

Assignment #7

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Goal

1. Discover the earliest point of ignition for an ideal ramjet.
2. Analyze specific thrust curves for a variety of area ratios.



Governing Equations and Methodologies

Part 1:

$$\frac{T}{\dot{m}} = (1 + f)V_4 - V_0 = V_0 \left((1 + f) \frac{V_4}{V_0} - 1 \right) = a_0 M_0 \left((1 + f) \frac{a_4 M_4}{a_0 M_0} - 1 \right)$$

$$\frac{T}{\dot{m}} = a_0 M_0 \left((1 + f) \sqrt{\frac{T_4}{T_0}} - 1 \right) = a_0 M_0 \left((1 + f) \sqrt{\frac{T_{3t}}{T_0 \left(1 + \frac{\gamma - 1}{2} M_0^2 \right)}} - 1 \right)$$

$$f = \frac{T_{3t} - T_{2t}}{\frac{Q_R}{c_p} - T_{3t}} = \frac{T_{3t} - T_0 \left(1 + \frac{\gamma - 1}{2} M_0^2 \right)}{\frac{Q_R}{c_p} - T_{3t}}$$



Governing Equations and Methodologies

Part 2:

$$1 + \frac{\gamma-1}{2} M_0^{*2} = \left(\frac{T_{3t}}{T_0} \right)^{1/3} \quad (1)$$

$$\frac{A_4}{A^*} = \frac{1}{M_4} \left[\frac{2 + (\gamma-1)M_4^2}{\gamma+1} \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (2)$$

$$P_0 \pi_d \pi_b \pi_n \frac{\left(1 + \frac{\gamma_0-1}{2} M_0^2 \right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma_4-1}{2} M_4^2 \right)^{\frac{\gamma}{\gamma-1}}} = P_4 \quad (3)$$

$$\frac{T}{\dot{m}} = \frac{T}{\dot{m}_{\text{momentum}}} + \frac{T}{\dot{m}_{\text{pressure}}} \quad (4)$$



Governing Equations and Methodologies

Part 2 (cont):

Momentum Specific Thrust:

$$\begin{aligned}
 \frac{T}{\dot{m}_{mom}} &= (1+f)V_4 - V_0 \\
 &= a_0 M_0 \left[(1+f) \frac{V_4}{V_0} - 1 \right] = a_0 M_0 \left[(1+f) \frac{M_4 \sqrt{T_4}}{M_0 \sqrt{T_0}} - 1 \right] \\
 &= a_0 M_0 \left[(1+f) \frac{M_4 \sqrt{T_{3t}}}{M_0 \sqrt{T_0}} \frac{1}{\left(1 + \frac{\gamma-1}{2} M_4^2\right)^{\frac{1}{2}}} - 1 \right]
 \end{aligned} \tag{5}$$

Pressure Specific Thrust:

$$\begin{aligned}
 &= \frac{(P_4 - P_0)A_4}{\dot{m}} = (1+f)P_0 \left(\frac{P_4}{P_0} - 1 \right) \frac{A_4}{A^* \rho^* V^*} = (1+f)P_0 \left(\frac{P_4}{P_0} - 1 \right) \frac{A_4 \sqrt{RT^*}}{A^* P^* \sqrt{\gamma}} \\
 T_t^* &= T^* \left(\frac{\gamma+1}{2} \right) = T_{3t} \rightarrow T^* = \frac{T_{3t}}{\left(\frac{\gamma+1}{2} \right)} \\
 \frac{P^*}{P_0} &= \frac{\left(1 + \frac{\gamma-1}{2} M_4^2 \right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}}} \\
 &= (1+f) \left(\frac{P_4}{P_0} - 1 \right) \frac{A_4}{A^*} \sqrt{\frac{RT_{3t}}{\gamma}} \frac{\left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(1 + \frac{\gamma-1}{2} M_0^2 \right)^{\frac{\gamma}{\gamma-1}}}
 \end{aligned} \tag{6}$$



Governing Equations and Methodologies

Part 3:

