CPSC 420: Advanced Algorithms Design and Analysis

2022W2

Hello, my name is:

	email	Office hours
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TAs

David Bromley

Dylan Brown

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Seyed Ali Tabatabaee

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Access everything from Canvas

`https://canvas.ubc.ca/courses/106386`

Piazza

Gradescope

HWK

Course Work

No late work; may be flexible with advance notice

30%	Homework (~ 4)
40%	Examlets (in class Jan 27, Feb 17, Mar 17, Apr 5)
30%	Final exam (TBA)

Must pass the weighted combo of exams to pass the course.

Homework Collaboration

You may discuss problems in a group of fellow 420 students of size at most three. However you must write your submission on your own. At the top of the first page of your submission, you must acknowledge all resources that you used, including books, websites, and students who helped. Submissions missing this statement will not be graded.

Books

Algorithms by Erickson `https://jeffe.cs.illinois.edu/teaching/algorithms/`

Algorithm Design by Kleinberg & Tardos
Introduction to Algorithms by Cormen, Leiserson, Rivest & Stein
The Art of Computer Programming V1-4 by Knuth

All available online or online via the UBC library.

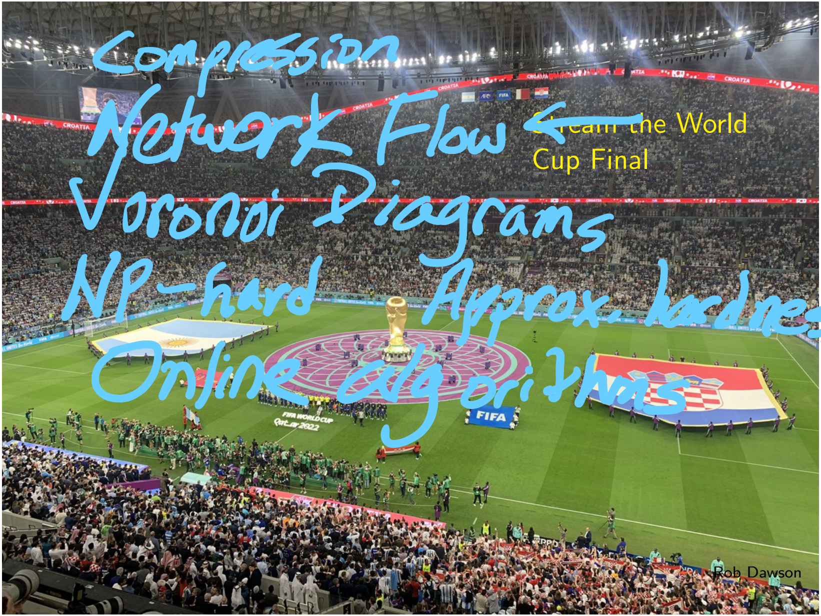
Today's announcements:

- HW1 available soon, due Jan 19, 23:59
- Examlet 1 on Jan 27 in class.

Today's Plan

- What is this course about?
- Introduce convex hulls

What is this course about?



Topics

- Geometric algorithms: Convex hulls, Voronoi diagrams
- Linear programming: Network Flow, Zero-sum games
- Online algorithms: Page replacement
- Dynamic programming
- NP-hardness: Approximation algorithms, Hardness of approximation
- Compression: Huffman, Lempel-Ziv
- (maybe) Fast Fourier Transform
- (maybe) Quantum computation

Goals of the Course

- Become more familiar with algorithm design techniques
- Explore optimizing other measures of an algorithm's performance (compression, paging, ...)
- Determine when improving an algorithm is hopeless
- Explore ways to find approximate solutions (and when that's hopeless)

Making Salad Dressing

$1A + 1B$
$= 12\%$ 28.5%

Given two bottles of salad dressing:

	oil	vinegar	other stuff
Bottle A	15%	36%	
Bottle B	9%	21%	

Can we make a dressing with

Mixture X	13%	31%	yes	?
Mixture Y	12%	30%	no	?

$1A + 2B = 13, 31$

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If we add another bottle?

Bottle C	12%	33%

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Problem

- Given a set of points $P = \{p_1, p_2, \ldots, p_n\}$ in $[0, 100] \times [0, 100]$
- Output a representation of the set of mixtures we can produce.

