Math 675 Spring 2026:
Homework 1 Unit ?
Due Tuesday, Janurary 3rd, 2026

Question 1:

For each system of differential equations in a-d below

- Plot the change vector using the points $(1,1),(1,-1),(-1,-1),(-1,1)$
- Use the vector field to classify the equilibrium point(s) as stable, unstable, a saddle point, an unstable spiral, a stable spiral, or a center.

(a) $x' = -x, \quad y' = -5y$
Coupled: No;
EPs: Set $x' = 0, y' = 0$

$$\begin{cases} -x = 0 \\ - 5y = 0 \end{cases} \rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

Thus, the EP is $(0,0)$

(x,y)	$(x',y') = (-x,-5y)$
$(1,1)$	$(-(1),-5(1)) = (-1,-5)$
$(1,-1)$	$(-(1),-5(-1)) = (-1,5)$
$(-1,-1)$	$(-(-1),-5(-1)) = (1,5)$
$(-1,1)$	$(-(-1),-5(1)) = (1,-5)$

(b) $x' = 4x - y, \quad y' = 2x + y$
Coupled: Yes;
EPs: Set $x' = 0, y' = 0$

$$\begin{cases} 4x - y = 0 \\ 2x + y = 0 \end{cases} \rightarrow \begin{cases} 4x = y \\ 2x + y = 0 \end{cases} \rightarrow$$

$$\begin{cases} 4x = y \\ 2x + 4x = 0 \end{cases} \rightarrow \begin{cases} 4x = y \\ 6x = 0 \end{cases} \rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases}$$

Thus, the EP is $(0,0)$

(c) $x_1' = x_2, \quad x_2' = -2x_1 - 3x_2$
Coupled: Yes;
$$\begin{cases} \quad - \quad x_2 = 0 \\ -2x_1 - 3x_2 = 0 \end{cases} \to \begin{cases} \qquad x_2 = 0 \\ -2x_1 - 3x_2 = 0 \end{cases} \to$$
$$\begin{cases} \qquad x_2 = 0 \\ -2x_1 \qquad = 0 \end{cases} \to \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$
Thus, the EP is $(0,0)$

(d) $x' = 5x + 2y, \quad y' = -17x - 5y$
Coupled: Yes;
EPs: Set $x' = 0, y' = 0$
$$\begin{cases} 4x - y = 0 \\ 2x + y = 0 \end{cases} \to \begin{cases} 4x \quad = y \\ 2x + y = 0 \end{cases} \to$$
$$\begin{cases} 4x \quad = y \\ 2x + 4x = 0 \end{cases} \to \begin{cases} 4x = y \\ 6x = 0 \end{cases} \to \begin{cases} y = 0 \\ x = 0 \end{cases}$$
Thus, the EP is $(0,0)$

Question 2:

Let $a \in \mathbb{R}$ be a parameter. Consider the system of differential equations

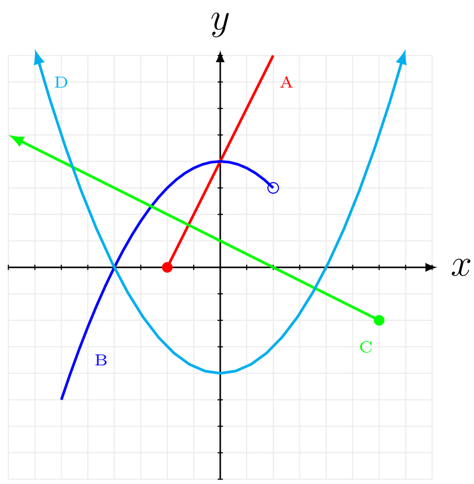
$$x'(t) = 2x, \qquad y'(t) = ay$$

(a) Use separation of variables to find the explicit solution for this system.

(b) Sketch the phase portrait for this system for the following values of a: -1, 0, 1, 2. Do not use technology for this part.

(c) Now use tech (Mathematica, Python, etc) to plot a complete vector field and some trajectories.

(d) Use the vector field to classify the equilibrium point(s) as stable, unstable, a saddle point, an unstable spiral, a stable spiral, or a center.



Question 3:

Let $a : R ß R$ be a continuous function. Consider the first order ODE

$$x' = a(t)x$$

(a) Find a formula involving integrals for the solution of this system.

$$\frac{dx}{dt} = a(t)x(t) \longrightarrow \int \frac{1}{x(t)} dx = \int a(t)dt$$

$$\ln |x|$$

$$\int x dx$$

(b) Prove that your formula gives the general solution of this system. Hint: follow the sketch of proof done on the first day of class