Assignment #3

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Goal

1. Plot velocity, position and acceleration as a function of time for pre-apogee rocket launch.
2. Calculate the associated acceleration, velocity and position vectors with respect to time for a variety of wind and launch angle conditions.

Assumptions

1. Side wind at 0 mph, 5 mph, and 10 mph and follow non-linear wind variation with altitude.
2. 0 degree and 5 degree launch angle and change due to windcocking.
3. Nonlinear density modeling conditions.
4. Rocket mass is 18.86 kg, motor mass is 1.76 kg, rocket diameter is 0.157 m and CD = 0.32, side CD = 1.

Governing Equations and Methodologies

The governing equations can be divided into four unique stages;

Phase 1 encompases motor start to launch rail clearance:

$$m\frac{dV}{dt} = T - mg - \frac{C_D \rho V^2 A}{2}; m = m(t), V = V(t) \qquad \Delta V = \left(\frac{T - mg - \frac{C_D \rho V^2 A}{2}}{m}\right)\Delta t$$

$$\frac{dm}{dt} = -K;\ \Delta m = -K\Delta t; m = m_0\ at\ t = 0$$

Where K=0.826, m(0)=18.86

Convergence modeling:

$$V = V + \Delta V$$

$$\boldsymbol{Z = Z_0 + V\Delta T + \frac{\Delta V}{2}\Delta t}$$

$$m = m - \Delta m$$

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Governing Equations and Methodologies

Phase 2 encompasses launch rail clearance to end of windcocking:

Flight dynamics:

$$m\frac{dV}{dt} = T - mgcos\theta - \frac{C_D \rho V^2 A}{2}; m = m(t), V = V(t)$$

$$\Delta V = \left(\frac{T - mg\ Cos\theta - \frac{C_D \rho V^2 A}{2}}{m}\right)\Delta t$$

$$V = V + \Delta V \quad \Rightarrow \quad V_Z = V\ Cos\theta;\ V_X = V\ Sin\theta$$

Where Θ corresponds the instantaneous angle between velocity vectors in the horizontal (x) and vertical (z) directions.

Governing Equations and Methodologies

Phase 2 encompasses launch rail clearance to end of windcocking:

Time length specification

$$\Delta t = t_2 - t_1 = 2\tau;\ \tau \sim \left(\frac{W}{\frac{T}{m} - g}\right)\frac{n^2}{(n-1)}$$

Tau indicates the half the magnitude of time in which windcocking will end. Generally, this will inform the domain scale of the non-linear theta calculation.

Non-Linear Theta Calculation:

$$\omega = \alpha t = \alpha_0\left(1 - \frac{t_f - t_1}{2\tau}\right)(t_f - t_1);\ \omega = 0\ at\ t_f = t_1$$

$$\theta = \theta + \Delta\theta;\ \Delta\theta = \omega\Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\theta = 0\ at\ t = t_1\ or\ \theta = \theta_0\ the\ launch\ angle\ at\ t_1$$

The instantaneous angle between velocity vectors in the horizontal (x) and vertical (z) directions [theta] can be determined non-linearly for more accurate modeling as opposed to interpolating. This is based on the instantaneous angular velocity and acceleration of the body.

