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Coordinated Landing of a Quadrotor on a Skid-Steered Ground Vehicle in the Presence of Time Delays

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***Abstract*— This work presents a control technique to autonomously coordinate a landing between a quadrotor UAV and a skid-steered UGV. Local controllers to feedback linearize the models are presented, and a joint decentralized controller is developed to coordinate a rendezvous for the two vehicles. The effects of time delays on closed loop stability are examined using a Retarded Functional Differential Equation (RFDE) formulation of the problem, and delay margins are determined for particular closed loop setups. Simulation results are presented, which demonstrate the feasibility of this approach for autonomous outdoor coordinated landing.**

***Index Terms*— Mobile robotics, autonomous landing, skid-steered vehicle, quadrotor, time delays.**

I. Introduction

Many unmanned air vehicles (UAVs) are designed to be small and manoeuvrable, and as a result have a limited payload. The trade-off for having a small, manoeuvrable vehicle often lies in having limited battery life. In order to increase the range of a UAV on a mission, the idea of pairing the UAV with an unmanned ground vehicle (UGV) that has a greater possible range has been put forward. Another advantage arising from pairing the two different types of autonomous vehicles is in being able to gain multiple perspectives during a mission. In order to have a fully autonomous system that pairs a UAV with a UGV, one must ensure that the vehicles have the ability to rendezvous autonomously and perform coordinated landings. This problem is examined in this paper, and a novel coordinated control approach for a quadrotor UAV and a skid-steered ground vehicle in the presence of time delays in the communications is presented. The skid-steered ground vehicle is useful in practice, in that it can operate successfully on a wide variety of terrains.

In [1], Oh *et al.* propose a tethered approach for autonomously landing a helicopter on a moving platform such as a ship deck. The tether is used to relay information between the platform and the vehicle, which is impractical and range limiting for small UAVs. Lange *et al.* [2] present an autonomous landing algorithm for quadrotor helicopters on a fixed platform using a known target pattern. Saripalli and Sukhatme [3] extend this to a moving platform, and rely on Kalman filtering for relative positioning and behaviour based control for automated landing. In [3], no proof of stability is given, the ground vehicle is assumed to have simple dynamics and the helicopter is maintained close to hover, which limits performance of the overall system. Voos and Bou-Ammar [4] present a novel approach to landing a quadrotor on a moving platform in which the quadrotor tracks the moving platform. The platform is assumed to move freely at constant velocity, while the quadrotor must reach it to complete the rendezvous and landing.

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In developing the coordinated control strategy presented in this work, nonlinear motion models for each vehicle will be used, along with feedback linearizing controllers to render the input-output dynamics of each vehicle linear. A coordinated controller that incorporates the state of both vehicles will be designed in order to drive the two vehicles together from an arbitrary initial state. With this approach the controller actuates both vehicles, instead of holding one fixed while coordinating a landing. To ensure that the approach is feasible, the effects of time delays in the communications must be assessed. The problem is presented in Retarded Functional Differential Equation (RFDE) form, and existing time delay stability results are applied to determine the maximum delay margin for which the closed loop remains stable. This sort of time delay analysis requires linear dynamics, accomplished by the feedback linearizing controllers around each plant.

In Section II the components of the system are presented, including the dynamic models of the UAV and UGV, their associated controllers, and the coordinated controller to drive both vehicles together. Section III develops the stability analysis for this system in the presence of time delays. The coordinated controller is re-developed for the case with time delays. Simulation results are presented in Section IV. Finally, conclusions are given and opportunities for future work are discussed in Section V.

II. Coordinated Controller Design

In order to develop accurate controllers to ensure a successful rendezvous, suitable dynamic models of the vehicles must be used. Initially, the quadrotor dynamics and controller will be presented, followed by the UGV dynamics and controller. Finally, the coordinated controller will be presented.

A. Quadrotor Dynamics

The following simplified nonlinear model of the quadrotor dynamics from Lee *et al.* [5] is used,

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} u_1 \sin\theta \\ -u_1 \sin\phi \\ u_1 \cos\phi\cos\theta - g \\ u_2 l \\ u_3 l \\ u_4 \end{bmatrix} \tag{1}$$

where $[x, y, z]^T$ represent the position of the quadrotor in the inertial frame, $[\phi, \theta, \psi]^T$ represent the 3-2-1 Euler angles, l is the distance of each rotor from the centre, u_i, $i \in \{1, 2, 3, 4\}$ represent a remapping of the thrust inputs such that u_1 corresponds to the total thrust while u_2, u_3, and u_4 corresponds to the control inputs for the roll, pitch, and yaw moments, respectively. Finally, g is the gravitational constant. With the quadrotor dynamics, a state vector may be defined and feedback linearization can be applied in order to yield linear dynamics from the system inputs to the outputs. The quadrotor state vector is defined as follows,

$$\begin{aligned}&[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T \\ &= [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T \end{aligned} \tag{2}$$

