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A Novel PID Controller Gain Tuning Method for a Quadrotor Landing on a Ship Deck using the Invariant Ellipsoid Technique

Chun Kiat Tan[1]* and Jianliang Wang[2]

[1]School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore (cktan5@ntu.edu.sg) * Corresponding author
[2]School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore (ejlwang@ntu.edu.sg)

Abstract: Quadrotors are useful in many applications. It is controlled by the thrust of each rotors. The altitude and attitude control is often achieved by PID controller due to its simplicity. Many methods such as the Ziegler Nichols method, Cohen Coon method and loop shaping methods are available to tune the gain with different objectives such as minimizing overshoot or settling time. However, few techniques are able to obtain an optimal controller gain for a system under persistent bounded disturbance. The invariant ellipsoid method based on the invariant set theory is developed to formulate an optimization problem to obtain such an optimal controller gain. The technique is applied to a ship deck landing problem of a quadrotor. The heave motion of the ship deck represents a persistent disturbance acting on the quadrotor which is required to perform a landing operation while maintaining small relative speed with the ship deck. Numerical simulation is performed to demonstrate the ability of the calculated gain to reject disturbance as compared to gain obtained by loop shaping method.

Keywords: Quadrotor, ship deck landing, PID control, invariant ellipsoid, invariant set theory.

1. INTRODUCTION

The use of unmanned aerial vehicles (UAVs) has gained popularity in areas such as surveillance in recent years. Mission-specific UAV has resulted in various UAV types and designs. In particular, the multi-rotor or more commonly the quadrotor has been useful in surveillance missions. Vertical take-off and landing allows the quadrotor to carry out missions despite the lack of open space and its hovering capability allows better stabilization and imaging.

Control of the quadrotor is achieved by changing the thrust on the rotors. By collectively increasing the thrust on the rotors, the total thrust hence the altitude of the quadrotor can be controlled. The pitch and roll is controlled by differentially increasing and decreasing the thrust on oppositely located rotors. The yaw is controlled by differentially increasing the rotation speed of oppositely rotating rotors. Control mixing is used to obtain the inputs to the rotors given the conventional thrust, roll, pitch and yaw commands.

PID controllers are commonly used to control quadrotors. The main reasons for the use of PID controllers are:

- simple structure,
- easy hardware implementation,
- easily tunable even without a model,
- and good range of performance.

Because of this, the use of PID controller has been prevalent in many quadrotor applications [1–6].

The selection of PID controller gain is thus critical to the performance of the quadrotor. In most cases, the gain is tuned manually during flight tests. However, a good initial point for the gain is always desirable. Methods such as the Ziegler Nichols method [7], Cohen Coon method [8] and loop shaping methods are commonly used. These PID controller tuning algorithms are often specific to certain situations. Objectives such as minimum overshoot and settling time are often the guidelines to tune the controller gain.

However there are few PID controller tuning algorithm targeted at the rejection of persistent disturbance. In this case, the disturbance acting on the system is persistent and L_∞ bounded. For a linear system, bounded input bounded output (BIBO) stability implies a bounded response.

In this paper, we propose a PID controller tuning algorithm that minimizes the output bound for a system subjected to persistent L_∞ bounded disturbance. The invariant set theory is used to develop the invariant ellipsoid method that results in an optimization problem to minimize the output bound. The invariant ellipsoid method is based on the general theory developed by [9–11].

To demonstrate the PID controller tuning algorithm, the algorithm is applied to the problem of a quadrotor landing on a ship heaving in the sea. Ship-wave interactions result in a pseudorandom heave motion, furthermore landing operation required a low relative speed between the quadrotor and the ship so as to achieve a safe landing. This problem has been investigated by [12, 13]. A PID controller is designed to control the landing operation of the quadrotor. The ship deck heave motion acts as a persistent and L_∞ bounded disturbance on the system.

The rest of this paper is organized as follows. Section 2 gives the dynamic model of the quadrotor and section 3 describes the PID controller used for the landing oper-

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ation. The PID tuning algorithm using the invariant ellipsoid method is given in section 4. Numerical simulation using Matlab and Simulink is presented in section 5 followed by some concluding remarks in section 6.

2. QUADROTOR DYNAMIC MODEL

In this paper, the heave control of the quadrotor is of primary concern. The attitude stabilization of the quadrotor is beyond the scope of this paper. Therefore the translational dynamics of the quadrotor will be described in the absence of the rotational dynamics.

