Backstepping Control of Each Channel for a Quadrotor Aerial Robot

Yu yali
engineering training center
Harbin engineering university
Harbin, china
yuyali2000@126.com

Jiang changhong
electric& electronics engineering college
changchun university of technology
changchun, china
jiang_changhong@126.com

Wu haiwei
School of Electrical and Information Engineering
BeiHua University
Jilin, china
becky0@126.com

Abstract—This paper has presented a model of vertical take-off and landing(VTOL) unmanned air vehicle. It has four rotors, so as quadrotor aerial robot. It can hover or fly at low speed indoor or unwinded outdoor. The quadrotor aerial robot is a highly nonlinear, multivariable, strongly coupled and under-actuated subsystem because it has 6-DOF but only has four inputs. Nonlinear mathematical model of quadrotor aerial robot which is based on Newton-Euler formalism is deduced in the paper, also described nonlinear control strategy for it. Backstepping of nonlinear controller design was based on constructing Lyapunov function for closed-loop system. The controller guarantees stability, tracking performance and robotness of the system. The effective control design scheme is shown through nonlinear simulations.

Keywords-VTOL; quadrotor aerial robot; Backstepping; Newton-Euler formalism

Ⅰ. INTRODUCTION

Quadrotor aerial robot is a mechatomc system which is an unmanned VTOL(vertical take-off and landing) helicopter. The main frame of this structure is a cross intersection rigid body, cross intersection poles have been composed of two diagonals of a square. Four rotors has been respectively configured at the four end of the cross intersection (the four ends of square). Each rotor has been drived by a motor. Our models as shown in Fig.1. Rotors pair which at the end of the same diagonal has the same rotation direction, that means motor pair (1,3) rotates clockwise and the other pair (2,4) rotates counter-clockwise in order to balance the moments of the total system.

Fig. 1. our model of quadrotor aerial robot

The cross intersection structure compared with the conventional helicopter have some advantages as follows: The size and orientation of the thrust vector amplitude of the main propeller have been changed by alter its rotation speed, blade pitch angle and period thread pitch, consequently the motion attitude of the conventional helicopter have been regulated. Tail rotor banlances the torque of the main rotor. The quadrotor aerial robot have the simplified rotor mechanics. In spite of elastic fiber has been choosed for the manufacture material of blade, air resistance can let rotor has a certain deformation, but in fact the deformtion can be neglected because the change is very tiny. So we can think of that the rotors have fixed pitch angle, the only method to alter its attitude and position is through changing speed of the rotors.

Quadrotor aerial robot can generate 6-DOF movement in the inertia frame through changing the motors rotational speed. Including translational motion along three coordinate axis(surge, sway and heave) and rotational motion around three axis(roll, pitch and yaw). Vertical motion of z-axis is achieved by increasing(or decreasing) speed of four motors altogether with the same quantity. When the total of thrust equal to the self-weight, the quadrotor aerial robot change to a hoverable robot.The change of pitch angle is achieved by a difference thrust between the front and the rear rotors and simultaneously to maintain the total thrust while the change of roll angle result from differences between the left and right rotor by the same way, respectively. Yaw rotation can by achieved by the difference in the counter-torque between each pair (1,3 and 2,4) of rotors. And maintaining the total thrust unchanged to aviod the up-down motion.

Ⅱ. DYNAMIC MODELING OF A QUADROTOR AERIAL ROTOR

Defined two main reference frames:

The earth fixed inertial reference frame $E(O^e, X^e, Y^e, Z^e)$, which describes geography horizontal at O^e point. look O^e as original point the vector $\zeta = [x, y, z]^T$ and $\eta = [\phi, \theta, \psi]^T$ denote respectively translational positions and attitude angles of the quadrotor(frame E). the altitude angles $\{\phi, \theta, \psi\}$ are respectively called pitch angle $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$, roll angle $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ and yaw angle $(-\pi < \psi < \pi)$.