Procedure:

- 1) Find Area ratio that corresponds to highest specific thrust after relaxing $P_0=P_4$ condition
- 2) Recalculate lowest possible M_0 to find earliest ignition time
- 3) Plot M_0 (flight mach number) vs Specific Thrust for chosen area ratio

$$P_{1t} = P_a \left(1 + \frac{(\gamma - 1)}{2} M_0^2 \right)^{\gamma/(\gamma-1)}$$

Equation used in lecture to describe earliest in flight mach number in to avoid shock at nozzle exit



Specific Thrust vs Flight Mach Number for Ideal Ramjet

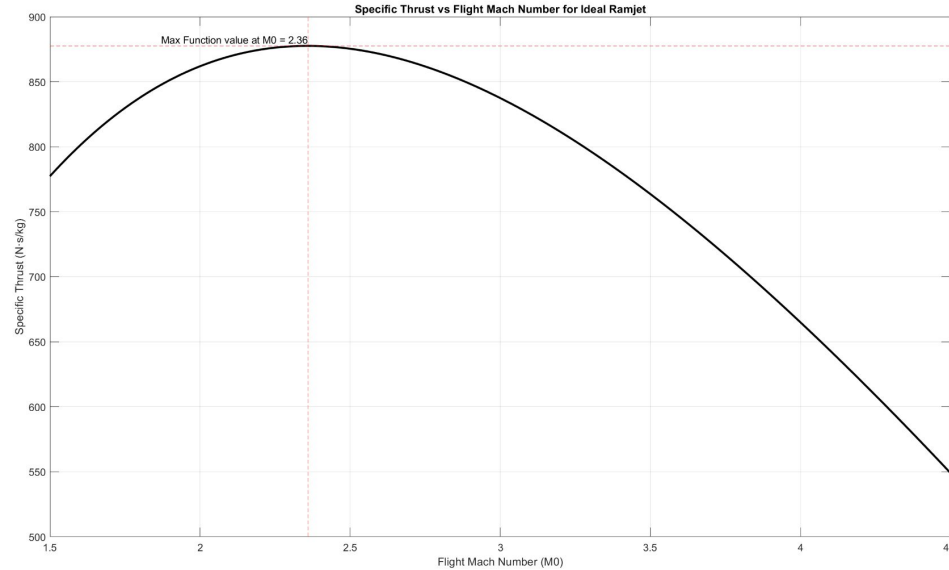


Figure 0. Specific thrust vs Flight Mach Number (M_0) for infinitely varying ideal ramjet. The maximum function value corresponds to ~ 878 Ns/kg @ an M_0 of 2.36, implying that the maximum possible thrust of any ideal ramjet design resides at a mach flight number of 2.36.



