Assignment #4

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Goal

1. Plot velocity and position of three body problem exiting with an instantaneous velocity from Low Earth Orbit (LEO) and entering into Low Mars Orbit (LMO).
2. Determine the family of solutions in which to apply a instantaneous velocity upon spacecraft in LEO to maintain 'safe' entry into martian atmosphere.

Assumptions

1. Instantaneous velocity applied during low earth orbit in radial direction.
2. Instantaneous velocity applied in away from planets center to slow down entry into low mars orbit.
3. One dimensional coordinate system.
4. Nonlinear gravitational field (changing with distance).
5. Not accounting for planet rotation around the sun so distance between earth and mars is constant.
6. Coordinate system starts at Earth’s center (0,0).

Governing Equations and Methodologies

The governing equations can be divided into one unique stage:

$$V_i = V_{LEO} + \Delta V_{LEO}$$

$$V_{LMO} = V_f + \Delta V_{LMO}$$

Differential equation governing net gravitational acceleration magnitude for unit distance away from center of Earth (reference point).

$$m\frac{dV}{dt} = -mg_E + mg_M$$

$$g_E = g_{o,E}\frac{R_{0,E}^2}{(r)^2}$$

$$g_M = g_{o,M}\frac{R_{0,M}^2}{(r - R_0^M)^2}$$

Here, r is a function of time.

Governing Equations and Methodologies

To describe the tangential velocity from low earth orbit at radius R from center of planet:

$$V_t = \sqrt{g_0 \frac{R_0^2}{R}} \quad , \quad V_{Leo} = \sqrt{g_0 \frac{R_{0E}^2}{R}} \qquad V_{Lmo} = \sqrt{g_0 \frac{R_{0M}^2}{[d\text{-}R]}}$$

Additionally, ΔV_{LMO} is a function of ΔV_{LEO} that is initialized as a domain spanning from 3300 m/s to 4000 m/s (justification for this domain is present in subsequent slides).

Determining Proper dV LEO

To determine the proper dV Lmo one must analyze subsequent dV Lmo value.

Based on non-convergent calculations below 3230 m/s, dV Leo must not be below this value (it will not escape orbit). Thus, the starting domain value of dV Leo is 3300 m/s.

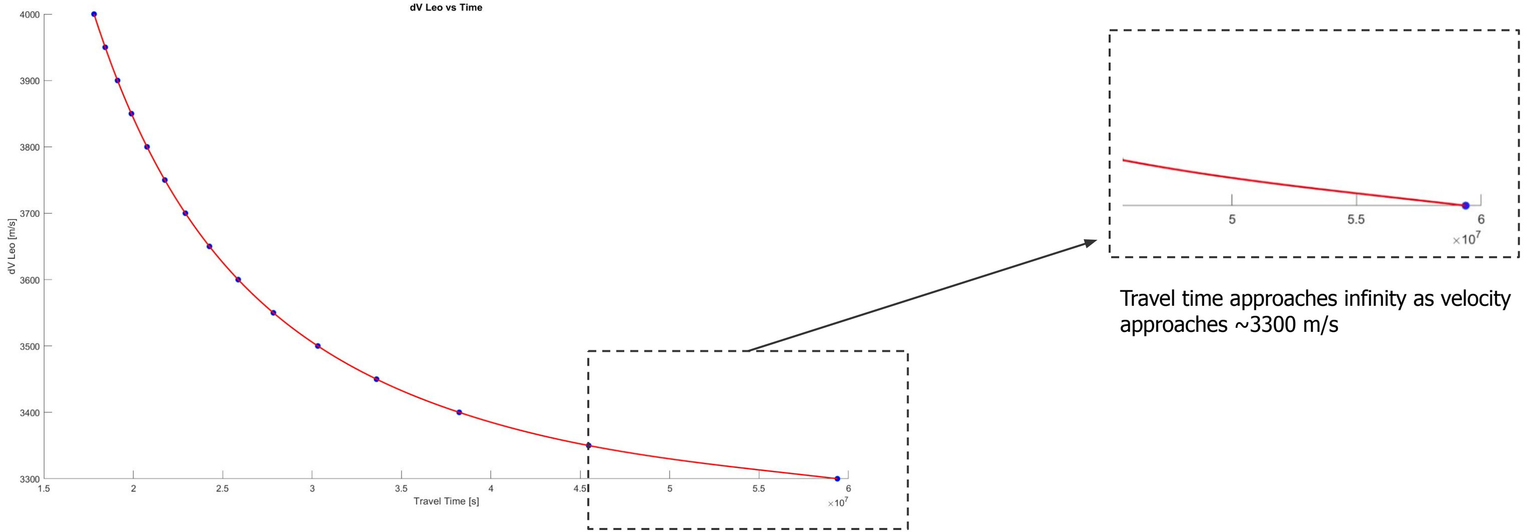
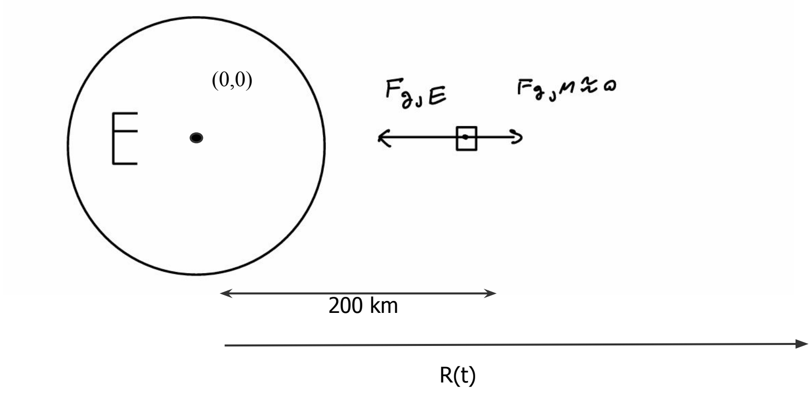


