Assignment #4

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Goal

1. Plot velocity and position of three body problem exiting with an instantaneous velocity from Low Earth Orbit (LEO) and entering into Low Mars Orbit (LMO) with constant thrust.
2. Slow down spacecraft with reverse thrusting to achieve smooth entrance into LMO.

Assumptions

1. Instantaneous escape velocity applied during low earth orbit in radial direction.
2. One dimensional coordinate system.
3. Nonlinear gravitational field (changing with distance).
4. Not accounting for planet rotation around the sun so distance between earth and mars is constant.
5. Coordinate system starts at Earth's center (0,0).
6. No time constraint, so the time increase due to reverse thrusting is justified.

Governing Equations and Methodologies

The governing equations can be divided into two unique approaches:

Differential equation governing net gravitational acceleration magnitude for unit distance away from center of Earth (reference point) with constant thrusting.

$$\frac{dV}{dt} = -g_{0E}\frac{R_{0E}^2}{R^2} + \frac{T_0}{m} - \frac{T_0}{m^2}\frac{dm}{dt}\frac{(R - R_{0E})}{V} + g_{0M}\frac{R_{0M}^2}{(R_{EM} - R)^2}$$

Differential equation governing net gravitational acceleration magnitude for unit distance away from center of Earth (reference point) with thrust as a function of distance from the sun.

$$\frac{dV}{dt} = -g_{0E}\frac{R_{0E}^2}{R^2} + \frac{T_0}{m}\frac{R_{ES}}{(R_{ES} + R)} - \frac{T_0}{m}\frac{R_{ES}}{(R_{ES} + R)^2}(R - R_{0E}) - \frac{T_0}{m^2}\frac{dm}{dt}\frac{R_{ES}}{(R_{ES} + R)}\frac{(R - R_{0E})}{V} + g_{0M}\frac{R_{0M}^2}{(R_{EM} - R)^2}$$

Here, r is a function of time.

Governing Equations and Methodologies

The initial radial velocity must be imposed as a magnitude of the escape velocity from earth.

Here, it can be described for both cases as:

$$V_{Escape} = [2 * g_{Earth} * R_{Earth}]^{1/2}$$

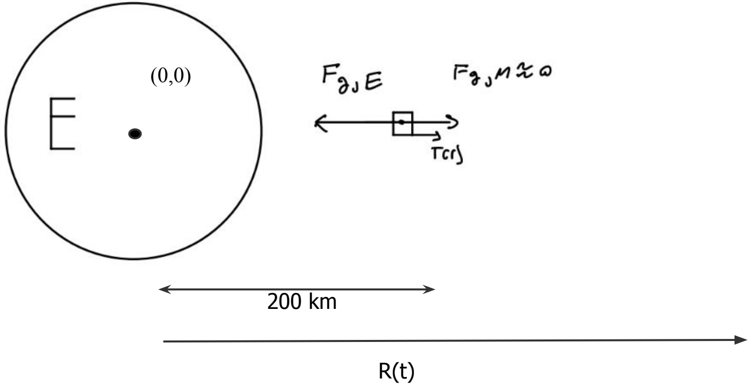
Change in Mass in Relation to Thrust

The mass flux (or time derivative of the mass function) can be related to the thrust in the following way. This allows for continuous plotting of mass as a function of thrust, which depending on the governing acceleration equation can depend on r.

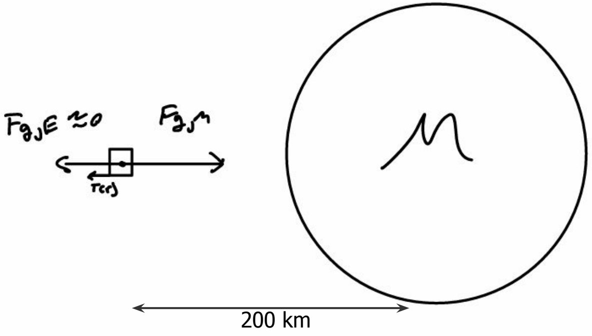
$$\frac{\mathrm{dm}}{\mathrm{d}t} = \frac{T(r)}{I_{sp}}$$

Free Body Diagrams & Coordinate System

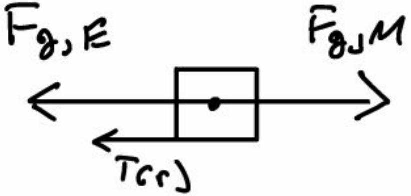
When in LEO:



When in LMO:



During transition from Earth to Mars:



T(r) can be negative (as shown) or positive depending on if thrust direction change is active