The equations of motion of the quadrotor is formulated in the inertial reference frame, the translational kinematics is

$$\dot{p}^{(i)} = v^{(i)}, \tag{1}$$

where $p^{(i)}$ and $v^{(i)}$ are position and velocity of the quadrotor in the inertial reference frame.

The translational dynamics is given by

$$m\dot{v}^{(i)} = -R_{bi}^T T\hat{e}_3 + mg\hat{e}_3, \tag{2}$$

where m is the mass of the quadrotor, g is the gravitational acceleration, $\hat{e}_3$ is the unit vector in the z axis in the body reference frame pointing downwards and R_{bi} is the rotation matrix from the body reference frame to the inertial reference frame. In this formulation, we have assumed minor forces acting on the quadrotor such as the aerodynamic forces to be insignificant.

3. CONTROLLER DESIGN

In this section, the ship deck landing control problem is formulated. This is followed by a PID controller synthesis for the altitude control that lands the quadrotor onto a heaving ship deck.

3.1 Decoupled Vertical and Planar Dynamics

The vertical and planar dynamics are decoupled. The vertical dynamics is closely related to the tracking of the ship deck heave motion and maintaining the relative position between the quadrotor and the ship deck.

The thrust vector in the inertial reference frame is

$$\begin{aligned} T^{(i)} &= -R_{bi}^T T\hat{e}_3 \\ &= -\begin{pmatrix} \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ \cos\phi\cos\theta \end{pmatrix} T, \end{aligned} \tag{3}$$

where ϕ, θ, ψ are the roll, pitch and yaw angles respectively.

The z component of Eq. (2) can be rewritten as

$$\ddot{h} = T\cos\phi\cos\theta - mg, \tag{4}$$

where $h = -Z$ is the height of the quadrotor.

Letting $h_1 = h$, $h_2 = \dot{h}_1$ and $u = T\cos\phi\cos\theta - mg$, Eq. (2) can be rewritten as

$$\dot{h}_1 = h_2, \tag{5a}$$

$$\dot{h}_2 = u. \tag{5b}$$

It is noted that $T = \frac{u+mg}{\cos\phi\cos\theta}$ is limited by mechanical bounds thus the virtual control u is also limited. In most cases, ϕ and θ is small and bounded therefore it is reasonable to assume $-mg \leq u \leq T_{max} - mg$, where T_{max} is the maximum achievable thrust.

In this problem, the quadrotor is required to track the heave motion of the ship deck (d). The tracking error is given by $e_1 = h_1 - d - H$, where $H > 0$ is the commanded height above the ship deck. The tracking error dynamics becomes

$$\dot{e}_1 = e_2, \tag{6a}$$

$$\dot{e}_2 = u - \ddot{d} - \ddot{H} = u_{pid} - \ddot{d}, \tag{6b}$$

where $u_{pid} = u - \ddot{H}$. This can be further expressed in the linear state space form as

$$\dot{e} = Ae + Bu_{pid} + Dw, \tag{7}$$

where $e = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $D = -\begin{pmatrix} 0 \\ \Delta \end{pmatrix}$ and $\Delta w(t) = \ddot{d}(t)$ for $\|w\|_\infty = 1$ and all $t \geq 0$. The L_∞ signal norm $\|x(t)\|_\infty = a$ is defined as $\{x(t) : \|x(t)\|_2 < a \text{ for all } t \geq 0\}$ and $\|x(t)\|_2$ is the Euclidean norm at time t. The parameter Δ can be estimated from the historical data of the ship deck heave motion at various sea states.

3.2 PID Controller

Eq. (7) shows the equation for a typical double integrator in the presence of external disturbance. A PID controller is proposed such that

$$u_{pid} = K_i \int e_1 dt + K_p e_1 + K_d \dot{e}_1, \tag{8}$$

where K_i, K_p and K_d are the gains of the PID controller. A schematic of the closed-loop system is shown in Fig. 1.

Letting $\tilde{e}_1 = \int e_1 dt$, $\tilde{e}_2 = e_1$ and $\tilde{e}_3 = e_2$, Eq. (7) becomes

$$\dot{\tilde{e}} = \tilde{A}\tilde{e} + \tilde{B}\tilde{u}_{pid} + \tilde{D}\tilde{w}, \tag{9}$$

where $\tilde{e} = \begin{pmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_3 \end{pmatrix}$, $\tilde{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $\tilde{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\tilde{D} = -\begin{pmatrix} 0 \\ 0 \\ \tilde{\Delta} \end{pmatrix}$, $\tilde{u}_{pid} = \dot{u}_{pid}$ and $\tilde{\Delta}\tilde{w}(t) = \Delta\dot{w}(t)$ for all $\|w\|_\infty = \|\tilde{w}\|_\infty = 1$.

