Proceedings of the American Control Conference
Philadelphia, Pennsylvania • June 1998

Singularity-Free Regulation of Underwater Vehicle-Manipulator Systems

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Abstract—**In this paper a sliding mode approach for the regulation problem of Underwater Vehicle-Manipulator Systems (UVMS) is developed. Based on the body-fixed and joint-space coordinates dynamic model, a control law is derived which avoids the inversion of the system Jacobian, thus overcoming the occurrence of kinematic singularities. Further, to avoid representation singularities of the orientation, attitude control of the vehicle is achieved through a quaternion based error. A Lyapunov-like analysis is used to prove the error convergence. Finally, numerical simulation shows the performance of the control scheme in a practical case study involving a full-dimensional task.**

1. Introduction

Several authors have dealt with coordinated control of Underwater Vehicle-Manipulator Systems (UVMS) which is a particularly challenging problem being these systems highly nonlinear and coupled.

In [1] an adaptive macro-micro control law is proposed based on the end-effector and vehicle position and attitude errors; however, the end-effector desired trajectory should be carefully planned in order to avoid kinematic singularities. In addition, the parameter adaptation law requires inversion of the system Jacobian. In [2] a feedback linearization approach is proposed following derivation of a detailed dynamic model in matrix form including the most important hydrodynamic terms. On the other hand, in the case of underwater systems, dynamic parameters are poorly known and time-varying depending on the environmental conditions. In [3] a discrete adaptive control strategy has been proposed; this has been applied in simulation on the surge motion of a vehicle carrying a three-link planar manipulator. Finally, [4] describes several interesting experimental results showing the interaction between an underwater vehicle and a single-link manipulator and the effectiveness of dynamic compensation actions.

The vehicle attitude control problem has been addressed among the others in the paper [5] which extends the work in [6] and [7] to obtain a singularity-free tracking control of an underwater vehicle based on the use of the unit quaternion.

Inspired by the work in [5], a new control law is proposed in this paper for the regulation problem of a UVMS. To overcome the occurrence of kinematic singularities, the control law is expressed in body-fixed and joint-space coordinates so as to avoid inversion of the system Jacobian. Further, to avoid representation singularities of the orientation, attitude control of the vehicle is achieved through a quaternion based error. The resulting control law is very simple and requires limited computational effort. A Lyapunov-like analysis is used to prove the error convergence. Finally, numerical simulation shows the performance of the control scheme in a practical case study involving a full-dimensional system, i.e. a 6 DOF vehicle plus the manipulator arm.

2. Dynamic modelling

The equations of motion of a UVMS, schematically represented in Figure 1, can be written in body-fixed reference frame in the form [2]:

$$M(q)\dot{\zeta} + C(q,\zeta)\zeta + D(q,\zeta)\zeta + g(q,\eta_2) = \tau, \qquad (1)$$

where $\zeta = [\nu_1^T \, \nu_2^T \, \dot{q}^T]^T$, ν_1 is the (3×1) vector of vehicle linear velocity expressed in the body-fixed reference frame, ν_2 is the (3×1) vector of vehicle angular velocity expressed in the body-fixed reference frame, q is the $(n \times 1)$ vector of joint positions being n the number of joints, $\eta = [\eta_1^T \, \eta_2^T]^T$, $\eta_1 = [x, y, z]^T$ is the (3×1) vector of vehicle position coordinates in a earth-fixed reference frame, $\eta_2 = [\phi, \theta, \psi]^T$ is the (3×1) vector of vehicle Euler-angle coordinates in a earth-fixed reference frame, $M(q)$ is the $((6+n) \times (6+n))$ mass matrix, $C(q,\zeta)\zeta$ is the $((6+n) \times 1)$ vector of Coriolis and Centripetal terms, $D(q,\zeta)\zeta$ is the $((6+n) \times 1)$ vector of friction and hydrodynamic damping terms (e.g., Drag, Lift, and Vortex Shedding generalized forces), $g(q,\eta_2)$ is the $((6+n) \times 1)$ vector of gravitational and buoyant generalized forces, τ is the $((6+n) \times 1)$ vector of forces and moments acting on the vehicle as well as of the joint torques.

Thrusters and control surfaces provide forces and moments on the vehicle according to a nonlinear relation. A simplified relationship can be expressed through the linear mapping [8]

$$\tau = Bu, \qquad (2)$$

where B is a $((6+n) \times p)$ matrix, and u is the $(p \times 1)$ vector of control inputs. In the remainder we will assume $p \geq (6+n)$ and B being full-rank.

It must be noted that vehicle attitude dependence is present in the body-fixed dynamic model (1) only through the term $g(q,\eta_2)$. Therefore, to obtain a dynamic model free of representation singularities it is necessary to replace the Euler-angle description of orientation in η_2 with a suitable attitude representation (e.g., Euler parameters as in [9]).

It can be proven that:

- the inertia matrix of the total system is positive definite and symmetric, i.e. $M(q) = M^T(q) > O$.
- the damping matrix is positive definite, i.e. $D(q,\zeta) > O$.
- the matrix $H = \dot{M}(q) - 2C(q,\zeta)$ is skew-symmetric, i.e. $\zeta^T H \zeta = 0$.

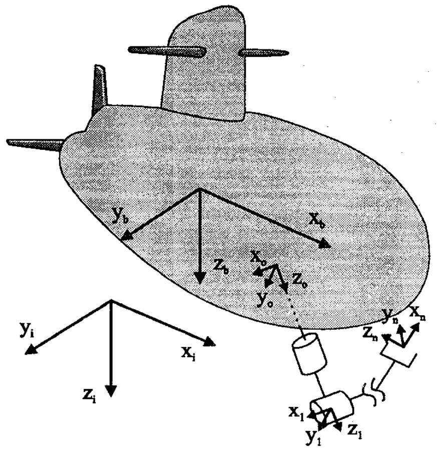

Figure 1 — Underwater Vehicle-Manipulator System.

3. Attitude Error Representation

By defining the mutual orientation between two frames of common origin in terms of the rotation matrix

$$\boldsymbol{R}_{\boldsymbol{r}}(\delta) = \cos\delta \boldsymbol{I} + (1 - \cos\delta)\boldsymbol{r}\boldsymbol{r}^T - \sin\delta \boldsymbol{S}(\boldsymbol{r}), \qquad (3)$$

where δ is the angle and $\boldsymbol{r}$ is the (3×1) unit vector of the axis expressing the rotation needed to align the two frames, $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{S}(\cdot)$ is the matrix operator performing the cross product between two (3×1) vectors, we can define the unit quaternion as

$$\mathcal{Q} = \{\eta, \boldsymbol{\varepsilon}\}, \qquad \eta = \cos\frac{\delta}{2}, \qquad \boldsymbol{\varepsilon} = \sin\frac{\delta}{2}\boldsymbol{r}, \qquad (4)$$

where $\eta \geq 0$ for $\delta \in [-\pi, \pi]$. This restriction is necessary for the uniqueness of the quaternion associated to a given matrix, in that the two quaternions $\{\eta, \boldsymbol{\varepsilon}\}$ and $\{-\eta, -\boldsymbol{\varepsilon}\}$ represent the same orientation, i.e. the same rotation matrix.

The unit quaternion satisfies the condition

$$\eta^2 + \boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon} = 1. \qquad (5)$$

Let now define ${}^I\boldsymbol{R}_B$ as the (3×3) rotation matrix from the body-fixed frame to the earth-fixed frame, which is also described by the quaternion $\mathcal{Q}$, and ${}^I\boldsymbol{R}_d$ the (3×3) rotation matrix from the frame expressing the desired vehicle orientation to the earth-fixed frame, which is also described by the quaternion $\mathcal{Q}_d = \{\eta_d, \boldsymbol{\varepsilon}_d\}$. One possible choice for the rotation matrix necessary to align the two frames is

$$\widetilde{\boldsymbol{R}} = {}^B\boldsymbol{R}_I\,{}^I\boldsymbol{R}_d, \qquad (6)$$

where ${}^B\boldsymbol{R}_I = {}^I\boldsymbol{R}_B^T$. The quaternion $\widetilde{\mathcal{Q}} = \{\tilde{\eta}, \tilde{\boldsymbol{\varepsilon}}\}$ associated with $\widetilde{\boldsymbol{R}}$ is easily obtained by

$$\begin{aligned} \tilde{\eta} &= \eta\eta_d + \boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}_d \\ \tilde{\boldsymbol{\varepsilon}} &= \eta\boldsymbol{\varepsilon}_d - \eta_d\boldsymbol{\varepsilon} + \boldsymbol{S}(\boldsymbol{\varepsilon}_d)\boldsymbol{\varepsilon} \end{aligned} \qquad (7)$$

