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Adaptive motion/force control of uncertain nonholonomic mobile manipulator with estimation of unknown external force

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Abstract This paper describes a stable adaptive motion/force control of uncertain nonholonomic mobile manipulator with the consideration of external force. As it is well known, unexpected external force makes the motion of the system unstable since there are no fixed points in the stationary coordinate. Here, a novel adaptive control scheme is utilized to estimate and compensate the unknown external force exerted to the end-effector even if the parameters of the system are uncertain. The important advantages of this approach are to achieve estimation without the requirement of force-sensing feedback and the knowledge of the system dynamic model. The update laws for the force and the parameters are derived from a Lyapunov function to guarantee the control system stability. Furthermore, a unified operational space dynamic formulation is presented to solve the problem of redundancy. As a result, the desired end-effector and platform trajectories are simultaneously tracked with a perfect coordination between the two subsystems. Therefore, the proposed controller proves that it can not only guarantee the stability, but also the tracking performance of the system in the task space. The effectiveness of the proposed algorithm is evaluated through extensive simulations and they demonstrate the stability, tracking trajectories and feasibility in estimating the external force and the dynamic uncertainties.

Keywords Nonholonomic mobile manipulator · External force · Adaptive control · Force estimation and compensation · Lyapunov theory · Redundancy resolution · Task space

1 Introduction

The mobile manipulator considered in this study is a multibody system, composed by a manipulator mounted on a nonholonomic mobile platform. This system has an infinite motion area and can perform various tasks efficiently by utilizing mobility and manipulation functions. To expand the feasible applications of such system, it is necessary to control not only

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the position, but also the interaction force between the manipulators and the environment. Many researchers have investigated the motion/force control problem of mobile manipulators. Despite the diversity of these approaches, it is possible to classify most of the design procedures as based on two major approaches: (i) motion tracking and force regulation [1–5]; (ii) motion tracking and force rejection [6–13]. In the first approach, the interaction force between the robot and the environment must be accommodated rather than suppressed to comply with the environmental constraints. To achieve this goal, a combination of motion and force control loops are required to drive the mobile manipulator and to keep the end-effector in contact with the environment. In the second approach, the constraint force is considered as a disturbance and must be suppressed since it destabilizes the overall system. These forces may be due to interaction with a work piece, collision with obstacles, etc.

To deal with uncertainties and disturbances, several works have been reported for the motion tracking and the force regulation of mobile manipulators using adaptive and robust control strategies. In [1], adaptive control is proposed for the trajectory/force control of mobile manipulator subjected to holonomic and nonholonomic constraints with unknown inertia parameters. The proposed algorithm is based on a suitable reduced dynamic model, defined reference signals and mixed tracking errors. It not only ensures that the entire state of the system to converges asymptotically to the desired trajectory, but also ensures the convergence of the constraint force asymptotically to the desired force. In [2, 3], an adaptive robust force/motion control strategies are presented for mobile manipulator under both holonomic and nonholonomic constraints in the presence of uncertainties and disturbances. The proposed control is robust not only to parameter uncertainties such as mass variations, but also to external ones such as disturbances. In [4], force/motion tracking controller is investigated for a nonholonomic mobile manipulator with unknown parameters and disturbances under uncertain holonomic constraints. First, the system is transformed into a reduced chained form. Then, a robust adaptive force/motion control with hybrid variable signals is proposed to compensate the parametric uncertainties and suppress the bounded disturbances. In [5], a motion/force control problem of a class of constrained mobile manipulator with unknown dynamics is considered. An adaptive recurrent neural network controller is proposed to deal with the unmodeled system dynamics, in the presence of holonomic and nonholonomic constraints. The proposed control strategy guarantees that the system motion converges asymptotically to the desired manifold while the constraint force remains bounded. In addition, an adaptive method is proposed to identify the contact surface.

To the best of our knowledge, a few control methods have been proposed for solving the problem of force rejection and motion tracking for mobile manipulators. In [6], a dynamically consistent and decoupled partitioning between the external task space and the internal null space is developed for the mobile manipulator. The independent controllers developed within each decoupled space facilitate the active internal reconfiguration and resolve the problem of redundancy at the dynamic level. Specifically, two variants of null-space controllers are implemented to improve the external force rejection. Similarly, the same authors formulate in [7] a dynamic-level control routine for the mobile manipulator that allows for independent dynamic control. The primary task consists of controlling the motion and/or the force interaction of the end-effector with the attached payload/external environment. The significant force disturbance acting on the end-effector is damped out under the influence of the hybrid-impedance controller. However, the proposed controller assumes perfect knowledge of the system dynamics which is difficult in real applications. In [8], a robust acceleration control based on the disturbance observer is employed to realize the desired motion of the mobile manipulator, even if an unknown external force is applied to the end-effector. The impact force is well suppressed by the manipulator's quick motion and the following force is determined by the vehicle motion. However, the proposed approach is applied to the joint space of the robot and a joint acceleration feedback is needed. The same authors propose in [9] an impedance control design for the mobile manipulator in the presence of external force acting on the work space. For the purpose of keeping the robot stable, the vehicle's desired position is determined so that the robot's ZMP (zero-moment point) may come as close as possible to the center of the supporting polygon. The ZMP is the point on the ground where the sum of all the moments of the active forces is equal to zero. Using this method, it is difficult to obtain the smooth motion of configuration when a quick response is required. The ZMP approach was also used in [10] to keep the mobile manipulator inside the stable region using a potential method. However, all the above works do not take into account the online compensation of the external force.

It should be noted that in the aforementioned force rejection schemes, the mobile manipulator is controlled in joint space and the problem of redundancy is not treated or resolved. Moreover, it considers a conventional force algorithm that relies on the precise knowledge of the complex system dynamics. None of the above works has coped with the online estimation and compensation of the external force in the presence of uncertain robot dynamics. However, for practical applications, the dynamic parameters are difficult to be obtained and the robot should be controlled in the task space. Consequently, the system performance is degraded and the instability may incur in the presence of an excessive external force and/or uncertainties. Recently, some researchers have successfully incorporated adaptive control for the online estimation of the external force, acting to the end-effector of robot manipulator operating in an unknown or changing environment [11, 13]. Motivated by these works, an adaptive approach is proposed for estimating and compensating unexpected external force and uncertainties for the nonholonomic mobile manipulator. The problem of redundancy is resolved using an extended task space formulation. It allows the robot to track a given trajectory in the task space when undesirable force occurs, even if the system dynamic is unknown a priori. Moreover, it preserves the safety of the robot and/or the environment in practical applications. The novelty of the approach presented here can be understood considering the following points:

1. A new adaptive algorithm is proposed to estimate and compensate, both external force and uncertainties, which increase the safety of the robot or/and the environment. Unlike of the most aforementioned works that guarantee only the convergence of the position tracking error to zero, the proposed adaptive methods can ensure that both, the position and the velocity errors converge asymptotically to zero. The convergence of the estimated parameter of their real values is also proved.
2. While most of the works in the literature [14–16] use a force/torque sensor to detect external force and since this sensor is expensive, affected by noise and complicated to implement. In the proposed work, the needs of a force/torque sensor and/or the environment model are not required. Also, the estimation of the unexpected external force acting on the end-effector is achieved using only position and velocity measurements, which are usually provided by simple encoders. Consequently, the control input does not require measurement of the force or the joint acceleration, and thus avoids noisy joint force/torque feedback.
3. Previous knowledge of the robot dynamics is not necessary because the developed controller learns the robot dynamics online using regression formulation, ensuring the relevance of this approach when the robot navigates in a real environment.
4. An integrated dynamic modeling method is proposed for the purpose of resolving the problem of redundancy and driving the robot in the operational space. The derived model

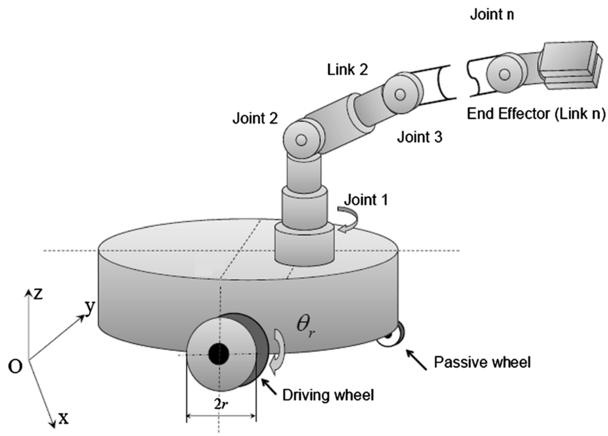

Fig. 1 Scheme of the mobile manipulator

exploits the extra degrees of freedom to better accomplish the assigned task with a perfect coordination between the mobile platform and the onboard manipulator.

2 System description

Consider the mobile manipulator shown in Fig. 1. It consists of a multi-link manipulator mounted on a differential-driven mobile platform. The mobile platform is equipped with two driving wheels mounted on the same axis and a passive front wheel. The motion and orientation are achieved by independent actuators, e.g., DC motors providing the necessary torques to the rear wheels.

2.1 Kinematic model

The mobile manipulator configuration is defined by a vector q of n independent coordinates, called generalized coordinates and may be separated into two sets as

$$q = [q_1 \quad q_2 \quad \dots \quad q_n]^T = \left[q_p^T \quad q_b^T\right]^T, \tag{1}$$

where $q_p \in \Re^{n_p}$ represents the generalized coordinates of the mobile platform, and $q_b \in R^{n_b}$ represents the generalized coordinates of the onboard manipulator. Note that $n = n_p + n_b$.

The mobile manipulator is subject to m non-integrable and independent velocity constraints called nonholonomic constraints and can be expressed as

$$A(q)\dot{q} = 0, \tag{2}$$

where $A(q) \in \Re^{m \times n}$ is the matrix of nonholonomic constraints.

The effect of these constraints can be viewed as a restriction of the dynamics on the manifold Ω_n where $\Omega_n = \{(q, \dot{q}) \in \Re^n / A(q)\dot{q} = 0\}$.