The body fixed reference frame $B(O^b, X^b, Y^b, Z^b)$, which describes reference frame attached to the airial robot. Three translation velocities and three rotation velocities are respectively: $\mathrm{v} = [\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3]^T$, $\omega = [\omega_1, \omega_2, \omega_3]^T$. So (V, Ω) is the velocitied vectors of body frame, and $(\dot{\zeta}, \dot{\eta})$ is the velocitied vectors of earth frame.

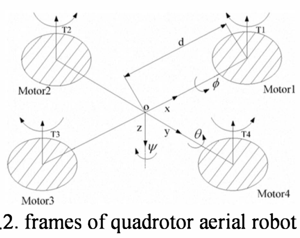

Fig.2. frames of quadrotor aerial robot

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According the relative airflow analysis conclusion of the rotor and blade, we know that when the quadrotor have forward velocity, the rotors not only under the thrust T which is produced by the configuration rotor shaft and drag force D but also under the torque Q and rolling moment L. All the forces and moments are derived from a combination of blade element theory and momentum theorem[1][2], which are relativly in proportion to ω^2 :

$$T = \frac{1}{2}\rho A C_T R^2 \omega^2 = b\omega^2 \tag{1}$$

$$D = \frac{1}{2}\rho A C_D R^2 \omega^2 = m\omega^2 \tag{2}$$

$$Q = \frac{1}{2}\rho A C_Q R^2 \omega^2 = n\omega^2 \tag{3}$$

$$L = \frac{1}{2}\rho A C_R R^2 \omega^2 = e\omega^2 \tag{4}$$

Where C_T , C_D , C_Q , C_R is respectively the thrust, drag, torque and rolling moment aeriodynamic coefficients, ρ is density of air, R is radius of the blade and $A = \pi R^2$ is rotor disk area, b, m, n ,e denote respectively the simplificated coefficients of the force and mement.

The real input of quadrotor aerial robot is speed of four rotors, but for convenient model calculating, the input has been changed to the form which is more appropiate for calculating, so we can obtained new artificial input variable as follows:

$$\begin{cases} u_1 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ u_2 = b(\omega_4^2 - \omega_2^2) \\ u_3 = b(\omega_3^2 - \omega_1^2) \\ u_4 = d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \end{cases} \tag{5}$$

Add a new input variable U_5[3]:

$$u_5 = \omega_2 + \omega_4 - \omega_1 - \omega_3 \tag{6}$$

Use the earth frame as the reference frame, total force and total moment of the quadrotor aerial robot is represented by the table 1as follows:

Table 1. Formulas of Forces and moments of quadrotor aerial robot

Force moments		Formulas
F	Thrust	$F = R[0,0,F_3]^T$
	Friction force	$F_a = [f_x, f_y, f_z]^T = [K_t\dot{x}, K_t\dot{y}, K_t\dot{z}]^T$
	Gravity force	$F_g = (0,0,mg)^T$
M	Torque	$\tau = [\tau_1, \tau_2, \tau_3]^T$
	Friction moment	$M_a = [M_{ax}, M_{ay}, M_{az}]^T = [K_r\dot{\phi}, K_r\dot{\theta}, K_r\dot{\psi}]^T$
	Gyroscopic effect	$M_g = [M_{gx}, M_{gy}, M_{gz}]^T = \sum_{i=1}^{4} I_r(\Omega \times e_z)(-1)^{i+1}\omega$

Where K_t , K_r denote respectively aerodynamic coefficients of force and moment. $F = [0 \ \ 0 \ \ F_3]^T$, $\tau = [\tau_1, \tau_2, \tau_3]^T$ denote respectively the vector formula of total forces and moments of four rotors in the quadrotor aerial robot system(use body frame as reference frame), derived process of that is as follows:

In the reference frame $B(O^b, X^b, Y^b, Z^b)$, the forces F produced by the quadrotor aerial robot are:

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \tag{7}$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} d(F_2 - F_4) \\ d(F_3 - F_1) \\ c\sum_{i=1}^{4}(-1)^{i+1}F_i \end{bmatrix} = \begin{bmatrix} du_2 \\ du_3 \\ cu_4 \end{bmatrix} \tag{8}$$

