Math 675 Spring 2026:
Homework 1 Unit ?
Due Tuesday, Janurary 3rd, 2026

Question 1:

Consider the uncoupled system:

$$x' = 3x, y' = -y$$

a) Solve the system explicitly for general initial conditions $(x(0), y(0)) = (x_0, y_0)$.

$$\begin{aligned} x'(t) &= 3x \\ \frac{dx}{dt} &= 3x \\ \int \frac{1}{x}dx &= \int 3dt \\ \ln|x| &= 3t + c_1 \\ e^{\ln|x|} &= e^{3t+c_1} \\ x &= e^{3t}e^{c_1} \\ \text{Solution: } x(t) &= c_1e^{3t} \\ \text{IC: } x(0) = x_0 &= c_1 \\ \text{Final Solution: } x(t) &= x_0e^{3t} \end{aligned}$$

$$\begin{aligned} y'(t) &= -y \\ \frac{dy}{dt} &= -y \\ \int \frac{1}{y}dy &= \int -1dt \\ \ln|y| &= -t + c_2 \\ e^{\ln|y|} &= e^{-t+c_2} \\ y &= e^{-t}e^{c_2} \\ \text{Solution: } y(t) &= c_2e^{-t} \\ \text{IC: } y(0) = y_0 &= c_2 \\ \text{Final Solution: } y(t) &= y_0e^{-t} \end{aligned}$$

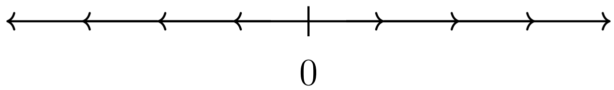
b) Identify all equilibrium points of the system.

$$\begin{cases} 3x = 0 \\ -y = 0 \end{cases} \rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

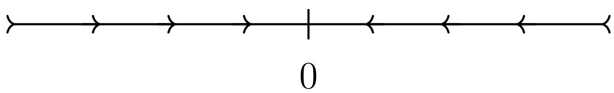
So like before, the only equilibrium point is at the origin $(0, 0)$.

c) Using your explicit solution, determine the behavior of solutions as $t \to +\infty$:
Given out solutions from before: $x(t) = x_0 e^{3t}$ and $y(t) = y_0 e^{-t}$

i) initial conditions on the x-axis
Using just $x = x_0 e^{3t}$, As $t \to +\infty$,
If $x_0 > 0$ then $x(t) \to +\infty$ and if $x_0 < 0$ $x(t) \to -\infty$.

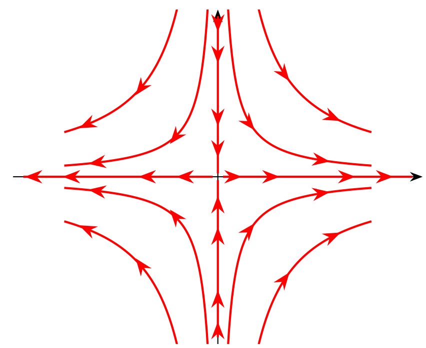


ii) initial conditions on the y-axis
Using just $y = y_0 e^{-t}$, As $t \to +\infty$,
If $y_0 > 0$ then $y(t) \to 0$ and if $y_0 < 0$ $y(t) \to 0$.



iii) initial conditions not lying on either axis

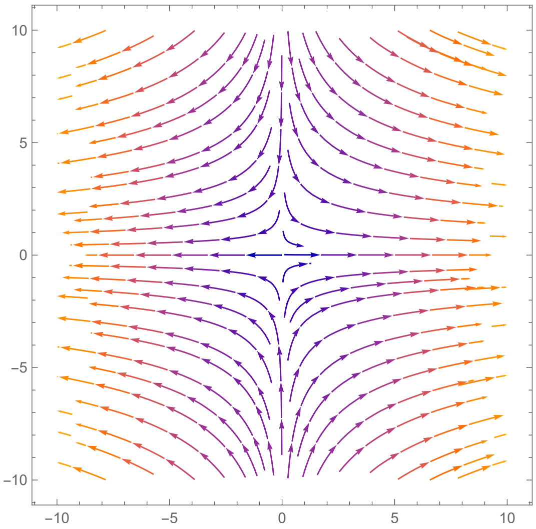
- If $t \to +\infty$ then solutions approach the positive x-axis
- If $t \to -\infty$ then solutions approach the negative x-axis



d) We can see from the behavior of the solutions as $t \to +\infty$ and $t \to -\infty$ that the equilibrium point at the origin is a saddle point. This is because along the x-axis, the solutions diverge away from the origin, while along the y-axis, the solutions converge towards the origin.

We can also see this as we take limits to positive and negitive infinity of $x(t) = x_0 e^{3t}$ and $y(t) = y_0 e^{-t}$ similar to what we have above,
As $t \to +\infty$, $x(t) \to +\infty$ and $y(t) \to 0$,
As $t \to -\infty$, $x(t) \to -\infty$ and $y(t) \to 0$.
Because the limits for $y(t)$ are both 0 yet the sign of infinity changes for $x(t)$, this tells us that this equilibrium point is a saddle point

e) Now we want to use Mathematica to plot the full phase portrait of this system.



Question 2:

Consider the uncoupled system:

$$x' = 3x, y' = -y$$

a) 2