The details of the control design are omitted here, but the interested reader is referred to [5] for more information. Note that in the model (1) the state ψ is decoupled and its controller may be designed separately. The outputs for the input-output feedback linearizing controller are chosen as $y_1 = x$, $y_2 = y$, $y_3 = z$. With this choice of outputs, one must apply dynamic extension. The control input u_1 must be augmented with two integrators. This yields the additional states z_1 and z_2, where $z_1 = u_1$, $\dot{z}_1 = z_2$, $\dot{z}_2 = w_1$, and w_1 is an artificial control input that is integrated twice to yield u_1. Then, the feedback linearizing controller for the three position states is given as [5],

$$\begin{bmatrix} w_1 \\ u_2 \\ u_3 \end{bmatrix} = A^{-1}(x) \left(-B(x) + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right) \tag{3}$$

Application of this controller to the plant yields the linear dynamics from the input to the output $[y_1^{(4)}, y_2^{(4)}, y_3^{(4)}]^T = v$. Definitions of $A(x) \in R^{3\times3}$ and $B(x) \in R^3$ can be found in [5]. The auxiliary input $v \in R^3$ can be chosen to ensure asymptotic tracking of a desired reference trajectory $y_d = [y_{1d}, y_{2d}, y_{3d}]^T$, as follows,

$$v = y_d^{(4)} - K_1 e^{(3)} - K_2 \ddot{e} - K_3 \dot{e} - K_4 e \tag{4}$$

where $e = y - y_d$ and the matrices K_1, K_2, K_3, and K_4 are diagonal constant gain matrices. However, in this work the feedback linearized dynamics for x and y that consists of integrators will be used directly in designing the joint decentralized controller. A tracking controller of the form given in (4) will be used to control the height of the quadrotor. Additionally, one must also choose the control law for the state ψ. Choosing,

$$u_4 = \ddot{\psi}_d - k_{\psi 1}(\dot{\psi} - \dot{\psi}_d) - k_{\psi 2}(\psi - \psi_d) \tag{5}$$

where ψ_d is the desired angle for ψ yields asymptotically stable yaw tracking error dynamics.

B. Skid-Steered Ground Vehicle Model

When working with UGVs, it is common to use two dimensional planar models. The modeling problem for general motion becomes quite difficult when the system is considered to move in three dimensions. Yu *et al.* [6] present a three dimensional dynamic model of a skid-steered ground vehicle for the special case of straight-line motion, but do not present a general three dimensional model. Sharaf *et al.* [7] develop a 6 DOF dynamic model of a 4 wheeled ground vehicle, but do not present a control strategy for the model.

Here we make use of a kinematic model of the skid-steered ground vehicle based on work in [8]. The UGV platform in use in the WAVELab at the University of Waterloo provides velocity inputs, so as a result a kinematic model is appropriate. Early experimental work has shown that with a kinematic model and the controller that will be presented below, very reasonable tracking performance can be obtained.

In order to present the kinematic model of the vehicle used in this work, define the state $q = [X, Y, \theta]^T$, where X and Y represent the position of the vehicle centre of mass (COM) in the global coordinates and θ represents the angle of the body-fixed frame in the global coordinates. Note that the body-fixed frame is attached at the vehicle COM. Then, the vehicle kinematic model is,

$$\underbrace{\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} \cos\theta & x_{ICR}\sin\theta \\ \sin\theta & -x_{ICR}\cos\theta \\ 0 & 1 \end{bmatrix}}_{S(q)} \underbrace{\begin{bmatrix} v_x \\ \omega \end{bmatrix}}_{v_c} \tag{6}$$

where $v_c = [v_x, \omega]^T$ represents the velocity of the vehicle COM expressed in local coordinates, and x_{ICR} is the x projection of the instantaneous centre of rotation (ICR) of the vehicle into the body-fixed frame. This projection always lies on the centre line of the vehicle and relates the angular velocity to the y component of velocity expressed in the local coordinates as $\omega = -v_y/x_{ICR}$ [9]. It is the local x projection of the point about which the vehicle rotates at any given instant. In practice, due to the skid steering, the location of the ICR will move [9]. However, in this work a constant approximation of the x projection of the ICR into the local coordinate frame is used and will be called x_0. For any curvilinear motion with a skid-steered vehicle, the x projection of the ICR will be non-zero. By using a constant non-zero x_0 in the controller, an operational constraint may be imposed on the vehicle that limits lateral skidding [8].