It is well known that Maggi's equation is an efficient quasi-velocity method for deriving the equations of motion for a general multibody system with n generalized coordinates and m nonholonomic constraints. The method consists of selecting n independent quasi-velocities such that m of them span the constraint space. The other $(n - m)$ quasi-velocities are chosen appropriately [25]. Hence, nonholonomic constraints (2) can be written in terms of virtual displacement δq_j as

$$\sum_{j=1}^{n} A_{ij}(q,t)\delta q_j = 0, \quad i = 1,\ldots,m. \tag{3}$$

Under the assumption that the m nonholonomic constraints are independent, it is possible to represent m of the δq_j's in terms of the rest denoted by $\delta \eta_j$ as follows:

$$\begin{bmatrix} A_{1,1} & \ldots & A_{1,m} \\ \vdots & \ddots & \vdots \\ A_{m,1} & \ldots & A_{m,m} \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \vdots \\ \delta q_m \end{bmatrix} = - \begin{bmatrix} A_{1,m+1} & \ldots & A_{1,n} \\ \vdots & \ddots & \vdots \\ A_{m,m+1} & \ldots & A_{m,n} \end{bmatrix} \begin{bmatrix} \delta \eta_{m+1} \\ \vdots \\ \delta \eta_n \end{bmatrix}. \tag{4}$$

The matrix on the left-hand side of Eq. (4) is invertible due to independence, thus

$$\begin{bmatrix} \delta q_1 \\ \vdots \\ \delta q_m \end{bmatrix} = - \begin{bmatrix} A_{1,1} & \ldots & A_{1,m} \\ \vdots & \ddots & \vdots \\ A_{m,1} & \ldots & A_{m,m} \end{bmatrix}^{-1} \begin{bmatrix} A_{1,m+1} & \ldots & A_{1,n} \\ \vdots & \ddots & \vdots \\ A_{m,m+1} & \ldots & A_{m,n} \end{bmatrix} \begin{bmatrix} \delta \eta_{m+1} \\ \vdots \\ \delta \eta_n \end{bmatrix}. \tag{5}$$

It is easy to deduce that Eq. (5) can be written in the following form:

$$\delta q_i = \sum_{j=m+1}^{n} S_{ij}(q,t)\delta \eta_j, \quad i = 1,\ldots,m. \tag{6}$$

Equivalently, in matrix form it can be written as

$$\dot{q} = S(q)\dot{\eta}. \tag{7}$$

Equation (7) is generally called a kinematic model of the nonholonomic mobile manipulator. If we suppose that the manipulator is mounted on a type (2, 0) wheeled mobile platform with two driving wheels actuated independently, then

$$\dot{\eta} = \begin{bmatrix} \dot{\theta}_r & \dot{\theta}_l & \dot{\theta}_1 & \ldots & \dot{\theta}_{n_b} \end{bmatrix}^T, \tag{8}$$

where $\dot{\theta}_r$ and $\dot{\theta}_l$ are the angular velocities of right and left wheel, respectively, and $\dot{\theta}_{n_j}$ is the angular velocity of the jth joint manipulator for $j = 1,\ldots,n_b$.

It is easy to verify from (2) and (7) that the following relation holds:

$$S^T(q)A^T(q) = 0. \tag{9}$$

2.2 Dynamic model

The dynamics of the mobile manipulator can be derived using Lagrange's formulation as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = Q_i + \sum_{j=1}^{n} \lambda_i A_{ji}, \quad i = 1,\ldots,n, \tag{10}$$

where $L = K - P$ is the Lagrangian function, which is the difference between the kinetic and the potential energy of the system, and $Q_i \in \Re^n$ is the generalized force.

The kinetic energy of the whole mobile manipulator system is

$$K = \sum_i \frac{1}{2}\big(v_{ci}^T m_i v_{ci} + \omega_i^T I_i \omega_i\big), \tag{11}$$

where m_i is the mass of part i, v_{ci} is the velocity at the center of mass of part i, I_i is the moment of inertia of part i and ω_i is the inertial angular velocity of part i.

Because of the planar motion, the potential energy of the mobile platform is zero. Therefore, the potential energy of the whole system contains only the part of the manipulator

$$P = \sum_i m_i g^T l_{ci}, \tag{12}$$

where l_{ci} is the coordinates of the center of mass of part i and g is the vector at the direction of gravity in an inertial frame.

After calculating (10), (11) and (12), the dynamic model of the mobile manipulator can be written in the matrix form as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau + E(q)F + A^T(q)\lambda, \tag{13}$$

where

$q \in \Re^n$	are the generalized coordinates for the mobile manipulator;
$M(q) \in \Re^{n\times n}$	is a symmetric positive definite inertia matrix;
$C(q,\dot{q}) \in \Re^{n\times n}$	is the centripetal and Coriolis force matrix;
$G(q) \in \Re^n$	is the gravitational force vector;
$A(q) \in \Re^{m\times n}$	is the kinematic constraint matrix;
$\lambda \in \Re^m$	is the Lagrange multiplier vector;
$B(q) \in \Re^{n\times(n-m)}$	is the input transformation matrix;
$\tau \in \Re^{n-m}$	is the input torque vector;
$F \in \Re^s$	is the task space force exerted to the end-effector;
$E(q) \in \Re^{n\times s}$	is the transform matrix between the task space force and the generalized coordinates.

The dynamic equation (13) exhibits the following properties which will be employed during the subsequent control development and stability analysis.

Property 1 The inertia matrix $M(q)$ is symmetric, positive definite and satisfies the following inequality:

$$M_{\min} \le \big\|M(q)\big\| \le M_{\max}$$

with $M_{\min}, M_{\max} > 0$ being known constants.

Property 2 $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, that is to say,

$$y^T\big(\dot{M}(q) - 2C(q,\dot{q})\big)y = 0 \quad \forall y \in \Re^n.$$

Property 3 There exists a vector $\boldsymbol{\Psi}$ with components depending on the unknown dynamic robot parameters (masses, moments of inertia, etc.) such that

$$M(q)\ddot{\chi} + C(q,\dot{q})\dot{\chi} + G(q) = Y_1(q,\dot{q},\dot{\chi},\ddot{\chi})\boldsymbol{\Psi} \quad \forall \dot{\chi},\ddot{\chi} \in \Re^n,$$

where $Y_1(q,\dot{q},\dot{\chi},\ddot{\chi})$ is called the regressor matrix, which contains a known function.

2.3 Reduced model

Using a kinematic model, we can obtain a reduced feasible-space (unconstrained) dynamic equation of motion. Substituting (7) and its derivative into (13), and then multiplying by $S^T(q)$. The constraint term $A^T(q)\lambda$ is eliminated by virtue of (9) and the reduced dynamics of the mobile manipulator can be expressed as

$$M_c(q)\ddot{\eta} + C_c(q,\dot{q})\dot{\eta} + G_c(q) = B_c(q)\tau + E_c(q)F, \tag{14}$$

where

$$\begin{aligned} M_c(q) &= S^T(q)M(q)S(q); \\ C_c(q,\dot{q}) &= S^T(q)\big[C(q,\dot{q})S(q) + M(q)\dot{S}(q)\big]; \\ G_c(q) &= S^T(q)G(q); \\ B_c(q) &= S^T(q)B(q); \\ E_c(q) &= S^T(q)E(q). \end{aligned}$$

Remark 1 The reduced dynamic form (14) is more appropriate for the controller design, as the constraint λ has been eliminated from the dynamic equation in generalized form (13). We also note that Eq. (14) reflects only the feasible motions allowable by satisfying the constraints.

Remark 2 Since Properties 1 and 2 are invariant under changes of coordinates, so Eq. (14) has similar properties as Eq. (13).

Remark 3 In practice, the system is subject to parametric and nonparametric uncertainties. Parametric uncertainties are caused by inaccurate measurement, time-varying parameters caused by loading or unloading of some objects, etc. Nonparametric uncertainties may be caused by unmodeled dynamics of the system such as castor wheels, non-ideals in the mechanical system such as backlash, viscous and dynamic frictions.

3 Extended task space formulation and redundancy resolution

The task r performed by the end-effector is a function of the mobile manipulator configuration as

$$r = f(q), \tag{15}$$

where $r \in R^s$ and $f(.)$ is a nonlinear transformation obtained through a geometric analysis.

Differentiating the above equation and using (7), the end-effector task velocity is given by

$$\dot{r} = \frac{\partial f(q)}{\partial q}\dot{q} = \frac{\partial f(q)}{\partial q}S(q)\dot{\eta} = J(q)\dot{\eta}, \tag{16}$$

where $J(q) \in R^{s\times(n-m)}$ is the Jacobian matrix which describes the relation between the ρ-dimensional joint velocity vector ($\rho = n - m$) and the s-dimensional end-effector task velocity vector $\dot{r}$.

The mobile manipulator is kinematically redundant when it possesses more degrees of freedom than it is needed to execute a given task, i.e., $\rho > s$. One of the advantages of robot redundancy is the potential to use the extra degrees of freedom ($l = \rho - s$) to satisfy additional task requirements [17], optimization of performance criteria [18], etc.

In this paper, a unified extended approach is proposed in dynamic level to combine the desired end-effector motion and a set of kinematic functions, which reflect the desired additional task that will be performed due to the redundancy. The resulting extended differential kinematic equations in the task space are then solved to obtain the required mobile manipulator's motion in dynamic level.

Let's consider a set of p task-related kinematic functions given by

$$y = g(q). \tag{17}$$

The additional task variables (17) can be expressed in the velocity form as

$$\dot{y} = \frac{\partial g(q)}{\partial q}\dot{q} = \frac{\partial g(q)}{\partial q}S(q)\dot{\eta} = Z(q)\dot{\eta}. \tag{18}$$

By using (15) and (17), the s-dimensional main task of the end-effector motion and the p-dimensional additional task give the $\rho \times 1$ extended task X as

$$X = \begin{bmatrix} r \\ y \end{bmatrix} = \begin{bmatrix} f(q) \\ g(q) \end{bmatrix} = h(q). \tag{19}$$

Now, Eqs. (16) and (18) are combined to obtain the extended differential kinematic model of the whole mobile manipulator system as

$$\dot{X} = \begin{bmatrix} \dot{r} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} J(q) \\ Z(q) \end{bmatrix}\dot{\eta} = J_e(q)\dot{\eta}, \tag{20}$$

where $J_e(q) \in \Re^{\rho\times\rho}$ is the extended Jacobian matrix which is supposed to be square and invertible. Hence, from (20), we can obtain

$$\begin{aligned} \dot{\eta} &= J_e^{-1}\dot{X}, \\ \ddot{\eta} &= J_e^{-1}\ddot{X} - J_e^{-1}\dot{J}_e J_e^{-1}\dot{X}. \end{aligned} \tag{21}$$

Substituting (21) into (14) and left multiplying by J_e^{-T}, we obtain the dynamics of the mobile manipulator in the extended task space as

$$M_x(q)\ddot{X} + C_x(q,\dot{q})\dot{X} + G_x(q) = B_x(q)\tau + E_x(q)F, \tag{22}$$

where

$$\begin{aligned} M_x(q) &= J_e^{-T}(q)M_c(q)J_e^{-1}(q); \\ C_x(q,\dot{q}) &= J_e^{-T}(q)\big[C_c(q,\dot{q}) - M_c(q)J_e^{-1}(q)\dot{J}_e(q)\big]J_e^{-1}(q); \\ G_x(q) &= J_e^{-T}(q)G_c(q); \\ B_x(q) &= J_e^{-T}(q)B_c(q); \\ E_x(q) &= J_e^{-T}(q)E_c(q). \end{aligned}$$

Remark 4 The extended formulation for the redundancy resolution was previously developed by H. Seraji [19–21], where two variants of the additional task were considered. The first study considered the end-effector orientation relative to the world frame, and the manipulator elbow angle between the upper arm and forearm as the additional task variables. The second study considered the end-effector orientation and the platform orientation as the additional task variables. The control law for the overall system was designed at the kinematic level. A more efficient formulation adopted in this work considers the end-effector motion relative to the world frame and the platform motion relative to the world frame as the additional task variables. This approach guarantees that the mobile platform brings the robotic arm into a preferred configuration that supports the arm to perform the complex tasks more efficiently. Moreover, the control law for the overall system is designed at the dynamic level, which is suitable for practical applications.

