Assignment #6
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Goal

1. Plot exit flow Mach number for varying exit area (Ae) to throat area (A*).
2. Plot coefficient of thrust vs ambient pressure for a exit area to throat ratio of 69:1.

Governing Equations and Methodologies

Part A:

Using the equation:

$$\frac{A_e}{A^*} = \frac{\left(\sqrt{\frac{\gamma^*}{\gamma_e}}\right)\left(\frac{1}{M_e}\right)\left(1 + \frac{\gamma_e - 1}{2}M_e^2\right)^{\frac{\gamma_{e+1}}{2(\gamma_e - 1)}}}{\left(\frac{\gamma^* + 1}{2}\right)^{\frac{\gamma^* + 1}{2(\gamma^* - 1)}}}$$

We first estimate γ^* with an appropriate guess ($\gamma^* \sim 1.3$)

Calculate $T^* = \frac{2T_0}{\gamma^* + 1}$, $c_p^* = \kappa\left[A + B\left(\frac{T^*}{1000}\right) + C\left(\frac{T^*}{100}\right)^2 + D\left(\frac{T^*}{1000}\right)^3 + E\left(\frac{1000}{T^*}\right)^2\right]$

$c_v^* = c_p^* - R$

And update γ^* as $\gamma^* = \frac{c_p^*}{c_v^*}$

This will continue until γ^* converges .

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Governing Equations and Methodologies

Part B:

Suppose Ae/A* = 69:1

1. Make an initial guess for γ_e
2. Solve for Me using formerly mentioned Ae/A* equation

3. Calculate exit temperature $\quad T_e = T_0 \left[1 + \left(\frac{\gamma_e - 1}{2}\right) M_e^2\right]^{-1}$
4. Calculate Cp and Cv at exit:
$$\left(c_p\right)_e = \kappa \left[A + B\left(\frac{T_e}{1000}\right) + C\left(\frac{T_e}{100}\right)^2 + D\left(\frac{T_e}{1000}\right)^3 + E\left(\frac{1000}{T_e}\right)^2\right]$$
$$\left(c_v\right)_e = \left(c_p\right)_e - R$$

5. Finally, update γ_e until solution converges.

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Governing Equations and Methodologies

Part B: Continued

    6. Calculate speed of sound, velocity and pressure at the exit using the subsequent equations.

$$a_e = \sqrt{\gamma_e R T_e}$$

$$V_e = M_e a_e$$

$$p_e = p_0 \left[1 + \left(\frac{\gamma_e - 1}{2} \right) M_e^2 \right]^{-\frac{\gamma_e}{\gamma_e - 1}}$$

Governing Equations and Methodologies

Part C:

To establish an actual thrust value (momentum thrust), use the equation,

$$Momentum\ Thrust = \dot{m}V_e$$

Where,

$$\dot{m} = \rho A_e V_e, \ \rho = P_e/(RT_e)$$

(The subscript 'e' denotes the exit condition)

Governing Equations and Methodologies

Part D:

To calculate varying thrust coefficient to ambient pressure:

1. Initialize a range of pressures P_a=[0 kPa to 101.325 kPa]
2. Using the total thrust equation:

$$Total\ Thrust = \dot{m}V_e + (P_e-P_a)A_e$$

3. Thrust coefficient equation:

$$C_T = Total\ Thrust\ /\ (P_0A*)$$

Where P_0 is given as 206.4 [bar] or 20640 [kPa]

Mach Number versus Area Ratio Variation (Part A)
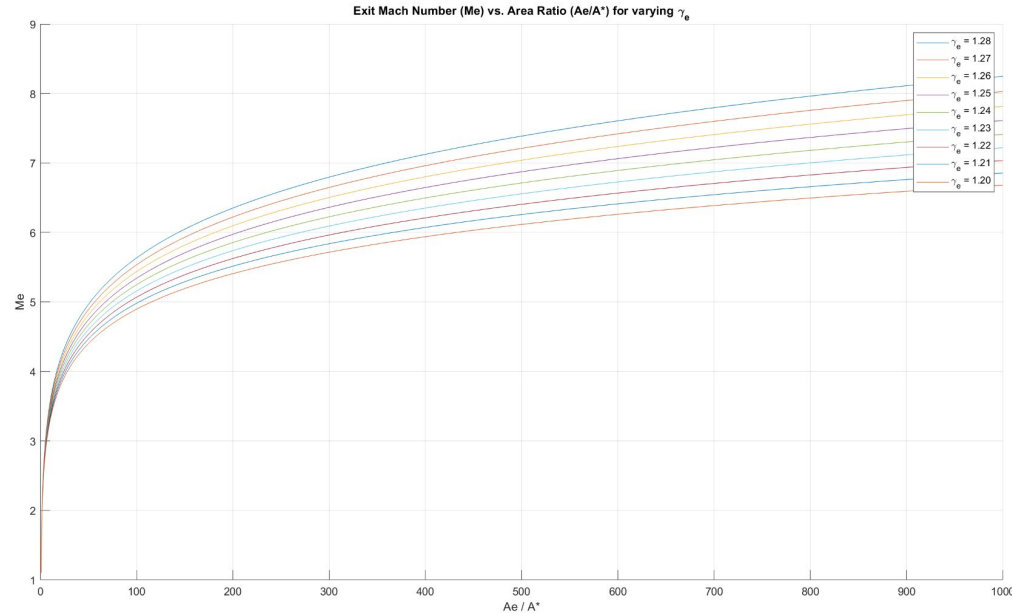

Figure 0. Mach number variance as a function of Ae/A* (ratio of cross-sectional exit area to throat area).

