Math 115E Activity 21

Chapter 7: Polynomials
Multiplicities

Multiplicities of Polynomials

Reminder: *Given a function $f(x)$ and knowing that points are of the form (x, y),*

- *The x-intercepts are when $y = 0$, the coordinate points of the form $(x, 0)$ on the function*
- *The y-intercept is when $x = 0$, the coordinate points of the form $(0, f(0))$ on the function*

Definition: *Given a polynomial $f(x)$, the number of times a given term $(x - c)$ appears in the factored form of $f(x)$ is called the* ***multiplicity***

Example: If we have the polynomial: $g(x) = (x-1)(x-2)^4(x+3)^3(x+4)^2$

- Then we can say the following solutions are $x = 1, x = 2, x = -3, x = -4$
- Now, notice that: $x = 1$ has a multiplicity of 1, and $x = 2$ has a multiplity of 4
  $x = -3$ has a multiplicity of 3, and $x = -4$ has a multiplity of 2
- The only y-intercepts is at $f(0) = (0-1)(0-2)^4(0+3)^3(0+4)^2 = (-1)(-2)^4(3)^3(4)^2 = -6912$
- The multiple x-intercepts are solutions of $0 = f(x)$ which are $(1, 0), (2, 0), (-3, 0), (-4, 0)$

For the following problems, x-intercepts, their multiplicities, and the y-intercepts

#1 $f(x) = (x-2)^3(x-1)$

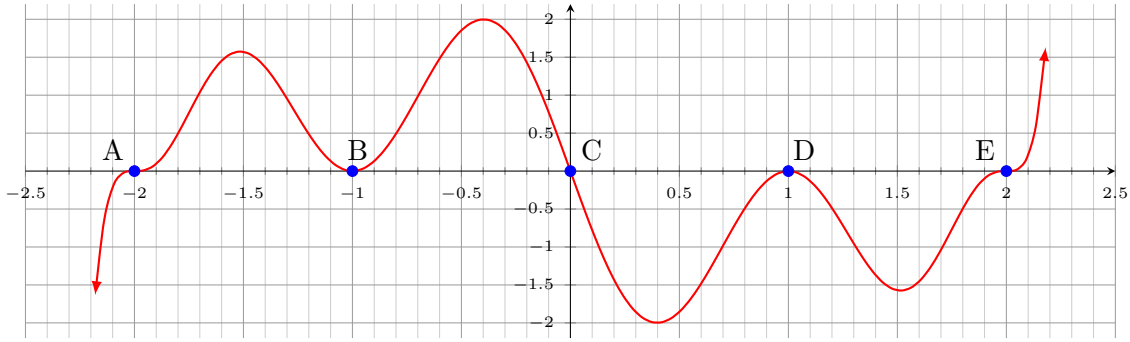
#2 $f(x) = (x+1)^2(x-1)$

#3 $f(x) = (x^2-4)(x+3)^3$

#4 $f(x) = (x-1)^4(x^2+3)(x+2)^2$

#5 $f(x) = x(x^2-2)(x-3)^3(x+4)^2$

Graphing Multiplicities



We have 5 intercepts for the following function as described on the above graph assume their multiplities are the smallest that they can be.

Let's find the multiplities of each intercept and their value

- Intercept A:
- Intercept B:
- Intercept C:
- Intercept D:
- Intercept E:

What do we think the smallest degree of the polynomial will be?

Sketch graph of the following function: $f(x) = \frac{1}{40}x(x+1)(x-2)^2(x-3)^2$
Use the end behavior, x intercepts, and the multiplicities of the following function: