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Time-Varying LQ Control for Autonomous Soft Landing of Small-scale Helicopter

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***Abstract* - This paper addresses soft landing of miniature helicopter issue and suggests an optimal strategy with appropriate design parameters for a typical soft landing. The time-varying gains and time-varying quadratic performance index LQ control for autonomous soft landing of miniature helicopter examines and simulate. Simulation results have proved that the designed controller is suitable.**

***Index Terms* - *Optimal control, linear-quadratic control, time-varying*，quadratic performance index .**

I. INTRODUCTION

In recent years, helicopters are increasingly used in military and civilian applications Due to their ability to hover, fly in very low altitudes and within confined spaces; there have been considerable literatures and reports on to the design of autonomous flight systems for helicopters with advanced control theories in past decades. Recent study on autonomous helicopters ranges from system modeling and identification, controller design to sensor integration and hardware implementation. With respect to autonomous landing, precision landing technology has been most studied and reported ([5], [6], [7]); however, the soft landing is less addressed, which is another important faculty to meet special demands for task such as intelligent surveillance. The concept of soft landing is similar to those of lunar exploration vehicle which require the force and torque exerted on surface of moon is as small as possible. With this kind of special capability, the miniature helicopters can land on the satellite silently without being detected. According to the principle of momentum, velocities of all directions should be “exactly” equal to zeros when helicopters touch down to surface. In addition, it is essential that the helicopter lands vertically on the surface. However; it is a challenge to design a low-level controller for autonomous soft landing since the dynamics of the helicopter is strongly nonlinearities and inherent instabilities. Since that miniature helicopter model is considered as a full MIMO system, the first systematic technique to control t is the linear quadratic regulator (LQR) design methodology. LQR control has been applied successfully to aeronautical control problems, mainly due to guaranteed robust asymptotic stability of the closed-loop. To improve LQR control performance, a linear-quadratic (LQ) controller with time-varying instead of constant gains is supposed in Ref [1], [2], which showed that time-varying gains LQ has less settling time and overshoot compared with constant gains one. However, Ref [2] indicated that the care must be taken to handle the high feedback gains near the terminal time.

The time-varying gains LQ control for autonomous soft landing of miniature helicopter examines and simulate in this paper and we discusses the implementation to reduce high feedback gains near the terminal time with time-varying quadratic performance index (QPI) in LQR .

This paper is organized as follows. In section Ⅱ the linear model of miniature helicopter are introduced. In section Ⅲ the time-varying gains and time-varying quadratic performance index LQR control law is deduced and simulation results is analyzed . Finally, the conclusions and directions for future study are presented in Section Ⅳ.

Ⅱ HELICOPTER MATHEMATICAL MODEL

The full mathematical model of a helicopter, including flexibility of the rotors and fuselage, dynamics of the actuators and combustion engine, is very complex. In most cases, the helicopter is considered as a rigid body, whose inputs are forces and torques applied to the center of mass and whose outputs are the linear position and velocity of the center of mass, as well as the rotation angles and angular velocities. The simulating mathematical model is based on the eleventh linear model which is proposed in [3] [4].The miniature helicopter uses a two-bladed main rotor with a Bell-Hiller stabilizer bar. The Bell-Hiller stabilizer bar is a secondary rotor consisting of a pair of paddles connected to the rotor shaft through an unrestrained teetering hinge. It receives the same cyclic control input as the main rotor do but it has a slower response than the main blades and is also sensitive to airspeed and wind gust. The 11th order model of the miniature helicopter with x and u defined in Tables I as follows:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t) \qquad (1)$$

Where

x = [u, v, p, q, φ, θ, a, b, w, r, rfb]

u = [δ lat, δ lon, δ col, δ ped]

TABLE I

MODEL STATES AND ITS DESCRIPTION

States& controls	Description
u	x –velocity
v	y -velocity
w	z -velocity
p	Roll Angular Rate
q	Pitch Angular Rate
r	Yaw angular rate
rfb	Yaw rate feedback
a	Longitudinal Flapping Angle
b	Lateral Flapping Angle
φ	Roll Euler Angle
θ	Pitch Euler Angle
δ lat	Lateral Cyclic Deflection
δ lon	Longitudinal Cyclic Deflection
δ col	Pedal Control Input
δ ped	Collective Control Input