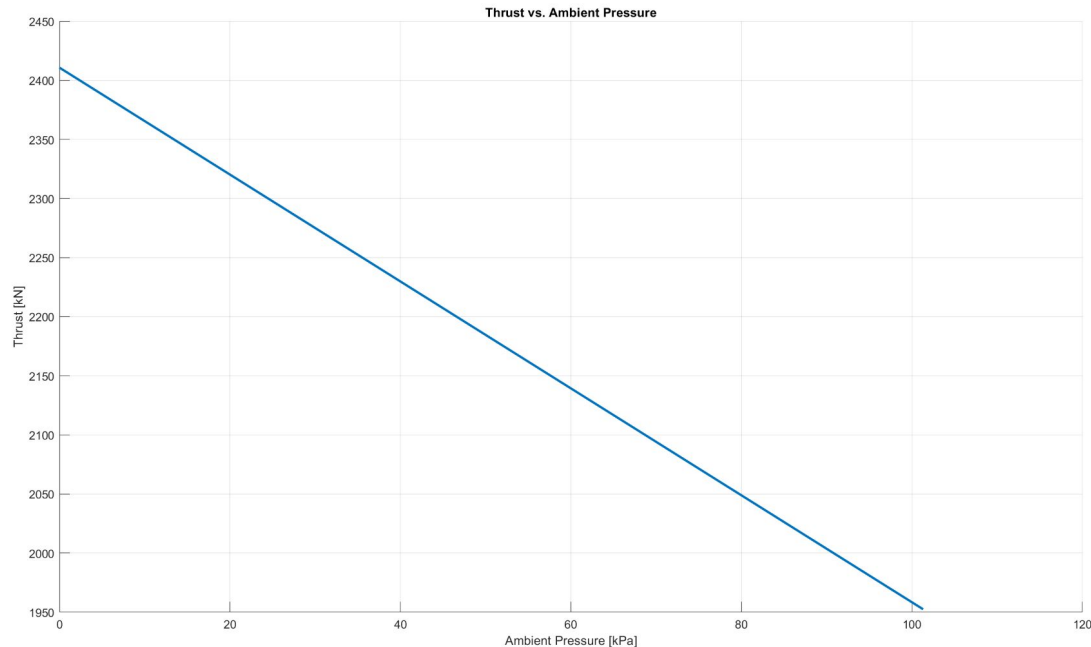# Thrust versus Ambient Pressure (Part D)

Figure 1. Thrust of Nozzle as it varies from vacuum to sea level pressure conditions [0 kPa, 101 kPa]. Momentum thrust is constant, but the difference between exit pressure and ambient pressure varies as the magnitude of the latter changes. **Momentum Thrust is calculated as 2.34 [MN]**

Coefficient of Thrust versus Ambient Pressure (Part D)
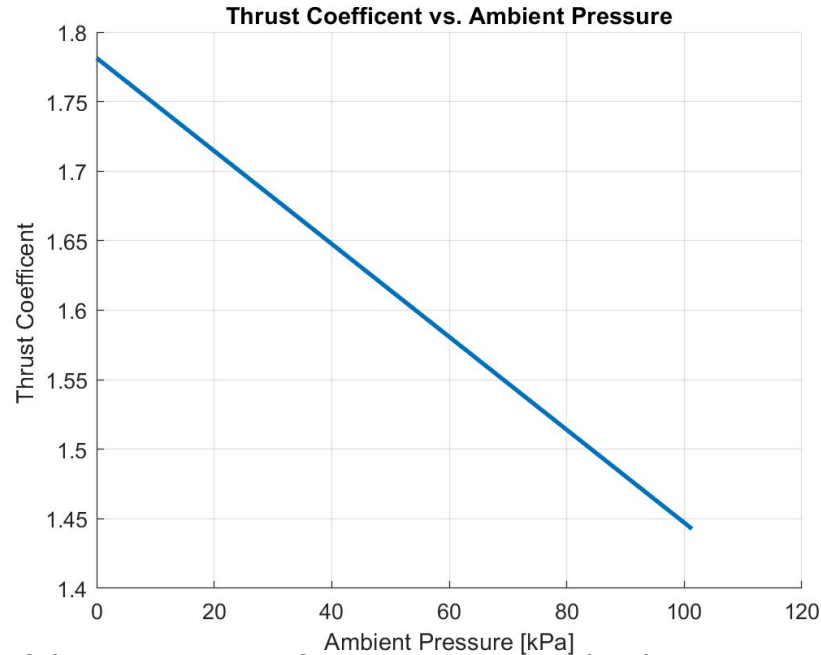

Thrust Coefficent vs. Ambient Pressure

Figure 2. Coefficient of thrust as it varies from vacuum to sea level pressure conditions [0 kPa, 101 kPa]. Momentum thrust is constant, but the difference between exit pressure and ambient pressure varies as the magnitude of the latter changes.

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Conclusions

- Mach number varies nonlinearly to area ratio, Thrust varies linearly with ambient pressure, and coefficient of thrust varies linearly with ambient pressure.
- Momentum Thrust is calculated as 2.345 [MN].