Including Angular Acceleration Term in Non-Constant Theta Calculations

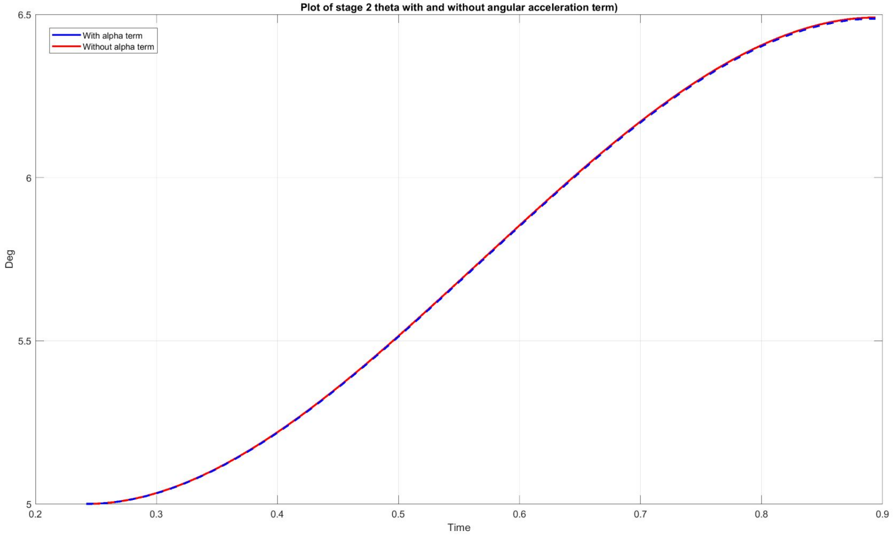

Figure 1. Theta vs time plot for 6.70m/s-0 degree launch conditions. Subsequent analysis is true for all observed launch rail states and non-zero wind conditions. Interestingly, theta vs time with and without incorporating angular acceleration leads nearly to the same result. However we can observe that the curve without angular acceleration maintains a slightly higher value for all values of t. This can be explained by the presence of a negative angular acceleration due to the increasing torque of the wind force applied in the opposite direction of motion. For accuracy, theta will be calculated with the alpha term.

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Governing Equations and Methodologies

Phase 3 encompasses end of windcocking to motor burn-out:

Note: For 0 mph wind conditions this stage is bypassed, opting to extend the second stage to motor burnout (t~[0.242s, 2.128s]).

For all non-zero wind conditions (5 mph, 10 mph):

Flight dynamic equations remain exactly the same from stage 2. However, theta is a constant value.

$$m\frac{dV}{dt} = T - mgcos\theta - \frac{C_D\rho V^2 A}{2}; m = m(t), V = V(t)$$

$$\Delta V = \left(\frac{T - mg\ Cos\theta - \frac{C_D\rho V^2 A}{2}}{m}\right)\Delta t$$

$$V = V + \Delta V \Rightarrow V_Z = V\ Cos\theta;\ V_X = V\ Sin\theta$$

Where Θ corresponds the instantaneous angle between velocity vectors in the horizontal (x) and vertical (z) directions.

Governing Equations and Methodologies

Phase 4 encompasses motor burn-out to apogee:

At this point there is no thrust from the motor. Thus,

$$m\frac{dV}{dt} = -\frac{C_D \rho V^2 A}{2};$$

$$\Delta V = -\left(\frac{\frac{C_D \rho V^2 A}{2}}{m}\right)\Delta t\ ; t_f = t_f + \Delta t$$

$$V = V + \Delta V;\ V_z = VCos\theta;\ V_X = VSin\theta$$

Additionally, theta is non-constant and dependent on instantaneous velocity values. Modeled by,

$$\theta = tan^{-1}\left(\frac{V_X}{V_Z}\right)$$

Governing Equations and Methodologies

For all stages, the following flight conditions are imposed:

Non-Linear Wind Conditions:

$$\frac{W_Z}{W_0} = \left(\frac{Z}{Z_0}\right)^{1/7}$$

Non-Linear Air Density Variations:

$$\frac{\rho}{\rho_0} = \left(\frac{(T_0 - Bz)}{T_0}\right)^{\frac{g}{RB}-1}; \frac{g}{RB} = 5.26$$

