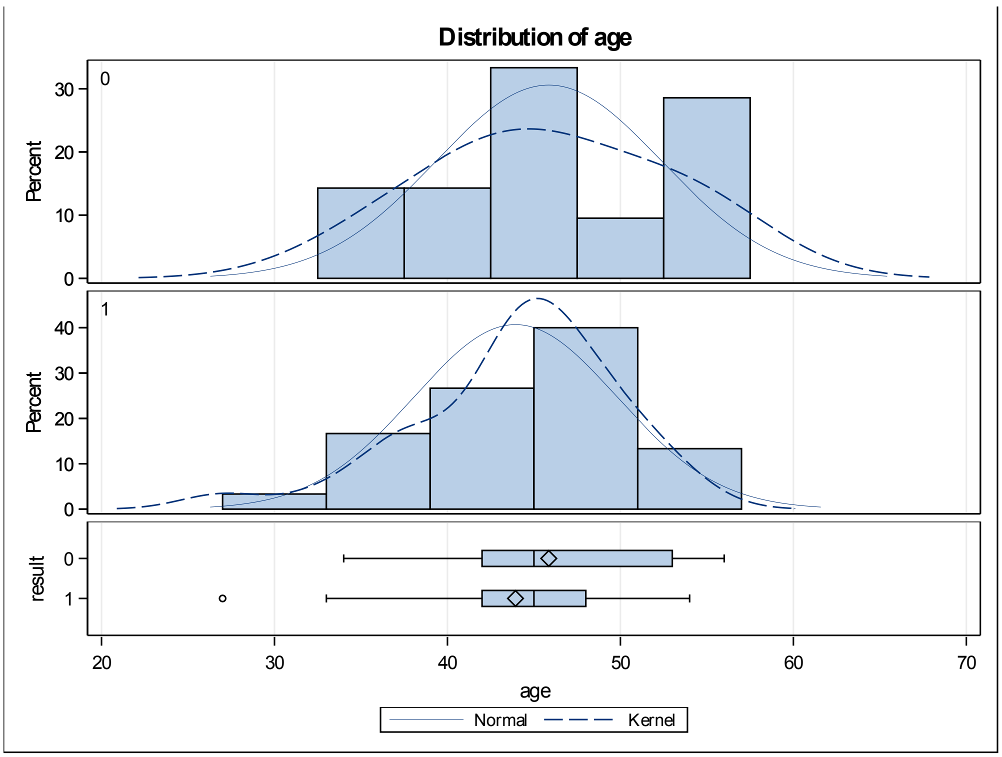
Question 1)

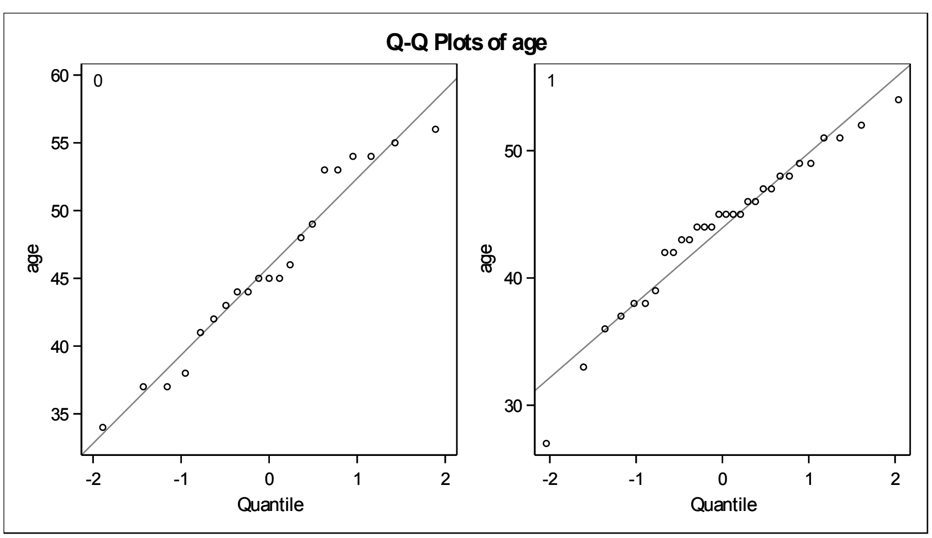
Part A – Assumptions Test

Normality

Judging from the sufficiently large sample size (n = 51) and an assessment of the distributions and QQ-plots for age using proc t-test (Figure 1), this two sample test should be **robust** to deviations from normality.