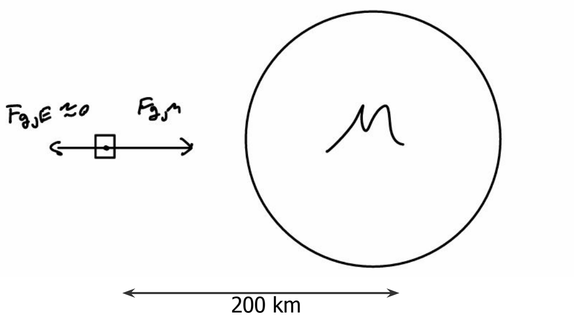
Figure 1. dV LEO vs total travel time for spacecraft to reach Mars LMO

Free Body Diagrams & Coordinate System

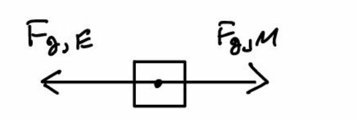
When in LEO:



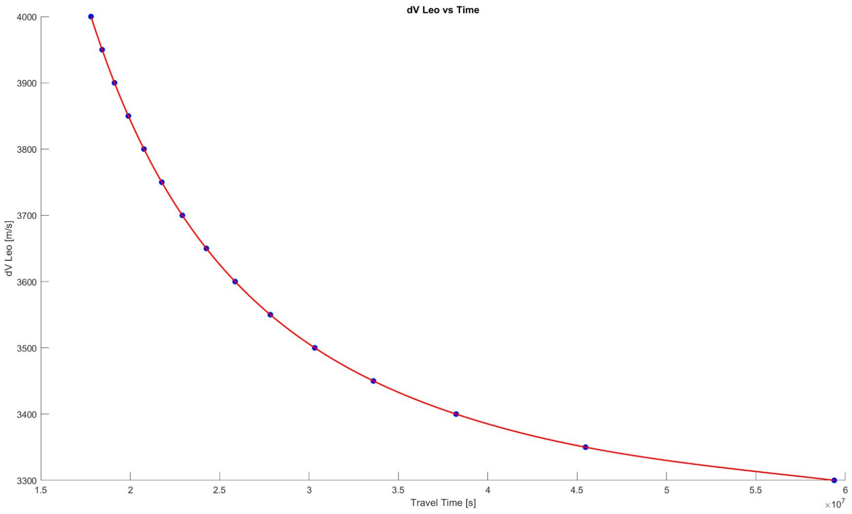
When in LMO:



During transition from Earth to Mars:



dV LEO and dV LMO Family of Solutions



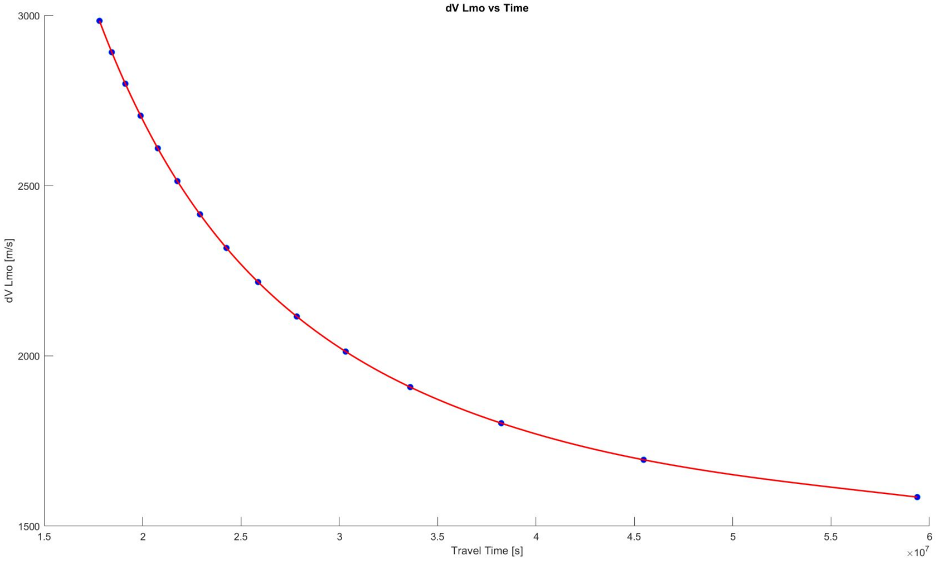


Figure 2. Recommended family of solutions for dV Leo and dV Lmo. These values are chosen based on the magnitude of travel time and resulting dV Lmo values expected. That is to say, dV Leo values below 3300 m/s are either too slow to reach mars (taking 3x the time relative to solutions like 3800m/s) or they cannot escape the gravity of earth. dV Leo values higher than 4000 m/s pose significant reorientation threats during LMO convergence as they require speeds higher than ~3000m/s to 'cancel' tangential velocity magnitude. More thorough justification for the chosen dV Leo domain will be present in the subsequent slide.

Alternative Case: dV Leo = [3300, 5000] m/s

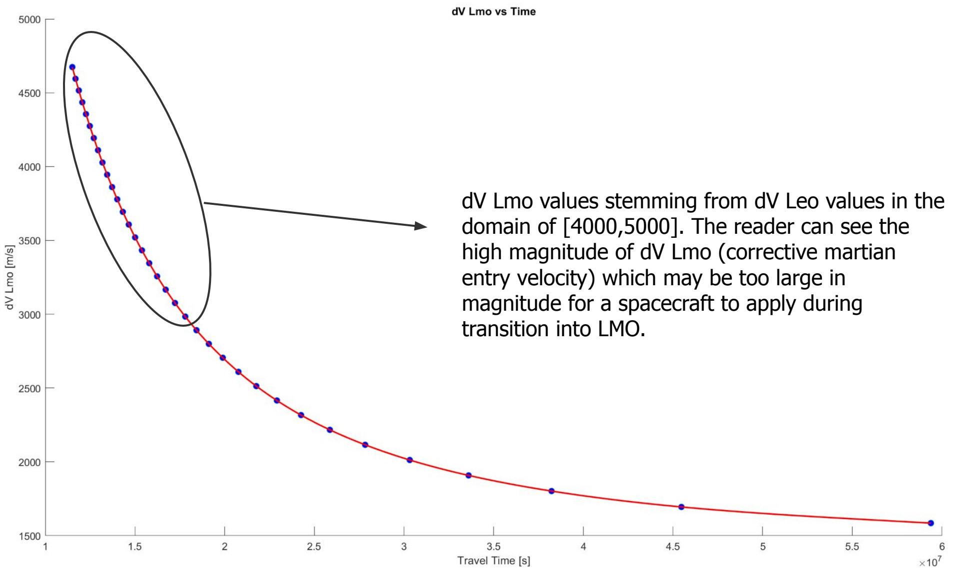


Figure 3. Hypothetical family of solutions of dV Lmo for values of dV Leo = [3300, 5000]. dV Lmo values above ~3000 m/s are to high to apply near instantaneous velocity correction during orbit entry. Thus, dV Leo should be restricted to a domain of [3300, 4000] m/s.