Due to the invariance under changes of coordinates, the dynamic model of the mobile manipulator in the extended task space still has the following properties [22].

Property 4 The inertia matrix in task space $M_x(q)$ is symmetric, positive definite, and satisfies the following inequality:

$$m_1 \leq \left\| M_x(q) \right\| \leq m_2,$$

where m_1 and m_2 are positive constants.

Property 5 A skew-symmetric relationship exists between the time derivative of the inertia matrix in the task space and the centripetal matrix in the task space as follows:

$$y^T \big(\dot{M}_x(q) - 2C_x(q, \dot{q})\big) y = 0 \quad \forall y \in \Re^\rho.$$

Property 6 From Remark 2, there still exists a vector $\boldsymbol{\Psi}$ with components depending on the unknown dynamic robot parameters (masses, moments of inertia, etc.) such that

$$M_x(q)\ddot{\chi} + C_x(q, \dot{q})\dot{\chi} + G_x(q) = Y(q, \dot{q}, \dot{\chi}, \ddot{\chi})\boldsymbol{\Psi} \quad \forall \dot{\chi}, \ddot{\chi} \in \Re^\rho,$$

where $Y(q, \dot{q}, \dot{\chi}, \ddot{\chi})$ is called the regressor matrix, which contains a known function.

4 Control design

In this section, three different strategies are proposed for the trajectory tracking control of nonholonomic mobile manipulator, when an external force is exerted to the end-effector. In the first controller, the external disturbing force is rejected using a kind of force sensor mounted at the tip of the end-effector. This sensor is used to detect the feedback information from the environment. However, the use of force sensor has some issues, such as its high cost and the mounting difficulties. Hence, in the second controller, the force sensor is replaced by an adaptive estimator to estimate the external force by using position and velocity measurements. Finally, in order to handle the uncertainties in the robot model, a new scheme with two adaptive estimators is proposed to cope with both dynamic parameters in the robot model and the external force acting on the end-effector. The proposed adaptive controllers do not require acceleration or force measurements, but they use only the standard proprioceptive sensors (joint encoders).

Let $X_d(t)$ be the extended desired trajectory in the task space and $\dot{X}_d(t)$ and $\ddot{X}_d(t)$ be the desired extended velocity and acceleration, respectively. Define the tracking errors as

$$\begin{aligned} e(t) &= X_d(t) - X(t), \\ \dot{e}(t) &= \dot{X}_d(t) - \dot{X}(t). \end{aligned} \tag{23}$$

For the controller design, define the following new variables:

$$\begin{aligned} \dot{X}_r &= \dot{X}_d + \Lambda e, \\ \ddot{X}_r &= \ddot{X}_d + \Lambda \dot{e}, \end{aligned} \tag{24}$$

where Λ is a positive definite matrix.

Define also the filtered tracking error as

$$s = \dot{X}_r - \dot{X} = \dot{e} + \Lambda e. \tag{25}$$

The control objective is specified as follows: Given a desired extended trajectory $X_d(t)$ under an unknown external force disturbance $F(t)$, determine a control law, such that for any $(X(0), \dot{X}(0)) \in \Omega$, X, $\dot{X}$ converge to a manifold Ω_d specified as Ω, where

$$\Omega_d = \big\{(X, \dot{X})/X = X_d, \dot{X} = \dot{X}_d\big\}.$$

In order to solve the previous problem, we make the following assumptions:

Assumption 1 The desired extended trajectory $X_d(t)$ and its first and second derivatives $\dot{X}_d(t)$, $\ddot{X}_d(t)$ are bounded.

Assumption 2 The parameters of the mobile manipulator are in known compact sets. Thus, the matrices M_x, C_x, G_x, B_x and E_x are all bounded.

Assumption 3 The external force $F(t)$ and its first derivative $\dot{F}(t)$ are both bounded. In addition, we suppose that $F(t)$ has a constant value in the steady state, i.e., $\lim_{t\to\infty} \|\dot{F}(t)\| = 0$.

Assumption 4 Using Property 6 and Assumption 2, it follows that the vector of the unknown parameter $\boldsymbol{\Psi}(t)$ and its first derivative $\dot{\boldsymbol{\Psi}}(t)$ are both bounded. In addition, we suppose that $\boldsymbol{\Psi}(t)$ has a constant value in the steady state, i.e., $\lim_{t\to\infty} \|\dot{\boldsymbol{\Psi}}(t)\| = 0$.

Remark 5 In real applications, the velocity and acceleration of the robot are always limited by the motors. Moreover, the parameters of the mobile manipulator cannot be obtained accurately, but its range can be estimated. Therefore, it is reasonable to assume that the unknown parameters of the robot are in a known compact set.

Remark 6 Since the mobile manipulator is a large inertial system, it is insensitive to fast time-varying disturbances and uncertainties. Thus, it is reasonable to suppose that $\lim_{t\to\infty} \|\dot{F}(t)\| = 0$ and $\lim_{t\to\infty} \|\dot{\boldsymbol{\Psi}}(t)\| = 0$.

Two very well-known lemmas will be used in the proof of theorems in this section. The first one is Barbalat's lemma and the other one is a corollary of Barbalat's lemma. Both lemmas are taken from [23] and are given below for the sake of completeness.

Lemma 1 (Barbalat's lemma) *If* $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ *exists and is finite, and* $f(t)$ *is a uniformly continuous function, then* $\lim_{t\to\infty} f(t)=0$.

Lemma 2 *If* $f, \dot{f}\in L_\infty$ *and* $f\in L_p$ *for some* $p\in[1,\infty)$, *then* $f(t)\to 0$ *as* $t\to\infty$.

In Lemma 2 the L_p norm of a function $f(t)$ is used. It is defined as $\|f\|_p = (\int_0^\infty |f(\tau)|^p d\tau)$ where $|\cdot|$ denotes the vector (scalar) length. If the above integral exists (is finite), the function $f(t)$ is said to belong to L_p. Limiting p to infinity provides a very important class of functions, namely the space L_∞ of bounded functions.

4.1 Trajectory tracking controller design based on force measurement

Under the assumption that the dynamics of the mobile manipulator are known and the real force F is perfectly measured by a force sensor mounted at the tip of the end-effector, $\lim_{t\to\infty} = \tilde{F} = F - \hat{F} = 0$, where $\hat{F}$ is the measured force and $\tilde{F}$ is the force measurement error. Consider the following control law:

$$\tau_x = B_x\tau = M_x\ddot{X}_r + C_x\dot{X}_r + G_x + Ks - E_x\hat{F}, \tag{26}$$

where K is a positive definite matrix.

Substituting (26) in (22), the closed-loop dynamics is obtained as

$$M_x\dot{s} + (C_x + K)s = 0. \tag{27}$$

The stability of the closed-loop system (27) is given by the following theorem.

Theorem 1 *Consider the mechanical system described by* (19), (20), *and* (22) *under Assumptions* 1, 2 *and* 3. *Assume that the dynamic parameters are known and the measured external force is* $\hat{F}$. *Using the control law* (26) *with appropriate values of* Λ *and* K, *the following results are guaranteed*:

(1) *The tracking errors* e *and* $\dot{e}$ *converge to zero as* $t\to\infty$.
(2) *All the signals in the closed-loop system are bounded for all* $t\ge 0$.

Proof Consider the Lyapunov candidate function

$$V = \frac{1}{2}s^T M_x s + e^T\Lambda K e. \tag{28}$$

The time derivative of V can be obtained as

$$\dot{V} = s^T\left(M_x\dot{s} + \frac{1}{2}\dot{M}_x s\right) + 2e^T\Lambda K\dot{e}. \tag{29}$$

Substituting the closed-loop dynamic (27) in the above equation, we have

$$\dot{V} = s^T\left(-C_x s - Ks + \frac{1}{2}\dot{M}_x s\right) + 2e^T\Lambda K\dot{e}. \tag{30}$$

Applying the skew-symmetricity Property 4, we have

$$\dot{V} = -s^T Ks + 2e^T\Lambda K\dot{e}. \tag{31}$$

Using $s = \dot{e} + \Lambda e$ yields

$$\dot{V} = -e^T \Lambda K \Lambda e - \dot{e}^T K \dot{e} \leq 0. \tag{32}$$

The last inequality implies that $V(t)$ is decreasing, and, since it is a positive function, we can conclude that $V(t)$ is bounded, i.e., $V \in L_\infty$. Hence, all the signals of V are bounded, that is, $e, \dot{e}, s \in L_\infty$.

Based on Assumption 1, we can prove from (23) and (24) that the signals $X, \dot{X}, \dot{X}_r, \ddot{X}_r$ are all bounded, i.e., $X, \dot{X}, \dot{X}_r, \ddot{X}_r \in L_\infty$. In addition, due to the boundedness of the robot parameters (Assumption 2) and external force (Assumption 3), it is easy to prove that the control law (26) is also bounded. That is to say, all the signals in the closed loop system are bounded. Furthermore, from the closed loop dynamics (27) and Assumption 2, we can conclude that $\dot{s} \in L_\infty$, hence it follows that $\dot{e}, \ddot{e} \in L_\infty$ and, consequently, $\ddot{V} \in L_\infty$. Finally, using the fact that $\ddot{V}$ is bounded, we can apply Barbalat's lemma to conclude that $\dot{V} \to 0$ as $t \to \infty$, which implies that the tracking errors $e, \dot{e} \to 0$ as $t \to \infty$. The proof is complete. □

4.2 Trajectory tracking controller design based on force estimation

In practical applications, the use of a force sensor to detect external force may have some drawbacks, such as its high costs, its sensitivity to noise and shock, the mounting problem, etc. To overcome these difficulties, an adaptive method to estimate the external force without the need of force sensor is proposed in this subsection.

Under the assumption that the dynamics of the mobile manipulator are known and an external force is applied to the end-effector, consider the following control law:

$$\tau_x = B_x \tau = M_x \ddot{X}_r + C_x \dot{X}_r + G_x + K s - E_x \hat{F}, \tag{33}$$

where $\hat{F}$ is the estimated vector of the external force.

The adaptation law for the external force is given by

$$\dot{\hat{F}} = -K_f^{-T} E_x^T s. \tag{34}$$

Substituting (33) in (22), the closed-loop dynamics is obtained as

$$M_x \dot{s} + (C_x + K)s + E_x \tilde{F} = 0, \tag{35}$$

where $\tilde{F} = F - \hat{F}$ is the vector of the external force error, which is the difference between the real and the estimated values.

The stability of the closed-loop system (35) is given by the following theorem.