Flight CONOPS

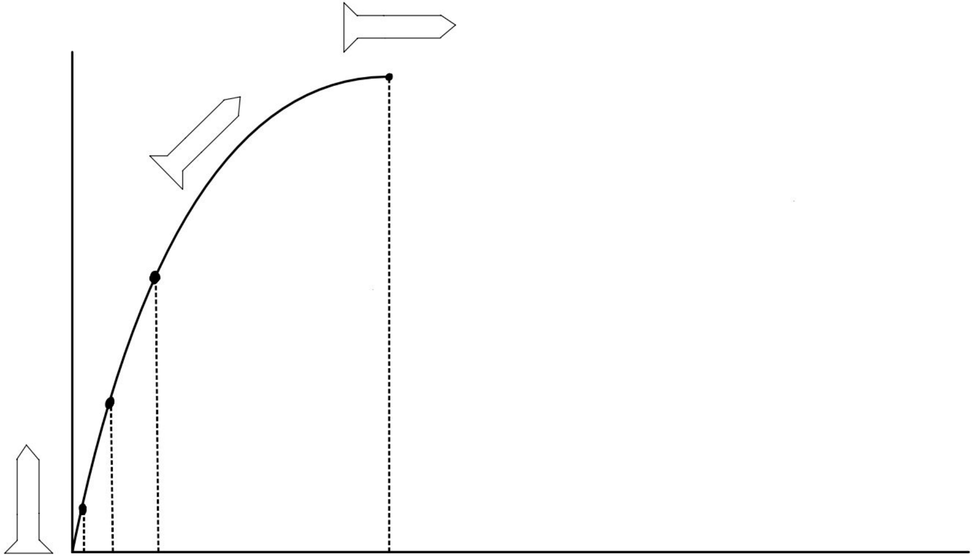
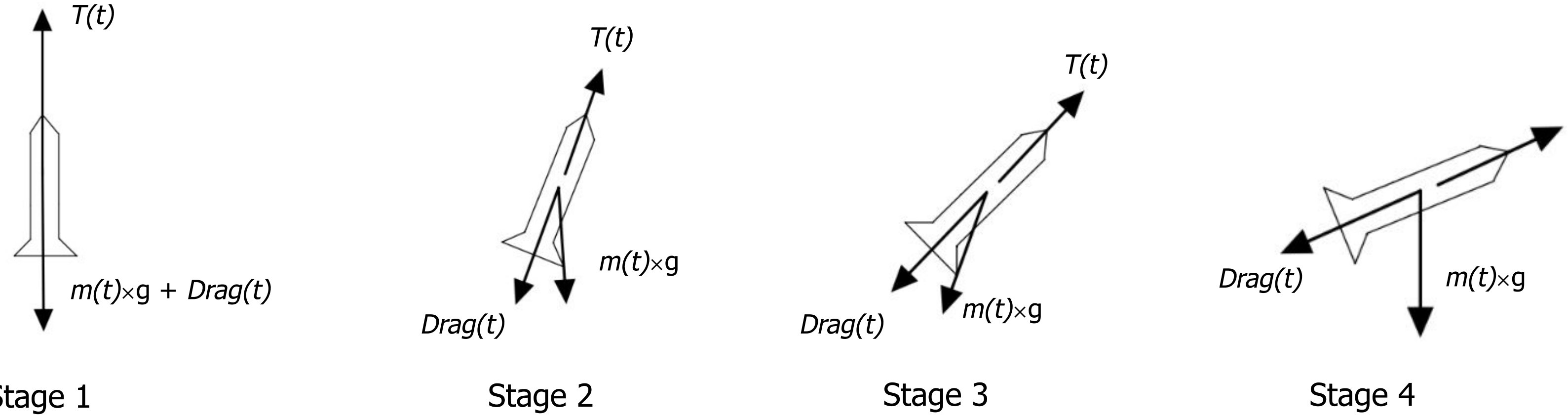


Figure 3. Indicates flight CONOPS for hypothetical rocket launch. Stages progress subsequently from left to right [1,2,3,4].

Free Body Diagram of Individual Stages



Note: Theta values for each stage follow the distribution given in slide 17

Thrust Data

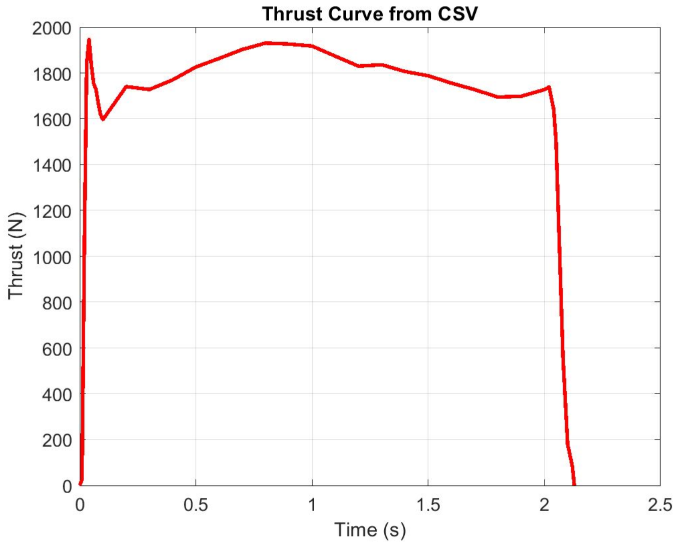


Figure 2. Thrust data taken from https://www.thrustcurve.org/motors/Cesaroni/3660L1720-P/ which indicates motor thrust at precise time values. These will be used in simulation calculations.

