Assignment #2

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Goal

1. Plot velocity, position and acceleration as a function of time for post-apogee rocket launch.
2. Calculate the maximum load directed onto the heaviest section of the rocket and ensure the safety factor is *above* 2 for quick link and 4 for shock cord.
3. Determine the optimal deployment altitude based on how well it meets criteria 1, and 2.

Assumptions

1. Side wind at 0m/s ➡ No drift concerns.
2. 0 degree AOA.
3. Polynomial/Exponential modeling conditions.
4. Rocket landing mass at 17.10 kg, rocket diameter is 0.157 m and CD = 0.32.
5. Maximum parachute diameter = 2.44 m , and cd = 2.2.
6. ‘Two sections of masses 10.48 kg and 6.62 kg and the sections align vertically post deployment with the same form CD’.

Governing Equations and Methodologies

The governing equations can be divided into three unique phases of launch.

Phase 1 encompases pre-main-deployment;

$$CD_R \times A_R \times \rho(x) \times \frac{v(t)^2}{2} - mg = m\frac{dv}{dt}$$

Phase 2 encompasses parachute inflation dynamics and subsequent impact upon rocket system characteristics;

$$CD_R \times A_R \times \rho(x) \times \frac{v(t)^2}{2} + CD_P(t) \times A_P(t) \times \rho(x) \times \frac{v(t)^2}{2} - mg = m\frac{dv}{dt}$$

Phase 3 encompasses full parachute deployment.

$$CD_R \times A_R \times \rho(x) \times \frac{v(t)^2}{2} + CD_P \times A_P \times \rho(x) \times \frac{v(t)^2}{2} - mg = m\frac{dv}{dt}$$

Governing Equations and Methodologies cont.

The governing equations can be divided into three unique phases of launch.

Phase 1 implies a constant drag coefficient and cross-sectional area of just rocket body.

$$A(t) = A_{max} \qquad C_D = C_{D,f}$$

Area and CD converge to their final values for rocket (CD=.32)

Phase 2 implies a changing drag coefficient and cross-sectional area for parachute.

$$A(t) = A_{max}\left(1 - e^{-\frac{4t}{t_f}}\right)$$

$$C_D(t) = 1 + (C_{DPf} - 1)\left(\frac{t}{t_f}\right)^2$$

Exponential model for varying cross-sectional area and Cd with respect to time

$$A(t) = A_{max}\left(\frac{t}{t_f}\right)^2$$

$$C_D(t) = 1 + (C_{DPf} - 1)\left(\frac{t}{t_f}\right)^2$$

Polynomial model for varying cross-sectional area and Cd with respect to time

Phase 3 implies a constant drag coefficient and cross-sectional area for both parachute and rocket body.

$$A(t) = A_{max} \qquad C_D = C_{D,f}$$

Area and CD converge to their final values for rocket (.32) and parachute (2.2)

Governing Equations and Methodologies cont.

Time constant for exponential calculation:

$$\tau = \frac{t_f}{4}$$

Parachute fill time (*n*=12):