Theorem 2 *Consider the mechanical system described by* (19), (20), *and* (22) *under Assumptions* 1, 2 *and* 3. *Assume that the dynamic parameters are known and an external unknown force is applied to the end-effector. Using the control law* (33) *and the adaptation law for the external force* (34) *and with appropriate values of* Λ, K *and* K_f, *the following results are guaranteed*:

(1) *The tracking errors* e *and* $\dot{e}$ *converge to zero as* $t \to \infty$.
(2) *All the signals in the closed-loop system are bounded for all* $t \geq 0$.
(3) *Moreover, if all columns of* $E_x(q)$ *are independent, then the force estimation error* $\tilde{F}$ *converges to zero as* $t \to \infty$.

Proof Consider the Lyapunov candidate function

$$V = \frac{1}{2} s^T M_x s + e^T \Lambda K e + \frac{1}{2} \tilde{F}^T K_f \tilde{F}^T. \tag{36}$$

The time derivative of V can be obtained as

$$\dot{V} = s^T \left(M_x \dot{s} + \frac{1}{2} \dot{M}_x s \right) + 2e^T \Lambda K \dot{e} + \dot{\tilde{F}}^T K_f \tilde{F}. \tag{37}$$

Substituting the closed-loop dynamic (35) in the above equation, we have

$$\dot{V} = s^T \left(-C_x s - K s - E_x \tilde{F} + \frac{1}{2} \dot{M}_x s \right) + 2e^T \Lambda K \dot{e} + \dot{\tilde{F}}^T K_f \tilde{F}. \tag{38}$$

Applying the skew-symmetricity Property 4, we have

$$\dot{V} = -s^T K s + 2e^T \Lambda K \dot{e} + \left(\dot{\tilde{F}}^T K_f - s^T E_x \right) \tilde{F}. \tag{39}$$

Using the adaptation law for the external force (34) and under Assumption 3, we have

$$\dot{V} = -s^T K s + 2e^T \Lambda K \dot{e}. \tag{40}$$

Using $s = \dot{e} + \Lambda e$ yields

$$\dot{V} = -e^T \Lambda K \Lambda e - \dot{e}^T K \dot{e}. \tag{41}$$

The last inequality implies that $V(t)$ is decreasing, and, since it is a positive function, we can conclude that $V(t)$ is bounded, i.e., $V \in L_\infty$. Hence, all the signals of V are bounded, that is, $e, \dot{e}, s, \tilde{F} \in L_\infty$. Based on Assumption 1, we can prove from (23) and (24) that the signals $X, \dot{X}, \dot{X}_r, \ddot{X}_r$ are all bounded, i.e., $X, \dot{X}, \dot{X}_r, \ddot{X}_r \in L_\infty$. In addition, due to the boundedness of the external force F (Assumption 3), we can obtain that $\hat{F}$ and its first derivative $\dot{\hat{F}}$ are also bounded, i.e., $\hat{F}, \dot{\hat{F}} \in L_\infty$. Now, Assumption 2 can be utilized to show that the control law (33) with the adaptation law (34) are also bounded. That is to say, all the signals in the closed-loop system are bounded. Furthermore, from the closed-loop dynamics (31) and Assumption 2, we can conclude that $\dot{s} \in L_\infty$, so it follows that $\dot{e}, \ddot{e} \in L_\infty$ and, consequently, $\ddot{V} \in L_\infty$. If we use the fact that $\ddot{V}$ is bounded, we can apply Barbalat's lemma to conclude that $\dot{V} \to 0$ as $t \to \infty$, which implies that the tracking errors $e, \dot{e} \to 0$ as $t \to \infty$. Finally, from (35), we have $E_x \tilde{F} = 0$, and, if all columns of E_x are independent, then $\tilde{F} = 0$. This concludes the proof. □

4.3 Trajectory tracking controller design based on force and parameter estimation

In developing the above control law, the parameters of the robot dynamics are supposed to be known precisely. However, in practice, the robot dynamic model is subjected to various uncertainties previously described in Remark 3. To solve the problem of unknown parameters, adaptive schemes are investigated to deal with unknown and/or time-varying parameters.

Consider $\hat{M}_x, \hat{C}_x$ and $\hat{G}_x$, the estimated values of M_x, C_x and G_x, respectively. Hence, the control law in (33) and (34) is modified to be

$$\tau_x = B_x \tau = \hat{M}_x \ddot{X}_r + \hat{C}_x \dot{X}_r + \hat{G}_x + K s - E_x \hat{F}. \tag{42}$$

Using the regression Property 5, the above control law is given by

$$\tau_x = B_x\tau = Y(q,\dot{q},\dot{X}_r,\ddot{X}_r)\hat{\boldsymbol{\Psi}} + Ks - E_x\hat{F}, \tag{43}$$

where $\hat{\boldsymbol{\Psi}}$ is the estimated vector of the parameter uncertainties.

The adaptation law for the external force is given by

$$\dot{\hat{F}} = -K_f^{-T}E_x^T s. \tag{44}$$

The adaptation law for the uncertainties is given by

$$\dot{\hat{\boldsymbol{\Psi}}} = K_u^{-T}Y^T s. \tag{45}$$

Substituting (43) in (22), the closed-loop dynamics is obtained as

$$M_x\dot{s} + (C_x + K)s + E_x\tilde{F} = Y(q,\dot{q},\dot{X}_r,\ddot{X}_r)\tilde{\boldsymbol{\Psi}}, \tag{46}$$

where $\tilde{\boldsymbol{\Psi}} = \boldsymbol{\Psi} - \hat{\boldsymbol{\Psi}}$ is the dynamic uncertainties error, which is the difference between the estimated and the real values. The stability of the closed-loop system (46) is given by the following theorem.

Theorem 3 *Consider the mechanical system described by* (19), (20), *and* (22) *under Assumptions* 1, 2 *and* 3. *Assume that the dynamic parameters are unknown and/or time-varying and an external unknown force is applied to the end-effector. Using the control law* (43), *the adaptation law for the external force* (44) *and the adaptation law for parameter uncertainties* (45) *and with appropriate values of* Λ, K, K_f *and* K_u, *the following results are guaranteed*:

(1) *The tracking errors* e *and* $\dot{e}$ *converge to zero as* $t \to \infty$.
(2) *All the signals in the closed-loop system are bounded for all* $t \geq 0$.
(3) *Moreover, if all columns of* $Z(q,\dot{q},\dot{X}_r,\ddot{X}_r) = [-E_x(q)\ \ Y(q,\dot{q},\dot{X}_r,\ddot{X}_r)]$ *are independent, then the force and parameter estimation errors converge to zero as* $t \to \infty$.

Proof Consider the Lyapunov candidate function

$$V = \frac{1}{2}s^T M_x s + e^T\Lambda K e + \frac{1}{2}\tilde{F}^T K_f\tilde{F} + \frac{1}{2}\tilde{\boldsymbol{\Psi}}^T K_u\tilde{\boldsymbol{\Psi}}. \tag{47}$$

The time derivative of V can be obtained as

$$\dot{V} = s^T\left(M_x\dot{s} + \frac{1}{2}\dot{M}_x s\right) + 2e^T\Lambda K\dot{e} + \dot{\tilde{F}}^T K_f\tilde{F} + \dot{\tilde{\boldsymbol{\Psi}}}^T K_u\tilde{\boldsymbol{\Psi}}. \tag{48}$$

Substituting the closed-loop dynamic (46) in the above equation, we have

$$\dot{V} = s^T\left(Y\tilde{\boldsymbol{\Psi}} - E_x\tilde{F} - Ks - C_x s + \frac{1}{2}\dot{M}_x s\right) + 2e^T\Lambda K\dot{e} + \dot{\tilde{F}}^T K_f\tilde{F} + \dot{\tilde{\boldsymbol{\Psi}}}^T K_u\tilde{\boldsymbol{\Psi}}. \tag{49}$$

Applying the skew-symmetricity Property 4, we have

$$\dot{V} = -s^T Ks + 2e^T\Lambda K\dot{e} + \left(\dot{\tilde{F}}^T K_f - s^T E_x\right)\tilde{F} + \left(\dot{\tilde{\boldsymbol{\Psi}}}^T K_u + s^T Y\right)\tilde{\boldsymbol{\Psi}}. \tag{50}$$

Using the adaptation laws (44) and (45), it follows under Assumption 3 that

$$\dot{V} = -s^T K s + 2e^T \Lambda K \dot{e}. \tag{51}$$

Using $s = \dot{e} + \Lambda e$ yields

$$\dot{V} = -e^T \Lambda K \Lambda e - \dot{e}^T K \dot{e}. \tag{52}$$

The last inequality implies that $V(t)$ is decreasing, and, since it is a positive function, we can conclude that $V(t)$ is bounded, i.e., $V \in L_\infty$. Hence, all the signals of V are bounded, that is, $e, \dot{e}, s, \tilde{F}, \tilde{\boldsymbol{\Psi}} \in L_\infty$. Based on Assumption 1, we can prove from (23) and (24) that the signals $X, \dot{X}, \dot{X}_r, \ddot{X}_r$ are all bounded, i.e., $X, \dot{X}, \dot{X}_r, \ddot{X}_r \in L_\infty$. In addition, due to the boundedness of the external force F (Assumption 3), we can obtain that $\hat{F}$ and its first derivative $\dot{\hat{F}}$ are also bounded, i.e., $\hat{F}, \dot{\hat{F}} \in L_\infty$. In the same way, due to Assumption 4, we can obtain that $\hat{\boldsymbol{\Psi}}$ and its first derivative $\dot{\hat{\boldsymbol{\Psi}}}$ are also bounded, i.e., $\hat{\boldsymbol{\Psi}}, \dot{\hat{\boldsymbol{\Psi}}} \in L_\infty$. Now, Assumptions 2 and 4 can be utilized to show that the control law (43) with the adaptation laws (44) and (45) are also bounded. That is to say, all the signals in the closed-loop system are bounded. Furthermore, from the closed loop dynamics (46) and Assumptions 2 and 4, we can conclude that $\dot{s} \in L_\infty$, so it follows that $\dot{e}, \ddot{e} \in L_\infty$ and, consequently, $\ddot{V} \in L_\infty$. If we use the fact that $\ddot{V}$ is bounded, we can apply Barbalat's lemma to conclude that $\dot{V} \to 0$ as $t \to \infty$, which implies that the tracking errors $e, \dot{e} \to 0$ as $t \to \infty$. Finally, from (46), we have $Y(q, \dot{q}, \dot{X}_r, \ddot{X}_r)\tilde{\boldsymbol{\Psi}} - E_x \tilde{F} = 0$ or, equivalently, $[-E_x(q) \;\; Y(q, \dot{q}, \dot{X}_r, \ddot{X}_r)]\left[\begin{smallmatrix} \tilde{F} \\ \tilde{\boldsymbol{\Psi}} \end{smallmatrix}\right] = 0$, if all columns of $Z(q, \dot{q}, \dot{X}_r, \ddot{X}_r) = [-E_x(q) \;\; Y(q, \dot{q}, \dot{X}_r, \ddot{X}_r)]$ are independent, and then $\tilde{F} = 0$ and $\tilde{\boldsymbol{\Psi}} = 0$. This completes the proof. □

5 Simulation results

To validate the theoretical concepts and demonstrate the performance of the proposed controllers given by Theorems 1, 2 and 3, a comprehensive simulation study is carried out using a redundant mobile manipulator ($\rho = 4$) operating in a two-dimensional task space ($s = 2$) as shown in Fig. 2. It is composed of a 2-degree of freedom planar manipulator mounted on a type (2; 0) wheeled mobile platform. The physical parameters for the system can be found in the Appendix.