Figure 1:





Part A – Assumptions Test (cont.)

Independence

On the basis of the verbal phrasing of the study description, which indicates the use of random sampling, the assumption of independence for this two sample test should **hold**.

Variance

On the basis of the results of the initial proc t-test (Figure 2), which indicates roughly equal means and standard deviations, the assumption of equal variance for this two sample test should **hold**.

T-test Assessment

On the basis of the evidence above, the use of a t-test in this case is **appropriate**.

Figure 2:

result	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	21	45.8571	6.5214	1.4231	34.0000	56.0000
1	30	43.9333	5.8835	1.0742	27.0000	54.0000
Diff (1-2)		1.9238	6.1519	1.7503		

Part B – Complete Analysis

1) Problem

To test for age discrimination against the elderly, a one-sided, two sample t-test was run to test the ***null-hypothesis (H_0)*** that: there is **no difference in the mean** age of workers who were fired vs. those not fired:

$(\mu_F - \mu_{NF} = 0)$.

This time, the ***alternative-hypothesis (H_A)*** would be that: the **mean age of those fired is greater** than that of those who were not: $(\mu_F - \mu_{NF} > 0)$.

2) Assumptions

As stated above, an initial assessment of the available data indicated that the **assumptions** of normality, independence, and variance **hold** for this two sample t-test.

3) Test

For a sample size of *n* = 51, the **critical value** at the $\propto =$ .05 level with *df* = 49 would be:
$\mathbf{t_{crit}} = \mathbf{1.678}$

For this given sample, the **test statistic** would be: $\mathbf{t_{stat}} = \frac{\bar{x}_F - \bar{x}_{NF}}{s_p(\sqrt{\frac{1}{n_F} + \frac{1}{n_{NF}}})} \approx \frac{\mathbf{45.8571 - 43.9333}}{\mathbf{6.1519(.28452)}} \approx$

$\mathbf{1.0991}$, which is somewhat far from the critical value.

4) Conclusion

Assuming the null-hypothesis and equal variance **(Pooled)** for this one-sided test, statistical software determined that the **p-value** for such a sample would be **roughly half of the double-sided** p-value of .2771, or: $\mathbf{p_{val} = .1386}$, which is larger than the $\propto =$ .05 threshold.

On this basis, one would **fail to reject** (FTR) the **null-hypothesis** (H_0), determining that: there is **not statistically significant evidence** to suggest that the mean age of those fired is greater than that of those who were not: $(\mu_F - \mu_{NF} > 0)$. →[$\mathbf{p_{val} = .1386}$].

According to the results of the **one-sided**, two sample t-test, the **95% confidence interval** for the difference in means, assuming equal variance (**Pooled**), would be:

$-1.0107 \leq \mu_F - \mu_{NF} \leq \infty$, or: **[-1.01**, **∞]**.

Using the **two-sided** confidence interval, for intuitive clarity, the difference in means, assuming equal variance (**Pooled**), would be:

$-1.5936 \leq \mu_F - \mu_{NF} \leq 5.4413$, or: **[-1.594**, **5.441]**.

Noting that the interval contains zero, this is consistent with the above data indicating there is **no statistically significant evidence of a greater mean age** for those who were fired.

5) Scope of Inference

Given the evidence for this observational study, inferences regarding the difference in mean ages **cannot** be drawn to the population being examined, and as assignment to groups was not randomized, inferences regarding causal relations also **cannot** be drawn.

