Effect of Gini Index and Average Income on Equilibrium Effective Rent Price

**Abstract**

**Introduction**

**Objective**

The purpose of this study is to determine equilibrium prices for several tiers of housing quality (defined in terms of both physical and geographical benefits) in a closed pure rental market consisting of an equal number of XXX (referred to as housing) and XXX (referred to as households).[[1]](#footnote-1)[[2]](#footnote-2) Our aim is to develop a model for use in a theoretical analysis of a fixed stock of both housing and households. This analysis is then compared with real world data to establish its usefulness in explaining some aspect of actual consumer real estate price movements.

This study does not expect to explain or predict real estate prices movements on a micro level, as housing is a very heterogeneous good which immense differentiation on the individual level. Housing “value” is a complicated function of different types of amenities which can be broadly described as “housing services.” This is further complicated by the fact that individual households differ in their assessment (or “valuation”) of “housing services” provided by their choice of housing. Instead, we focuses on real estate prices on a macro level by segmenting housing and households into quantiles based on their average “value” and average income, respectively. We rank the total utility received by each household quantile from each housing quantile at the prevailing market prices. Total utility is a function of housing “value” and consumption, the latter depending on residual income after savings and rent is deducted. Based on the combined calculated utilities of all household quantiles for each housing quantile, rent prices will be iteratively adjusted until market equilibrium is reached.

At this point it would be prudent to illustrate the differences in the concepts of “value” and “price,” as this study uses them. “Value” is an arbitrary number placed upon a housing unit to be used specifically for comparison purposes with other housing units[[3]](#footnote-3). On the other hand, “price,” is a real dollar amount, measured in $(USD) per sqft, which is determined by supply (housing units) and demand (households).

To successfully move from a micro level to macro level analysis, this notion of “value” for housing and “valuation” for households must be defined. Let’s start with “value”. On a micro level, housing value is a complicated function of many attributes, such as size, age, condition, schools, environment, community, amenities, and much more. To establish a simpler macro level definition, this study proposes considering housing value as a function of location and size, only. The concept of location is assumed to encompass all other non-size related aspects of housing. The concept of size is assumed to encompass sqft, lot size, number of bedrooms, and other related concepts. The full stock of housing will then be segmented into quantiles based on location, with a proxy for “value” assigned to each quantile. Appropriate proxies for “value” will be determined later with different methods used for theoretical vs empirical applications. As sizes can vary within each location, this study will assume a constant price per sqft ratio with a location. Prices will be determined in terms of price per sqft instead of a total price allowing us to price the location as a whole.

This method relies heavily on an assumption of feature commonality within locations. To support this assertion, consider how a household would search for specific “housing services” like condition, schools, or amenities in their search for a home. If a households wants to be close to the beach they are narrowing there search by location. If the households desires schools of a certain caliber, they are confining themselves to specific locations. Even housing conditions can be correlated with location. Housing condition is a function of age, maintenance, and upgrades. Housing ages will be similar within a location because developers will tend to build housing units in clusters on available land. Maintenance and upgrades, on a macro level, will be consistent within a location because competitive forces over time will force households to keep their housing units up to par with their neighborhood in an effort to attract potential buyers. Size, on the other hand, unequivocally varies within as single location. To align the feature of size into the philosophy of this study, housing “value” and “pricing” will be measured and considered on a “per sqft” basis. Size based features like, bedrooms per sqft and lot size per sqft as assumed to scale linearly with sqft in a particular location. This allows us to narrow our macro level analysis to a feature of “average sqft” which is assumed to be another location specific feature. As an example, a household deciding to start a family may choose to move out of the city, with a small average sqft, to a suburb, with a higher average sqft. This again has been simplified to a location specific search.

Now moving on to the concept of “valuation” for households. We have thus argued for grouping all housing into quantiles ranked by there “value,” which is determined by their location. This allows us to apply some homogeneity to a very heterogeneous housing supply. This will not help us if households are allowed heterogeneous opinions, or “valuations,” of housing “value.” As households, unlike housings, are not master planned by developers, nor immobile, applying homogeneity to them is admittedly less straight forward. A less theoretical and more rigorous study would be forced to open up the concept of “value” into its components (size, age, condition, schools, environment, community, amenities, etc.) to allow for targeted utility calculation for households. For example, households with children may value good schools while non-child bearing households may not. Younger households may value nightlife amenities while older households may not. These differences in opinions throw a wrench in our attempt to simplify housing into quantiles ranked solely by a single proxy number for “value.” Here is where our study is going to take some “theoretical license.” We are going to assume consistent household “valuation” across the income spectrum. At least in terms of the average household of each quantile. This will serve as a theoretical starting point. When the model is extrapolated to specific areas, accommodations for differences in households will be made.

**Methodology**

First, household structure and behavior must be defined. Households will be separated into quantiles based on increasing average “income.” Household quantiles are defined from an overall average income and Gini Index as independent variables applying to all households as a whole.[[4]](#footnote-4)[[5]](#footnote-5) It can be shown in the “Model” section, that these variable selections constrain the average income of each household quantile. Each household in each quantile is assumed to earn the quantiles average income. The number of quantiles and total number of households will be exogenous variables to the model.