The simulation was conducted using MATLAB software, and three experiments are presented: (i) to show the applicability of the extended approach for solving the problem of redundancy, (ii) to illustrate the effectiveness of the adaptive approach for the estimation and the rejection of external disturbance force and uncertainties. In all numerical simulations, we consider a sinusoidal trajectory as the main task for the end-effector and a straight line trajectory as the additional task of the mobile platform. The resulting extended desired task is given by

$$X_d(t) = \begin{bmatrix} r_d(t) \\ y_d(t) \end{bmatrix} = \begin{bmatrix} 0.2t + 0.3 \\ 0.5 + 0.25\sin(0.2\pi t) \\ 0.2t \\ 0 \end{bmatrix}.$$

Let $q(t) = [x_c \;\; y_c \;\; \phi \;\; \theta_r \;\; \theta_l \;\; \theta_1 \;\; \theta_2]^T$ be the vector of the generalized coordinates of the mobile manipulator and $\dot{\eta}(t) = [\dot{\theta}_r \;\; \dot{\theta}_l \;\; \dot{\theta}_1 \;\; \dot{\theta}_2]^T$ the vector of the joint velocities. We set the initial configuration as $q(0) = [0 \;\; 0 \;\; \pi/2 \;\; 0 \;\; 0 \;\; \pi/4 \;\; -\pi/4]^T$ and the initial joint

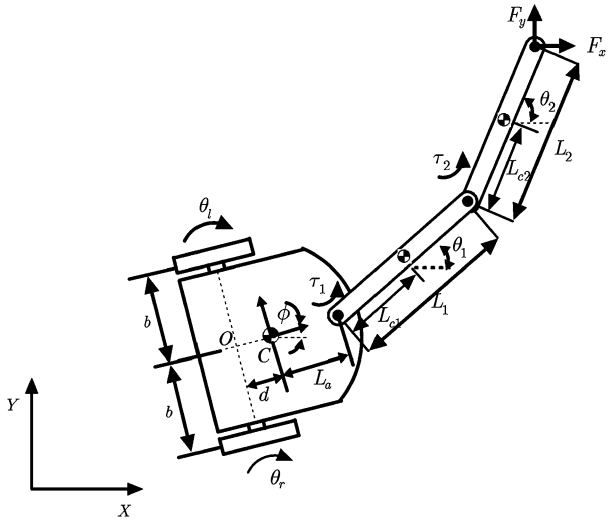

Fig. 2 A 2-DOF planar mobile manipulator

velocities as $\dot{\eta}(t) = [0 \ \ 0 \ \ 0 \ \ 0]^T$. The initial values of the estimated force and parameters are all set to zero. The gains are selected as $\Lambda = 2I_{4\times4}$, $K = 100I_{4\times4}$, $K_f = 5 \cdot 10^{-3} I_{2\times2}$ and $K_u = 2I_{8\times8}$. All the simulations are conducted under the same initial conditions, control gains and environmental constraints.

In order to show the necessity of using the force information in the control law, we present a comparative study between the trajectory tracking controller without force compensation and the trajectory tracking controller with force measurement. A constant external force disturbance $F = [50 \ \ 20]^T$ is adopted and the simulation results are shown in Fig. 3. It is clear that the mobile manipulator has failed to track the desired trajectories through the trajectory tracking controller without force compensation (Fig. 3-a1) and a large tracking errors appear due to the external force disturbance (Figs. 3-b1 and 3-c1). In contrast, under the trajectory tracking controller with force measurement, the convergence to the desired trajectories can be easily obtained (Fig. 3-a2) and the significant tracking errors for both position and velocity are successfully attenuated (Figs. 3-b2 and 3-c2). Hence, it is concluded that the force information feedback is essential to attain good control performance. However, as we described in the previous section, the use of force sensor may have some drawbacks (high costs, sensitivity to noise and shock, etc.). So, we use the trajectory tracking controller with force estimation given in Theorem 2. This controller can estimate the unknown external force using only joints encoders' information, which removes the need for force sensor. The simulation results in the presence of the constant external force disturbance are shown in Fig. 4. It can be seen that the good tracking performances are obtained with the proposed adaptive controller despite the fact that the value of the applied force is not available. This is due to the efficiency of the estimation mechanism, which guarantees the convergence of the estimated force to the real one, within a small transient phase (Fig. 4). The estimated value is then included in the control law to ensure the rejection of the disturbing force and avoid its undesirable impact, which allows the mobile manipulator to track the desired trajectories accurately. This result proves that a measured force provided generally by a force sensor is not needed in the control law. However, the adaptive algorithm can estimate this force by using only position and velocity measurements, thus the good tracking performance is maintained.

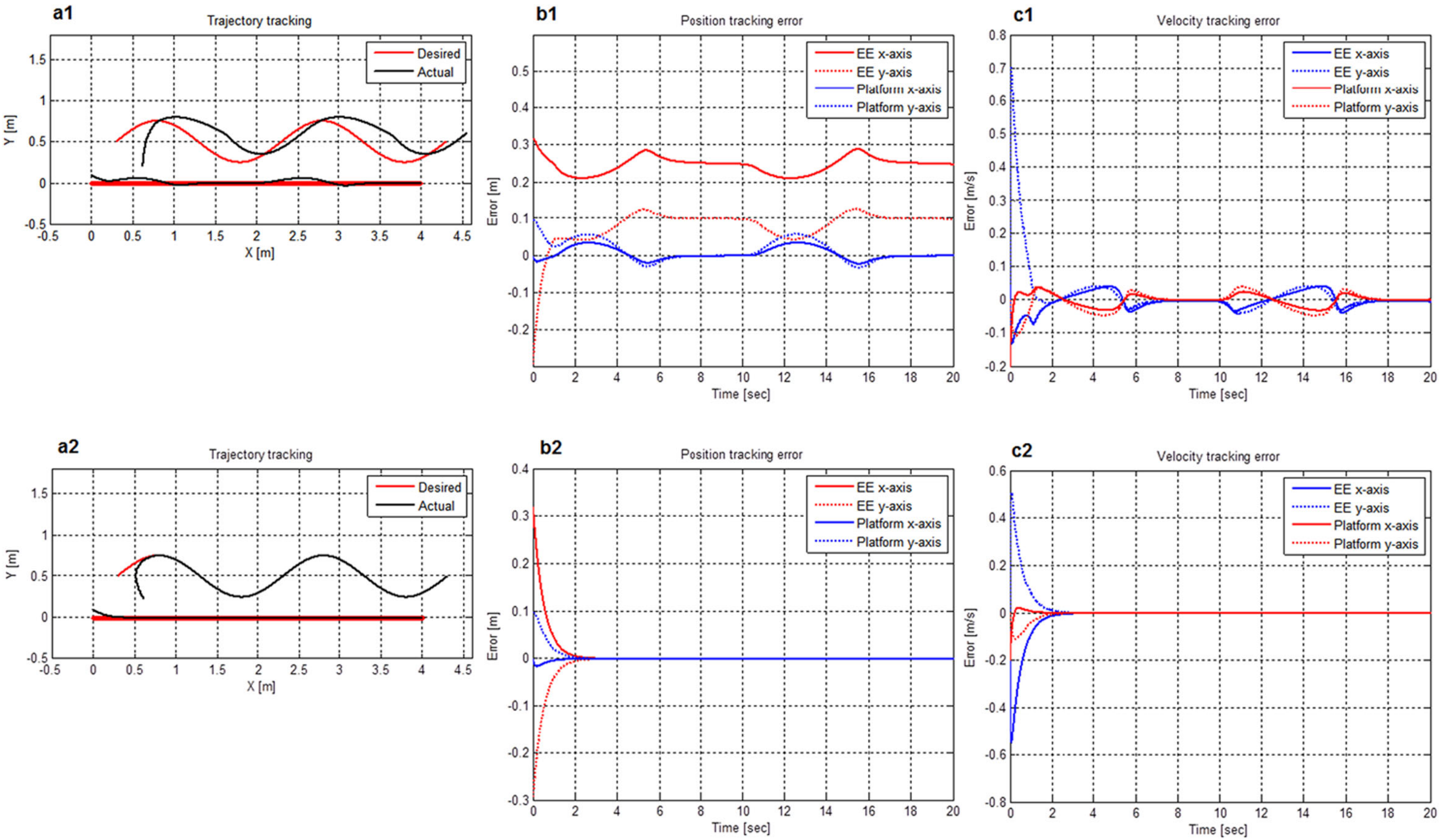

Fig. 3 Comparison of performance tracking under a constant external force disturbance: without force compensation (**a1**, **b1** and **c1**) and with force measurement (**a2**, **b2** and **c2**)