Linear Thrust with No Thrust Direction Change

Constant Thrust Mass, Change in Mass, and Thrust v Time: No Thrust Direction Change

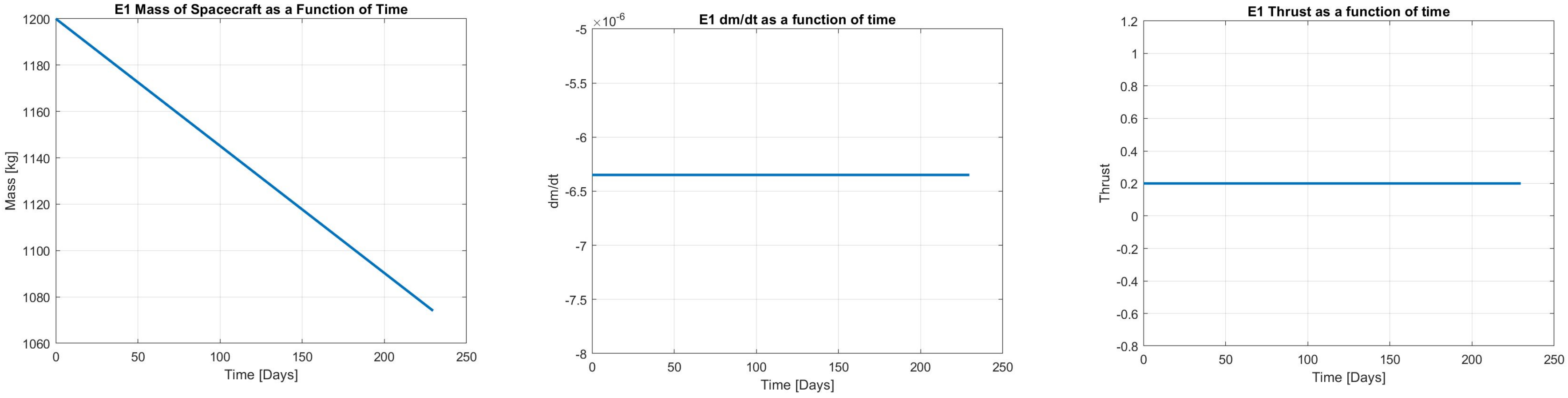


Figure 1. Mass, dm/dt, and thrust as a function of time for motion equation 1 (E1). Because thrust is constant and has no dependance on solar energy, dm/dt is constant and small as mass decreases from 1200 kg to ~1070 kg in ~230 days. Here, no thrust direction change is present.

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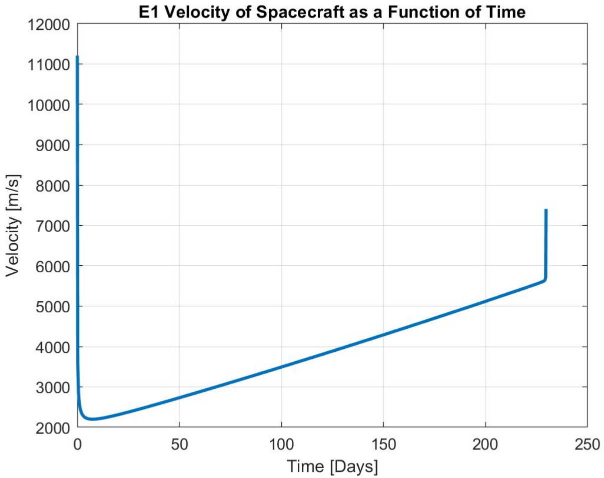
Constant Thrust: Velocity and Position Plots: No Thrust Direction Change



