Exercise 2.1 Forces and their direction

The questions in this section will help you use vectors in diagrams and calculations and use the vector nature of forces to solve problems.

> **TIP**
>
> To solve problems of this kind, first draw a diagram and then use trigonometry.

1 A force, F, has a magnitude of 6.0 N and is directed at an angle of 40° above the horizontal. Find

a the horizontal component of F.

b the vertical component of F.

2 Vector **X** has components: $\mathbf{X}_{\text{horizontal}} = 5$ N and $\mathbf{X}_{\text{vertical}} = 4$ N. Calculate

a the magnitude of **X**.

b the angle **X** makes with the horizontal.

3 Force F has components of (5, 3), Force G has components of (−3, 4) and Force H has components of (−2, −7). All three forces act on a point body.

a Draw a scale diagram to show that the net force on the point body is zero.

b Show, algebraically, that the net force on the point body is zero.

4 An object has three forces acting on it simultaneously:

An upwards vertical force of 6 N, a left-to-right force of 3 N and another, unspecified force.

If the object is in equilibrium, find the

a magnitude of the unspecified force.

b direction of the unspecified force.

5 Figure 2.1 shows a mass hanging on a string at a particular moment.

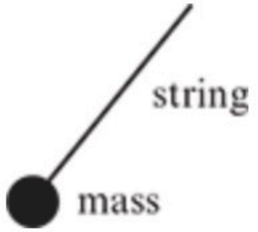

Figure 2.1

- **a** Copy the diagram and add to it all of the forces acting on the mass.
- **b** Explain why the mass is not in equilibrium.

6 A picture hangs on a wall by two strings. Each string makes an angle of 35° to the vertical. The tension in each string is 60 N.

- **a** Draw a labelled free-body force diagram for the picture.
- **b** Calculate the weight of the picture.

7 Figure 2.2 is a free-body force diagram showing three forces acting on a body.

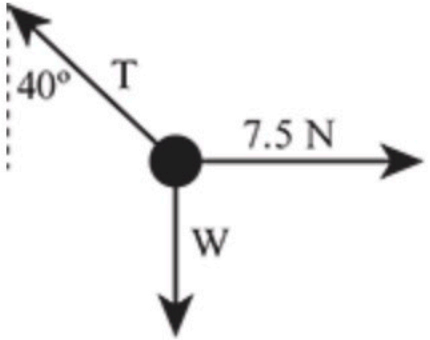

Figure 2.2

If the body is in equilibrium, find

- **a** the value of T.
- **b** the value of W.

8 Figure 2.3 shows how the length of a spring changes as it is stretched by a force.

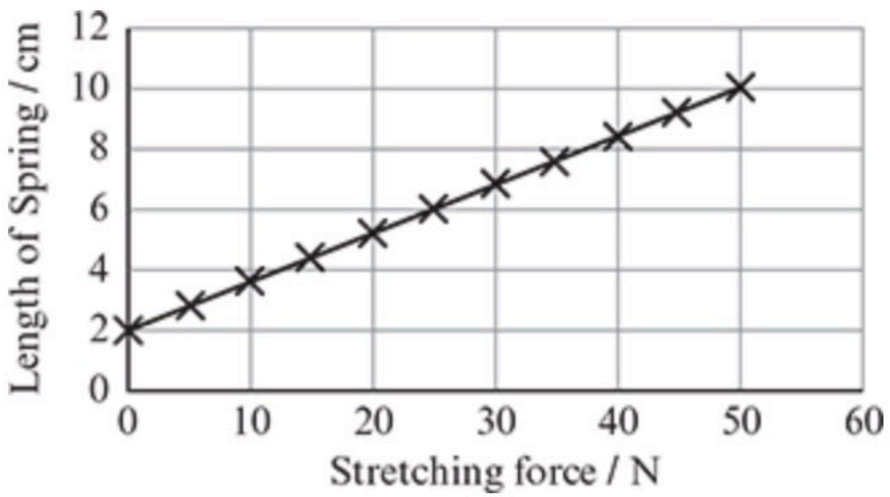

Figure 2.3

- **a** Explain how the graph shows that the spring obeys Hooke’s law.
- **b** State the unstretched length of the spring.
- **c** Use the graph to determine the spring constant of the spring.

9 Figure 2.4 shows a free-body force diagram for a book resting on an inclined slope. The book is not moving.

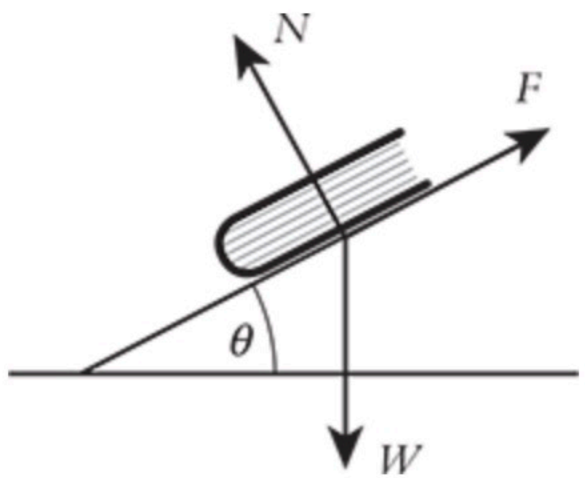

Figure 2.4

- **a** By taking components of the forces along the slope and perpendicular to the slope, determine how F is related to N.
- **b** How does your answer to **a** relate to the coefficient of static friction, μ_S?
- **c** Explain why the book will eventually slip down the slope if the angle of the slope to the horizontal is gradually increased.