Since the quaternion associated with $\widetilde{\boldsymbol{R}} = \boldsymbol{I}$ (i.e. representing a null orientation error) is $\widetilde{\mathcal{Q}} = \{1, \boldsymbol{0}\}$, it is sufficient to represent the attitude error as

$$\boldsymbol{e}_o = \tilde{\boldsymbol{\varepsilon}}. \qquad (8)$$

Finally, it is necessary to consider the *quaternion propagation equation*

$$\begin{aligned} \dot{\tilde{\eta}} &= -\frac{1}{2}\tilde{\boldsymbol{\varepsilon}}^T\tilde{\boldsymbol{\nu}}_2 \\ \dot{\tilde{\boldsymbol{\varepsilon}}} &= \frac{1}{2}\tilde{\eta}\tilde{\boldsymbol{\nu}}_2 + \frac{1}{2}\boldsymbol{S}(\tilde{\boldsymbol{\varepsilon}})\tilde{\boldsymbol{\nu}}_2, \end{aligned} \qquad (9)$$

where $\tilde{\boldsymbol{\nu}}_2 = \boldsymbol{\nu}_{2,d} - \boldsymbol{\nu}_2$ is the angular velocity error expressed in body-fixed frame.

4. Proposed Control Law

The proposed control law is

$$\boldsymbol{u} = \boldsymbol{B}^{\dagger}[\boldsymbol{K}_D\boldsymbol{s} + \hat{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{\eta}_2) + \boldsymbol{K}\,sign(\boldsymbol{s})], \qquad (10)$$

where $\boldsymbol{B}^{\dagger}$ is the pseudoinverse of matrix $\boldsymbol{B}$, $\boldsymbol{K}_D$ is a positive definite matrix of gains, $\hat{\boldsymbol{g}}(\boldsymbol{q}, \boldsymbol{\eta}_2)$ is the estimate of gravitational and buoyant forces, $\boldsymbol{K}$ is a positive definite matrix, and $sign(\boldsymbol{x})$ is the vector function whose i-th component is

$$sign(\boldsymbol{x})_i = \begin{cases} 1 & \text{if } x_i \geq 0 \\ -1 & \text{if } x_i < 0. \end{cases} \qquad (11)$$

In (10), $\boldsymbol{s}$ is the $\big((6+n) \times 1\big)$ sliding manifold defined as follows

$$\boldsymbol{s} = \boldsymbol{\Lambda}\begin{bmatrix} {}^B\boldsymbol{R}_I\tilde{\boldsymbol{\eta}}_1 \\ \boldsymbol{e}_o \\ \tilde{\boldsymbol{q}} \end{bmatrix} - \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \\ \dot{\boldsymbol{q}} \end{bmatrix} = \boldsymbol{y} - \boldsymbol{\zeta}, \qquad (12)$$

with $\boldsymbol{\Lambda} > \boldsymbol{O}$, $\tilde{\boldsymbol{\eta}}_1 = [x_d - x, y_d - y, z_d - z]^T$, $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ where the subscript d denotes desired values for the relevant variables.

5. Stability of the Control Law

In this section we will show that the proposed control law is asymptotically stable in a Lyapunov sense. We consider the function

$$V = \frac{1}{2}\boldsymbol{s}^T\boldsymbol{M}(\boldsymbol{q})\boldsymbol{s}, \qquad (13)$$

that is positive definite being $\boldsymbol{M}(\boldsymbol{q}) > \boldsymbol{O}$.

Differentiating V with respect to time yields

$$\dot{V} = \frac{1}{2}\boldsymbol{s}^T\dot{\boldsymbol{M}}\boldsymbol{s} + \boldsymbol{s}^T\boldsymbol{M}\dot{\boldsymbol{s}} \qquad (14)$$

that, taking into account (1), (12) and the skew-symmetry of $\boldsymbol{H}$, can be rewritten as

$$\dot{V} = -\boldsymbol{s}^T\boldsymbol{D}\boldsymbol{s} + \boldsymbol{s}^T[\boldsymbol{M}\dot{\boldsymbol{y}} - \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}\boldsymbol{y} + \boldsymbol{D}\boldsymbol{y} + \boldsymbol{g}]. \quad (15)$$