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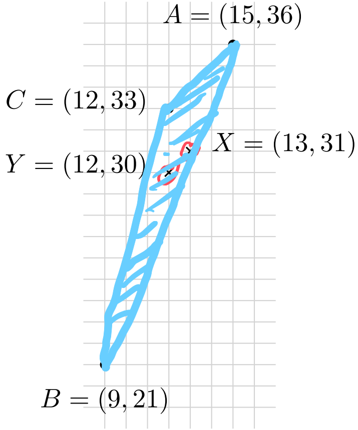

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Making mixtures

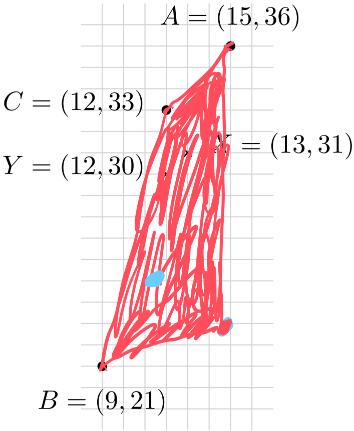
A **mixture** (aka **convex combination**) of points $p_1, \ldots, p_n$ is

$$\sum_{i=1}^{n} \alpha_i p_i$$

where $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$.

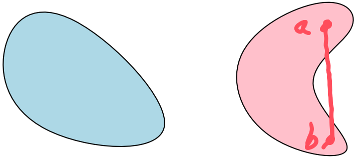
For $n = 2$ points, the set of convex combinations is the line segment between them.

For $n = 3$, it's the triangle with those points as vertices.

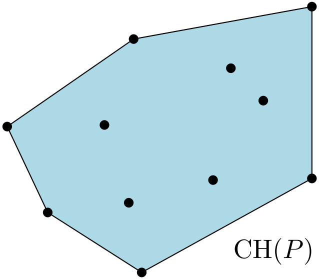


Convex Hull

A set S is **convex** if for all $a, b \in S$ the segment $\overline{ab}$ is in S.



The **convex hull** of a set P of points is the intersection of all convex sets that contain P.



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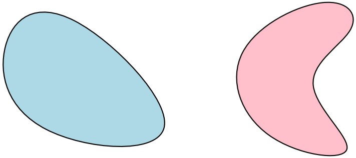
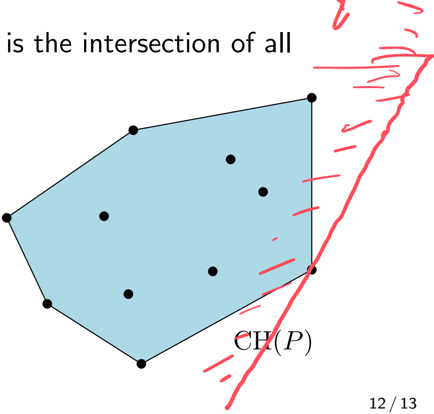


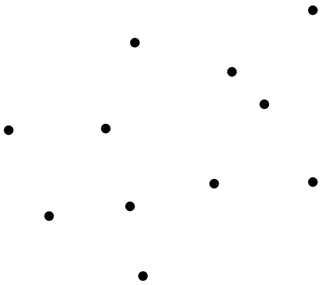
halfplane

The **convex hull** of a set P of points is the intersection of all convex sets that contain P.

A point $p \in P$ is on the boundary of CH(P) iff there exists a line ℓ through p with all P on one side of ℓ.



Jarvis March (Gift-wrapping)



Idea: Tie a string to a point $p_1 \in P$ that is on the $CH(P)$. Rotate a taut string around the points until it "bends" at the next point on $CH(P)$. Keep going until back to p_1.

Turn test

Path $a \to b \to c$ makes a left turn at b iff

$$\det \begin{pmatrix} a_x & a_y & 1 \\ b_x & b_y & 1 \\ c_x & c_y & 1 \end{pmatrix} > 0$$

$$a_x b_y - a_y b_x + a_y c_x - a_x c_y + b_x c_y - c_x b_y > 0$$