Where d is the distance from the epicenter of a quadrotor to the rotor axis and c is the drag factor.

$$R = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + \sin\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{9}$$

Using the kinematic equations and Newton's law, we can list following equations:

$$\dot{\zeta} = \upsilon_e \tag{10}$$

$$m\dot{\upsilon}_e = \sum F_{tal} \tag{11}$$

$$\dot{\eta} = \Omega_e \tag{12}$$

$$I_f\dot{\Omega}_e + \Omega_e \times I_f\Omega_e = \sum T_{tal} \tag{13}$$

Using dynamic equations, $\sum F_{tal}$ and $\sum T_{tal}$ can be calculated:

$$\sum F_{tal} = -F_e + F_a + F_g \tag{14}$$

$$\sum T_{tal} = T - T_a - T_g \tag{15}$$

Where $I_f = \lfloor I_x, \ I_y, \ I_z \rfloor$ is the total inertia matrix of quadrotor aerial robot.

Using equations (7), (11)and(14), the translation equations are given by:

$$\begin{cases} \ddot{x} = -\frac{u_1}{m}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) + \frac{K_t}{m}\dot{x} \\ \ddot{y} = -\frac{u_1}{m}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) + \frac{K_t}{m}\dot{y} \\ \ddot{z} = -\frac{u_1}{m}\cos\theta\cos\phi + \frac{K_t}{m}\dot{z} + g \end{cases} \tag{16}$$

And using equations(8), (13)and(15), the rotation equations are given by:

$$\begin{cases} \ddot{\phi} = \left(\frac{I_x - I_y}{I_x}\right)\dot{\theta}\dot{\psi} - \frac{I_r}{I_x}\dot{\theta}u_5 - K_r\dot{\phi} + \frac{d}{I_x}u_2 \\ \ddot{\theta} = \left(\frac{I_x - I_z}{I_y}\right)\dot{\phi}\dot{\psi} - \frac{I_r}{I_y}\dot{\phi}u_5 - K_r\dot{\theta} + \frac{d}{I_y}u_3 \\ \ddot{\psi} = \left(\frac{I_y - I_x}{I_z}\right)\dot{\phi}\dot{\theta} - K_r\dot{\psi} + \frac{c}{I_z}u_4 \end{cases} \tag{17}$$

III. CONTROL STRATEGY

From equation (16) and (17), because quadrotor aerial robot has 6-DOF but only has four inputs, it is a highly nonlinear, multivariable, strongly coupled and under-actuated subsystem. The hoverable or low speed rotational quadrotor has been controlled under the research environment of indoor or unwinded outdoor. So that the quadrotor aerial rotor model has neglected the disturbance of wind or dynamic friction. Its model has been simplified and repartitioned the following channels[4]: $y-\phi$ channel, $x-\theta$ channel, z and ψ channel.

Now take the $y-\phi$ channel for example, Backstepping controller design of the quadrotor aerial robot has been described.

At first, defined state variable:

$$\begin{aligned} y_1 &= y \\ y_2 &= \dot{y} \\ y_3 &= \phi \\ y_4 &= \dot{\phi} \end{aligned} \tag{18}$$

From equation (16) and (17), we can neglected the coupling effect from pitch angle θ to y-axis. So it can be assumed that θ is a low angle. Dynamical model of the $y-\phi$ channel is:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{u_1}{m}\cos\psi \sin x_3 \\ \dot{y}_3 &= y_4 \\ \dot{y}_4 &= \left(\frac{I_z - I_y}{I_x}\right)\dot{\theta}\dot{\psi} - \frac{I_r}{I_x}\dot{\theta}u_5 + \frac{d}{I_x}u_2 \end{aligned} \tag{19}$$

Design step of backstepping controller are:

First step:

defined a tracking error $z_1 = y_1 - y_{1d}$, then its time derivative is:

$$\dot{z}_1 = \dot{y}_1 - \dot{y}_{1d} = y_2 - \dot{y}_{1d} = z_2 + \alpha_1 + f_1 \tag{20}$$

Where $z_2 = y_2 - \alpha_1$ is a tracking error, α_1 is undetermined virtual input.