Plugging (10) into (15) gives

$$\begin{aligned} \dot{V} = &-\boldsymbol{s}^T(\boldsymbol{D} + \boldsymbol{K}_D)\boldsymbol{s} \\ &+ \boldsymbol{s}^T[\boldsymbol{M}\dot{\boldsymbol{y}} + (\boldsymbol{C} + \boldsymbol{D})\boldsymbol{y} + \tilde{\boldsymbol{g}} - \boldsymbol{K}\,sign(\boldsymbol{s})] \end{aligned} \qquad (16)$$

that, in view of positive definiteness of $\boldsymbol{K}_D$ and $\boldsymbol{D}$, can be upper bounded as follows

$$\begin{aligned}\dot{V} \leq & -\lambda_{min}(\boldsymbol{K}_D+\boldsymbol{D})\|\boldsymbol{s}\|^2 - \lambda_{min}(\boldsymbol{K})\|\boldsymbol{s}\| \\ & + \|\boldsymbol{M}\dot{\boldsymbol{y}} + (\boldsymbol{C}+\boldsymbol{D})\boldsymbol{y} + \tilde{\boldsymbol{g}}\|\|\boldsymbol{s}\|,\end{aligned} \tag{17}$$

where λ_{min} denotes the smallest eigenvalue of the corresponding matrix.

By choosing $\boldsymbol{K}$ such that

$$\lambda_{min}(\boldsymbol{K}) \geq \|\boldsymbol{M}\dot{\boldsymbol{y}} + (\boldsymbol{C}+\boldsymbol{D})\boldsymbol{y} + \tilde{\boldsymbol{g}}\|, \tag{18}$$

the time derivative of V is negative definite and thus $\boldsymbol{s}$ tends to zero asymptotically.

If an estimate of the dynamic parameters in (1) is available, we might consider the control law

$$\boldsymbol{u} = \boldsymbol{B}^{\dagger}[\boldsymbol{K}_D\boldsymbol{s} + \hat{\boldsymbol{g}} + \hat{\boldsymbol{M}}\dot{\boldsymbol{y}} + (\hat{\boldsymbol{C}}+\hat{\boldsymbol{D}})\boldsymbol{y} + \boldsymbol{K}sign(\boldsymbol{s})] \tag{19}$$

in lieu of (10). Starting from the function in (13) and plugging (19) in (15) gives

$$\begin{aligned}\dot{V} = & -\boldsymbol{s}^T(\boldsymbol{D}+\boldsymbol{K}_D)\boldsymbol{s} \\ & + \boldsymbol{s}^T[\widetilde{\boldsymbol{M}}\dot{\boldsymbol{y}} + (\widetilde{\boldsymbol{C}}+\widetilde{\boldsymbol{D}})\boldsymbol{y} + \tilde{\boldsymbol{g}} - \boldsymbol{K}sign(\boldsymbol{s})]\end{aligned} \tag{20}$$

that, in view of positive definiteness of $\boldsymbol{K}_D$ and $\boldsymbol{D}$, leads to negative definiteness of $\dot{V}$ if

$$\lambda_{min}(\boldsymbol{K}) \geq \|\widetilde{\boldsymbol{M}}\dot{\boldsymbol{y}} + (\widetilde{\boldsymbol{C}}+\widetilde{\boldsymbol{D}})\boldsymbol{y} + \tilde{\boldsymbol{g}}\|. \tag{21}$$

It is worth noting that condition (21) is weaker than condition (18) in that the matrix $\boldsymbol{K}$ must overcome the sole model parameters mismatching.

6. Stability of the sliding manifold

In the previous section we have demonstrated that the proposed control law guarantees convergence of $\boldsymbol{s}$ to the sliding manifold $\boldsymbol{s} = \boldsymbol{0}$. In this section we will show that, once the sliding manifold has been reached, the error vectors $\tilde{\boldsymbol{\eta}}_1$, $\boldsymbol{e}_o$, $\tilde{\boldsymbol{q}}$ converge asymptotically to the origin, i.e. that regulation of the system variables to their desired values is achieved.

By taking $\boldsymbol{\Lambda} = blockdiag\{\boldsymbol{\Lambda}_p, \boldsymbol{\Lambda}_o, \boldsymbol{\Lambda}_q\}$ where $\boldsymbol{\Lambda}_p$ is a (3×3) matrix, $\boldsymbol{\Lambda}_o$ is a (3×3) matrix, $\boldsymbol{\Lambda}_q$ is a $(n\times n)$ matrix, it is possible to *decouple* the stability analysis in 3 parts as follows.