Each household quantile will evaluate each housing quantile, and assign a utility to it. Household utility is calculated via a Cobb-Douglas Utility function, with constant returns to scale, which considers only housing “value” and household consumption as determinants. Housing “value” is defined by the proxy number assigned to each housing quantile. Consumption is defined by the residual cash flow after savings and rent are deducted from income. Marshallian demand, and associated total expenditures, for housing and consumption is assumed to follow a societal rule, where housing spending should be a set percent of total household expenditures. This is analogous to the traditional 30% guideline, which is arguably not following in actual societies. Nevertheless, for purposes of this study it is assumed that optimal household utility is achieved by following this guideline which is exogenous to the model. This guideline will define the Cobb-Douglas coefficients shown in the “Model” section. Household savings rates are treated as exogenous to the model and assumed to decrease from a set rate as household income decreases. This is to account for a tendency of poorer households to forfeit savings as they may see it necessary to achieve an adequate standard of living. Both the housing to total expenditures ratio and savings rate profile are treated as exogenous variables to the model.

Next, housing structure must be defined as our households need something to evaluate. Housing is separated into quantiles based on increasing average “value.” As utility is used solely for ranking purposes, actual numeric “value” proxies are not important. What is important is that this numeric proxy is higher for higher “value” housing. For purposes of pricing, each housing quantile must be assigned an average sqft. It is assumed that in the model, households purchase only the necessary amount of sqft, and do not attempt to splurge or save by purchasing more or less sqft, respectively, as necessary. It is also assumed that all housing in a given location has a representative diversity of sizes, thereby, not causing a household to look else ware, for size considerations. The purpose of this assumption is to take differences in size out of the utility maximization equation by saying, on average, these effects balance out.

This would be a good time to properly define and elaborate on several terms critical in working with Cobb-Douglas utility. We need to differentiate between housing and consumption expenditures, quantities, and prices, as well as clarify their role in the model. Housing and consumption “expenditures” are relatively straightforward. The households total monthly rent equates to total housing expenditures for purposes of this model. Total monthly consumption expenditures is whatever is left after savings and rent. This is spending on non-housing goods and services like food, transportation, entertainment, and etc. Housing and consumption quantities are units of housing and consumption services (or goods) which are acquired with the relative total expenditure. Prices can be computed by the ratio of total expenditures to total quantities. This may be obvious but it gives us a chance to property define what we are calculating in this study.

We are calculating utilities in respect to housing “value” and consumption. Housing “value” is an arbitrary numerical proxy used for ranking only and consumption is assumed to be worth dollar for dollar for what is spent on it.[[6]](#footnote-6) With incomes and prices known we could calculate for each household, optimum Marshallian demand of housing and consumption. Housing is not provided in continuous bundles like consumption, though. The housing available in our model is predefined in discrete quantiles. If households were allowed to calculate their optimum bundles of housing and consumption, we would no doubt have demands for intermediate housing bundles not available in our discrete model. Therefore, this study does things backwards. We ask each household to evaluate each housing (separately) and tell us what utility is derived from it and its associated non-housing consumption. When a household does this evaluation, it takes the current price per sqft (for the location/quantile) times the average sqft (for the location/quantile) as the total housing expenditure. The total consumption expenditure, associated with each choose of housing, is calculated by taking the total income, less total savings, less this total housing expenditure.

With this setup, we have a set of household quantiles, of increasing average income, assumed to belong to each individual household in the quantile and a set of housing quantiles, of increasing average “value” and average sqft assumed to belong to each individual housing in the quantile. The calculation consist of a two-dimensional process iterating through all household quantiles in an outer loop and then all housing quantiles in an inter loop. Before the calculation process begins, rent prices per sqft for each housing quantile must be assumed as a starting point. The process proceeds with each household quantile evaluating each housing quantile by calculating an overall utility. The result of a single iteration of this process will yield a “utility matrix” mapping the utility of each household quantile on one axis with each housing quantile on the other axis. Utility values are normalized along the household quantiles axis to ensure that the total utility value for each household quantile is one. The resulting matrix from this operation is called the “unit demand matrix.” It shows how the demand from each household quantile is ranked for each housing quantile. Each row (or column) of each household quantile demand ranking is multiplied by the total number of households for that particular quantile. This effectively scales the “unit demand matrix” up to an “overall demand matrix,” which basically allows us to account for differently sized household quantiles.[[7]](#footnote-7) Next, each column (or row) of each housing quantile is divided by the total number of housings for a particular quantile. This generates a one dimensional vector of demand to supply ratios for each housing quantiles. This is called the “demand supply vector,” which gives us the combined weight of each household quantiles demand fraction relative to the total supply of each housing quantile.[[8]](#footnote-8)

The purpose of the “demand supply vector” is to update the current rent prices of each housing quantile. For each housing quantile, current prices are multiplied by a ratio of the demand supply difference to the total supply. Implications of this are that price will increase when demand is higher than supply, price will decrease when demand is lower than supply, and price will remain unchanged when demand equals supply. This iterative process will continue until demand equals supply for each housing quantile. The result of this process will be the determination of equilibrium average rents for each housing quantile has a function of different selections of overall average income and Gini Index.

**Model**

**Analysis**

**Conclusions**

1. (Define ”Households” & “Housing”) [↑](#footnote-ref-1)
2. (Explain “Pure Rental Market”) [↑](#footnote-ref-2)
3. (Reference economic concept of “Utility”) [↑](#footnote-ref-3)
4. (Explain “Overall Average Income”) [↑](#footnote-ref-4)
5. (Explain “Gini Index”) [↑](#footnote-ref-5)
6. (Explain “Inflation Effect”) [↑](#footnote-ref-6)
7. (Explain “Matrix Rows & Columns”) [↑](#footnote-ref-7)
8. (Explain “Demand/Supply Threshold Methods”) [↑](#footnote-ref-8)