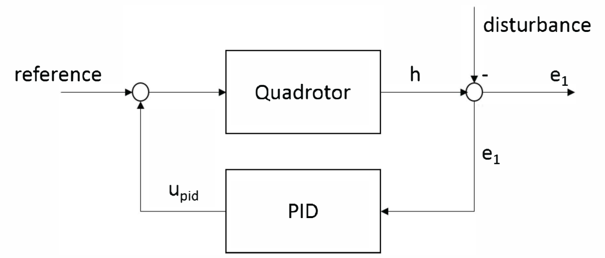

Fig. 1 A schematic of the closed-loop control system. The relative height between the quadrotor and the ship deck is feedback into the PID controller and the control inputs is augmented with the reference altitude before entering into the quadrotor to form a closed-loop.

With respect to Eq. (9), the PID controller becomes

$$\tilde{u}_{pid} = K_{pid}\tilde{e}, \tag{10}$$

where $K_{pid} = \begin{pmatrix} K_i & K_p & K_d \end{pmatrix}$ is a form of state feedback.

4. PID GAIN TUNING

The gain of the PID controller can be tuned by various algorithms. Each technique has their advantages and is most suited for the situation that it is designed for. In this paper, a PID tuning technique is proposed to optimize the controller performance using the invariant set theory.

4.1 Invariant Ellipsoid Method

The invariant set theory is used in the invariant ellipsoid method to develop the constraint equation for the optimization of the controller gain. The invariant ellipsoid method aims to obtain a minimal ellipsoid that is invariant in the presence of a L_∞ bounded disturbance.

Consider a Lyapunov function of the form

$$V = \tilde{e}^T Q \tilde{e}, \tag{11}$$

where $Q = Q^T > 0$ is to be determined. The time derivative of V is

$$\begin{aligned} \dot{V} \;&= \tilde{e}^T \left(QA + \tilde{A}^T Q + Q\tilde{B}K_{pid} + K_{pid}^T \tilde{B}^T Q \right) \tilde{e} \\ &\quad + 2\tilde{w}^T \tilde{D}^T Q \tilde{e}. \end{aligned} \tag{12}$$

For the response of the system to remain within an ellipsoid given by $V < 1$, the following condition must be satisfied, $\tilde{e}^T \left(QA + \tilde{A}^T Q + Q\tilde{B}K_{pid} + K_{pid}^T \tilde{B}^T Q \right) \tilde{e} < 0$ for all $\tilde{e}^T Q \tilde{e} > 1$ and $\tilde{w}^T \tilde{w} < 1$ at all $t \geq 0$. This ellipsoid is known as the invariant ellipsoid in the sense that once the state $\tilde{e}$ enters the ellipsoid, it will remains within the ellipsoid.

Concatenating the states as $x = \begin{pmatrix} \tilde{e} & \tilde{w} \end{pmatrix}^T$, the above conditions are written as

$$\begin{pmatrix} \tilde{A}^T Q + Q\tilde{A} + Q\tilde{B}K_{pid} + K_{pid}^T \tilde{B}^T Q & Q\tilde{D} \\ \tilde{D}^T Q & 0 \end{pmatrix} < 0, \tag{13}$$

for all

$$\begin{pmatrix} -Q & 0 \\ 0 & 0 \end{pmatrix} < -1, \tag{14}$$

and

$$\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} < 1, \tag{15}$$

where I is the identity matrix.

An execution of the S-procedure will reduce the conditions given by Eq. (13) to (15) to

$$\begin{pmatrix} \tilde{A}^T Q + Q\tilde{A} + Q\tilde{B}K_{pid} + K_{pid}^T \tilde{B}^T Q + \tau_1 Q & Q\tilde{D} \\ \tilde{D}^T Q & -\tau_2 I \end{pmatrix} < 0, \tag{16}$$

where $\tau_1 \geq \tau_2 > 0$.