$$t_f = \frac{K\, C_{DPf}\, D_{Pf}}{V_{deploy}} = n\frac{D_{Pf}}{V}$$

Flight CONOPS

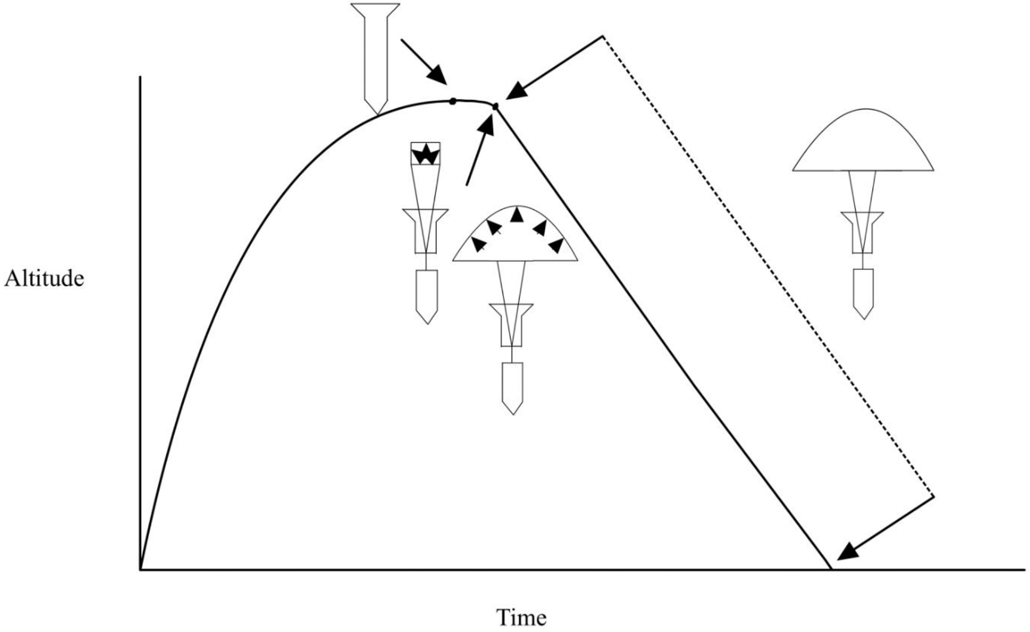

Figure 1. Indicates flight CONOPS for hypothetical rocket launch. Stages progress subsequently from left to right [1,2,3].

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Flight Characteristics [Polynomial Model]

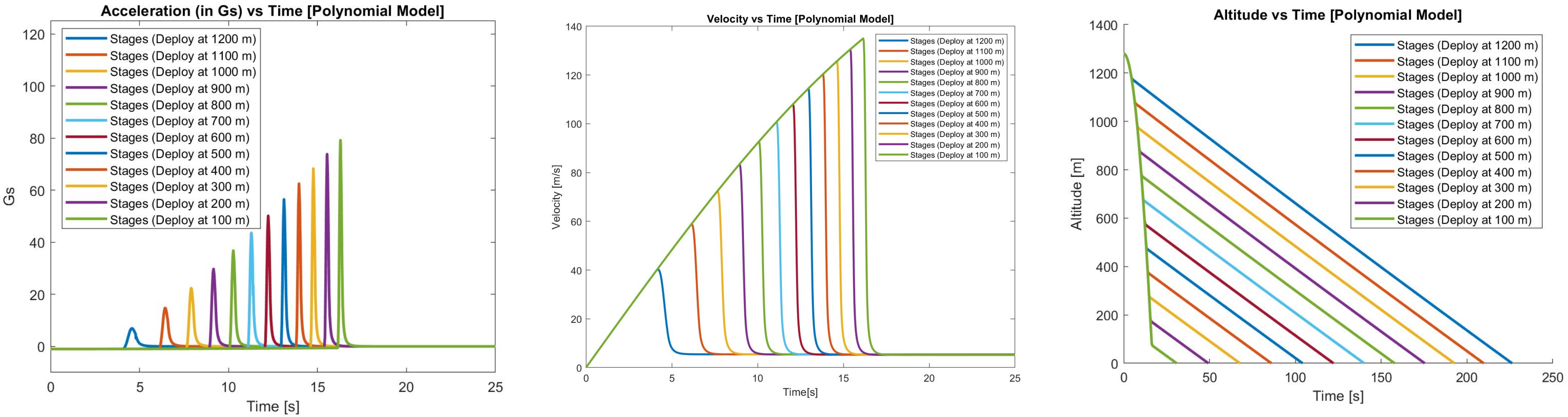

Figure 2. Flight plots of polynomial analytical model.

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Flight Characteristics [Exponential Model]