6.1. Vehicle position error dynamics

The vehicle position error dynamics on the sliding manifold is described by the equation

$$-\boldsymbol{\nu}_1 + \boldsymbol{\Lambda}_p{}^B\boldsymbol{R}_I\tilde{\boldsymbol{\eta}}_1 = \boldsymbol{0}. \tag{22}$$

Notice that the rotation matrix ${}^B\boldsymbol{R}_I$ is a function of the vehicle orientation.

By considering $V = \frac{1}{2}\tilde{\boldsymbol{\eta}}_1^T\tilde{\boldsymbol{\eta}}_1$ as Lyapunov function candidate and observing that $\dot{\boldsymbol{\eta}}_1 = {}^I\boldsymbol{R}_B\boldsymbol{\nu}_1$, it is easily obtained

$$\dot{V} = -\tilde{\boldsymbol{\eta}}_1^{T\,I}\boldsymbol{R}_B\boldsymbol{\Lambda}_p{}^B\boldsymbol{R}_I\tilde{\boldsymbol{\eta}}_1, \tag{23}$$

that is negative definite for $\boldsymbol{\Lambda}_p > \boldsymbol{O}$. Hence, $\tilde{\boldsymbol{\eta}}_1$ converges asymptotically to the origin.

6.2. Vehicle orientation error dynamics

The vehicle orientation error dynamics on the sliding manifold is described by the equation

$$-\boldsymbol{\nu}_2 + \boldsymbol{\Lambda}_o\boldsymbol{e}_o = \boldsymbol{0}. \tag{24}$$

Further, taking into account (9) with $\boldsymbol{\nu}_{2,d} = \boldsymbol{0}$ and (24), it can be recognized that

$$\dot{\tilde{\eta}} = \frac{1}{2}\boldsymbol{e}_o^T\boldsymbol{\Lambda}_o\boldsymbol{e}_o. \tag{25}$$

Let consider the Lyapunov function candidate

$$V = \boldsymbol{e}_o^T\boldsymbol{e}_o. \tag{26}$$

The time derivative of V is:

$$\dot{V} = 2\boldsymbol{e}_o^T\dot{\boldsymbol{e}}_o = -\boldsymbol{e}_o^T\tilde{\eta}\boldsymbol{\nu}_2 - \boldsymbol{e}_o^T\boldsymbol{S}(\boldsymbol{e}_o)\boldsymbol{\nu}_2, \tag{27}$$

where we have used (9) with $\boldsymbol{\nu}_{2,d} = \boldsymbol{0}$. Plugging (24) into (27) and taking $\boldsymbol{\Lambda}_o = \lambda_o\boldsymbol{I}_{3\times 3}$ with $\lambda_o > 0$, gives

$$\dot{V} = -\tilde{\eta}\lambda_o\boldsymbol{e}_o^T\boldsymbol{e}_o. \tag{28}$$

which is negative semidefinite with $\tilde{\eta} \geq 0$. It must be noted that, in view of (25), $\tilde{\eta}$ is a not-decreasing function of time and thus it stays positive when starting from a positive initial value.

The set R of all points $\boldsymbol{e}_o$ where $\dot{V} = 0$ is given by

$$R = \{\boldsymbol{e}_o = \boldsymbol{0}, \quad \boldsymbol{e}_o : \tilde{\eta} = 0\}; \tag{29}$$

from (5), however, it can be recognized that

$$\tilde{\eta} = 0 \quad \Rightarrow \quad \|\boldsymbol{e}_o\| = 1 \tag{30}$$

and thus $\dot{\tilde{\eta}} > 0$ in view of (25). Therefore, the largest invariant set in R is

$$M = \{\boldsymbol{e}_o = \boldsymbol{0}\} \tag{31}$$

and the invariant set theorem ensures asymptotic convergence to the origin.

6.3. Manipulator joint error dynamics

The manipulator joint error dynamics on the sliding manifold is described by the equation

$$-\dot{\boldsymbol{q}} + \boldsymbol{\Lambda}_q\tilde{\boldsymbol{q}} = \boldsymbol{0} \tag{32}$$

whose convergence to $\tilde{\boldsymbol{q}} = \boldsymbol{0}$ is evident taking $\boldsymbol{\Lambda}_q > \boldsymbol{O}$.

