

## Comprehensive Exam Notes

Here we go!

### Section 1: Graduate Algebra

### Section 2: Real Analysis

#### WEEK 1/2: Convergence

##### Learning Goal 1

State and check the definitions of pointwise and uniform convergence for sequences and series of functions.

##### Learning Goal 2

Apply the Weierstrass  $M$ -test.

##### Learning Goal 3

Prove and use the theorem relating continuity and uniform convergence.

##### Learning Goal 4

State and check the definition of uniform continuity.

#### WEEK 4: Fourier Series Part 1

##### Learning Goal 5

Describe how we derive the formulas for computing the coefficients of the Fourier series.

##### Learning Goal 6

Compute the Fourier series for simple examples. “Simple” here means that the integrals required to compute the coefficients don’t take forever to compute.

#### WEEK 4: MISSING FOURIER SERIES PART 2

#### WEEK 8: The Space $L^2$ : Part 1

##### Learning Goal 7

State the definition of  $L_2([a, b])$ . Check if a given function is or is not in  $L_2([a, b])$ .

##### Learning Goal 8

State and prove the Cauchy-Schwartz inequality and the Minkowski inequality.

#### WEEK 9: The Space $L^2$ : Part 2

##### Learning Goal 9

State, check, and use the definition of a norm.

##### Learning Goal 10

Identify complete normed vector spaces and explain why a space is or is not a complete normed vector space.

#### WEEK 10: Linear Transformations

##### Learning Goal 11

State the definition of a bounded linear operator. Use the definition in proofs.

### Section 3: Numerical Analysis

### Section 4: Complex Analysis

### Section 5: Topology

### Section 6: Differential Equations