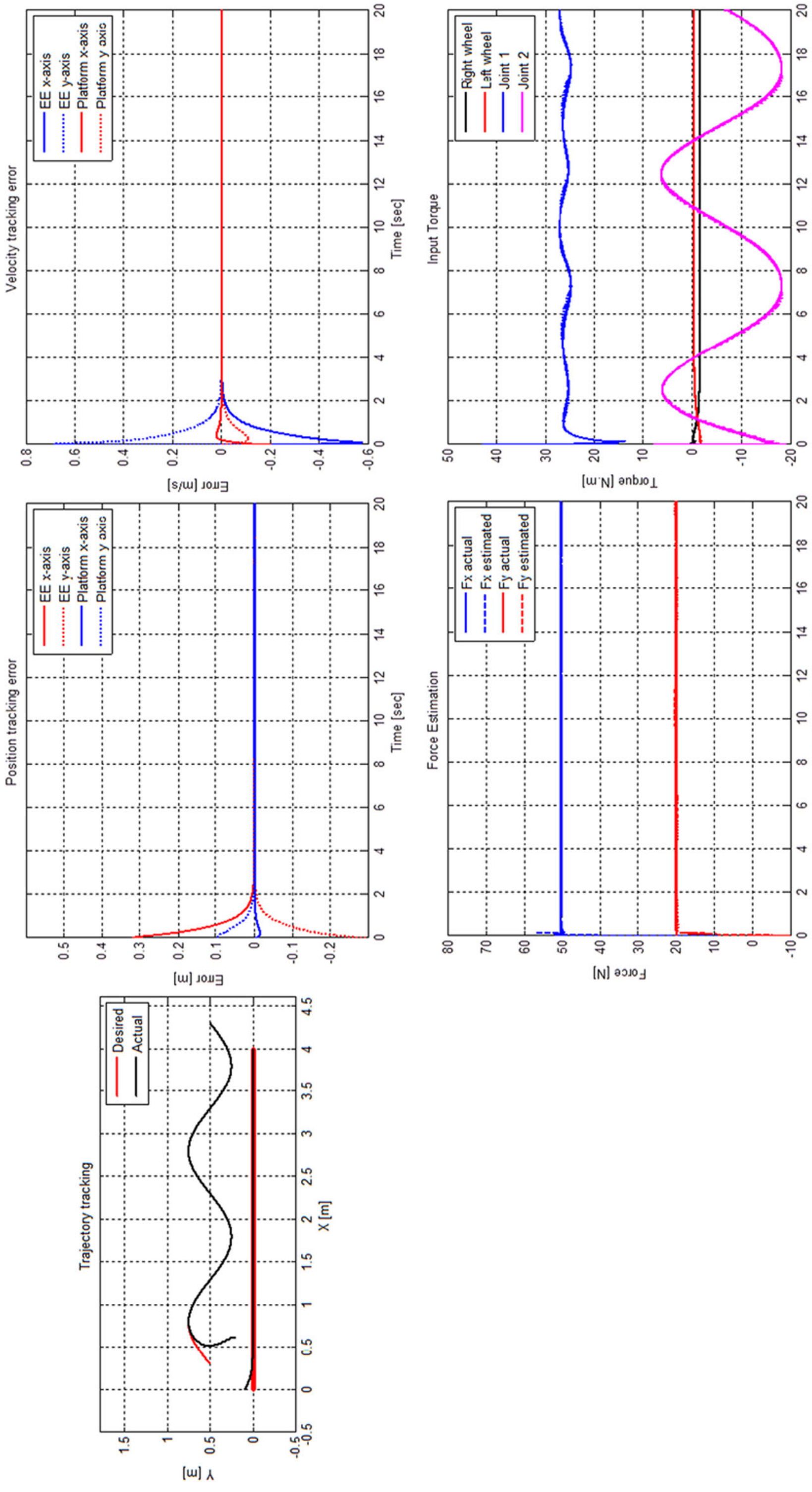

Fig. 4 Performances of the trajectory tracking controller based on force estimation under a constant external force disturbance

To further illustrate the robustness of the adaptive approach in the presence of various external force disturbances, we consider a sinusoidal external force and a square external force. As seen in Figs. 5 and 6 and under any type of time-varying applied force, the online adaptive scheme is still able to estimate the unknown external force very quickly and with a small error. Moreover, we can observe that the control input is changed properly to achieve better estimation. Hence, the disturbance is well rejected and the mobile manipulator system continues to perform remarkably well, which validates the effectiveness of the proposed adaptive trajectory tracking controller with force estimation.

However, the above mentioned adaptive law is derived under the assumption of the perfect knowledge of the system parameters, which is difficult to have in practical situations. Therefore, it is modified to estimate not only the unknown external force, but also the unknown dynamic parameters. Hence, we conduct the simulation with the trajectory tracking controller with force and parameter estimation given in Theorem 3. The tracking performances for various external force disturbances are illustrated in Figs. 7, 8, 9. It can be seen that the proposed controller provides a reasonable tracking capability even if the dynamic parameters of the system are completely unknown. In such case, the impact force is well estimated and suppressed, and the robot still follows desired task trajectories. The trajectory tracking errors are small enough, and all the unknown parameters converge to fixed values after a little fluctuation at the beginning of motion. It is also worth noting that the input torques generated by the adaptive controller using force estimation are very similar to the ones generated by the adaptive controller with force and parameter estimation. Therefore, we can affirm that the latter control scheme is insensitive to parameter variation since it was learned online during the task execution.

6 Conclusion

In this paper, we have presented adaptive control strategies for the estimation and rejection of external force disturbance acting on the end-effector of a nonholonomic mobile manipulator. Adaptive update law is employed to estimate the unmeasured external force and then utilized in the control law to reject it and maintain a good tracking performance. Unlike the other sensorless methods that need an accurate model of the robot, the knowledge of the robot parameters is not necessary in this work. All the signals of the system remain bounded even when an excessive external force is applied or/and the parameters of the robot are completely unknown. The stability of the proposed controller is proved by Lyapunov theory and the good performance can be concluded from the obtained results.

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Appendix

The kinematic and the dynamic parameters of the robot are listed in Tables 1 and 2, respectively [7].

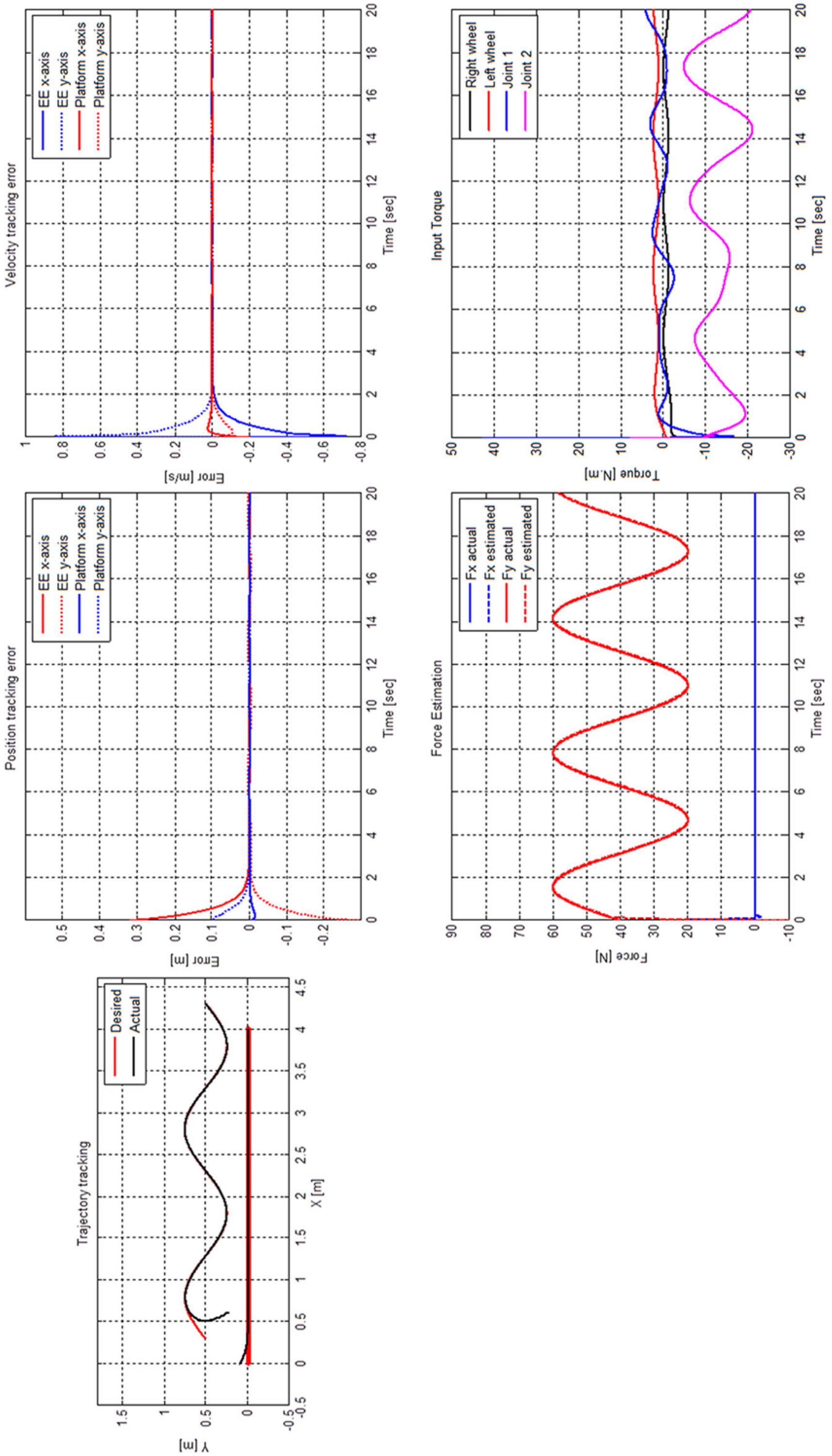

Fig. 5 Performances of the trajectory tracking controller based on force estimation under a sinusoidal external force disturbance

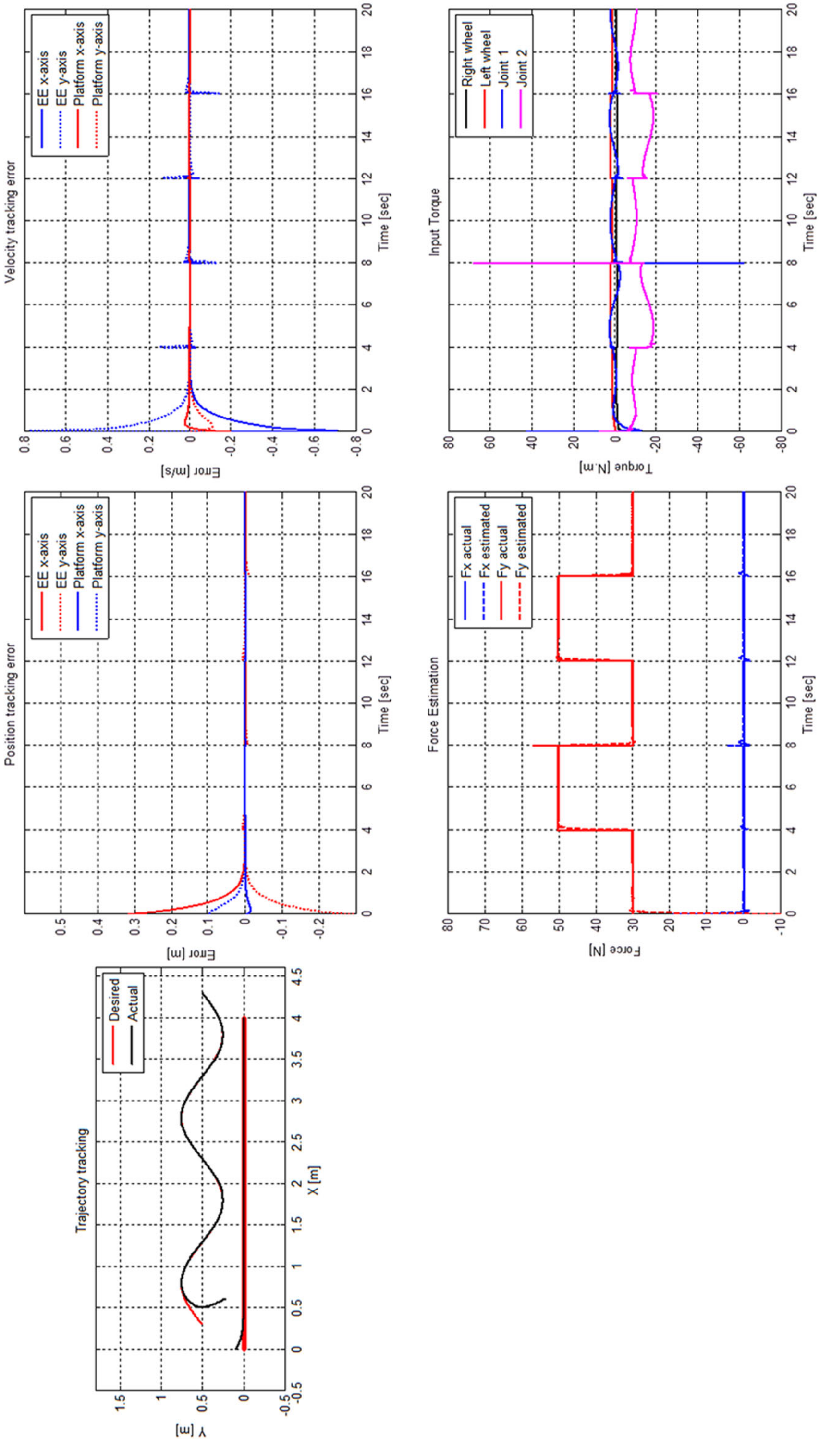

Fig. 6 Performances of the trajectory tracking controller based on force estimation under a square external force disturbance