7. Simulations

Numerical simulations have been performed in order to show the effectiveness of the proposed control law. The UVMS simulator, developed in MATLAB 4.2, SIMULINK 1.3 environment, is described in [10]. Modularity of the software allows the user to define the number of links and the structure of the manipulator arm (if any) as well as to change system and environmental parameters.

The vehicle data are taken from [11]; they refer to the experimental Autonomous Underwater Vehicle NPS AUV II. In this paper a two-link manipulator with rotational joints has been considered which is mounted under the vehicle body with the joint axes parallel to the fore-aft direction; since the vehicle inertia along that axis is minimum, this choice increases dynamic coupling between the vehicle and the manipulator. The length of each link is 1 m, the center of gravity is coincident with the center of buoyancy and it is supposed to be in the geometrical center of the link; each link is not neutrally buoyant. Links are cylindrical, thus hydrodynamic effects can be computed by simplified relations as in [2]. Dry and viscous joint friction is also taken into account.

As for the control law, we have considered implementation of (10); however, it is well known that the *sign* function would lead to chattering in the system. Practical implementation of (10), therefore, requires replacement of the *sign* function e.g. with the *sat* function

$$\boldsymbol{u} = \boldsymbol{B}^{\dagger}[\boldsymbol{K}_D \boldsymbol{s} + \hat{g}(\boldsymbol{q}, \boldsymbol{\eta}_2) + \boldsymbol{K} sat(\boldsymbol{s}, \varepsilon)], \qquad (33)$$

where the $sat(\boldsymbol{x}, \varepsilon)$ is the vector function whose i-th component is

$$sat(\boldsymbol{x}, \varepsilon)_i = \begin{cases} 1 & \text{if } x_i > \varepsilon \\ -1 & \text{if } x_i < -\varepsilon \\ \frac{x_i}{\varepsilon} & \text{otherwhise.} \end{cases} \qquad (34)$$

Convergence to the equilibrium of the UVMS under this different control law can be easily demonstrated starting from (13) following the guidelines in [12]. In detail, it is obtained that $\dot{V} < 0$ in the region characterized by $\|\boldsymbol{s}\| \geq \varepsilon$, while the sign of $\dot{V}$ is undetermined in the boundary layer characterized by $\|\boldsymbol{s}\| < \varepsilon$. This approach is well established in sliding mode control and does not represent a practical drawback since ε can be taken sufficiently small.

In the simulation $\boldsymbol{B}$ is supposed to be the identity matrix, meaning that direct control of forces and moments acting on the vehicle and joint torques is available. The control law parameters are

$$\boldsymbol{\Lambda}_o = \boldsymbol{\Lambda}_p = diag\{0.5, 0.5, 0.5\}, \quad \boldsymbol{\Lambda}_q = diag\{3, 2\}$$

and

$$\boldsymbol{K}_D = blockdiag\{10^4 \boldsymbol{I}_{6\times 6}, 3000, 500\}$$

$$\boldsymbol{K} = 1000 \boldsymbol{I}_{8\times 8}, \qquad \varepsilon = 0.1$$

We have considered a station keeping task for the vehicle in the initial location $\boldsymbol{\eta}_i = [0\,0\,0\,0\,0\,0]^T$ [m,rad] with the manipulator in the initial configuration $\boldsymbol{q}_i = [-\frac{\pi}{4}\ \frac{\pi}{2}]^T$ [rad]. The vehicle must be then kept still, i.e. $\boldsymbol{\eta}_d = \boldsymbol{\eta}_i$, while moving the manipulator arm to the desired final configuration $\boldsymbol{q}_f = [0\,0]^T$ [rad] according to a 5th order polynomial.

It should be noted that the vehicle orientation set point is assigned in terms of Euler angles; these must be converted into the corresponding rotation matrix so as to extract the quaternion expressing the orientation error from the rotation matrix computed as in (6). Remarkably, this procedure is free of singularities.

The obtained simulation results are reported in Figure 2 to 7 in terms of the time histories of the vehicle position, the vehicle control forces, the vehicle attitude expressed by Euler angles, the vehicle moments, the manipulator joint errors, and the manipulator joint torques, respectively.