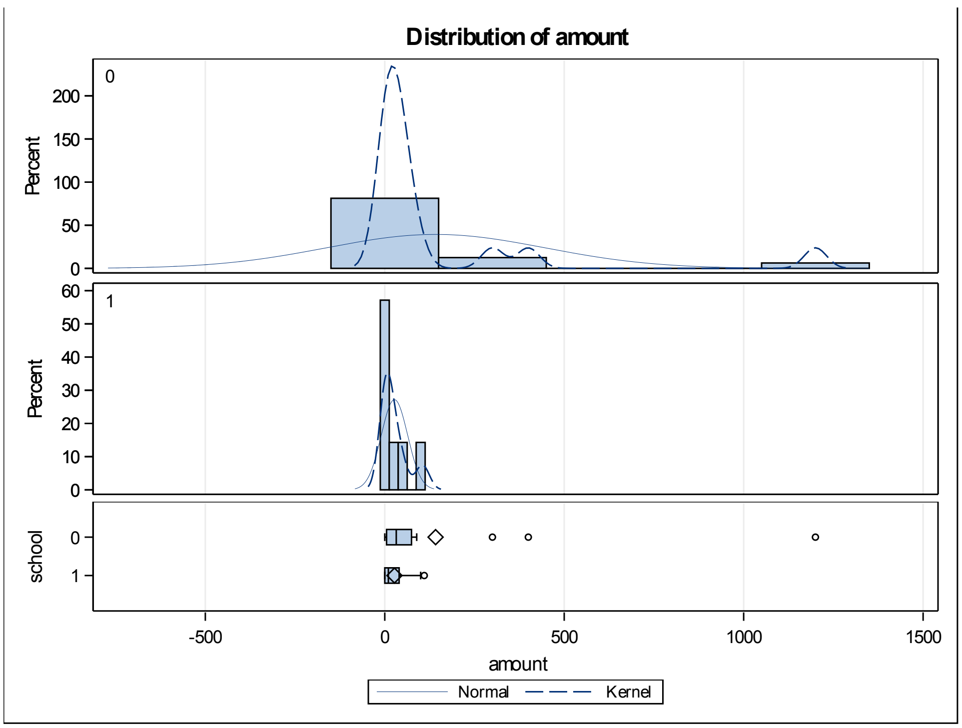
Question 2)

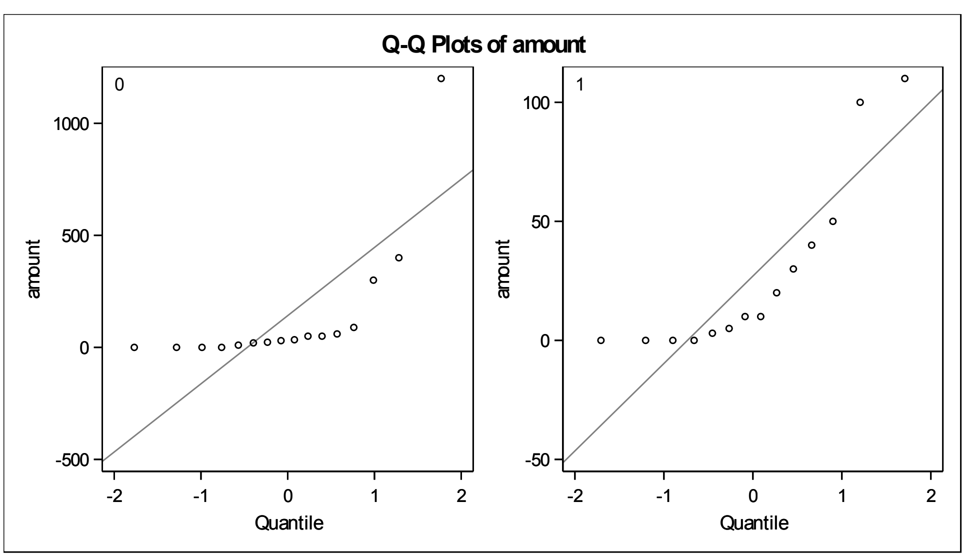
Part A – Assumptions Test

Normality

Although an assessment of the distributions and QQ-plots for dollar amount using proc t-test indicates right-skewed data for both groups (Figure 3), the moderately large sample size (n = 30) should ensure that this two sample test should be **sufficiently robust** to deviations from normality.

Figure 3:





Part A – Assumptions Test (cont.)

Independence

On the basis of the verbal phrasing of the study description, which indicates the use of polling two unrelated groups of Business Stat students from two disparate universities, the assumption of independence for this two sample test should **hold**.

Variance

On the basis of the results of the initial proc t-test (Figure 4), which indicates unequal means and standard deviations (the larger mean having also the larger standard deviation), the assumption of equal variance for this two sample test **fails to hold**. In this case, the **Satterthwaite** measure would be more appropriate.

T-test Assessment

On the basis of the evidence above, the use of a **t-test** in this case is **appropriate, but questionable**. Only on the basis of the sufficiently large sample size (n = 30) and the use of the Satterthwaite measure would a t-test be appropriate in this case. Preferably, a **permutation test** should be used.

As a further frame of reference, **natural-log transformations** of this data (removing records where amount = 0) do, in fact, yield appropriate data for a t-test (Figure 5).