Applying Schur formula to Eq. (16), we get

$$\begin{aligned} &\tilde{A}^T Q + Q\tilde{A} + Q\tilde{B}K_{pid} + K_{pid}^T \tilde{B}^T Q \\ &+\tau_1 Q + \tfrac{1}{\tau_2} Q\tilde{D}\tilde{D}^T Q < 0. \end{aligned} \tag{17}$$

Let $P = Q^{-1}$ and pre- and post-multiply Eq. (17) by P, Eq. (17) becomes

$$\begin{aligned} &P\tilde{A}^T + \tilde{A}P + \tilde{B}K_{pid}P + PK_{pid}^T \tilde{B}^T \\ &+\tau_1 P + \tfrac{1}{\tau_2} \tilde{D}\tilde{D}^T < 0. \end{aligned} \tag{18}$$

Observe that for a fixed τ_1 where P_1 and P^* are the minimal solution for $\tau_2 = \tau_2^{(1)} < \tau_1$ and $\tau_2 = \tau_2^* = \tau_1$ respectively. Since $\tau_2^{(1)} < \tau_2^*$, P_1 also satisfy $P_1\tilde{A}^T + \tilde{A}P_1 + \tilde{B}K_{pid}P_1 + P_1 K_{pid}^T \tilde{B}^T + \tau_1 P_1 + \frac{1}{\tau_2^*}\tilde{D}\tilde{D}^T < 0$. However P^* is the minimal solution, we can conclude that the minimal solution at a fixed τ_1 always occurs at $\tau_2 = \tau_1$.

As the minimal ellipsoid is of interest here, the condition $\tau_1 = \tau_2 = \alpha$ is imposed. Thus Eq. (18) becomes

$$\begin{aligned} &P\tilde{A}^T + \tilde{A}P + \tilde{B}K_{pid}P + PK_{pid}^T \tilde{B}^T \\ &+\alpha P + \tfrac{1}{\alpha} \tilde{D}\tilde{D}^T < 0. \end{aligned} \tag{19}$$

It is convenient to let $Y = K_{pid}P$ such that Eq. (19) is linear in variables P and Y.

$$P\tilde{A}^T + \tilde{A}P + \tilde{B}Y + Y^T \tilde{B}^T + \alpha P + \frac{1}{\alpha}\tilde{D}\tilde{D}^T < 0. \tag{20}$$

If Eq. (20) is satisfied, then the response will be bounded within an ellipsoid given by $\tilde{e}^T P^{-1} \tilde{e} \leq 1$.

Furthermore, it is reasonable to restrict the control action within the invariant ellipsoid. This can be done by considering $\|\tilde{u}_{pid}(t)\|_R^2 = \|K_{pid} e(t)\|_R^2 < u_{\max}^2$ at time t, defined as $\tilde{e}^T K_{pid}^T R K_{pid} \tilde{e} < u_{\max}^2$, where

$R = R^T > 0$ is an invertible scaling matrix. This condition is to be satisfied for all $\tilde{e}^T Q \tilde{e} < 1$. By performing S-procedure, the above condition is reduced to

$$K_{pid}^T R K_{pid} < \tau Q, \tag{21}$$

for $\tau \le u_{\max}^2$.

Since minimal ellipsoid is of interest, we set $\tau = u_{\max}^2$, pre- and post- multiplying by P and use the relation $Y = K_{pid}P$ to get

$$Y^T R Y < u_{\max}^2 P. \tag{22}$$

Applying of the Schur formula yields

$$\begin{pmatrix} P & Y^T \\ Y & u_{\max}^2 R^{-1} \end{pmatrix} > 0. \tag{23}$$

Therefore if Eq. (23) is satisfied, $\|\tilde{u}_{pid}(t)\|_R^2 = \|K_{pid}e(t)\|_R^2 < u_{\max}^2$ for all $\tilde{e}^T Q \tilde{e} < 1$ and all $t \ge 0$. In typical cases, $R = I$ where I is an identity matrix is imposed.

From the discussion thus far, we are able to formulate a minimization problem to obtain the optimal gain K_{pid} such that the invariant ellipsoid is minimized. It is noted that the smaller the size of matrix P, the smaller the invariant ellipsoid. Thus the trace of P is chosen as the objective function to be minimized.

The optimization problem is thus stated as follows:

$$\underset{P,Y}{\text{minimize}} \;\; \mathrm{trace(P)}$$

subject to

$$P\tilde{A}^T + \tilde{A}P + \tilde{B}Y + Y^T\tilde{B}^T + \alpha P + \frac{1}{\alpha}\tilde{D}\tilde{D}^T \le 0,$$

$$\begin{pmatrix} P & Y^T \\ Y & u_{\max}^2 R^{-1} \end{pmatrix} \ge 0,$$

$$P \ge 0.$$

In solving this optimization problem, the PID control gain K_{pid} is obtained such that the performance of the PID controller in the presence of a L_∞ bounded disturbance is optimized. The output of the system e_1 is restricted to within a minimal bound specified by a matrix P.