Altitude vs Time

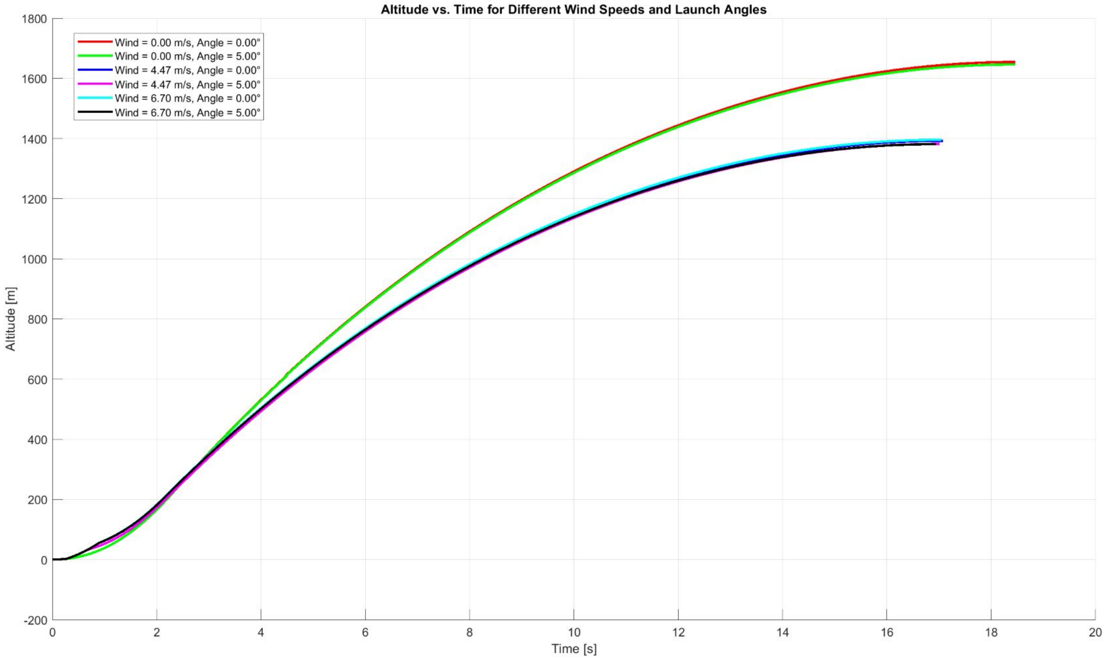


Figure 4. Altitude versus time for all wind and launch angle conditions. Conditions where launch angle = 0 deg are at the highest apogee values which agree with intuition, lending credit to the simulations accuracy. Wind speeds above 0 m/s all seem to converge at relatively the same apogee value with 6.70 m/s, 0 degrees being slightly higher than the subsequent apogee value of 4.47 m/s 0 degrees. This interesting result indicates that apogee time values, which are user imposed based on geometric interpretation may be slightly off. More likely is the conclusion that side winds of similar magnitude have a relatively small effect on apogee when compared with each other.

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Horizontal Displacement vs Time