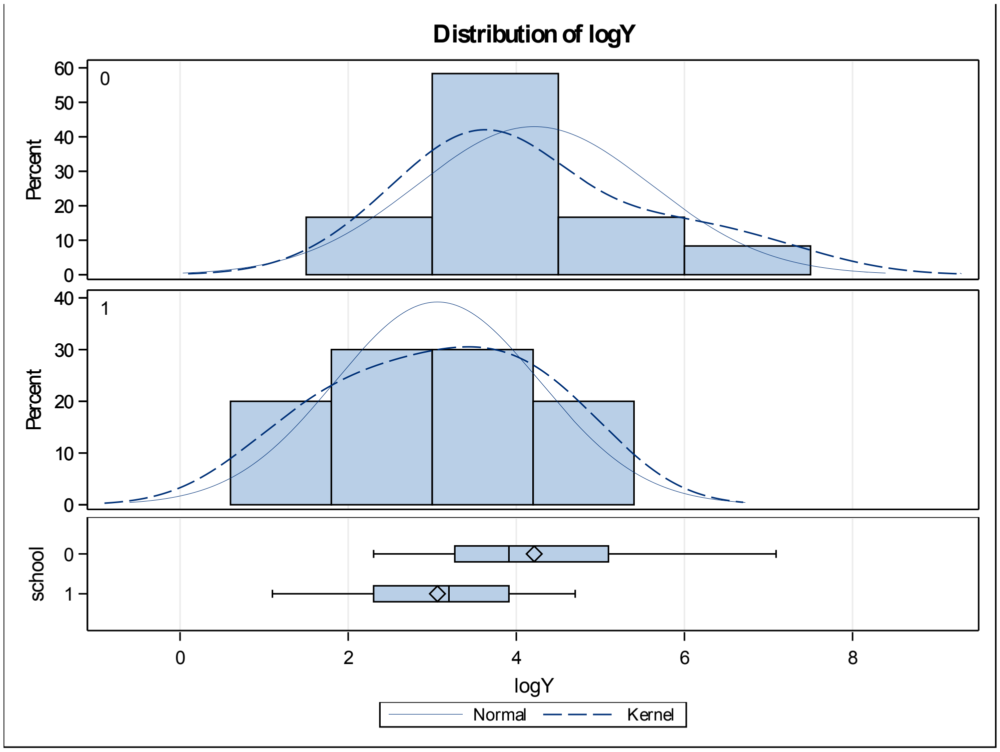
Figure 4:

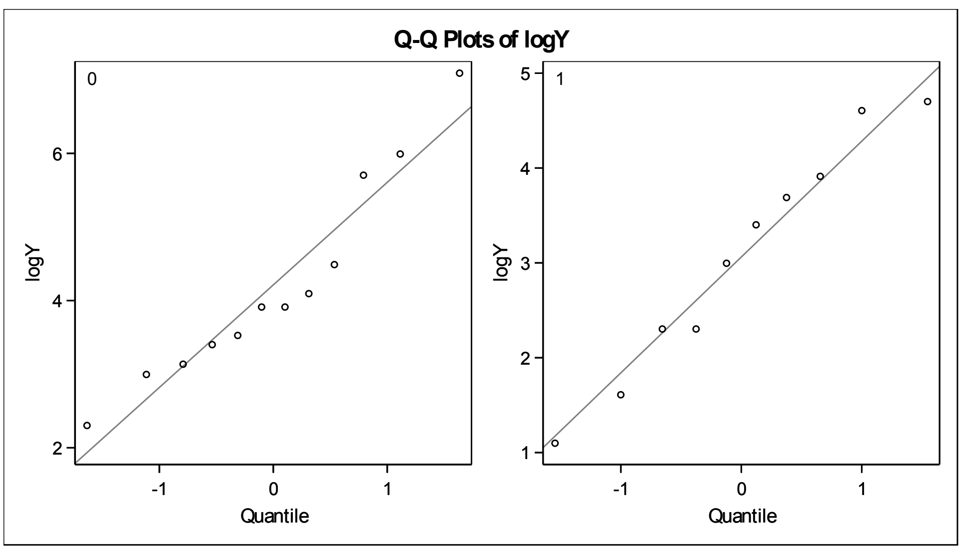
school	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	16	141.6	304.3	76.0670	0	1200.0
1	14	27.0000	36.7193	9.8136	0	110.0
Diff (1-2)		114.6	224.1	82.0131		

Figure 5:

school	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	12	4.2128	1.3934	0.4022	2.3026	7.0901
1	10	3.0617	1.2211	0.3861	1.0986	4.7005
Diff (1-2)		1.1511	1.3186	0.5646		

Figure 5 (cont.):





Part B – Complete Analysis

1) Problem

A two-sided, two sample t-test was run to test the ***null-hypothesis (H_0)*** that: there is **no difference in the mean** amount of pocket cash carried by students at SMU and Seattle U: $(\mu_{SMU} - \mu_{Seattle} = 0)$.