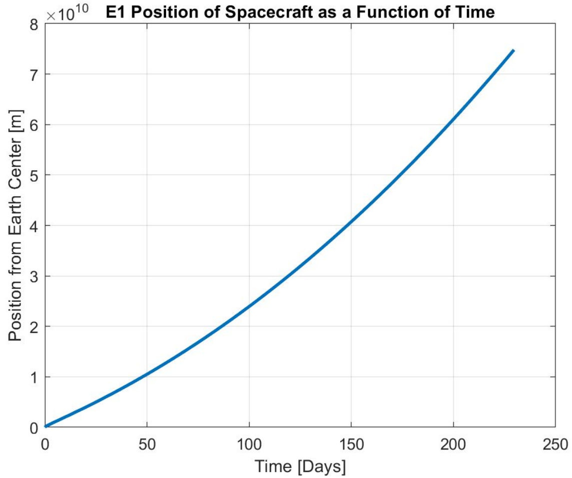
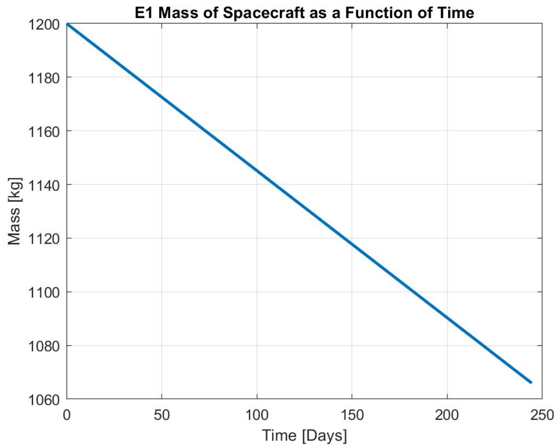
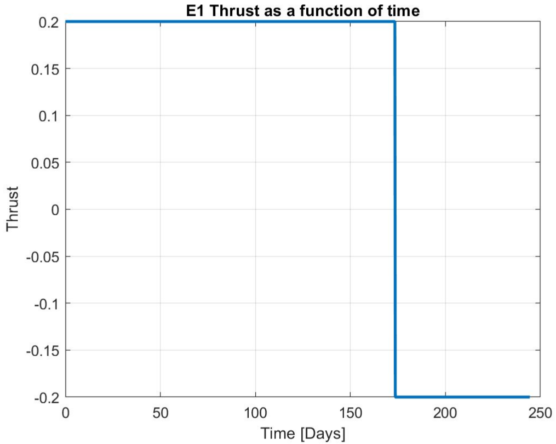


Figure 2. Velocity and position plots with respect to time. Here, it can be seen that entrance velocity is ~7500 at 200 km from martian surface with a ~230 day arrival time. This velocity value is too high for this mission's purposes and thus reverse thrusting is justified.

Linear Thrust with Thrust Direction Change

Constant Thrust Mass, Change in Mass, and Thrust v Time: Thrust Direction Change





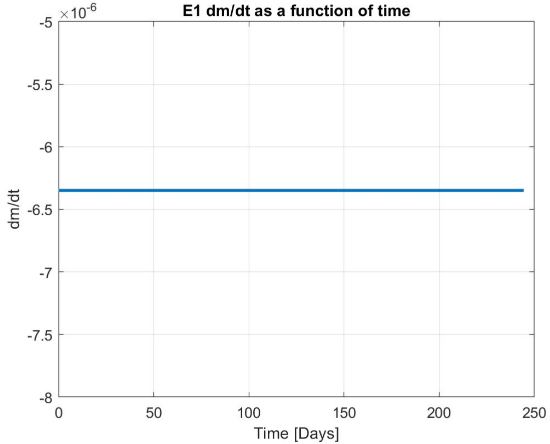


Figure 3. Mass, dm/dt, and thrust as a function of time for motion equation 1 (E1). Because thrust is constant and has no dependance on solar energy, dm/dt is constant and small as mass decreases from 1200 kg to ~1066 kg in ~245 days. Additionally, thrusting is assumed to change direction instantaneously, thus the change in thrust from 0.2 N to -0.2 N happens instantaneously.

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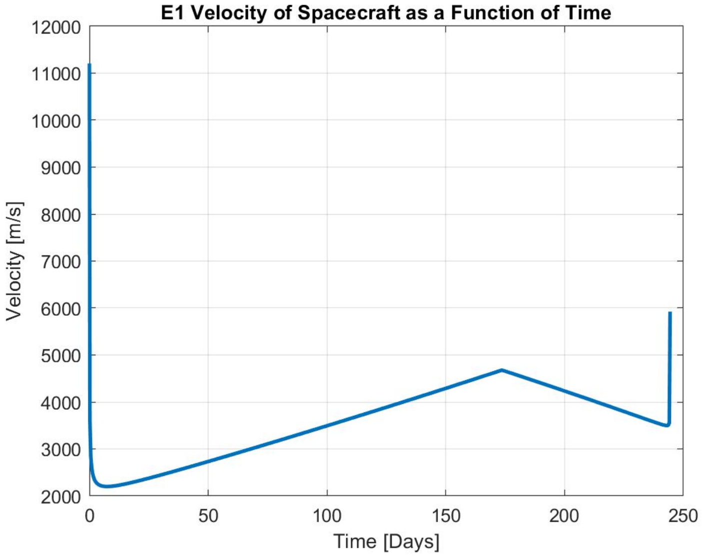
Constant Thrust: Velocity and Position Plots: Thrust Direction Change