Choose candidated Lyapunov function is $V_1 = \frac{1}{2} z_1^T z_1$

Its time derivative is:

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 (z_2 + \alpha_1 + f) \tag{21}$$

Introducing $\alpha_1 = -c_1 z_1 - f_1$, where $c_1 > 0$ is a regulatable parameter, then

$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2$, considering stable of the closed-loop system, coupling item $z_1 z_2$ has been neglected. Substituting α_1 to (20):

$$\dot{z}_1 = -c_1 z_1 + z_2 \tag{22}$$

Other variable respectively are:

$$\begin{aligned} f_1 &= \dot{z}_1 - y_2 = -\dot{y}_{1d} \\ \alpha_1 &= -c_1 z_1 - f_1 = -c_1(y_1 - y_{1d}) + \dot{y}_{1d} \\ z_2 &= y_2 - \alpha_1 = y_2 + c_1(y_1 - y_{1d}) - \dot{y}_{1d} \end{aligned} \tag{23}$$

Second step:

The time derivative of z_2 is:

$$\begin{aligned} \dot{z}_2 &= \dot{y}_2 + c_1(\dot{y}_1 - \dot{y}_{1d}) \\ &= \frac{u_1}{m}\cos\psi \sin y_3 + c_1 y_2 \\ &= z_3 + \alpha_2 + f_2 \end{aligned} \tag{24}$$

Where $z_3 = y_3 - \alpha_2$ is a tracking error, α_2 is undetermined virtual input.

Choose candidated Lyapunov function is $V_2 = \frac{1}{2}\sum_{i=1}^{2} z_i^T z_i$

Its time derivative is:

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 + z_1 z_2 + z_2 \dot{z}_2 \\ &= -c_1 z_1^2 + z_1 z_2 + z_2(z_3 + \alpha_2 + f_2) \end{aligned} \tag{25}$$

Introducing $\alpha_2 = -z_1 - c_2 z_2^2 - f_2$, where $c_2 > 0$ is a regulated parameter, then

$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2 + z_2 z_3$, considering stable of the closed-loop system, coupling item $z_2 z_3$ has been neglected. Substituting α_2 to (24):

$$\dot{z}_2 = -z_1 - c_2 z_2 + z_3 \tag{26}$$

Other variable respectively are:

$$\begin{aligned} f_2 &= \dot{z}_2 - y_3 = -\frac{u_1}{m}\cos\psi \sin y_3 + c_1 y_2 - y_3 \\ \alpha_2 &= -z_1 - c_2 z_2 - f_2 \\ &= -(1 + c_1 c_2)(y_1 - y_{1d}) - (c_1 + c_2) y_2 + y_3 \\ &\quad + \frac{1}{m} U_1 \cos\psi \sin y_3 \\ z_3 &= y_3 - \alpha_2 = (1 + c_1 c_2)(y_1 - y_{1d}) + (c_1 + c_2) y_2 \\ &\quad - \frac{1}{m} U_1 \cos\psi \sin y_3 \end{aligned} \tag{27}$$

Third step:

The time derivative of z_3 is:

$$\begin{aligned} \dot{z}_3 &= (1 + c_1 c_2)(\dot{y}_1 - \dot{y}_{1d}) + (c_1 + c_2)\dot{y}_2 - \frac{1}{m} U_1 \dot{y}_3 \cos\psi \cos y_3 \\ &\quad + \frac{1}{m} U_1 \dot{\psi} \sin\psi \sin y_3 \\ &= z_4 + \alpha_4 + f_3 \end{aligned} \tag{28}$$

Where $z_4 = y_4 - \alpha_3$ is a tracking error, α_3 is undetermined virtual input.