The ***alternative-hypothesis (H_A)*** would be that: there is, in fact, a statistically significant **difference in the mean**: ($\mu_{SMU} - \mu_{Seattle} \neq 0$).

2) Assumptions

As stated above, an initial assessment of the available data indicated that although the samples are independent and the sample size (n = 30) is sufficiently large to ensure robustness to the data's right-skewed deviation from normality, the assumption of equal variance does not hold for this two sample t-test, so the **Satterthwaite measure** should be used.

3) Test

For a sample size of *n* = 30, the **critical value** at the $\propto = .05$ level with *df* = 28 would be:
$\mathbf{t_{crit}} = \mathbf{2.048}$

For this given sample, the **test statistic** would be:

$\mathbf{t_{stat}} = \dfrac{\bar{x}_{SMU} - \bar{x}_{Seattle}}{s_p\left(\sqrt{\frac{1}{n_{SMU}} + \frac{1}{n_{Seattle}}}\right)} \approx \dfrac{\mathbf{141.6 - 27.0}}{\mathbf{224.1(.36596)}} \approx \mathbf{1.397}$, which is rather far from the critical value.

4) Conclusion

Assuming the null-hypothesis and unequal variance **(Satterthwaite)** for this two-sided test, statistical software determined that the **p-value** for such a sample would be $\mathbf{p_{val} = .1551}$, which is substantially larger than the $\propto = .05$ threshold.

On this basis, one would **fail to reject** (FTR) **the null-hypothesis** (H_0), determining that: there is **not statistically significant evidence** to suggest that the mean amount of pocket cash carried by students at SMU and Seattle U is not equal ($\mu_{SMU} - \mu_{Seattle} \neq 0$) → [$\mathbf{p_{val} = .1551}$].

According to the results of the two-sided, two sample t-test, the **95% confidence interval** for the difference in means, assuming unequal variance (**Satterthwaite**), would be:

$-48.395 \leq \mu_{SMU} - \mu_{Seattle} \leq 277.6$, or ≈ **[-48.4, 277.6]**.

Noting that the interval contains zero, this is consistent with the above data indicating there is **no statistically significant evidence of a difference** in mean amount for the groups observed.

a) Further Conclusion Notes

To confirm this result, one-thousand (1000) permutations were run, producing a **permutation p-value** of [$\mathbf{p_{val} \approx .1640}$].

As a further frame of reference: although a t-test on the natural-logarithm transformed data produced a ratio of medians of [**geometric mean** = 3.1618], the p-value (for both this t-test and a similar permutation test) remained above the $\propto = .05$ threshold for statistical significance.

5) Scope of Inference

Given the evidence for this observational study, inferences regarding the difference in mean amount **cannot** be drawn to the population being examined, and as polling responses were voluntary and assignment to groups was not randomized, inferences regarding causal relations also **cannot** be drawn.

Part C – Outlier Checking

The outlier (SMU, 1200) was checked for its influence. Since analyses with and without the outlier produced the same answer to the question of interest, final results were reported including the suspect observation. However, to create a frame of reference, a comparison was still made of the test results with and without the outlier. Although removing the outlier somewhat reduced the right-skewedness of the SMU data, along with its measures of mean and standard deviation (Figure 6), a comparison of the t-test results shows this has negligible effect on p-value, actually increasing its measure from **[.1551]** to **[.1855]** (Figure 7). For these reasons, the existence of the outlier has **no bearing** on the above decision regarding the appropriate t-tools, and the **outlier should be kept**.