The units are feet seconds and cent-radians for the states, and deci-inches for the controls. The additional four kinematics states(ϕ ,x, y, h)are related to the above mentioned states as follows:

$$\dot{\psi} = r, \dot{x} = u, \dot{y} = v, \dot{h} = -w$$

Where ϕ is the yaw angle, x and y are the horizontal coordinates of the center of mass, and h is the altitude. For soft landing problem, thus the model of the miniature helicopter including 12 states representing the velocities and position are re-written as follows

x = [u, v, p, q, φ, θ, ϕ ,w, r, x, y, h]

$$\boldsymbol{A} = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & -g & X_a & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 \\ L_u & L_v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & A_{\mathbf{b}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_r & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{lat} & A_{lon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{lat} & B_{lon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{col} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_{ped} & 0 \end{bmatrix}$$

Ⅲ Soft Landing LQR Control law design

A Time-varying LQ control law design

We suppose that helicopter model is deterministic with all state variables available. In the case of designing time-varying gains LQ control law for soft landing, the problem is to find time-varying control sequences to transfer states involving velocity and position to certain constraints group in finite time from any point. In detail, LQ is a full-state feedback control where x is the state vector and u is the control efforts designed to minimize the quadratic performance measure described by:

$$J = (1/\boldsymbol{2})\boldsymbol{x}^T(t_f)\boldsymbol{S}_f\boldsymbol{x}(t_f) + \mu^T\left[\boldsymbol{M}_f\mathrm{x}(t_f) - \psi\right] + (1/\boldsymbol{2})\int_{t_0}^{t_f} \boldsymbol{x}^T(k)\boldsymbol{Q}\boldsymbol{x}(k) + 2\boldsymbol{x}^T\boldsymbol{N}\boldsymbol{u} + \boldsymbol{u}^T(k)R\boldsymbol{u}(k)\,\mathrm{d}t \tag{3}$$

Subject to:

$$\dot{x} = Ax + Bu \qquad x(0) = x_0 \tag{4}$$

Where A, B, Q, N and R are given constant matrix. This is a linear-quadratic (LQ) Bolza problem with equality terminal constraints, its Hamiltonian is:

$$H = (1/\boldsymbol{2})\boldsymbol{x}^T(k)\boldsymbol{Q}\boldsymbol{x}(k) + 2\boldsymbol{x}^T\boldsymbol{N}\boldsymbol{u} + \boldsymbol{u}^T(k)R\boldsymbol{u}(k) + \lambda^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}) \tag{5}$$

The Euler-Lagrange equations are:

$$\begin{Bmatrix} \dot{x} \\ \dot{\lambda} \end{Bmatrix} = \begin{bmatrix} \mathrm{A - BR^{-1}N^T} & \mathrm{BR^{-1}B^T} \\ \mathrm{-Q + N^TR^{-1}N} & \mathrm{-A^T - NR^{-1}B} \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} \tag{6}$$

$$\mathrm{u} = -\mathrm{R}^{-1}\mathrm{N}^{\mathrm{T}}(\mathrm{x} + \mathrm{B}^{\mathrm{T}}\lambda) \tag{7}$$

with the two-point boundary conditions

$$x(0) = x_0, \lambda(t_f) = \boldsymbol{S}_f\mathrm{x}(t_f) - \boldsymbol{M}_f{}^T\mu \tag{8}$$

The boundary condition suggests a solution of the form

$$\lambda(t) = \boldsymbol{S}(t)\mathrm{x}(t) + \boldsymbol{M}(t)\mu \tag{9}$$

Where

$$\boldsymbol{S}(t_f) = \boldsymbol{S}_f, \qquad \boldsymbol{M}(t_f) = \boldsymbol{M}_f \tag{10}$$

Differentiate with respect to time to obtain equations:

$$\dot{\lambda}(t) = \dot{\boldsymbol{S}}(t)\mathrm{x}(t) + \boldsymbol{S}(t)\dot{\mathrm{x}}(t) + \dot{\boldsymbol{M}}(t) \tag{11}$$