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Spacecraft Position vs Time

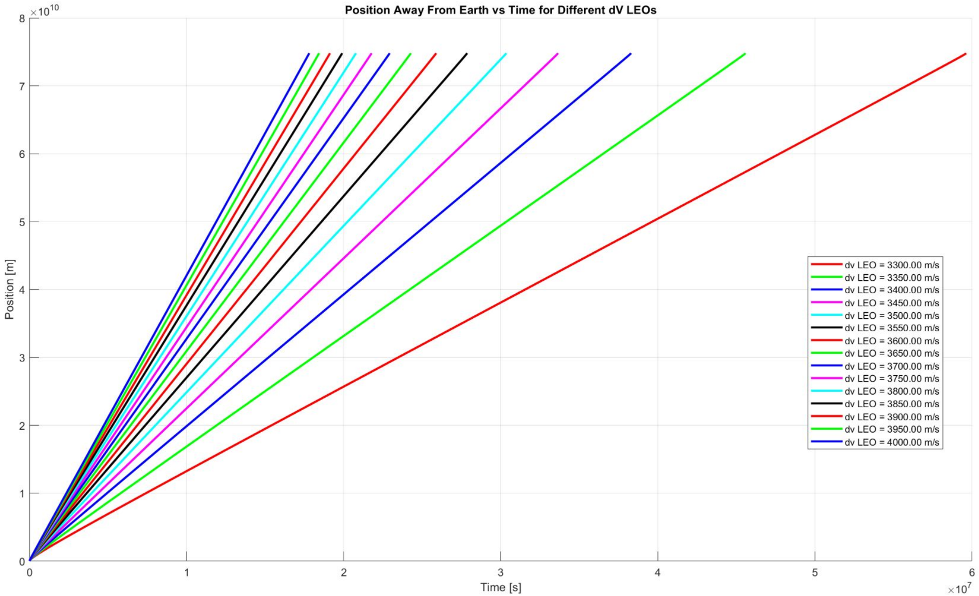

Figure 4. Spacecraft position vs time with legend. Although distance that craft travels remains exactly equal among all trials, total travel time shrinks rapidly as dV Leo increases.

Spacecraft Velocity vs Time

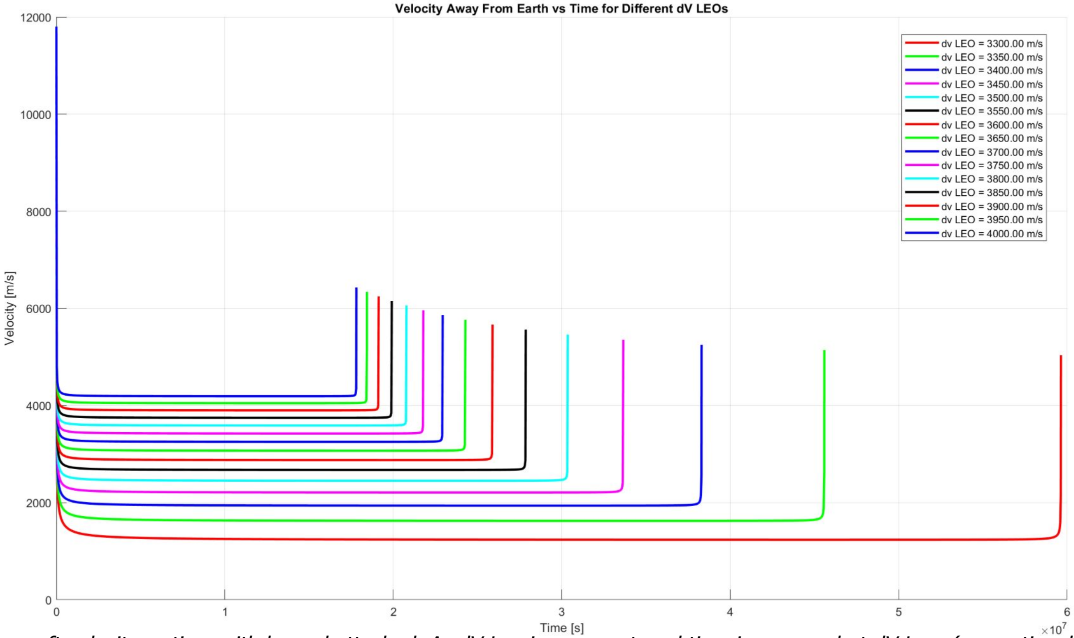

Figure 5. Spacecraft velocity vs time with legend attached. As dV Leo increases, travel time increases, but dV Lmo (proportional to the Y-Axis value) will increase exponentially. This makes interplanetary travel infeasible for dV Leo values higher than 4000 m/s as dV Lmo magnitudes approach infinity rapidly.