Figure 6:

school	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	15	71.0667	117.7	30.3824	0	400.0
1	14	27.0000	36.7193	9.8136	0	110.0
Diff (1-2)		44.0667	88.4804	32.8803		

Figure 7:

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	27	1.34	0.1913
Satterthwaite	Unequal	16.876	1.38	0.1855

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	28	1.40	0.1732
Satterthwaite	Unequal	15.499	1.49	0.1551

Question 3)

1) Problem

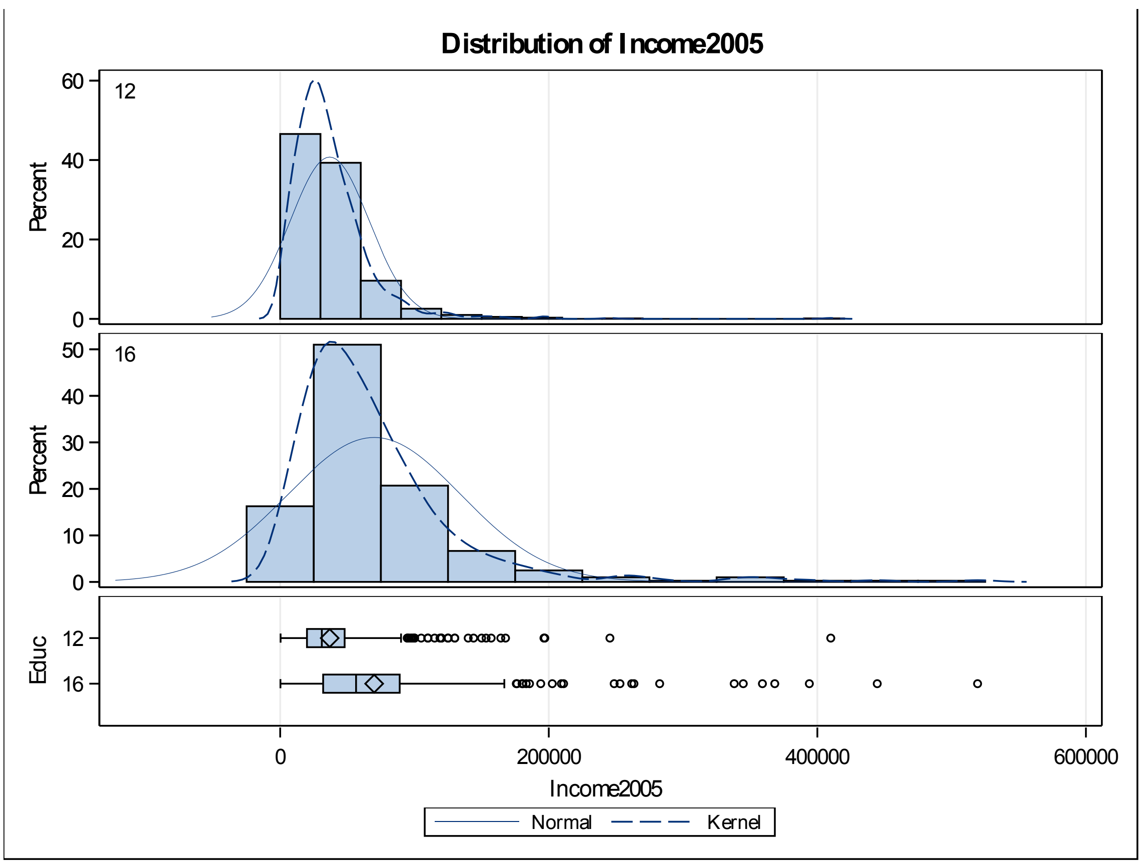
A two-sided, two sample t-test was run to test the ***null-hypothesis (H_0)*** that: the ratio of median incomes of those with 16 and 12 years of education is equal to one: $(M_{16} / M_{12} = 1)$, i.e., that the medians are equal: $(M_{16} = M_{12})$

The ***alternative-hypothesis (H_A)*** would be that: the ratio of median incomes for the groups is, in fact, unequal to one: $(M_{16} / M_{12} \neq 1)$, i.e., that the medians are unequal: $(M_{16} \neq M_{12})$.

2) Assumptions

An initial assessment of the verbal phrasing of the study design and the available data indicated that although the samples are independent, the sample size (n = 1426), though large, is not sufficient to ensure robustness to the data's extreme right-skewed deviation from normality (Figure 8). Likewise, the assumption of equal variance does not hold for this two sample t-test, and since the larger mean also has the larger standard deviation (Figure 9), a **logarithmic transformation** of the data (Figure 10) was used before executing the final t-test.