Choose candidated Lyapunov function is $V_3 = \frac{1}{2}\sum_{i=1}^{3} z_i^T z_i$

Its time derivative is:

$$\begin{aligned} \dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + z_3 \dot{z}_3 \\ &= -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 + z_3(z_4 + \alpha_3 + f_3) \end{aligned} \tag{29}$$

Introducing $\alpha_3 = -z_2 - c_3 z_3 - f_3$, where $c_3 > 0$ is a regulated parameter, then

$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4$, considering stable of the closed-loop system, coupling item $z_3 z_4$ has been neglected. Substituting α_3 to (28):

$$\dot{z}_3 = -z_1 - c_3 z_3 + z_4 \tag{30}$$

Other variable respectively are:

$$\begin{aligned} f_3 &= \dot{z}_3 - y_4 \\ &= (1 + c_1 c_2)(y_2 - y_{1d}) - (c_1 + c_2)\frac{1}{m} U_1 \cos\psi \sin y_3 \\ &\quad - \frac{1}{m} U_1 \dot{y}_3 \cos\psi \cos y_3 + \frac{1}{m} U_1 \dot{\psi} \sin\psi \sin y_3 - y_4 \\ \alpha_3 &= -z_2 - c_3 z_3 - f_3 \\ &= -(c_1 + c_3 + c_1 c_2 c_3)(y_1 - y_{1d}) - (2 + c_1 c_2 + c_2 c_3 + c_1 c_3) y_2 \\ &\quad + (c_1 + c_2 + c_3)\frac{1}{m} U_1 \cos\psi \sin y_3 + \frac{1}{m} U_1 \dot{y}_3 \cos\psi \cos y_3 \\ &\quad - \frac{1}{m} U_1 \dot{\psi} \sin\psi \sin y_3 + y_4 \\ z_4 &= y_4 - \alpha_3 \\ &= (c_1 + c_3 + c_1 c_2 c_3)(y_1 - y_{1d}) + (2 + c_1 c_2 + c_2 c_3 + c_1 c_3) y_2 \end{aligned}$$

$$-(c_1+c_2+c_3)\frac{1}{m}U_1\cos\psi\sin y_3-\frac{1}{m}U_1\dot{y}_3\cos\psi\cos y_3 \quad (31)$$
$$+\frac{1}{m}U_1\dot{\psi}\sin\psi\sin y_3$$

Fourth step:

Choose candidated Lyapunov function is: $V_3=\frac{1}{2}\sum_{i=1}^{3} z_i^T z_i$

$$\dot{V}_4=-c_1z_1^2-c_2z_2^2-c_3z_3^2+z_3z_4+z_4\dot{z}_4 \quad (32)$$

If $\dot{z}_4=-z_3-c_4z_4$, where $c_4>0$ is a regulated parameter, then

$\dot{V}_4=-c_1z_1^2-c_2z_2^2-c_3z_3^2-c_4z_4^2<0$, according to the Barbashin-Krasovskii theory[5], we can drawn the conclusion that the closed-loop system obtains asymptotic stable throuth above designed process.

Substituting $z_1 \sim z_4$ to $\dot{z}_4=-z_3-c_4z_4$. The controlled variable u_2 has been obtained.

Solving expression $\dot{z}_4=-z_3-c_4z_4$, then:

$$u_2=\frac{mI_x}{dU_1\cos\psi\cos\phi}\Big\{-\big[(1++c_1c_2)+c_4(c_1+c_3+c_1c_2c_3)(y-y_d)$$
$$-\big[(c_1+c_2)+c_4(2+c_1c_2+c_2c_3+c_1c_3)+(c_1+c_3+c_1c_2c_3)\big](\dot{y}-\dot{y}_d)$$
$$-\big[1+c_4(c_1+c_2+c_3)+(2+c_1c_2+c_2c_3+c_1c_3)\big]\frac{1}{m}U_1\cos\psi\sin\phi$$
$$-(c_1+c_3+c_3+c_4)\frac{1}{m}U_1\dot{\phi}\cos\psi\cos\phi$$
$$+(c_1+c_3+c_3+c_4)\frac{1}{m}U_1\dot{\psi}\sin\psi\sin\phi$$
$$+\frac{1}{m}U_1\dot{\phi}^2\cos\psi\sin\phi+2\frac{1}{m}U_1\dot{\phi}\dot{\psi}\sin\psi\cos\phi+\frac{1}{m}U_1\dot{\psi}^2\cos\psi\sin\phi\Big\}$$
$$-\left(\frac{I_z-I_y}{d}\right)\dot{\theta}\dot{\psi}-\frac{I_r}{d}\dot{\theta}u_5$$
(33)