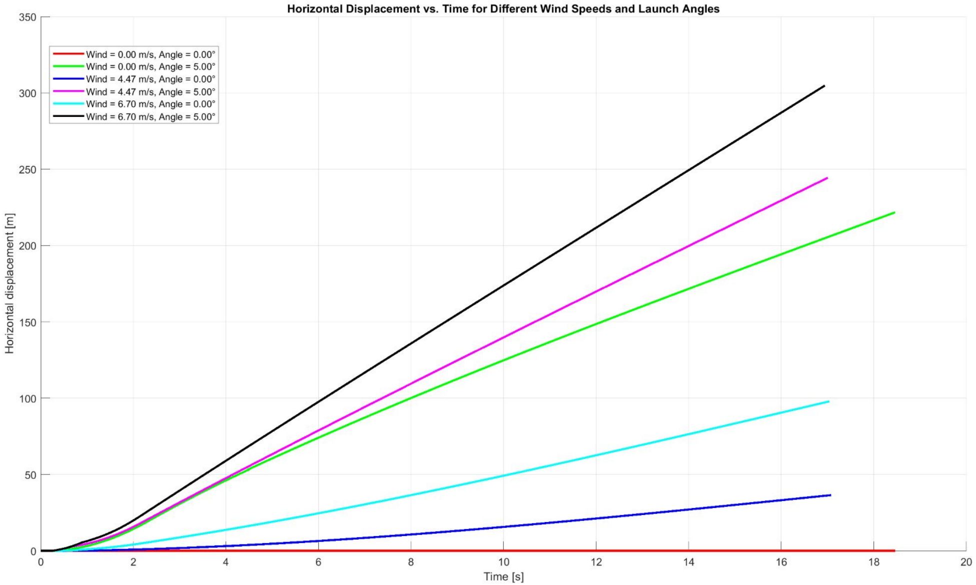

Figure 5. Horizontal displacement vs time for given wind and launch angle conditions. Data indicates that all 5 degree launches have a larger maximum horizontal displacement than their 0 degree counterparts. Geometric interpretation of the plot agrees with intuition, mainly that as side wind speeds and launch angle increase, maximum horizontal displacement increases. Additionally, data appears non-linear in the pre-burnout regime which seems accurate mainly due to the rockets rapid change in altitude (and thus wind speed conditions) during this time.