And substitute for $\dot{\lambda}$ and $\dot{\mathrm{x}}$ from (6), using (9) to eliminate λ ,

$$-\mathrm{\overline{A}^T}(\boldsymbol{S}\mathrm{x} + \mathrm{M}) - \mathrm{\overline{Q}x} = \dot{\boldsymbol{S}}\mathrm{x} + \boldsymbol{S}(\mathrm{\overline{A}x} - \mathrm{\overline{B}}(\boldsymbol{S}\mathrm{x} + \mathrm{M})) + \dot{\boldsymbol{M}} \tag{12}$$

Where

$$\mathrm{\overline{A} = A - BR^{-1}N^T, \overline{B} = BR^{-1}B^T, \overline{Q} = Q - N^TR^{-1}N} \tag{13}$$

Since x(t) $\neq$ 0,it follows that the minimization problem is translated to the solution of differential Riccati equation given by:

$$\dot{\boldsymbol{S}} = -\boldsymbol{S}\mathrm{\overline{A}} - \mathrm{\overline{A}^T}\boldsymbol{S} - \mathrm{\overline{Q}} + \boldsymbol{S}\mathrm{\overline{B}}\boldsymbol{S} \qquad \boldsymbol{S}(t_f) = \boldsymbol{S}_f \tag{14}$$

$$\dot{\boldsymbol{M}} = -(\mathrm{\overline{A}} - \mathrm{\overline{B}}\boldsymbol{S})^T\mathrm{M} \qquad \boldsymbol{M}(t_f) = \boldsymbol{M}_f \tag{15}$$

The solution for problem is combination of feedback and feed-forward control, which is similar to LQ follower:

$$\mathrm{u(t) = u_f(t) - K(t)x(t)} \tag{16}$$

$$\mathrm{u_f(t) = R^{-1}B^T}M(t) \tag{17}$$

$$\mathrm{K(t) = R^{-1}N^T(x + B^T}S) \tag{18}$$

B Time-varying QPI design

In the quadratic performance index (QPI), the weighting matrices Q and R are design parameters varied to obtain a specified closed-loop dynamic behavior, pole placement, penalizing control accuracy and control effort, respectively. Since QPI has Q and R in two bilinear forms, they need be positive semi-definite and positive definite, respectively. The Q and R matrices associated with the design were chosen diagonal. The diagonal entries in the Q and R weights were tuned to ensure that a good step response was achieved without saturating the control inputs. In particular, the velocity states and the controls were heavily penalized, to ensure an over damped step response. This is desirable, since it will prevent excitation of the un-modeled high frequency dynamics of the helicopter.

With respect to the soft landing, the weighting matrix R is time varying described by:

$$R(t)=\begin{cases}1.2R_0 & 0\le t\le 0.1t_f\\ R_0 & 0.1t_f\le t\le 0.9t_f\\ 1.5R_0 & 0.9t_f\le t\le t_f\end{cases}$$

C. Simulation and Implementation Results

Fig1-7 show simulation results with constant weighting matrix. Fig1 shows three linear velocities and angular velocity of yaw. The top is for three linear velocities and the bottom is for yaw angular rate. Their max value are 10, 3.6, 1.7and 0.85 respectively, which are indicated on the head of graph. Fig2 shows four controls. The top is for Longitudinal Cyclic and Pedal Control and the bottom is for Lateral Cyclic and Collective Control. Their max value are 5.4, 5.4, 17and 56 respectively, It is obvious that the controls vary greatly during the descend process, especially near the beginning and ending time.

The first plot of Fig. 3 shows the attitude angles versus time; the pitch angle θ and the roll angle φ vary greatly before come to zero, while the second plot shows the yaw angular ϕ stays close to zero. Fig3and fig4 shows the five feedback gains for Longitudinal Cyclic Deflection δ lon versus time. The gains are small at the beginning, then increase during the second half of landing process. The pitch angle feedback gains(red line in fig4) is reach peak near the ending while others decrease to zero.fig6 shows the position states versus time and fig7 is its 3-demension version which tell the real trace of the helicopter.

