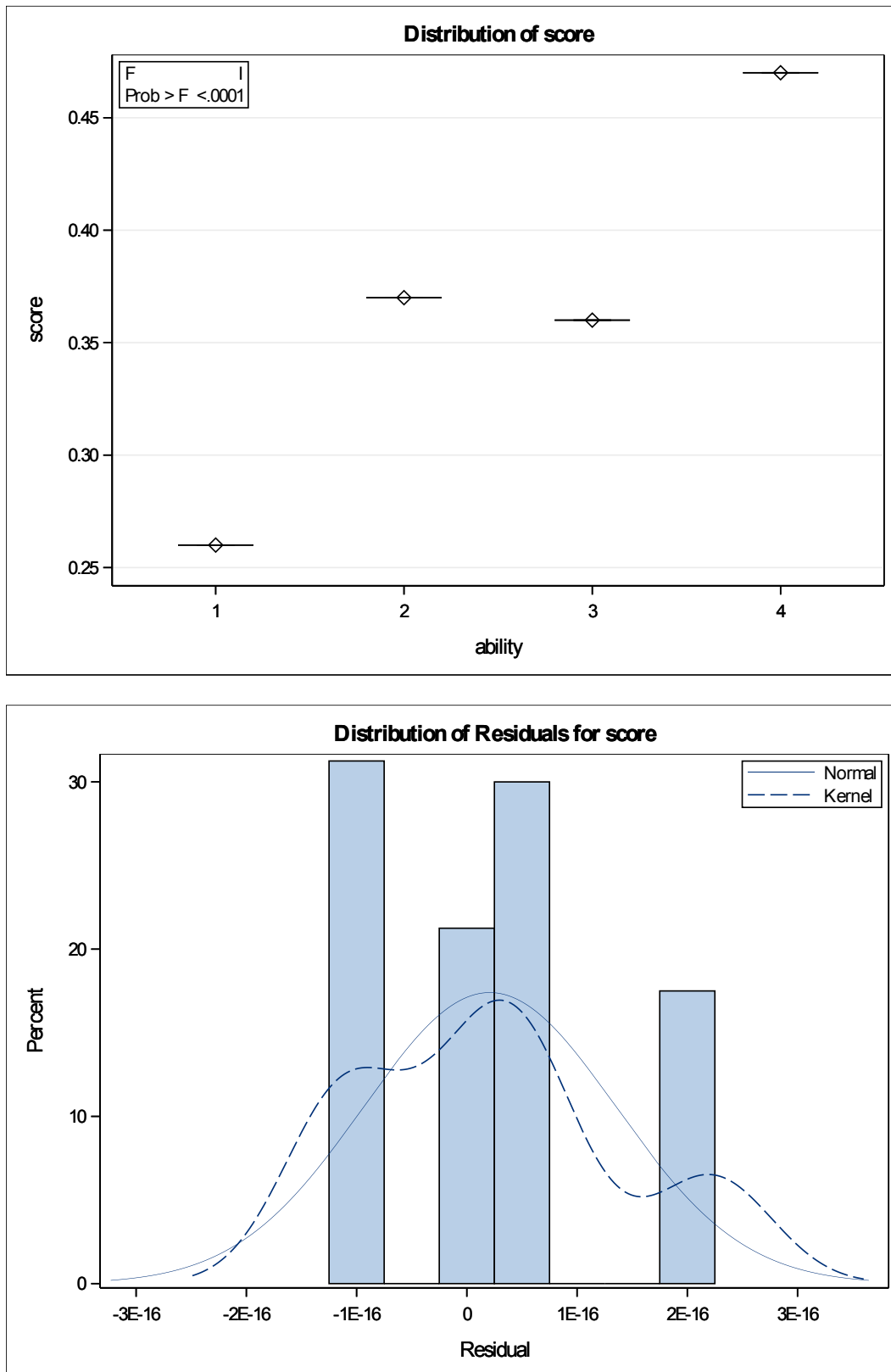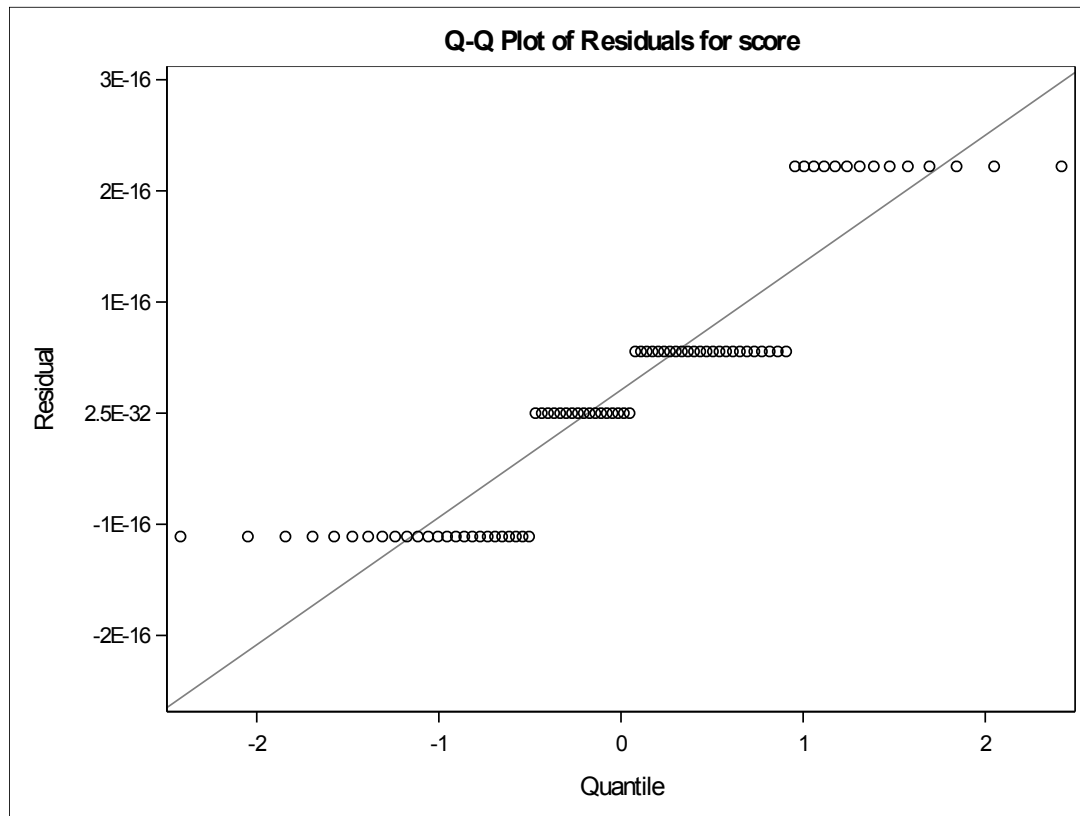
Figure 3:

Distribution of score

Distribution of Residuals for score

Figure 3 (Cont.):

Q-Q Plot of Residuals for score

3) Contrast Estimate and Confidence Interval

Because individual observations were not available for analysis, the mean value for each group was replicated by the number of observations per group (n = 17; 24; 25; 14), and the pooled standard deviation (from s = .14; .21; .17; .21) and contrast estimate was computed by hand:

$$S_p = \sqrt{\frac{(n_1-1)(s_1{}^2)+(n_2-1)(s_2{}^2)+(n_3-1)(s_3{}^2)+(n_4-1)(s_4{}^2)}{(n_1+n_2+n_3+n_4)-4}} = .18478 \qquad\qquad t_{crit,76} \approx 1.99$$

$$g = C_1\bar{x}_1 + C_2\bar{x}_2 + C_3\bar{x}_3 + C_4\bar{x}_4 = -3(.26) + -1(.37) + 1(.36) + 3(.47) = \boxed{.62}$$

$$SE(g) = S_p * \sqrt{\frac{C_1{}^2}{n_1} + \frac{C_2{}^2}{n_2} + \frac{C_3{}^2}{n_3} + \frac{C_4{}^2}{n_4}} = .18478 * 1.276298 = .235829$$

95% CI = .62 $\pm$ (1.99).235829 = **(.1507, 1.0893)** $t_{stat} = \frac{.62-0}{.235829} = 2.6290$

P-Val (One-sided) = **.00518**

Providing evidence of linear trend, the P-value and confidence interval produced by the contrast estimate indicate a statistically significant determination that the performance of low-ability students increases steadily with the ability of the best student in the group.