Figure 8:



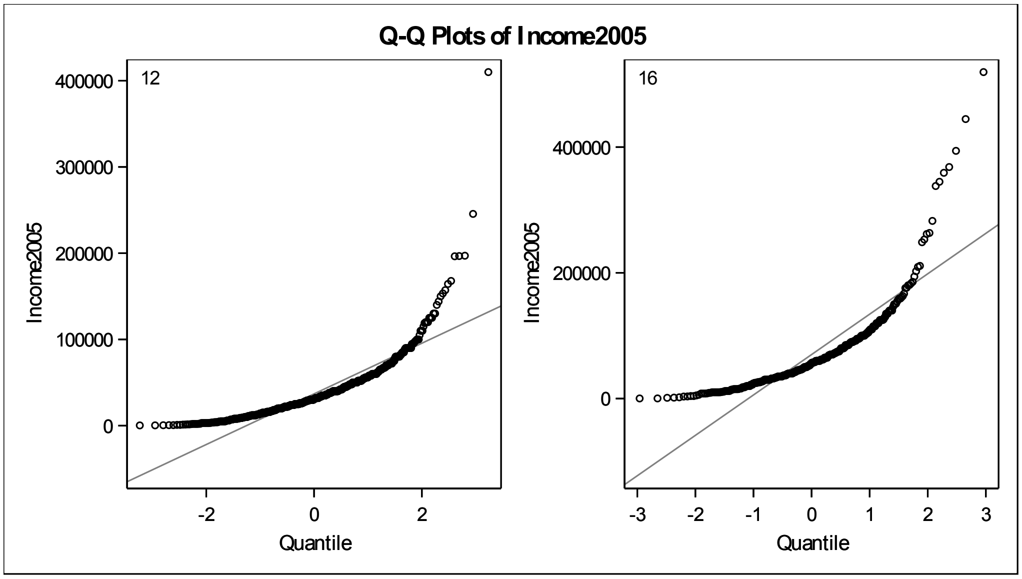


Figure 9:

Educ	N	Mean	Std Dev	Std Err	Minimum	Maximum
12	1020	36864.9	29369.7	919.6	300.0	410008
16	406	69997.0	64256.8	3189.0	200.0	519340
Diff (1-2)		-33132.1	42326.9	2483.8		

Figure 10:

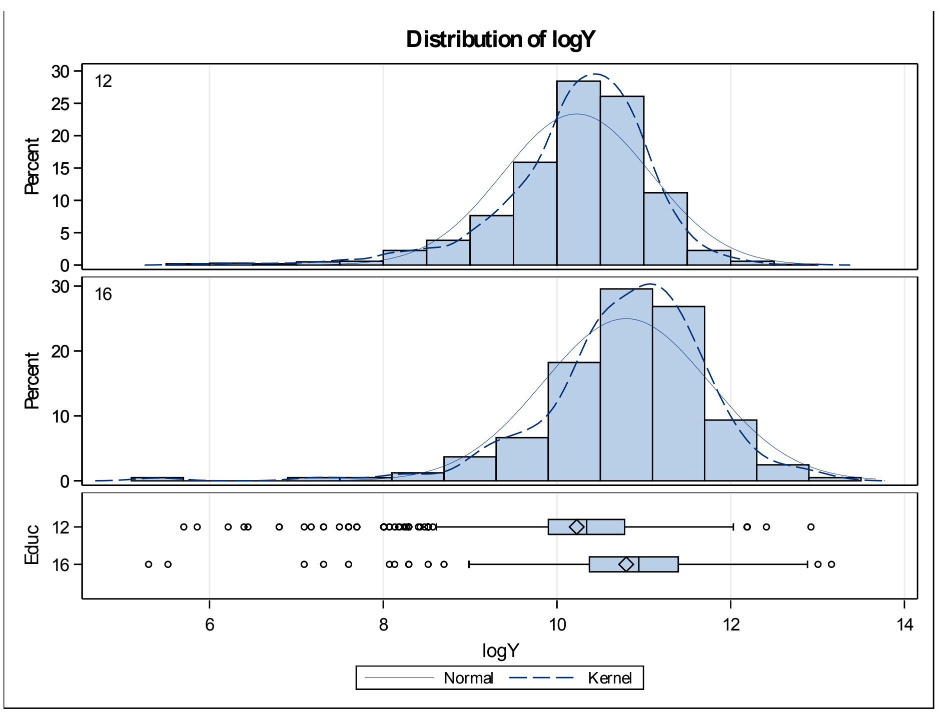
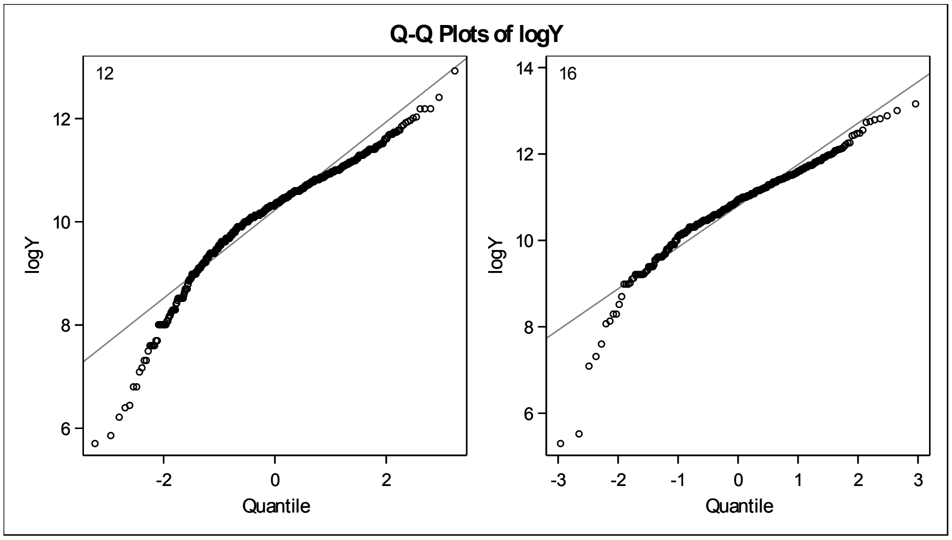