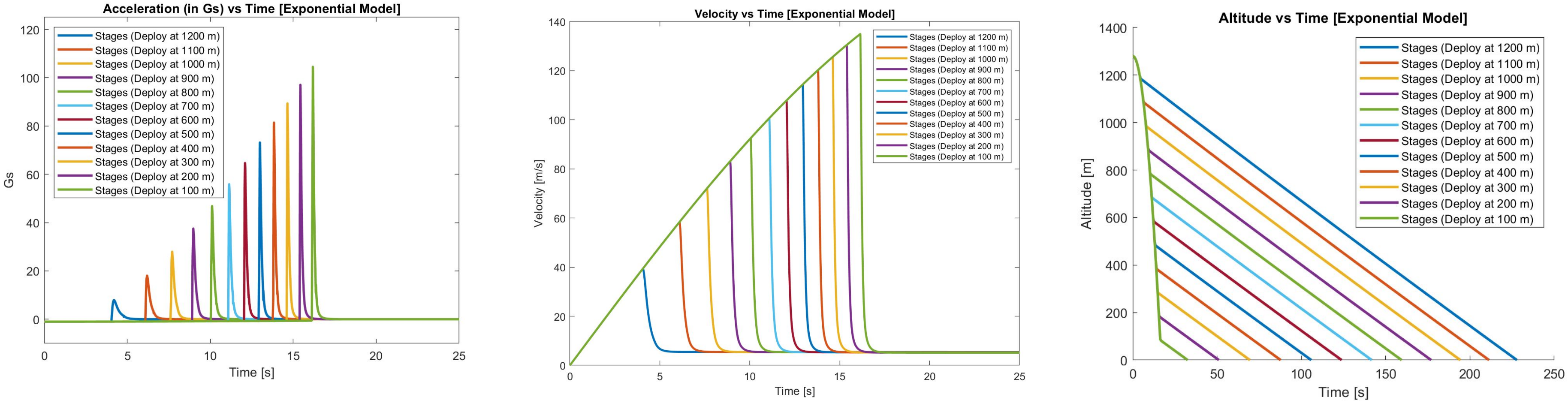

Figure 3. Flight plots of exponential analytical model.

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Shock Force Determination

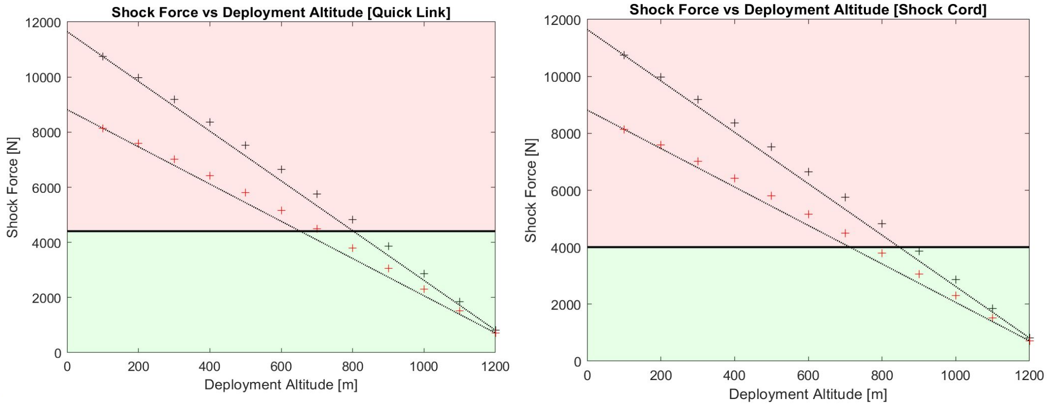


Figure 4. Red region indicates no fly zone. Green region indicates altitude domain analytically permitted for deployment. Both quick link failure (4400N) and shock cord failure (4000N).

Shock force vs deployment altitude for exponential model (black) and polynomial model (red). Designs for both <u>shock cord</u> and <u>quick link</u> are considered.

- 900 meter parachute deployment is the lowest permitted in both analytical models and agrees with Shock Cord or Quick Link SF consideration .
- This value will be used to in subsequent analyses as *minimum deployment value*.
- Trade off for 900m deployment in non-zero wind conditions is higher drift, but this is beyond the scope of this analysis.

Acceleration Characteristics at Deployment Altitude (900m)