Similarly, using Backstepping method other three channels controller respectively are:

$x-\theta$ channel Backstepping controller is :

$$u_3=\frac{mI_y}{dU_1\cos\psi\cos\theta}\Big\{\big[(1++c_5c_6)+c_8(c_5+c_7+c_5c_6c_7)(x-x_d)$$
$$+\big[(c_5+c_6)+c_8(2+c_5c_6+c_6c_7+c_5c_7)+(c_5+c_7+c_5c_6c_7)\big](\dot{x}-\dot{x}_d)$$
$$-\big[1+c_8(c_5+c_6+c_7)+(2+c_5c_6+c_6c_7+c_5c_7)\big]\frac{1}{m}U_1\cos\psi\sin\theta$$
$$-(c_5+c_6+c_7+c_8)\frac{1}{m}U_1\dot{\theta}\cos\psi\cos\theta$$
$$+(c_5+c_6+c_7+c_8)\frac{1}{m}U_1\dot{\psi}\sin\psi\sin\theta \quad (34)$$
$$+\frac{1}{m}U_1\dot{\theta}^2\cos\psi\sin\theta+2\frac{1}{m}U_1\dot{\theta}\dot{\psi}\sin\psi\cos\theta+\frac{1}{m}U_1\dot{\psi}^2\cos\psi\sin\theta\Big\}$$
$$-\left(\frac{I_z-I_x}{d}\right)\dot{\phi}\dot{\psi}-\frac{I_r}{d}\dot{\phi}u_5$$

Z channel Backstepping controller is:

$$u_1=\frac{m}{\cos\theta\cos\phi}\big[z_{11}+g+c_{11}(z_{12}-c_{11}z_{11})+c_{12}z_{12}-\ddot{z}_d \quad (35)$$

Where $c_{11}>0$, $c_{12}>0$ are all constants,

$$z_{11}=z-z_d$$
$$z_{12}=x_{12}+c_{11}z_{11}-\dot{x}_{11d}$$
$$=\dot{z}+c_{11}(z-z_d)-\dot{z}_d$$
$$=c_{11}(z-z_d)-(\dot{z}-\dot{z}_d)$$

ψ channel Backstepping controller is :

$$u_4=I_z\left[-z_9-\left(\frac{I_x-I_y}{I_z}\right)\dot{\theta}\dot{\phi}-c_9(z_{10}-c_9z_9)-c_{10}z_{10}+\ddot{\psi}\right] \quad (36)$$

Where, $c_9>0$, $c_{10}>0$ are all constants,

$$z_9=\psi-\psi_d$$
$$z_{10}=x_{10}+c_9z_9-\dot{x}_{9d}$$
$$=\dot{\psi}+c_9(\psi-\psi_d)-\dot{\psi}_d$$
$$=c_9(\psi-\psi_d)-(\dot{\psi}-\dot{\psi}_d)$$

In expression(33) ~ (36), $c_1 \sim c_{12}$ are regulated parameter which are all greater than zero.

Ⅳ. RESULT AND DISCUSSION

Our prototype aerial robot using the rotors of DraganflyerIII. The rotor of DraganflyerIII has the fixed pitch angle, diameter is 28cm, weight is 6g. lift coefficient k_t, k_d and I_x has been obtained form [6].