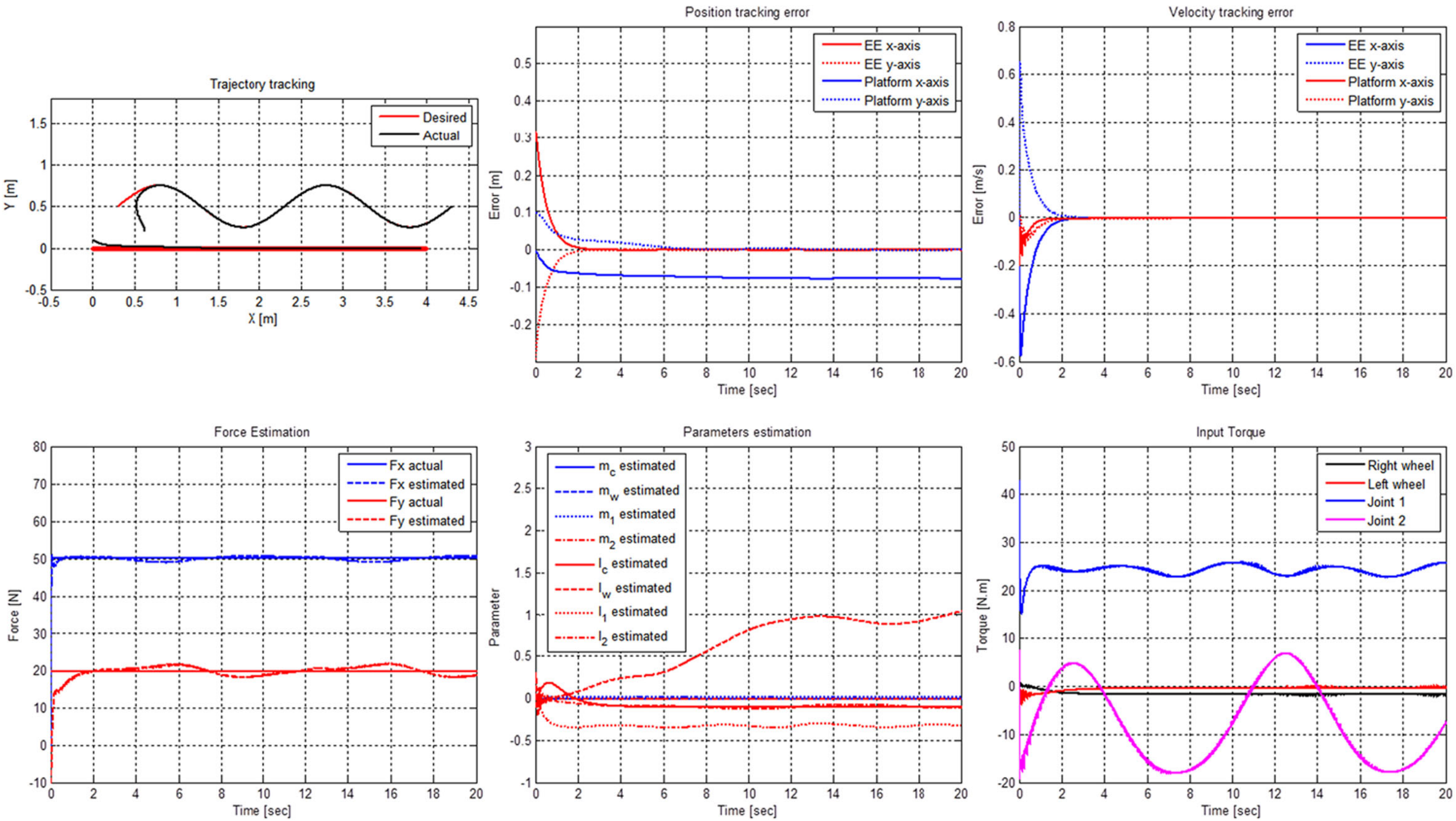

Fig. 7 Performances of the trajectory tracking controller based on force and parameter estimation under a constant external force disturbance

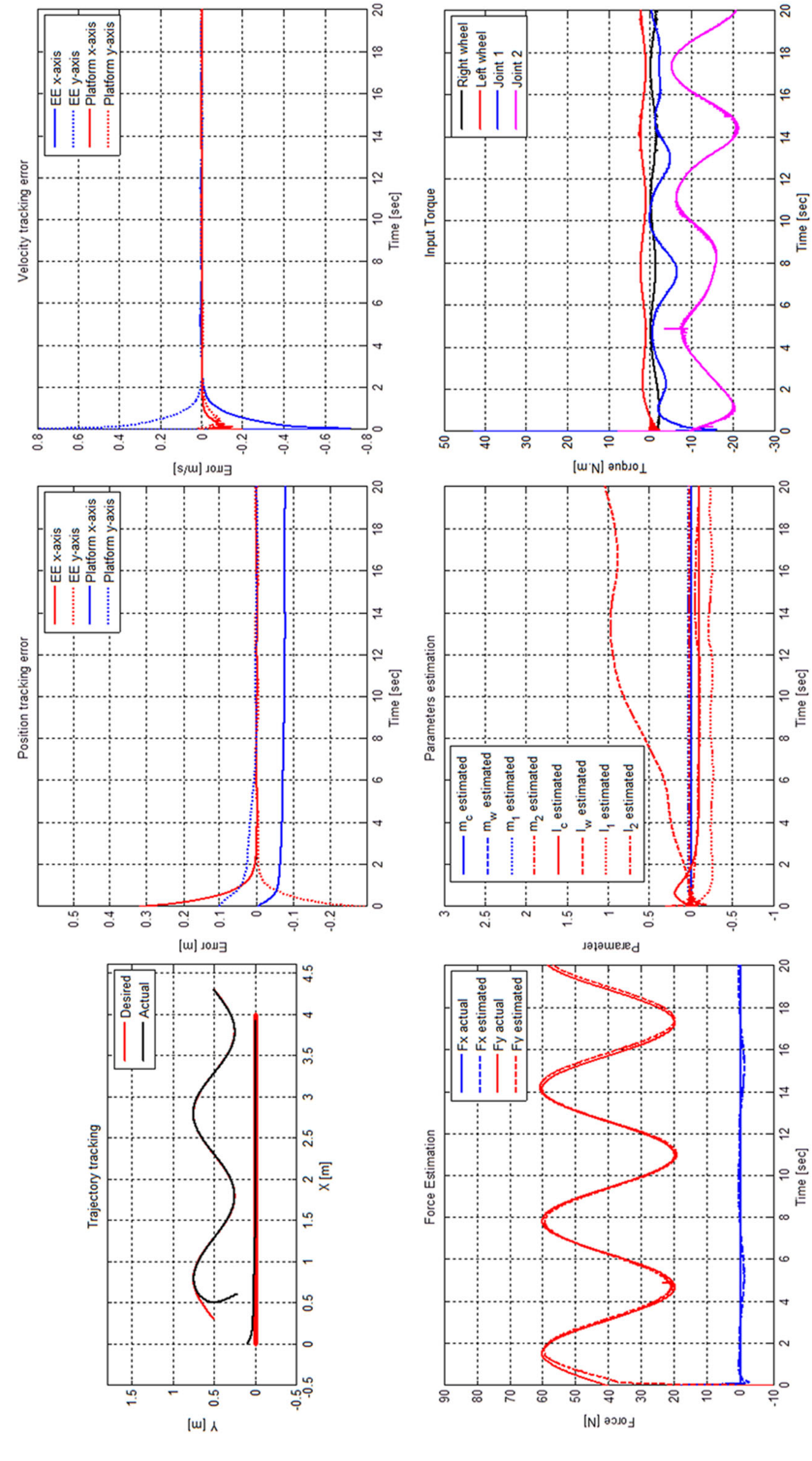

Fig. 8 Performances of the trajectory tracking controller based on force and parameter estimation under a sinusoidal external force disturbance

