Math 115E Activity 15

Chapter 5
Factoring Quadratics Part 2

How to factor quadratic equations

Quadratic factoring when a = 1:
When factoring, the form $x^2 + bx + c$
can be factored as $(x + m)(x + n)$
Start with real numbers m *and* n *so:*
they both multiply to c *and both add to* b
There is not a value in front of either x

Quadratic factoring when a ≠ 1:
When factoring, the form $\boldsymbol{a}x^2 + bx + c$
can be factored as $(px + m)(qx + n)$
Start with two real numbers such that:
multiply to $\boldsymbol{a} \cdot c$ *and yet add to* b
then we re-group the terms and factor

Example: We want to solve $3x^2 - 11x + 10$ by factoring

Step 1: Find the factors of $3 * 10 = 30$ that add up to -11, (Write the factors if needed)

Step 2: The factors of 30 are: $\pm(1, 30), \pm(2, 15), \pm(3, 10), \pm(5, 6)$,
so the pair that adds to -11 is -5 and -6

Step 3: Rewrite the quadratic: $3x^2 + (-6x - 5x) + 10 = 0$

Step 4: Regroup so that each group has a common factor: $(3x^2 - 6x) + (-5x + 10) = 0$

Step 5: Factor out a common term: $(3x)(x - 2) - 5(x - 2) = 0$

Step 6: Factor again with the $x - 2$ term: $(3x - 5)(x - 2) = 0$

Step 7: Solve for x, so we get $3x - 5 = 0$ and $x - 2 = 0$, giving us $x = \frac{5}{2}$ and $x = 2$

DONE: So starting with $3x^2 - 11x + 10 = 0$, we get $(3x - 5)(x - 2) = 0$

NOTE: If we aren't able to factor out a common term or we dont get the same expression in both parentheses in Step 5, then go back to Step 3 and swap the factor pair.

Factor the following quadratic equations

#1 $x^2 + 9x + 14 = 0$

#2 $x^2 - 8x + 7 = 0$

#3 $x^2 + x - 30 = 0$

#4 $3x^2 + 10x + 8 = 0$

#5 $2x^2 - 9x + 10 = 0$

#6 $2x^2 - 6x - 20 = 0$

#7 $9x^2 - 27x + 18 = 0$

#8 $4x^2 - 13x + 10 = 0$

#9 $2x^2 - 13x - 7 = 0$

#10 $4x^2 + 20x + 25 = 0$

#11 $3x^2 - 19x + 20 = 0$

#12 $8x^2 - 6x - 9 = 0$

The Quadratic Formula:
If we are given any polynomial,which may not be factorable, in the form $ax^2 + bx + c = 0$
We can solve this by using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
We will either get $0, 1$ *or* 2 *solutions for the* x*-values*

Example: We want to solve $3x^2 - 11x + 10$ by using the Quadratic Formula

Step 1: Identify the given coefficients: $a = 3, b = -11, c = 10$

Step 2: Plug these into the formula: $x = \dfrac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(10)}}{2(3)}$

Step 3: Simplify as much as we can $x = \dfrac{-11 \pm \sqrt{121 - 120}}{6} \longrightarrow x = \dfrac{11 \pm 1}{6}$

Step 4: Obtain the solutions: $x = \dfrac{11 \pm 1}{6}$, so $x_1 = \dfrac{11 + 1}{6} = \dfrac{12}{6} = 2$ and $x_2 = \dfrac{11 - 1}{6} = \dfrac{10}{6} = \dfrac{5}{3}$

DONE: So starting with $3x^2 - 11x + 10 = 0$, we get $x = \frac{5}{3}$ and $x = 2$, which is the same as before!

Solve the following quadratic equations

#1 $x^2 + 9x + 14 = 0$

#2 $x^2 + 4x + 2 = 0$

#3 $x^2 + 2x - 1 = 0$

#4 $x^2 + x - 30 = 0$

#5 $2x^2 - 6x + 3 = 0$

#6 $2x^2 - 9x + 10 = 0$

#7 $4x^2 - 13x + 10 = 0$

#8 $4x^2 + 20x + 25 = 0$

#9 $10x^2 + 10x - 10 = 0$

#10 $8x^2 - 6x - 9 = 0$

#11 $4x^2 - 12x + 3 = 0$

#12 $x^2 + 3x - 1 = 0$