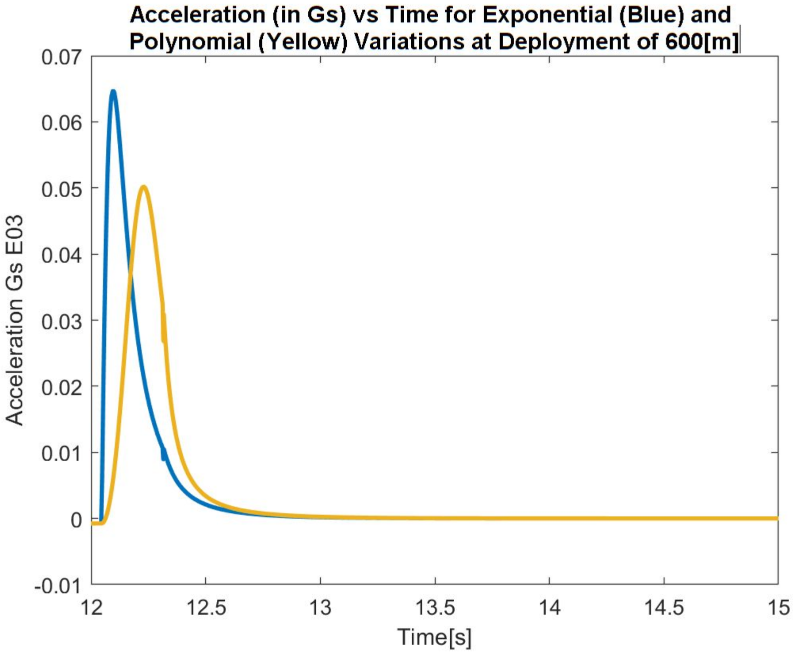
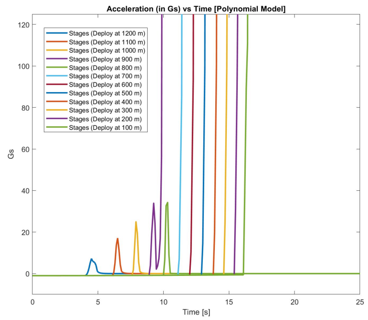


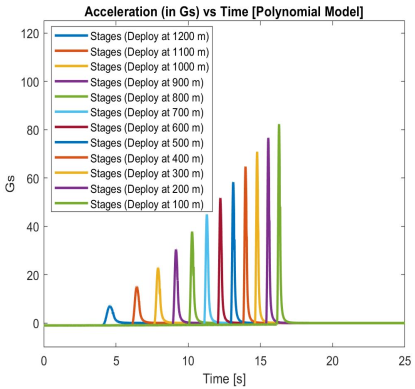
Figure 5. Acceleration vs time curves for exponential and polynomial analytical models.

Exponential model represents more aggressive acceleration curve with higher maximum acceleration. The value of max G has been accounted for in FS calculation.

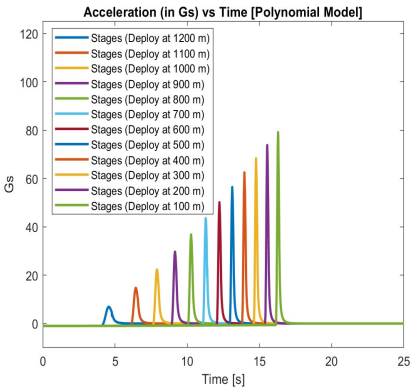
Convergence



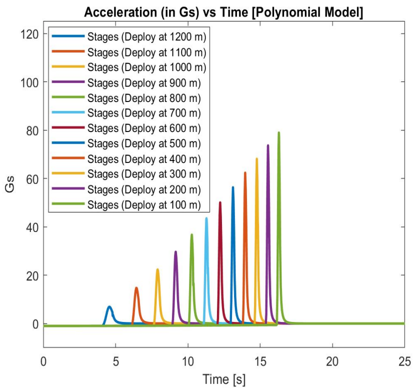
Time Step =0.1



Time Step =0.01



Time Step =0.001



Time Step =0.0005

Max Acceleration = inf

Max Acceleration = 82.3384G

Max Acceleration = 79.2088G

Max Acceleration = 79.0291G

$$\frac{\infty - 82.3384}{\infty} \times 100\% = NA$$

$$\frac{82.3384 - 79.2088}{82.3384} \times 100\% = 3.80\%$$

$$\frac{79.2088 - 79.0291}{79.2088} \times 100\% = .227\%$$

Converges!

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Conclusions

- Deploy at 900m to prevent structural damage.
    - Drift is not an issue given 0 m/s side winds and 0 degree AOA.
- Convergence happens between time steps of 0.001 and 0.0005
- Based on the previous analysis, *goals* 1, 2 and 3 have been meet.