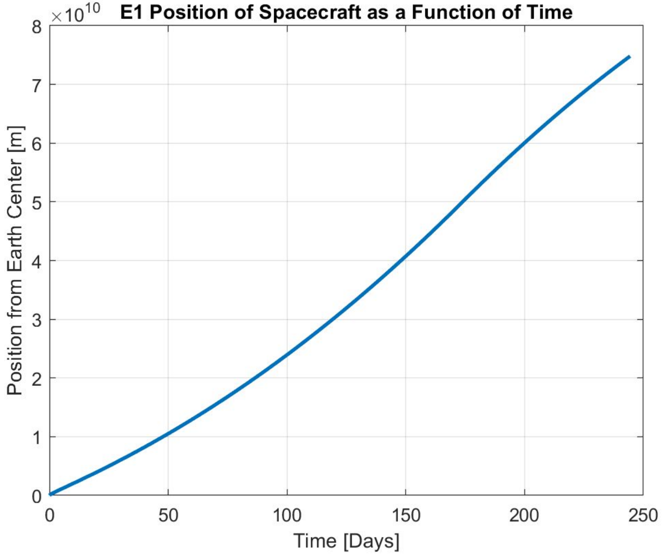


Figure 4. Velocity and position plots with respect to time. Here, it can be seen that entrance velocity is ~5923 m/s at 200 km from martian surface with a ~245 day arrival time. This velocity value iis much lower than the trial with no change in thrust direction. The point at which thrust is reversed (~173.611 days) was chosen through trial and error and represents a good balance between travel time and entrance velocity.

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Non-Linear Thrust with Thrust Direction Change

Non-linear Thrust Mass, Change in Mass, and Thrust v Time: Thrust Direction Change

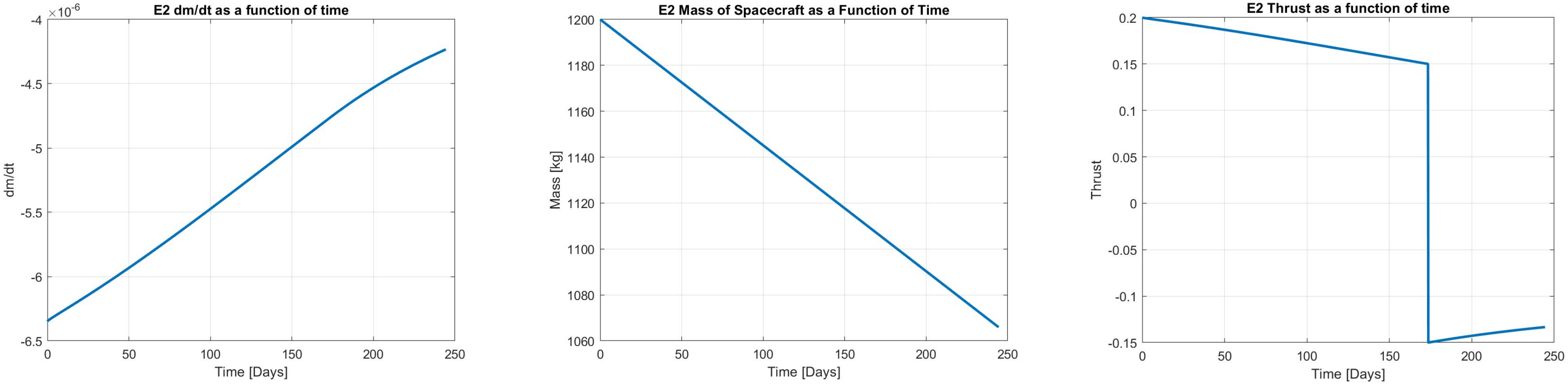
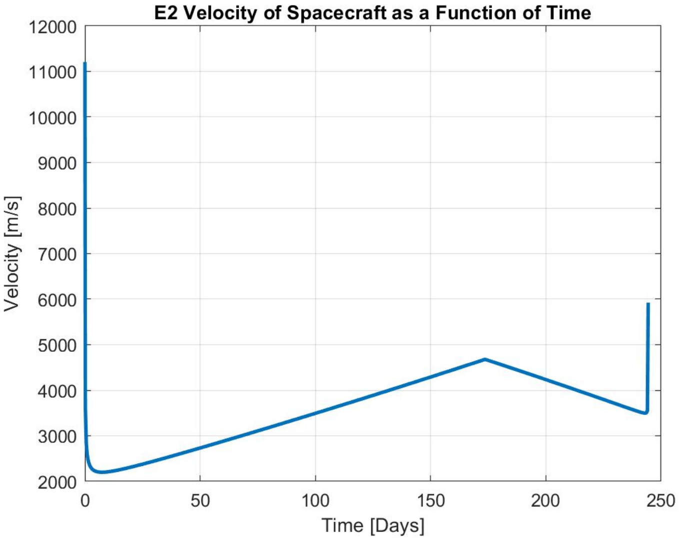