The QPI is 1161.6233 in the case of constant weighting matrices Q and R.

Compared with Fig2, Fig8 shows control is smaller than the former, especially near the ending of landing process. Fig9 indicates that the moving space of helicopter is larger than that in fig7, mainly because of the extension in x direction. The QPI is 34.55 in the case of time-varying weighting matrices Q and R.

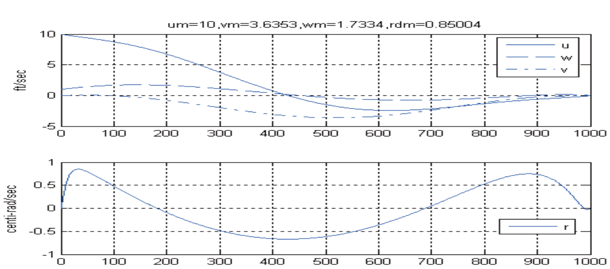

Fig1 velocities with constant weighting matrix

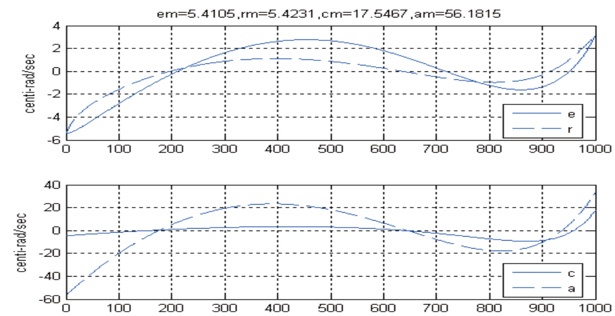

Fig2 controls with constant weighting matrix

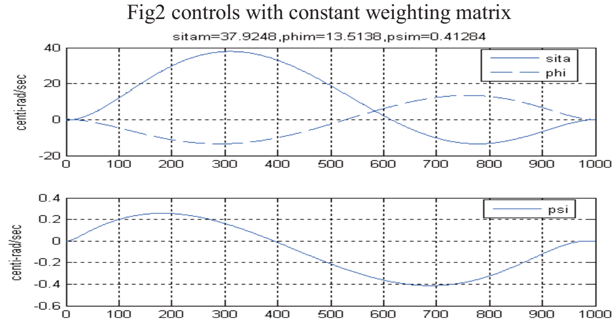

Fig3 angular with constant weighting matrix

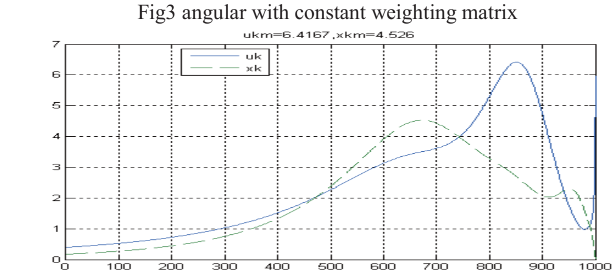

Fig4 feedback gains with constant weighting matrix(a)

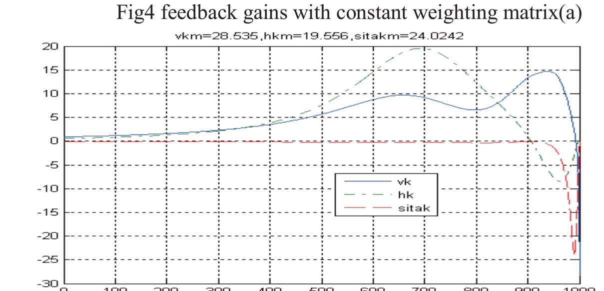

Fig5 feedback gains with constant weighting matrix (b)

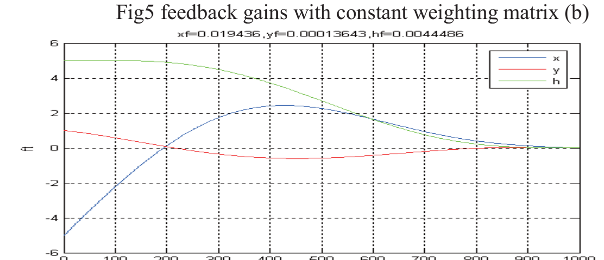

Fig6 position with constant weighting matrix (a)