4.2 Optimized Controller Gain

The optimization problem is solved using the Yalmip and Sedumi toolbox in Matlab. Figs. 2 and 3 show the variation of the optimal controller gain and $\mathrm{trace(P)}$ with different α values. The controller gain becomes too large to implement as $\mathrm{trace(P)}$ decreases and α increases, therefore a tradeoff gain value is selected.

The following controller gain is chosen, $K_i = -23.9$, $K_p = -32.0$ and $K_d = -13.0$ at $\alpha = 1$.

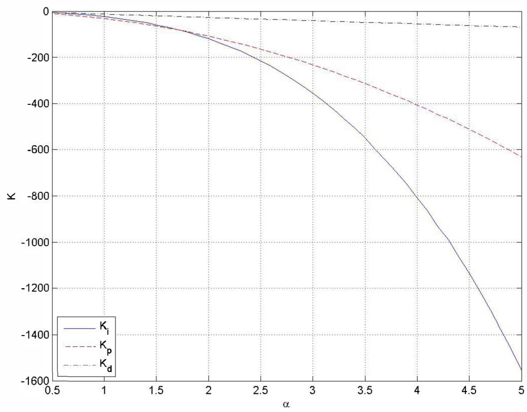

Fig. 2 The variation of the optimal controller gain with α. The controller gain becomes larger in magnitude with α.

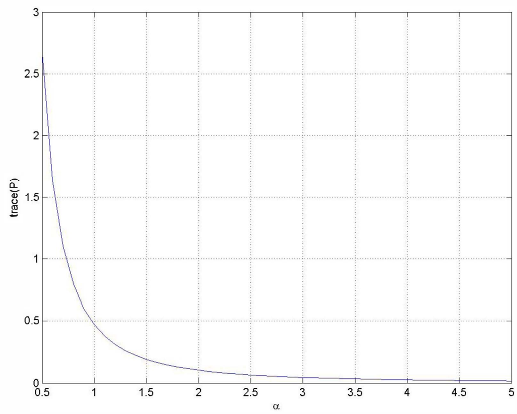

Fig. 3 The variation of the optimal $\mathrm{trace(P)}$ with α. The trace represents the size of the invariant ellipsoid, a smaller value indicates that the response is bounded more tightly to the equilibrium point. The trace decreases with α.

5. NUMERICAL SIMULATION

5.1 Simulation

The ship deck landing problem is numerically simulated using Matlab and Simulink. The quadrotor is commanded to hold an relative height of 10m above the ship deck for 50s and then commanded to land onto the landing pad located 1m above the ship deck. A smooth sigmoid function is chosen as the reference path,

$$H = H_0 + \frac{(H_f - H_0)}{1 + \exp\left(\frac{-6(2(t-t_0)-\Delta t)}{\Delta t}\right)}, \tag{24}$$

where H_0, H_f are the initial and final height, t_0 is the time when the height is commanded to decrease and Δt is the time duration of the descend. In this simulation, the following parameters are used. $H_0 = 10$, $H_f = 1$, $t_0 = 50$ and $\Delta t = 30$.

Furthermore the ship deck motion is simulated as

$$d = \sum_{j=1}^{N} A_j \sin(\omega_j t + \epsilon_j), \tag{25}$$

where d is the ship deck motion above the mean sea level, N is the number of harmonics to be modelled, A_j, ω_j and ϵ_j are the amplitude, frequency and phase of the j^{th} harmonic respectively. Fig. 4 shows the randomly generated ship deck motion by using a randomly generated initial phase.

The controller gain using the invariant ellipsoid method is compared with the baseline case that is chosen from the loop shaping method. The controller gain for the baseline case is $K_i = -0.2$, $K_p = -4.5$ and $K_d = -2.6$.

The simulation result is shown in Figs. 4 $\sim$ 8. The results for using the proposed controller gain and the baseline gain are plotted as new and classical respectively.

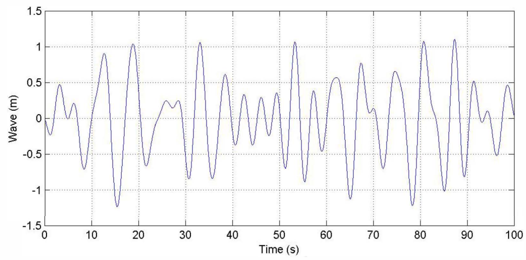

Fig. 4 Heave motion of the ship deck, with the mean sea level at $h = 0$.