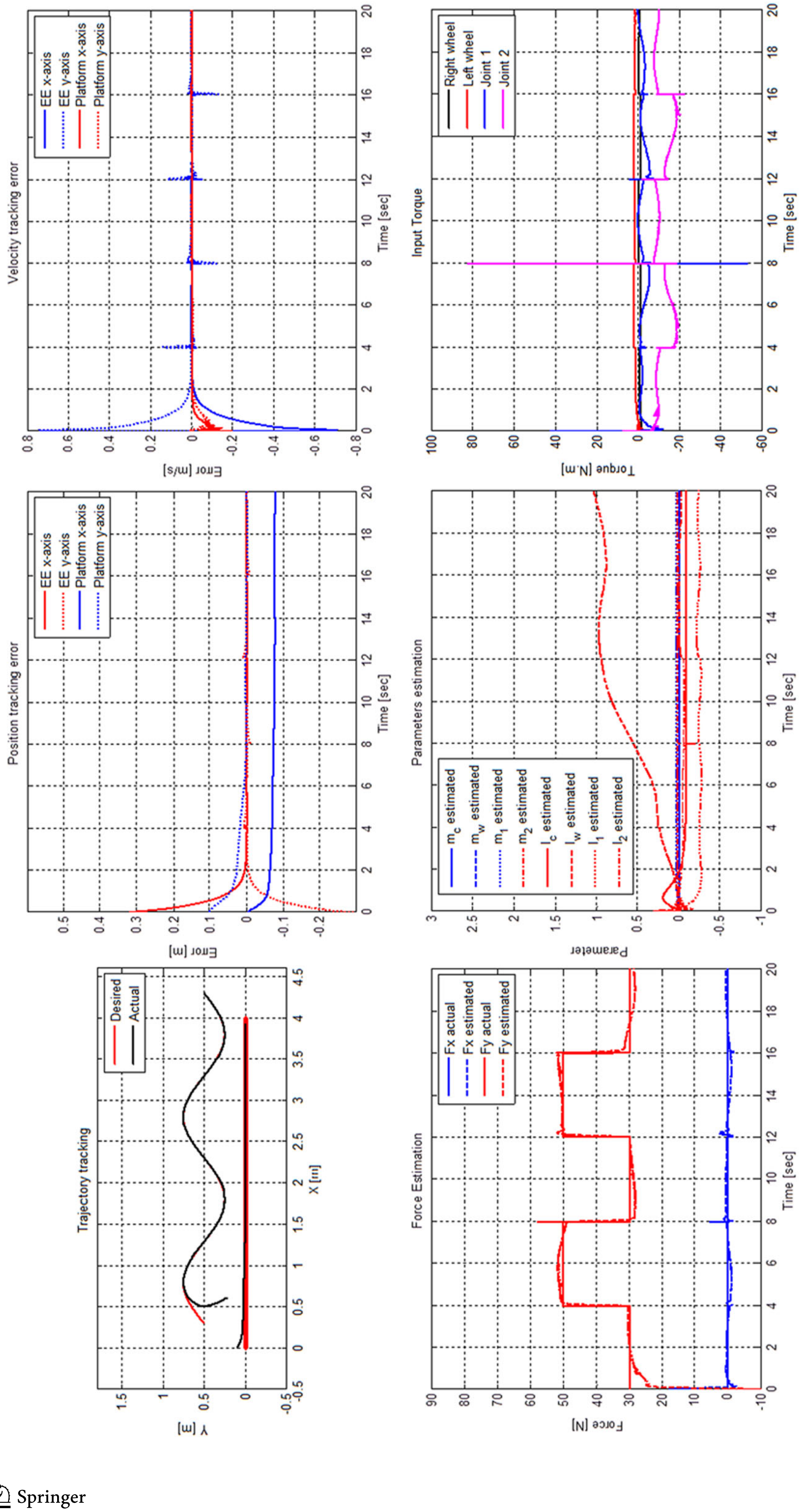

Fig. 9 Performances of the trajectory tracking controller based on force and parameter estimation under a square external force disturbance

Table 1 Kinematic parameters of the mobile manipulator

Parameter	Description	Value
r	Wheel radius	0.0508 m
$2b$	Distance between the two driving wheels	0.364 m
d	Distance from the wheel axis to the center of mass	0.116 m
L_1	Length of the link 1	0.514 m
L_2	Length of the link 2	0.362 m
L_a	Position of the base of the onboard manipulator	0.1 m
L_{c1}	Position of the center of mass of link 1	0.252 m
L_{c2}	Position of the center of mass of link 1	0.243 m

Table 2 Dynamic parameters of the mobile manipulator

Parameter	Description	Value
m_w	Wheel mass	0.159 kg
m_c	Mobile platform mass	17.25 kg
m_1	Link 1 mass	2.56 kg
m_2	Link 2 mass	1.07 kg
I_w	Wheel inertia about the wheel axis	0.0002 kg m^2
I_c	Mobile platform inertia about the center of mass	0.297 kg m^2
I_1	Link 1 inertia about the center of mass	0.148 kg m^2
I_2	Link 2 inertia about the center of mass	0.0228 kg m^2

The constraint matrix is given by

$$A(q) = \begin{bmatrix} -\sin(\phi) & \cos(\phi) & -d & 0 & 0 & 0 & 0 \\ -\cos(\phi) & -\sin(\phi) & -b & r & 0 & 0 & 0 \\ -\cos(\phi) & -\sin(\phi) & b & 0 & r & 0 & 0 \end{bmatrix}.$$

Let $S(q)$ be a basis of the null space of $A(q)$ given by

$$S(q) = \begin{bmatrix} \frac{r}{2b}(b\cos(\phi) - d\sin(\phi)) & \frac{r}{2b}(b\cos(\phi) + d\sin(\phi)) & 0 & 0 \\ \frac{r}{2b}(b\sin(\phi) + d\cos(\phi)) & \frac{r}{2b}(b\sin(\phi) - d\cos(\phi)) & 0 & 0 \\ \frac{r}{2b} & -\frac{r}{2b} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The end-effector position is considered as the main task, and the additional task is the mobile platform position. Then, the resulting extended task vector X is given by

$$X = h(q) = \begin{bmatrix} x_c + L_a\cos(\phi) + L_1\cos(\phi+\theta_1) + L_2\cos(\phi+\theta_1+\theta_2) \\ y_c + L_a\sin(\phi) + L_1\sin(\phi+\theta_1) + L_2\sin(\phi+\theta_1+\theta_2) \\ x_c + L_a\cos(\phi) \\ y_c + L_a\sin(\phi) \end{bmatrix}.$$

The extended Jacobian matrix is derived as

$$J_e(q) = \begin{bmatrix} c(b\cos(\phi) - (d+L_a)\sin(\phi)) & c(b\cos(\phi) + (d+L_a)\sin(\phi)) \\ c(b\sin(\phi) + (d+L_a)\cos(\phi)) & c(b\sin(\phi) - (d+L_a)\cos(\phi)) \\ c(b\cos(\phi) - (d+L_a)\sin(\phi)) & c(b\cos(\phi) + (d+L_a)\sin(\phi)) \\ c(b\sin(\phi) + (d+L_a)\cos(\phi)) & c(b\sin(\phi) - (d+L_a)\cos(\phi)) \end{bmatrix.$$
$$\left.\begin{matrix} -L_1\sin(\theta_1) & -L_2\sin(\theta_2) \\ L_1\cos(\theta_1) & L_2\cos(\theta_2) \\ 0 & 0 \\ 0 & 0 \end{matrix}\right].$$

Using the Lagrangian approach, we can obtain the standard form (13) with

$$M(q) = \begin{bmatrix} p_1 & 0 & -p_2\sin(\phi) & 0 & 0 & -p_3\sin(\theta_1) & -p_4\sin(\theta_2) \\ 0 & p_1 & p_2\cos(\phi) & 0 & 0 & p_3\cos(\theta_1) & p_4\cos(\theta_2) \\ -p_2\sin(\phi) & p_2\cos(\phi) & p_5 & 0 & 0 & p_8 & p_9 \\ 0 & 0 & 0 & p_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_7 & 0 & 0 \\ -p_3\sin(\theta_1) & p_3\cos(\theta_1) & p_8 & 0 & 0 & p_6 & p_{11} \\ -p_4\sin(\theta_2) & p_4\cos(\theta_2) & p_9 & 0 & 0 & p_{11} & p_{10} \end{bmatrix},$$

where

$$p_1 = m_c + m_1 + m_2; \qquad p_2 = L_a(m_1+m_2); \qquad p_3 = m_1L_{c1} + m_2L_1;$$
$$p_4 = m_2L_{c2}; \qquad p_5 = L_a^2(m_1+m_2) + I_c; \qquad p_6 = m_1L_{c1}^2 + m_2L_1^2 + I_1;$$
$$p_7 = m_\omega r^2 + I_\omega; \qquad p_8 = L_a(m_1L_{c1} + m_2L_1)\cos(-\phi+\theta_1);$$
$$p_9 = m_2L_aL_{c2}\cos(-\phi+\theta_2); \qquad p_{10} = m_2L_{c2}^2 + I_2;$$
$$p_{11} = m_2L_1L_{c2}\cos(\theta_1-\theta_2);$$
$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & c_1 & 0 & 0 & c_2 & c_3 \\ 0 & 0 & c_4 & 0 & 0 & c_5 & c_6 \\ 0 & 0 & 0 & 0 & 0 & c_7 & c_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_9 & 0 & 0 & 0 & c_{10} \\ 0 & 0 & c_{11} & 0 & 0 & c_{12} & 0 \end{bmatrix},$$

where

$$c_1 = -L_a(m_1+m_2)\cos(\phi)\dot{\phi}; \qquad c_2 = -(m_1L_{c1} + m_2L_1)\cos(\theta_1)\theta_1;$$
$$c_3 = -m_2L_{c2}\cos(\theta_2)\theta_2; \qquad c_4 = -L_a(m_1+m_2)\sin(\phi)\dot{\phi};$$
$$c_5 = -(m_1L_{c1} + m_2L_1)\sin(\theta_1)\theta_1; \qquad c_6 = -m_2L_{c2}\sin(\theta_2)\theta_2;$$
$$c_7 = -L_a(m_1L_{c1} + m_2L_1)\sin(-\phi+\theta_1)\theta_1; \qquad c_8 = -m_2L_aL_{c2}\sin(-\phi+\theta_2)\theta_2;$$
$$c_9 = L_a(m_1L_{c1} + m_2L_1)\sin(-\phi+\theta_1)\dot{\phi}; \qquad c_{10} = m_2L_aL_{c2}\sin(\theta_1-\theta_2)\theta_2;$$
$$c_{11} = m_2L_aL_{c2}\sin(-\phi+\theta_2)\dot{\phi}; \qquad c_{12} = -m_2L_1L_{c2}\sin(\theta_1-\theta_2)\theta_1;$$

$$G(q) = [0]_{5\times 1};$$

$$E(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -L_a\sin(\phi) - L_1\sin(\phi+\theta_1) - L_2\sin(\phi+\theta_1+\theta_2) & L_a\cos(\phi) + L_1\cos(\phi+\theta_1) + L_2\cos(\phi+\theta_1+\theta_2) \\ 0 & 0 \\ 0 & 0 \\ -L_1\sin(\phi+\theta_1) - L_2\sin(\phi+\theta_1+\theta_2) & L_1\cos(\phi+\theta_1) + L_2\cos(\phi+\theta_1+\theta_2) \\ -L_2\sin(\phi+\theta_1+\theta_2) & L_2\cos(\phi+\theta_1+\theta_2) \end{bmatrix};$$

$$B(q) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad F = \begin{bmatrix} F_x \\ F_y \end{bmatrix}, \qquad \tau = \begin{bmatrix} \tau_r \\ \tau_l \\ \tau_1 \\ \tau_2 \end{bmatrix}.$$

The unknown parameters of the mobile manipulator system are the mass and the moment of inertia. Hence, the vector of the unknown parameters is given by

$$\boldsymbol{\Psi} = \begin{bmatrix} m_c & m_\omega & m_1 & m_2 & I_c & I_\omega & I_1 & I_2 \end{bmatrix}^T.$$

The terms of the regressor matrix in Property 6 are given as [24]

$$Y_{ij} = \frac{\partial D_i(q,\dot{q},\dot{X}_r,\ddot{X}_r)}{\partial \boldsymbol{\Psi}_j}; \quad 1 \le i \le 4 \text{ and } 1 \le j \le 8,$$

where

$$D_i(q,\dot{q},\dot{X}_r,\ddot{X}_r) = \sum_{k=1}^{4} M_{x_{ik}}\ddot{X}_{r_k} + C_{x_{ik}}\dot{X}_{r_k} + G_{x_i}; \quad 1 \le i \le 4.$$

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