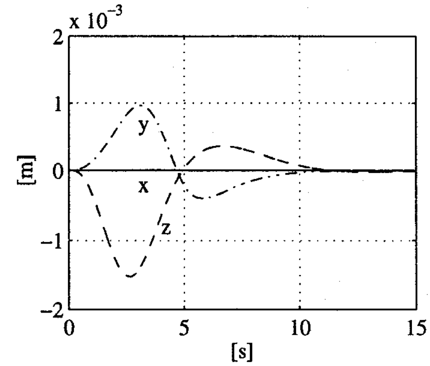

Figure 2 — Vehicle position.

Figure 2 shows that, as expected, the vehicle position is affected by the manipulator motion; however, the displacements are small and the target position is recovered after a transient.

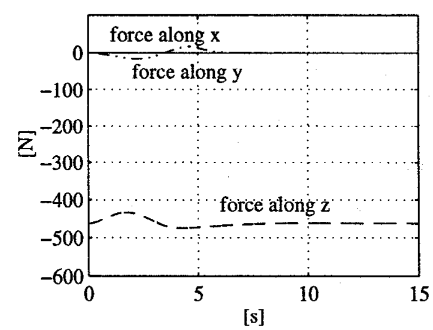

Figure 3 — Vehicle control forces.

From Figure 3, it can be recognized that at steady state the force along z is non null; this happens because the manipulator is not neutrally buoyant.

Figure 4 shows that the dynamic coupling is mostly experienced along the roll direction because of the chosen UVMS structure. This effect was intentional to test the control robustness.

From Figure 5 it can be recognized that vehicle control moments are zero at steady state; this happens because the center of gravity and the center of buoyancy of vehicle body and manipulator links are all aligned with the z-axis of the earth-fixed frame at the final system configuration.

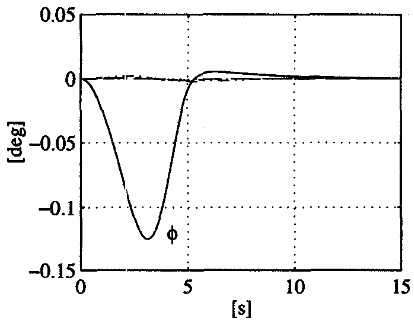

Figure 4 — Vehicle attitude.

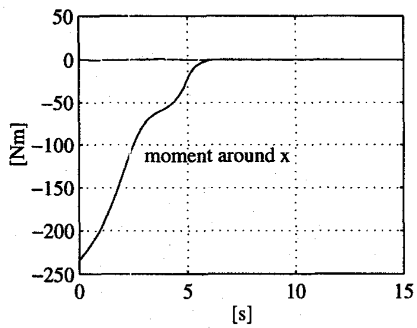

Figure 5 — Vehicle control moments.

Figure 6 and 7 show the time histories of manipulator joint errors and torques. It is worth noting that the initial value of the joint torques is non null because of gravity and buoyancy compensation, while they are null at steady state in view of the particular final system configuration. It can be recognized that control generalized forces are smooth while the task is successfully executed.

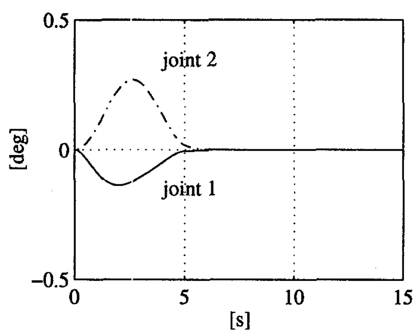

Figure 6 — Manipulator joint errors.

8. Conclusions

In this paper a control law for the regulation of Underwater Vehicle-Manipulator Systems (UVMS) has been proposed which is based on a sliding mode approach. The control law is free of representation singularities of the orientation due to the use of quaternions to realize the vehicle attitude control. In addition, since the manipulator control loop is closed on joint-space variables it is free of kinematic singularities. Two versions of the regulator have been proposed depending on the knowledge of the dynamic parameters and a stability analysis has been developed. The control technique has been finally tested in a simulation case study involving a full-dimensional system, i.e. a 6 DOF vehicle plus the manipulator arm.

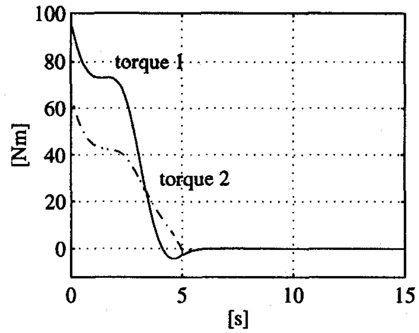

Figure 7 — Manipulator joint torques.

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