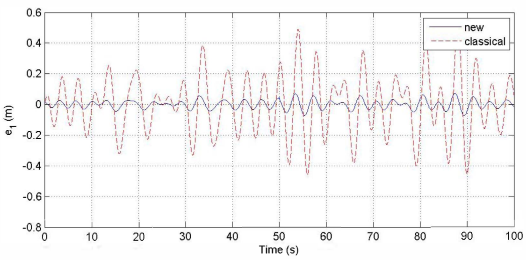

Fig. 5 Tracking error e_1 of the quadrotor, a small tracking error indicates that the quadrotor is able to track the ship deck motion.

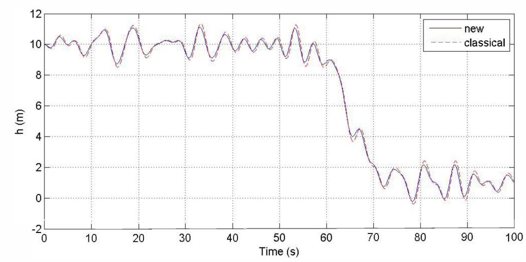

Fig. 6 Height of the quadrotor relative to the mean sea level.

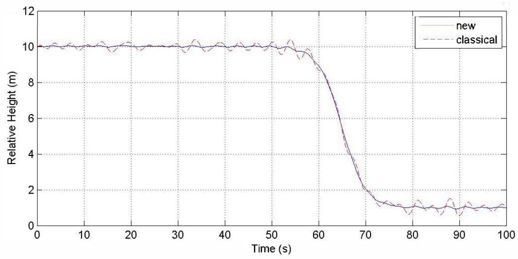

Fig. 7 Height of the quadrotor relative to the ship deck.

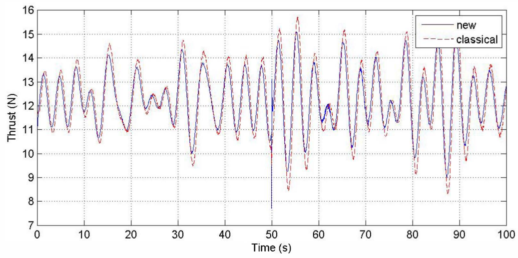

Fig. 8 Thrust applied by the quadrotor.

5.2 Discussion

From Fig. 5, it can be seen that the optimized controller gain performed better than the gain obtained from the classical loop shaping method. This is because the invariant ellipsoid is minimal for the case of optimized controller gain, therefore the tracking error e_1 is bounded more tightly to zero.

This has a positive consequence towards the landing mission on the ship deck. The quadrotor is able to track the relative height reference more closely, resulting in a small error in relative speed between the quadrotor and the ship deck. This reduces the possibility of damages due to collision. Fig. 7 highlights this effect.

Despite having a larger gain as compared to the baseline controller gain, the control action given by the thrust of the quadrotor is not significantly larger as shown in Fig. 8. This is because of the inclusion of a limit on the control action during the optimization of the controller gain.

It should be noted that the proposed controller gain works better in the presence of disturbance (in this case the ship deck motion). While in the case of minimal disturbance, the performance of traditionally tuned controller may approach that of the proposed controller gain. This is shown in Fig. 9 where the ship deck motion is set to zero.

6. CONCLUDING REMARKS

In this paper, a novel method is proposed to select an optimal PID gain for a ship deck landing system of a quadrotor in the presence of random ship deck motion. The invariant ellipsoid method uses the invariant set theory to obtain an optimization problem to solve for the PID controller gain. Numerical simulation is performed

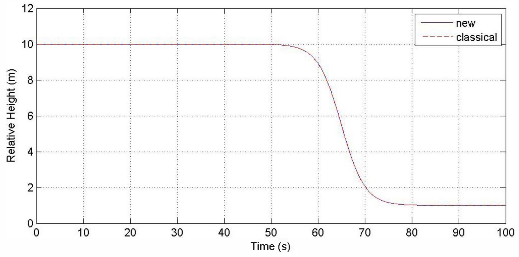

Fig. 9 Height of the quadrotor relative to the ship deck while the ship deck is not moving.

to evaluate the performance of the proposed controller gain as opposed to a controller gain obtained through loop shaping method. The proposed controller gain performed better in the presence of external disturbance. This method provides an alternative to select a PID controller gain in the presence of disturbance.

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