Figure 6. Mass, dm/dt, and thrust as a function of time for motion equation 2 (E1). Because thrust varies only slightly between the two models, the total approach time is nearly identical, yet the non-linear thrusting is about .004 days faster. Additionally, thrusting is assumed to change direction instantaneously, thus the change in thrust from 0.2 N to -0.2 N happens instantaneously. In total, the spacecraft reaches Mars in about 244.387 days.

Non-linear Thrust: Velocity and Position Plots: Thrust Direction Change



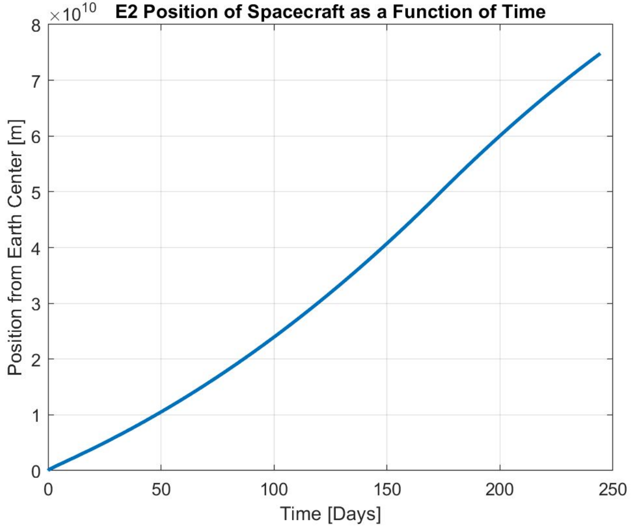


Figure 4. Velocity and position plots with respect to time. Here, it can be seen that entrance velocity is ~5915 m/s at 200 km from martian surface with a ~245 day arrival time. This velocity value is much lower than the trial with no change in thrust direction. The point at which thrust is reversed (~173.611 days) was chosen through trial and error and represents a good balance between travel time and entrance velocity. This value is very similar to the linear thrust with direction change partially because the ion thrusting force magnitude is so low, so subtle changes to its value will not substantially affect spacecraft motion. The total fight time is smaller relative to the previous trial, which be attributed to the decreased ion thrust magnitude in the reverse thrust phase (after t ~173.611 days) leading to a less ability to slow down the spacecraft during this period.

Convergence

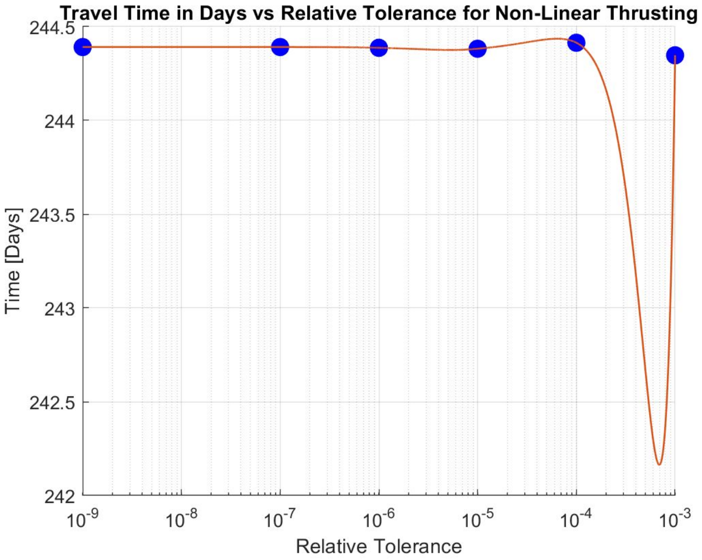


Figure 6. Convergence plot for total travel time in days vs relative tolerance. Convergence occurs at RelTol = 10^-7.

Conclusions

- Reversing thrust significantly decreases entrance velocity and facilitates smooth entrance into Martian atmosphere.
    - No change in thrust creates velocity values ~1700 m/s higher than changing thrust direction at 173.611 days.
- Thrust variance due to radial displacement from sun is not as significant as originally expected, with total time between *non-linear thrust with thrust direction change* and *linear thrust with thrust direction change* arrival times having a difference of 0.004 days (~6 minutes).
- Convergence occurs at RelTol = 1*10^-7 to balance simulation run time and computational accuracy.