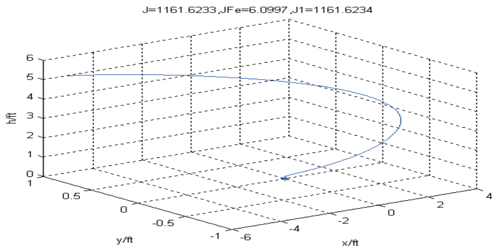

Fig7 position with constant weighting matrix(b)

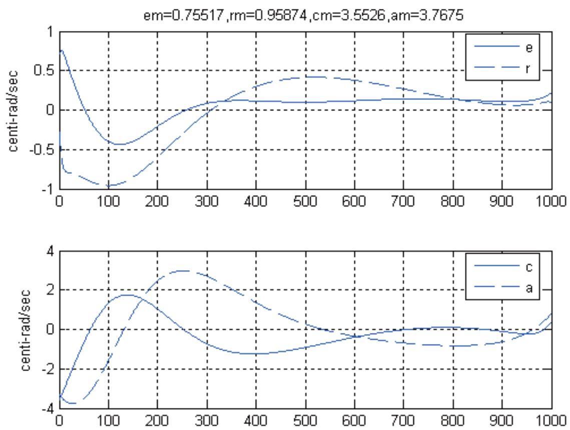

Fig8 controls with time varying weighting matrix

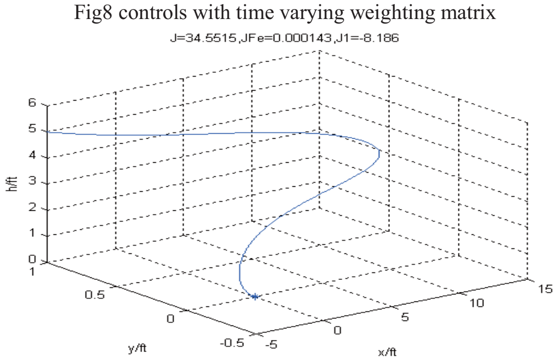

Fig9 position with time varying weighting matrix

Ⅳ CONCLUSIONS

Time-varying linear-quadratic controllers are now practical because of faster computers with large memory storage. They improve speed and accuracy for rapid maneuvers of aircraft, spacecraft, or robots compared to time-invariant controllers. The time-varying quadratic performance index LQ control is a sound way to handle the high feedback gains near the terminal time.

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REFERENCES

[1]. BRYSON, A. E., Dynamic Optimization, Addison-Wesley-Longman, Menlo Park, California, 1999.

[2]. BRYSON, A. E., Time-Varying Linear-Quadratic Control Journal of Optimization Theory and Application, 1999.3. Vol. 100. No.3. pp. 515-525

[3]B. Mettler, M. B. Tischler, and T. Kanade, System identification of small-size unmanned helicopter dynamics, Presented at the American Helicopter Society 55th Forum, May, 1999.

[4] Guowei Cai, Alvin K. Cai, Ben M. Chen and Tong H. Lee, Construction, Modeling and Control of a Mini Autonomous UAV Helicopter, Proceedings of the IEEE International Conference on Automation and Logistics Qingdao, China September 2008

[5] V. Gavrilets, A. Shterenberg, M. A. Dahleh and E. Feron, Avionics system for a small unmanned helicopter performing aggressive maneuvers, Proceedings of the 19th Digital Avionics Systems Conferences, pp. 1E2/1-1E2/7, Philadelphia, USA, 2000.

[6] S. Saripalli, J. F. Montgomery and G. S. Sukhatme, Visually guided landing of an unmanned aerial vehicle, IEEE Transactions on Robotics and Automation, Vol. 19, pp. 371-380, 2003.

[7] M. Musial, U. W. Brandenburg and G. Hommel, "Inexpensive system design: The flying robot MARVIN," Proceedings of the 16th International UAV Conference on Unmanned Air Vehicle Systems, pp 23.1C23.12, Bristol, UK, 2001.