Velocity in Z Direction vs Time

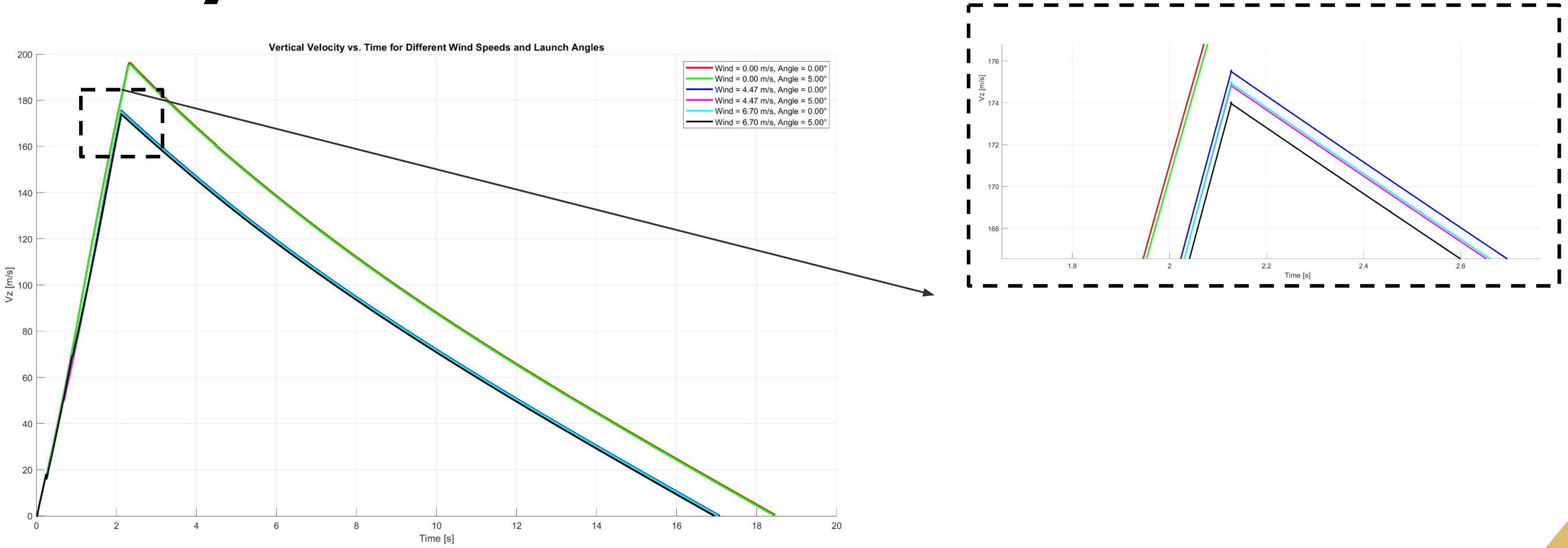

Figure 6. Indicates velocity in the z direction (vector direction normal to ground area) during time. Slight changes in Vz at t ~0.221 [s] correspond to launch rail clearance and could indicate velocity interpolation errors as launch rail exit should not significantly impact velocity flight data. Time beyond launch rail exit until motor burnout show a linear velocity profile within the wind speed and angle parameters. Wind speeds of 0 m/s are the highest in magnitude in this stage followed by 4.47 m/s-0 degrees, 6.70 m/s-0 degrees, 4.47m/s-5 degrees and finally 6.70m/s-5 degrees. Both launch angle increases and wind speed increases should correlate to a decrease in velocity as (in the former case) less energy is allotted in the vertical direction and (in the latter case) there is increased drag during flight.

Theta vs Time

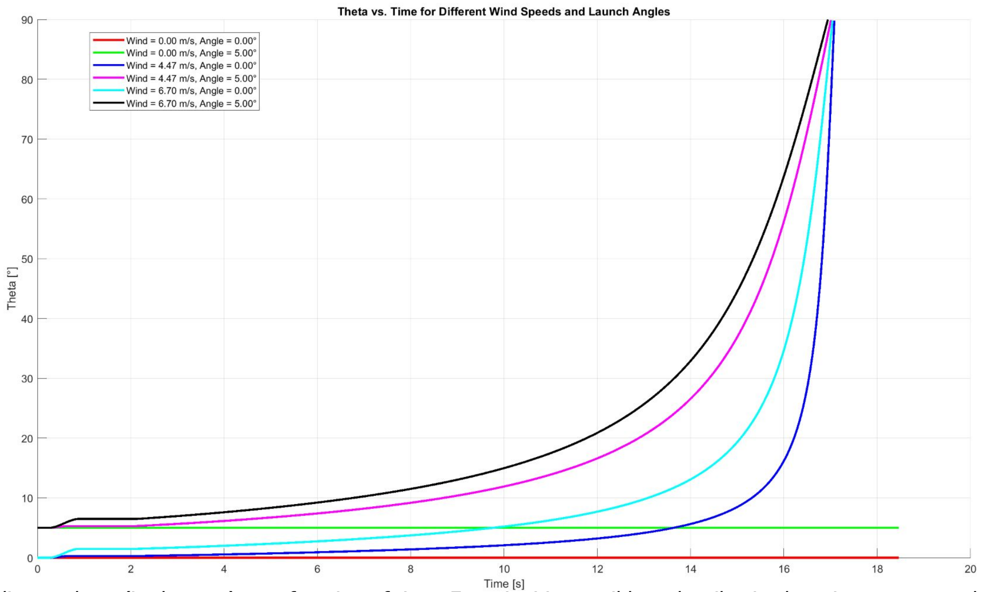

Figure 7. Indicates theta (in degrees) as a function of time. From ignition until launch rail exit, theta is constant at the value of the launch angle; from launch rail exit to end of windcocking a nonlinear relationship between theta and time is implemented (refer to slide 7); after, theta is constant until motor burnout at t = 2.128 [s]; finally theta increases following a arctangent relationship between vertical (numerator) and horizontal (denominator) as seen in slide 9.