10 Air freight is off-loaded from an aeroplane by allowing the containers to slide down an inclined ramp at an angle of 30° to the horizontal. The coefficient of static friction between the containers and the ramp is 0.45.

Show that the containers are able to slide.

11 A box of mass 25 kg is being pulled across a horizontal surface at a constant speed by a rope at an angle of 40° to the horizontal. The tension in the rope is 150 N.

Determine the coefficient of dynamic friction, μ_d, between the box and the horizontal surface. (g = 9.81 Nkg^{-1})

12 a State Hooke's Law.

b For springs that obey Hooke's Law, calculate the tension in a

i spring of spring constant 25 Nm^{-1} that has been extended by 30 cm.

ii spring of spring constant 0.30 Nm^{-1} that has been extended by 2.5 mm.

c The overall spring constant of three identical springs in series is 12 Nm^{-1}. Determine the spring constant of one spring.

d The overall spring constant of two springs in parallel is 50 Nm^{-1}. Determine the spring constant of one spring.

13 A children's toy comprises a plastic semi-sphere mounted on top of a spring of spring constant 5×10^3 Nm^{-1}. The total mass of the toy is 25 g. The toy is used by compressing the spring by 1 cm. When it is let go, the toy jumps into the air. (g = 10 ms^{-2})

a Calculate the force in the fully compressed spring.

b Calculate the initial acceleration of the toy as the spring starts its return to its uncompressed length.

c Once the spring has regained its uncompressed length, the toy experiences only its downwards acceleration due to gravity. The time for this to happen is 4.5 ms. If we consider that the initial acceleration upwards decreases linearly with the compressed length of the spring, show that the initial speed at which the toy moves upwards is 4.48 ms^{-1}.

d Calculate how high the toy can jump into the air.

TIP

Use your knowledge of an acceleration–time graph to find the change in velocity of the toy.

14 a State Archimedes' principle.

b Consider a cylinder of cross-sectional area A and height h immersed in a liquid of density ρ. The top of the cylinder is level with the surface of the liquid. The bottom of the cylinder is a distance h below the surface.

i Give an expression for the pressure in the liquid just underneath the cylinder at point P.

ii Give an expression for the upwards force on the underside of the cylinder due to the pressure in the liquid at a depth, h.

iii State an expression for the downwards force on the cylinder due to the atmospheric pressure, P_0.

iv Derive an expression for the buoyancy force; that is, the net upwards force on the cylinder due to the liquid.

v By finding an expression for the weight of the liquid displaced by the cylinder, show Archimedes' principle to be true.

vi If the cylinder just floats, what must its density be?

c A block of wood of density 1.1×10^3 kgm^{-3} has a mass of 100 kg. It is completely immersed in water.

i Determine the net force on the block of wood.

ii Does the block of wood float or sink?

15 Stokes' law states that the viscous drag force, F_d, acting on a sphere moving through a viscous fluid is $F_d = 6\pi\eta r v$, where η is the viscosity of the fluid, r the radius of the moving object and v the speed at which the object is moving through the fluid.

A sphere of lead of radius 1.5 mm falls at a constant velocity vertically through water of a viscosity 1.0×10^{-3} Nsm^{-2}.

a Draw a free-body force diagram of the sphere showing the forces acting on the sphere.

b By equating the weight of the sphere with its buoyancy force and its drag force, show that the terminal velocity of the sphere is given by the expression:

$$v_{terminal} = \frac{2r^2 g(\rho_{lead} - \rho_{water})}{9\eta},$$

where ρ_{lead} is the density of lead and ρ_{water} is the density of the fluid through which it is falling.

c Given that the density of lead is 1.14×10^4 kgm^{-3}, calculate the speed at which the sphere is falling. ($g = 9.81$ Nkg^{-1})

d In practice, such a sphere falling through water falls with a terminal velocity that is smaller than the value that Stokes' law predicts. Suggest a reason why this is.

e If the same sphere were to fall through water of a higher temperature, state what would happen to the terminal velocity of the sphere and suggest a reason to support your statement.

TIP

Remember that there are two upwards forces acting on a falling body in a fluid.

16 A typical family car, of mass 1.2×10^3 kg, driving along a road is subject to two forces that act in the opposite direction to its motion. These two forces are dynamic friction between the tyres and the road and a frictional drag force caused by the car moving through the air.

a **i** If the coefficient of dynamic friction, μ_k, is 0.02, calculate the dynamic friction force acting on the moving car. (g = 9.81 N kg^{-1})

ii How does this dynamic friction force change when the car's speed increases?

The frictional drag force, F_d, acting on the car is given by

$$F_d = \frac{1}{2} C A \rho v^2$$

where C is the drag coefficient (a value influenced by the design of the car), A is the cross-sectional area of the car, ρ is the density of the air and v is the speed of the car.

b For a typical family car, C = 0.3 and A = 2.0 m^2. Given that the density of air is 1.3 kg m^{-3}, calculate the speed at which frictional drag force is equal to the dynamic frictional force.

c Hence, determine the driving force from the engine necessary to maintain this constant speed.

d Most family cars are capable of travelling at 45 m s^{-1} (in most countries, this speed would be faster than the legal speed limit). With reference to the two frictional forces examined in this question, suggest why typical family cars are not designed to travel at speeds higher than this.

e Higher-performance cars and some high-end sports cars are designed to be able to travel at speeds higher than 45 m s^{-1}. Suggest how the design of these cars enables them to travel at such high speeds.