Figure 10 (cont.):



3) Test

For a sample size of *n* = 1426, the **critical value** at the $\propto = .05$ level with *df* = 1424 would be: $\mathbf{t_{crit}} = \mathbf{1.961}$

For this given sample, the **test statistic** would be:

$\mathbf{t_{stat}} \approx \mathbf{10.45}$, which is very far from the critical value.

4) Conclusion

Assuming the null-hypothesis and unequal variance **(Satterthwaite)** for this two-sided test, statistical software determined that the **p-value** for such a sample would be $\mathbf{p_{val} < .0001}$, which is much less than the $\propto = .05$ threshold.

On this basis, one would **reject the null-hypothesis** (H_0), determining that: there is **statistically significant evidence** to suggest that the ratio of median incomes of those with 16 and 12 years of education is unequal to one: $(M_{16} / M_{12} \neq 1)$, i.e., $(M_{16} \neq M_{12})$ → [$\mathbf{p_{val} < .0001}$].

As the study sought to determine whether those with 16 years of education had greater income, a **one-sided 95% confidence interval** for the median ratio, assuming unequal variance (**Satterthwaite**) and an estimated **multiplicative effect** of **1.7680**, would be:

$1.6161 \leq (M_{16} / M_{12}) \leq \infty$, or ≈ **[1.616, ∞]**.

For intuitive clarity, a **two-sided 95% confidence interval** for the median ratio, assuming unequal variance (**Satterthwaite**) and an estimated **multiplicative effect** of **1.7680**, would be:

$1.5884 \leq (M_{16} / M_{12}) \leq 1.9679$, or ≈ **[1.588**, **1.968]**, which is tantamount to saying that M_{12} is estimated as being between 50.81% and 62.97% of M_{16}.

Noting that the interval does not contain 1, this is consistent with the above conclusions, indicating there is **statistically significant evidence** to suggest that the median incomes are unequal for the groups observed ($M_{16} \neq M_{12}$).

5) Scope of Inference

Given the evidence for this observational study, inferences regarding the estimated difference in income **can indeed** be drawn to the subset of the population being examined. However, as participation in, and responses to the survey were voluntary and assignment to groups was not randomized, inferences regarding causal relations **cannot** be drawn.

Bonus Question)

In-State

Original Data	Percentage for Percentiles, given # of values	Z-score of Original Data	Z-Score percentiles assuming normal distribution, given column 2 values
1000	0	-.660	.2546
4000	25	-.473	.3197
5000	50	-.411	.3415
8000	75	-.224	.4106
40000	100	1.767	.9613

Out-of-State

Original Data	Percentage for Percentiles, given # of values	Z-score of Original Data	Z-Score percentiles assuming normal distribution, given column 3 values
3000	0	-1.214	.1122
8000	25	-.904	.1829
30000	50	.458	.6765
32000	75	.582	.7197
40000	100	1.077	.8591

From the above, one can conclude these data come from **non-normal** (R-skewed) **distributions**.