Secondly, the **Dunnett** multiple comparisons procedure was run with statistical software, using the (High-ability) group as the control, suggesting (Figure 4), on the basis of the limited (replicated) data, that a **statistically significant difference results from pairwise comparisons of every other group with the (High-ability) group**:

Figure 4:

Alpha	0.05
Error Degrees of Freedom	76
Error Mean Square	0
Critical Value of Dunnett's t	2.37207

Comparisons significant at the 0.05 level are indicated by *.**			
ability Comparison	Difference Between Means	Simultaneous 95% Confidence Limits	
2 - 4	-0.1000	-0.1000	-0.1000	***
3 - 4	-0.1100	-0.1100	-0.1100	***
1 - 4	-0.2100	-0.2100	-0.2100	***

ability	score LSMEAN	H0:LSMean=Control Pr > |t|
1	0.26000000	<.0001
2	0.37000000	<.0001
3	0.36000000	<.0001
4	0.47000000	

These results were verified with respect to those produced by a **Tukey-Kramer** procedure identifying figures for **all pairwise group comparisons**, output for which is included below:

Figure 5:

Alpha	0.05
Error Degrees of Freedom	76
Error Mean Square	0
Critical Value of Studentized Range	3.71485

ability	score LSMEAN	LSMEAN Number
1	0.26000000	1
2	0.37000000	2
3	0.36000000	3
4	0.47000000	4

Least Squares Means for effect ability
Pr > |t| for H0: LSMean(i)=LSMean(j)

Dependent Variable: score

i/j	1	2	3	4
1		<.0001	<.0001	<.0001
2	<.0001		<.0001	<.0001
3	<.0001	<.0001		<.0001
4	<.0001	<.0001	<.0001	

Comparisons significant at the 0.05 level are indicated by ***.

ability Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
4 - 2	0.1000	0.1000	0.1000	***
4 - 3	0.1100	0.1100	0.1100	***
4 - 1	0.2100	0.2100	0.2100	***
2 - 4	-0.1000	-0.1000	-0.1000	***
2 - 3	0.0100	0.0100	0.0100	***
2 - 1	0.1100	0.1100	0.1100	***
3 - 4	-0.1100	-0.1100	-0.1100	***
3 - 2	-0.0100	-0.0100	-0.0100	***
3 - 1	0.1000	0.1000	0.1000	***
1 - 4	-0.2100	-0.2100	-0.2100	***
1 - 2	-0.1100	-0.1100	-0.1100	***
1 - 3	-0.1000	-0.1000	-0.1000	***

As a final comparison and verification of results, the achievement test scores of the (**High-ability**) students (**Display 5.25**) were used to run another **Tukey-Kramer** procedure to assess all pairwise group mean differences in that data set, output for which is included below:

Figure 6:

Alpha	0.05
Error Degrees of Freedom	101
Error Mean Square	0
Critical Value of Studentized Range	3.69438

ability	score LSMEAN	LSMEAN Number
1	0.75000000	1
2	0.77000000	2
3	0.72000000	3
4	0.85000000	4

Least Squares Means for effect ability Pr > |t| for H0: LSMean(i)=LSMean(j) Dependent Variable: score				
i/j	1	2	3	4
1		<.0001	<.0001	<.0001
2	<.0001		<.0001	<.0001
3	<.0001	<.0001		<.0001
4	<.0001	<.0001	<.0001	

Comparisons significant at the 0.05 level are indicated by ***.				
ability Comparison	Difference Between Means	Simultaneous 95% Confidence Limits		
4 - 2	0.0800	0.0800	0.0800	***
4 - 1	0.1000	0.1000	0.1000	***
4 - 3	0.1300	0.1300	0.1300	***
2 - 4	-0.0800	-0.0800	-0.0800	***
2 - 1	0.0200	0.0200	0.0200	***
2 - 3	0.0500	0.0500	0.0500	***
1 - 4	-0.1000	-0.1000	-0.1000	***
1 - 2	-0.0200	-0.0200	-0.0200	***
1 - 3	0.0300	0.0300	0.0300	***
3 - 4	-0.1300	-0.1300	-0.1300	***
3 - 2	-0.0500	-0.0500	-0.0500	***
3 - 1	-0.0300	-0.0300	-0.0300	***

On the basis of the above information, there is statistically significant evidence to suggest that all group composition differences are associated with different levels of test performance.