Spacecraft Velocity vs Time for large dV Leo domain [3300, 20000]

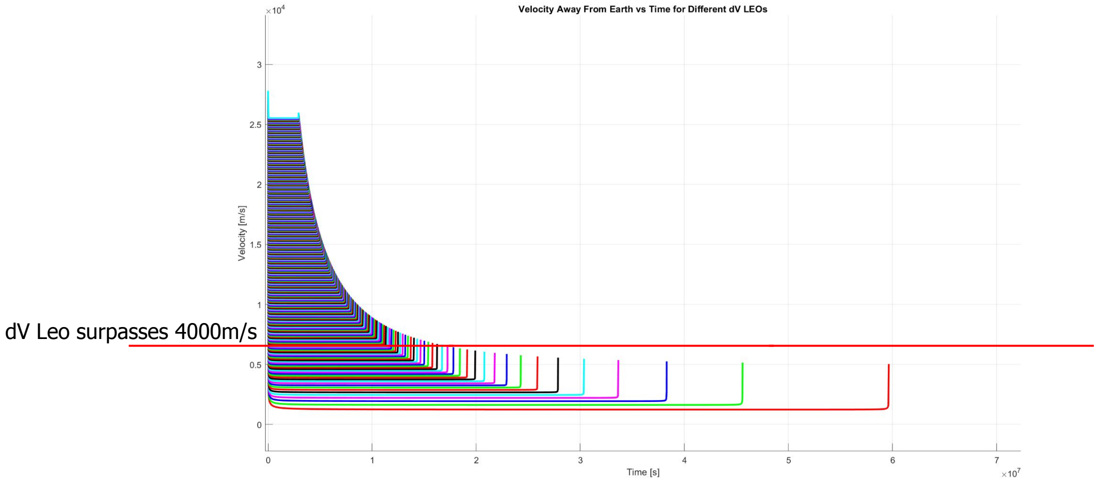

Figure 5. Spacecraft velocity vs time for a large dV domain of [3300,20000] m/s. Velocity value approaches infinity as travel time approaches 0. This response happens rapidly once dV Leo surpasses the ~4000 m/s regime. The subsequent dV Lmo values will be much to high for feasible system recovery once entering low mars orbit.

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Convergence

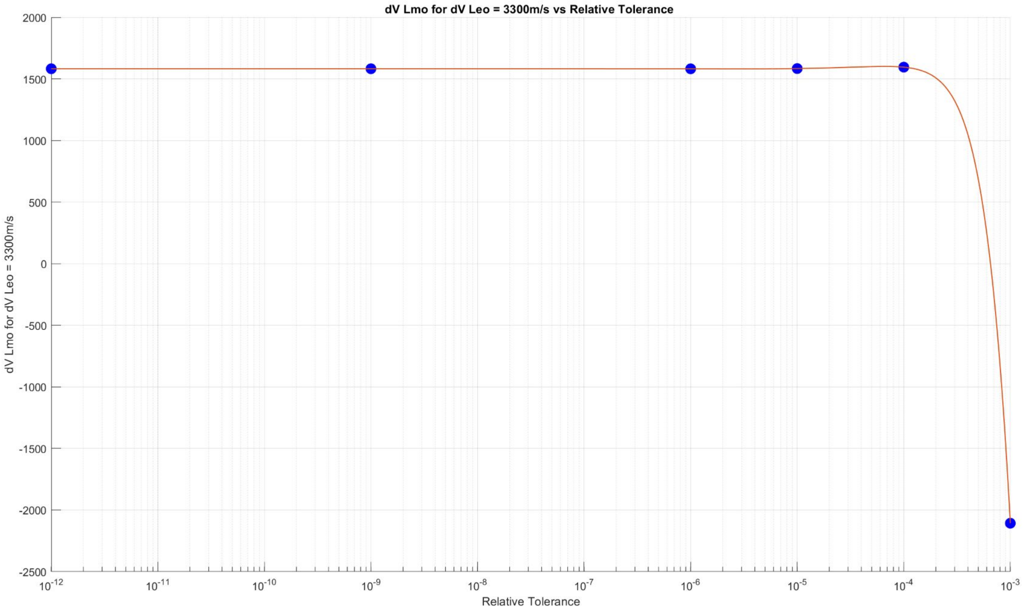

Figure 6. Convergence plot for dV Lmo when dV Leo = 3300 m/s. The relative tolerance is adjusted to log scale on the x-axis. Convergence achieved at reltol= 1E-9 as difference between reltol= 1E-9 and reltol= 1E-12 ~0%.

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Conclusions

- Convergence achieved at reltol=1E-09, which balances simulation accuracy and run time.
  - Relative tolerance values under E-03 significantly affect simulation results as model can no longer predict approach to martian atmosphere.
- Family of solutions lies between dV Leo 3300 m/s and 4000 m/s to balance between travel time and appropriately low dV Lmo corrective velocity values as explained in previous slides.