And the main coefficients of the prototype as following:

Table 2. main coefficients of the prototype

Coefficient	m	d	k_t	k_d	I_x	I_y	I_z	I_r
Unit	kg	m	Ns^2	Nms^2	kgm^2	kgm^2	kgm^2	kgm^2
Value	0.75	0.25	$3.13\,e^{-5}$	$7.5\,e^{-7}$	$19.688\,e^{-3}$	$19.681\,e^{-3}$	$3.938\,e^{-2}$	$6\,e^{-5}$

Now the performance of Backstepping controller has been analysed through three experiments: hovering at fixed-point, robust test.

1. hovering at fixed-point

For quadrotor aerial robot, Assuming coefficient of initial position, Euler angle and angular velocity are respectively 0(m), 20°. Controls aim is let quadrotor move from original point to point$(1\ 1\ 1)^T$(m) then hovering. There are 12 coefficients of the designed Backstepping controller, choosing it has direct influence on control performance. The coefficient more larger, performance of the Backstepping control capbility more better. But if the coefficient overlarge then it will not only leads to the controls parameters out of the physical realized range but also causes larger control error. From a lot of experiment, the regulated range of coefficient $c_1 \sim c_{12}$ is inter [0.5 3].

When assuming $c_1 \sim c_{12}$ identically equal, choosing respectively value of 0.8 and 2.5, through a lot of simulation experiments, the best controls parameter is 2. The simulation result as Fig2 and Fig3:

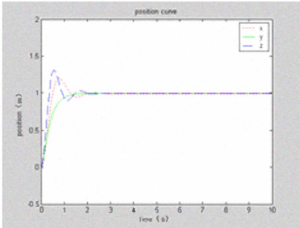

Fig 2. Position curve

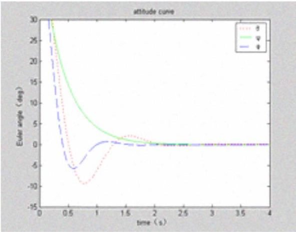

Fig 3. Euler angle curve

2. Robotness experiment

Same as hoverable experiment, quadrotor aerial robot initial states are all zero, the parameter of controller is 2. To prove the controller's robotness, adding gauss white noise to all state feedback variables.

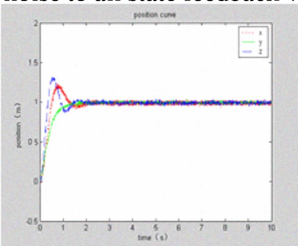

Fig 4. Position curve

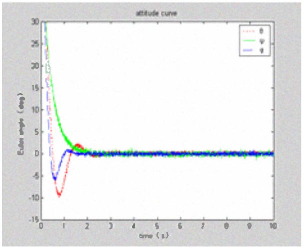

Fig 5. Euler angle curve

From Fig.4 and Fig 5, we can see that quadrotor aerial robot can well tracking the inputs commands after adding noise. When the system is stable, the position and attitude angle of the quadrotor aerial robot all fluctuated at range of ± 0.1m and $\pm 4°$. Having a slight variation with Position corrsponding to attitude mutation, quadrotor aerial robot revert to hoverable state under the control of Backstepping controller. Control performance of controller for the quadrotor aerial robot has been verified through the simulation experiment. It can realize the hoverable control at fixed point and obtain the performance target of smaller tracking error and shorter regulatable time, through regulating parameter of controller. Trajectory tracking experiment result illustrated that the controller has good tracking performance. Under the condition of short time mutating of the sensor noise and rotor speed, the simulation experiment result described Backstepping method provided certaion robotness, and overcomed lacking robotness in the traditional control method of closedloop system.

Ⅴ. CONCLUSIONS

This paper has implemented nonlinear control strategy for the quadrotor aerial robot. The nonlinear model of quadrotor aerial robot is based on Newton-Euler formalism. Backstepping of nonlinear controller design is based on constructing Lyapunov function for closed-loop system and guarantees stability, tracking performance and robotness. Effectiveness of the control design scheme is shown through nonlinear simulations.

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