Total Specific Thrust vs Flight Mach Number for Various A_4/A^* Ratios

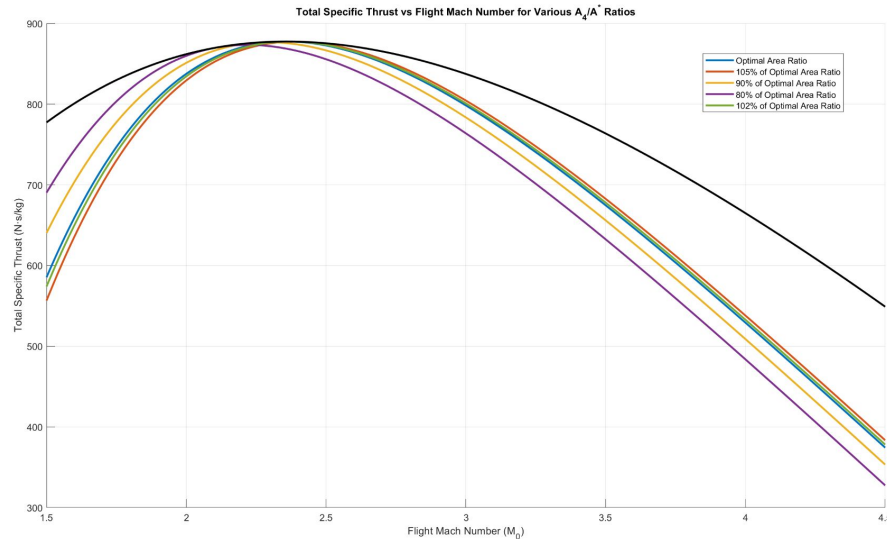


Figure 1. Specific thrust vs Flight Mach Number (M_0) for various A_4/A^* ratios. These values are calculated from the optimal area ratio (the area ratio at which the highest mach number occurred in the previous plot = 2.3115). When relaxing the condition $P_4 = P_0$, the highest specific thrust occurs at the optimal area ratio. This value, $A_4/A^* = 2.3115$ will be used in step 3 to compute the new plot and range for Total Specific Thrust Variance with M_0 .



Part 3

Maximum Specific Thrust for Each A_4/A^* Ratio:

$A_4/A^* = 2.3577$: Max Thrust = 877.53 N·s/kg at $M_0 = 2.37$

$A_4/A^* = 2.4270$: Max Thrust = 877.39 N·s/kg at $M_0 = 2.39$

$A_4/A^* = 2.0803$: Max Thrust = 876.67 N·s/kg at $M_0 = 2.29$

$A_4/A^* = 1.8492$: Max Thrust = 873.40 N·s/kg at $M_0 = 2.22$

$A_4/A^* = 2.3115$: Max Thrust = 877.56 N·s/kg at $M_0 = 2.36$

Table 0. Terminal output for Max thrust and optimal mach numbers for different area ratios ($A_4/A^* = 2.3115$ being the 'ideal' area ratio calculated in step 2).



Part 3

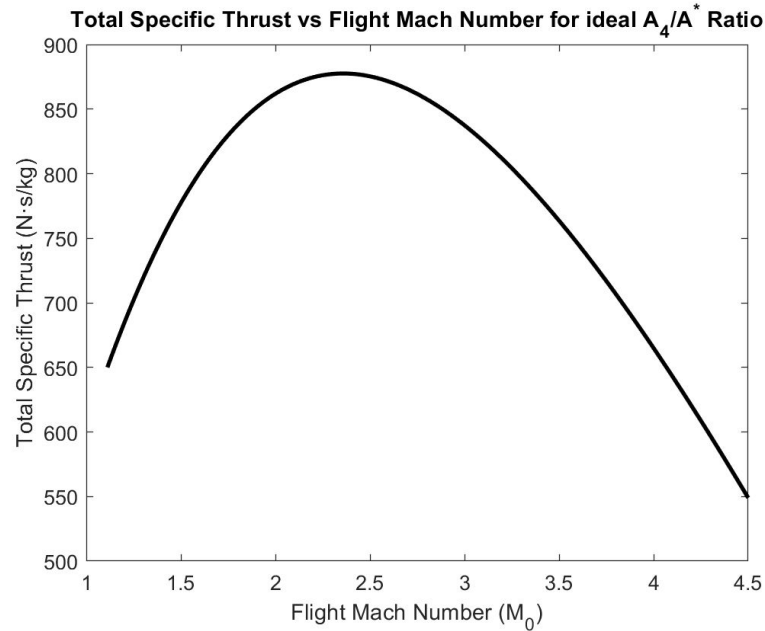


Figure 2. Flight Mach Number vs Total Specific Thrust (Normalized). Here, the starting point has been altered relative to the previous plot (now starting at 1.112 instead of 1.5) as 1.112 is the calculated flight mach number that corresponds to the earliest possible point to ignite the engine to avoid normal shocks.



Conclusions

1. The optimal area ratio after relaxing the $P_0 = P_4$ condition was found to be 2.3115.
2. Maximum specific thrust for this region occurs at $ST = 877.56$ Ns/kg
3. Earliest point to ignite this engine is at a flight mach number (M_0) of ~ 1.112 as calculated from in class equations (ref slide 6).

