Exercise 3.2 Conservation of mechanical energy

The following questions will consolidate your use of the conservation of mechanical energy principle.

1 Calculate

a the kinetic energy of a tennis ball of mass 58 g travelling at 25 ms^{-1}.

b the gravitational potential energy of a bird of mass 40 mg flying at a height of 15 m above the ground (g = 10 Nkg^{-1}).

c the elastic potential energy stored in a spring stretched by a force of 3.5 N to an extension of 1.5 cm.

2 The total mechanical energy of a body is the sum of its kinetic energy and its potential energy.

Calculate the total mechanical energy of a:

a 5 kg mass moving at 2 ms^{-1} along the ground.

b 4 kg mass sitting stationary on top of a cupboard at a height of 2 m.

c 3 kg mass moving horizontally through the air at a speed of 4 ms^{-1}, 5 m above the ground.

d spring of spring constant 18 Nm^{-1} stretched by a distance 8 cm, lying stationary on the floor.

e spring, of mass 20 g, stretched by 60 cm by a force of 5 N, moving horizontally at 2 ms^{-1} 1.8 m above the ground.

3 It is said that Isaac Newton was inspired to develop the theory of gravitation by watching an apple fall from a tree.

a Ignoring any effects due to air friction and taking g = 10 Nkg^{-1}, calculate the gravitational potential energy of a 100 g apple on a tree branch that is 6.0 m above the ground.

b If the apple falls to the ground, use equations of kinematics to calculate

i the time it would take to land on the ground.

ii the speed of the apple just before it hits the ground.

c Now use the conservation of mechanical energy to calculate the speed of the apple just before it hits the ground.

4 Figure 3.2 shows a large block of stone, of mass 3 × 10^3 kg, being pushed up a slope.

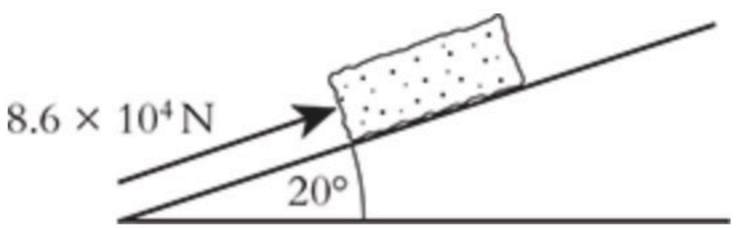

Figure 3.2

The slope is 15 m long. The force used to push the block of stone is 8.6 × 10^4 N.

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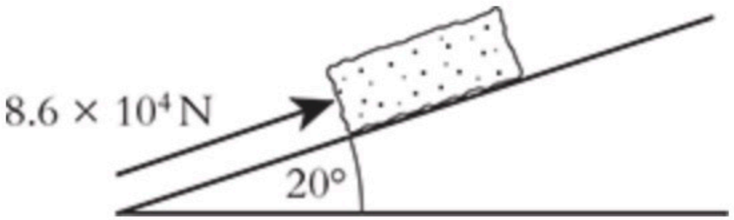

Figure 3.2

The slope is 15 m long. The force used to push the block of stone is 8.6×10^4 N.

a Calculate the work done in moving the block of stone.

b Calculate the gravitational potential energy gained by the block of stone.

c How do you account for the difference between your answers to **parts a** and **b**?

5 A teacher drops his cup of coffee from a height of 1.5 m. The mass of the cup of coffee was 0.45 kg. Ignoring any effects of air friction:

a Calculate the work done by the Earth's gravitational field on the cup of coffee.

b Without referring to the principle of conservation of energy, show that the kinetic energy of the cup of coffee just before it hits the ground equals the work done by the Earth's gravitational field on the cup of coffee.

c Verify that the gravitational potential energy lost by the cup of coffee is transferred into kinetic energy.

d When the cup of coffee hits the floor, it breaks and the pieces of the cup eventually stop moving. With reference to the principle of conservation of energy, suggest what has happened to the gravitational potential energy that the cup once had.

6 A children's toy comprises a plastic semi-sphere mounted on top of a spring of spring constant 5×10^3 Nm^{-1}. The total mass of the toy is 25 g. The toy is used by compressing the spring by 1 cm. When it is let go, the toy jumps into the air. (g = 10 ms^{-2})

a Calculate the total mechanical energy of the toy when the spring is fully compressed.

b By applying the conservation of mechanical energy, calculate the initial speed at which the toy moves upwards.

c Hence, calculate how high the toy can jump.

7 An athlete jumps on a trampoline. During a jump, the athlete, of mass 60.0 kg, reaches a height of 8.0 m above the trampoline bed, and when she lands back on the bed, the elastic surface deforms downwards by a distance of 0.7 m. Taking g = 10 Nkg^{-1} and ignoring any effects due to air friction, determine the spring constant of the trampoline bed.

TIP

In question 7, you will need to use SUVAT and the conservation of mechanical energy.

8 At Victoria Falls on the border between Zimbabwe and Zambia, there is a bridge over the Zambezi River, from which people bungee jump. Advertising literature states that the jumper falls a distance of 74 m before the stretched bungee cord halts their descent some 40 m or so above the river. The bungee cord is advertised as being 20 m long unstretched.

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a Taking $g = 10 \text{ Nkg}^{-1}$, and ignoring any effects due to air friction, calculate

i the speed of a 70 kg jumper at the moment the bungee cord just starts to stretch.

ii the time taken for the jumper to reach this point.

b The bungee cord stretches by 54 m.

i Calculate the average acceleration of the jumper during the stretching of the bungee cord.

ii Hence, calculate the time taken for the jumper to come to a stop at the bottom of his fall.

c Assuming that the bungee cord behaves according to Hooke's law, calculate the spring constant of the bungee cord.

9 A student wants to drive a spike into the ground by dropping a large mass onto it. If the necessary work done by the spike in being driven into the ground is 420 J, calculate the mass required by the student if he is going to drop the mass fom a height of 1.5 m. What assumptions have you needed to make?