

## Exercise 2.3 Circular motion

The questions in this exercise will help you improve your calculations and problem-solving involving circular motion.

- 1 The angular displacement made in one second is also called the angular speed,  $\omega$ .
  - a
    - i Write the equation to show how  $\omega$  is related to the time period,  $T$ , of rotation.
    - ii Write the equation to show how  $\omega$  is related to the frequency,  $f$ , of the rotations.
  - b Calculate the angular speed,  $\omega$ , of
    - i the Earth in its orbit around the Sun.
    - ii a children's carousel that makes one rotation in one minute.
    - iii the Moon that makes 13 rotations of the Earth in one year.
  - c The Singapore Flyer is a large Ferris wheel of radius 75 m, with observation pods for spectators on the circumference. It takes 30 minutes to make one complete rotation. Calculate the
    - i frequency of the Singapore Flyer's rotation.
    - ii angular speed of the Singapore Flyer.
    - iii linear speed at which one of the observation pods travels.
- 2
  - a Any body that is performing circular motion is accelerating. Explain why this statement must be true.
  - b When a body is rotating in a circular motion, in which direction **must** there be an unbalanced force?
  - c What is the name of this unbalanced force that produces circular motion?
  - d Write the equation for the necessary force,  $F$ , required for a body to make a circular motion. Be sure to note what each of the terms in the equation means.

### TIP

Use the equations for centripetal acceleration and centripetal force.

- 3
  - a Sketch graphs to show how the acceleration of a body in circular motion depends on the
    - i body's mass,  $m$ .
    - ii body's linear speed,  $v$ .
    - iii radius of the circle in which it is moving.

- b** Sketch graphs to show how the force on a body undergoing circular motion varies with the
- i** body's mass.
  - ii** body's linear speed.
  - iii** radius of the circle in which the body is moving.
- 4** The necessary centripetal force has to be provided by a real force occurring in the motion of an object. For each of the following examples, outline what the real force is that is providing the necessary centripetal force for circular motion.
- a** The Moon in its orbit around the Earth
  - b** An electron in its orbit around a nucleus
  - c** A proton in its orbit around the Large Hadron Collider at CERN
  - d** A car moving on an arc of a circle on a bend in a road
  - e** A ball attached to light string being rotated in a horizontal plane
- 5** In one model of a hydrogen atom, the electron is considered to orbit the nucleus at a distance of  $5.29 \times 10^{-11}$  m. The electrostatic force between the proton and the electron is  $8.23 \times 10^{-7}$  N and the mass of the electron is  $9.1 \times 10^{-31}$  kg. Calculate the speed of the electron in its orbit around the proton.
- 6 a** Consider a car travelling on a circular road at a constant speed.
- i** How does the friction force between the road and the car's tyres depend on the car's mass?
  - ii** Derive an expression for the maximum speed of the car around the circular road. Show that this must apply to all cars, large or small.
  - iii** Explain why the maximum speed at which a car can travel safely on a circular road is likely to be slower when the road is wet or icy.
- b** The exit road from a major motorway is usually an arc of a circle. The speed limit on this part of the road is always clearly shown as a motorist leaves the motorway.
- i** Explain why the speed limit on the exit road of the motorway is always lower than the speed limit on the carriageway of the motorway.
  - ii** If the average coefficient of friction between the exit road and a car's tyres is 0.75 and the radius of the circular arc is 80 m, calculate the maximum speed at which a car can travel safely on the exit road without skidding.
- c** The tyres of a car are worn. This has halved the coefficient of static friction between the road and the car's tyres. How has the maximum speed at which the car can travel safely around a bend changed?

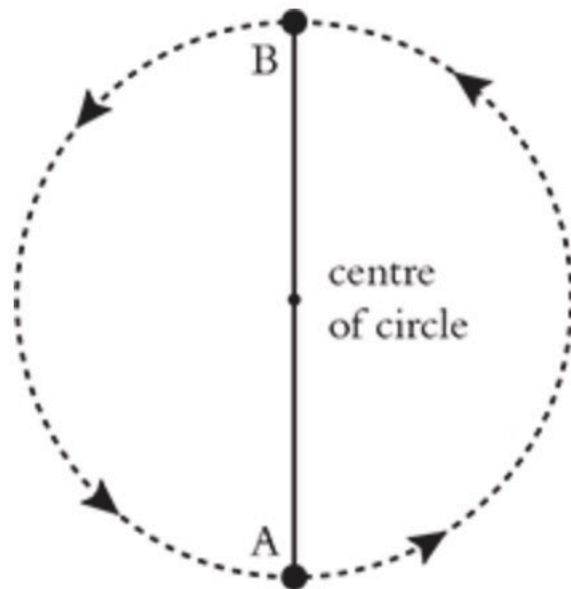
- 7** A student attaches a small ball of mass 100 g to the end of a string of length 60 cm. The student makes the ball execute circular motion in the horizontal plane with a frequency of 2.5 Hz.
- a** Calculate the
    - i** linear speed of the ball.
    - ii** tension in the string.
  - b** In fact, it is not possible for anyone to make a ball on the end of a string execute circular motion in an exactly horizontal plane.
    - i** Explain why this must be the case.
    - ii** Draw a free-body force diagram for such a ball on the end of a string being rotated.
  - c** Must the actual tension in the string be smaller, the same or larger than when the string is perfectly horizontal?
  - d** Which component of the tension is acting as the centripetal force?
  - e** What is the other component of the tension force doing?

**TIP**

Remember that it is the net unbalanced force that acts as the centripetal force.

- 8** Consider a cyclist on a banked section of a circular track.
- a** Draw a free-body force diagram for the cyclist. (You do not need to think about the friction force between the cycle's tyres and the track.)
  - b** Use your free-body force diagram to derive an expression for the angle,  $\theta$ , of the banked track in terms of the speed of the cyclist,  $v$ , and the radius of the circular track,  $r$ .
  - c** Explain why the angle of banking for an Olympic cyclist track is likely to be larger than that for an amateur cyclist track.

- 9 Figure 2.6 shows the path of a mass,  $m$ , which is being swung around by a light string in a vertical circle of radius  $r$ .



**Figure 2.6**

- Copy the diagram and add arrows to show the forces acting on the mass at position A.
- Derive an expression for the tension,  $T$ , in the string at position A.
- Derive an expression for the tension in the string at position B.
- What do you notice about the tension in the string as the mass moves in a vertical circle?