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Acceleration vs Time

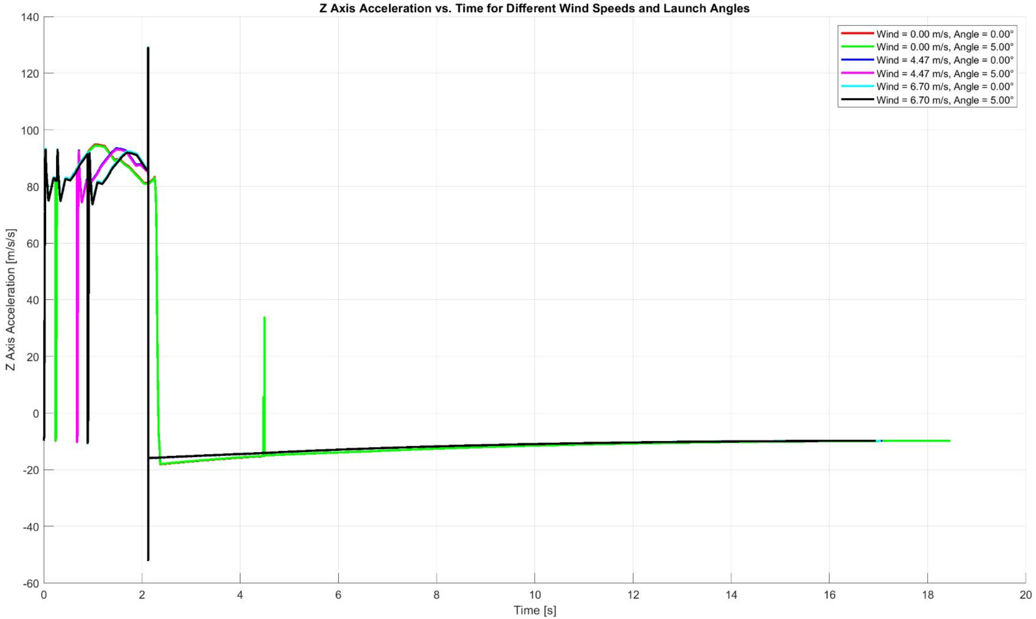

Figure 8. Z axis acceleration vs time plot for all flight conditions. Large deviations in acceleration data is caused by issues in 'gradient' matlab function interacting with transitional periods in stage regime (i.e. moving from stage 1 to stage 2). In general, instantaneous spikes to -9.81 m/s can be related to a zero drag and thrust acceleration quantities. This (and other data noise) is most likely due to a compilation error and should be ignored as analysis shows they do not exist long enough to significantly affect previous flight parameters. Acceleration is most interesting in stage 4 due to the visible decrease in drag profile due to a decreasing velocity.

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Convergence

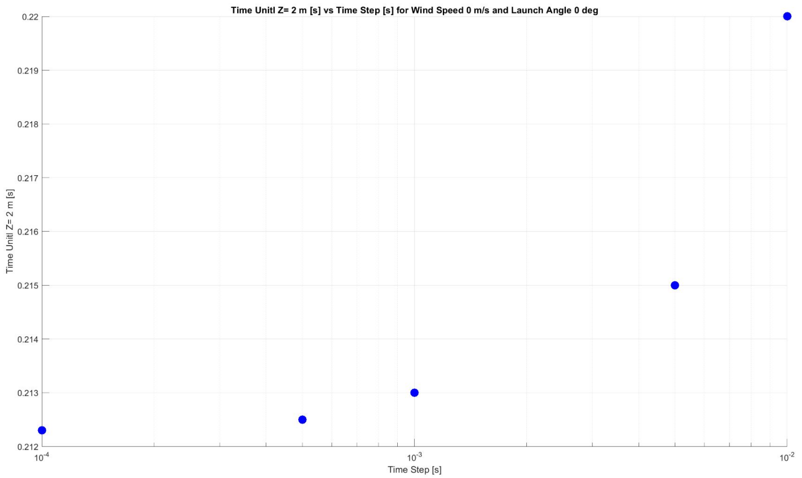

Figure 9. Convergence plot for previous simulation. The y-axis metric represents the time it takes for the rocket to clear the launch rail (Z = 2 [m]). This was chosen as convergence parameter mainly because it does largely depend on wind and launch angle conditions and is relatively linear when modelling (as opposed to later altitude, velocity and acceleration values). Simulations predict convergence at dt=0.0001, however suitable convergence exists at dt=0.001. In combination with the fact that dt = 0.001 leads to significantly lower simulation run times, this is the chosen value for the time step in the program file.

Conclusions

- Convergence achieved at dt = 0.001, which balances simulation accuracy and run time.
- Non-linear theta calculations provide accuracy in windcocking modelling.
  - Angular acceleration term does not significantly impact flight data value but is necessary for precise modelling.
- Side wind and launch angle significantly affect horizontal displacement.
  - Apogee varies the largest between 0 m/s wind velocity non-zero wind velocity.
- Position, velocity and acceleration profiles agree with theoretical expectations but need further validation through testing.