To design a controller for this model, input-output feedback linearization is used. In this work it is important to be able to control the position of the vehicle in the inertial frame, so X and Y are chosen as outputs. It is apparent from the model that this system has vector relative degree $\{1, 1\}$. Then, the input-output dynamics of the system are given as,

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \cos\theta & x_0\sin\theta \\ \sin\theta & -x_0\cos\theta \end{bmatrix} \begin{bmatrix} v_x \\ \omega \end{bmatrix} \tag{7}$$

The feedback linearizing controller for this set of dynamics is determined to be,

$$
\left[\begin{array}{c} v_x \\ \omega \end{array}\right] = \left[\begin{array}{cc} \cos\theta & \sin\theta \\ \frac{\sin\theta}{x_0} & -\frac{\cos\theta}{x_0} \end{array}\right] \left[\begin{array}{c} \bar{v}_x \\ \bar{v}_y \end{array}\right] \tag{8}
$$

Applying this controller to (7) yields $[\dot{X}, \dot{Y}]^T = [\bar{v}_x, \bar{v}_y]^T$. This set of dynamics will be used in designing the joint decentralized controller.

C. Joint Decentralized Control

We now present a control approach that has been developed to force the quadrotor and ground vehicle to rendezvous. This approach is not based on trajectory tracking, but instead is based on the exponential stabilization of a particular set of dynamics.

The goal of this control strategy is to drive the relative position error of the two vehicles to zero, along with the velocity of the vehicles with respect to an inertial frame. If only position error is stabilized, then the two vehicles may rendezvous, but continue to drive together at some velocity. As this controller is designed strictly to accomplish rendezvous and landing, the decision was made to ensure that the velocity of both vehicles is driven to zero. From there a tracking controller could be enabled on the ground vehicle to carry on with the rest of the mission while transporting the quadrotor. However, in the general landing case it is not a requirement that the velocities be driven to zero.

The controllers for each of the plants render them linear through feedback. Consider the closed loop quadrotor model $[y_1^{(4)}, y_2^{(4)}, y_3^{(4)}]^T = v$, where y_1 corresponds to motion along the x axis, y_2 corresponds to motion along the y axis, and y_3 corresponds to motion along the z axis. Also consider the feedback linearized model of the ground vehicle given by $[\dot{X}, \dot{Y}]^T = [\bar{v}_x, \bar{v}_y]^T$. The goal is to ensure that $y_1 - X = y_2 - Y = 0$, and also that $\dot{y}_1 = \dot{y}_2 = 0$. Since a kinematic model is used for the ground vehicle, there is no explicit velocity state for it. However, ensuring that the quadrotor velocity and the relative position of the vehicles is driven to zero will ensure that the ground vehicle will stop moving as well. In developing the controller, since the plants are decoupled, only the position along the x axis is considered here. The development for the y axis is identical. It is possible to extend this controller to the z state as well. However, from a practical and safety point of view it is best to command the quadrotor in z only once one is certain that the x and y positions of the vehicles are close to each other.

Defining the relative position of the vehicles as $e_1 = y_1 - X$, and including the remainder of the quadrotor states, the state vector of the dynamics to be stabilized may be expressed as $[e_1, \dot{y}_1, \ddot{y}_1, y_1^{(3)}]^T$. Then, the dynamics of the system are given as,

$$
\left[\begin{array}{c} \dot{e}_1 \\ \ddot{y}_1 \\ y_1^{(3)} \\ y_1^{(4)} \end{array}\right] = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} e_1 \\ \dot{y}_1 \\ \ddot{y}_1 \\ y_1^{(3)} \end{array}\right] + \left[\begin{array}{cc} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{array}\right] \left[\begin{array}{c} \bar{v}_x \\ v_1 \end{array}\right] \tag{9}
$$

Note that both the quadrotor and ground vehicle control inputs drive these dynamics. The rank of the controllability matrix is 4, meaning that it is completely controllable. The following control law will stabilize the system,

$$
\left[\begin{array}{c} \bar{v}_x \\ v_1 \end{array}\right] = -K \left[\begin{array}{c} e_1 \\ \dot{y}_1 \\ \ddot{y}_1 \\ y_1^{(3)} \end{array}\right] \tag{10}
$$

where $K \in R^{2\times 4}$. Using standard linear state feedback control techniques such as pole placement or LQR design, the closed loop poles of the above system may be arbitrarily placed, giving the desired response and ensuring the vehicles converge to each other at a suitable rate.

The controller given in (10) consists of the control signals for both plants. The signal $\bar{v}_x$ as computed in (10) is applied to the UGV, and likewise the term v_1 is applied to the quadrotor. Therefore, each plant requires knowledge of the state of the other in order to stabilize this system. Intuitively this is quite reasonable, as the goal is for each vehicle to drive toward the other. An additional inherent complication that arises from this setup is that of time delays. Depending on the communications network used and the distance between the vehicles, the delay could be significant. The next section will examine the stability of this closed loop system in the presence of time delays.

III. Time Delay Stability Analysis

Given that the control signal applied to each plant requires knowledge of the state of the other plant, and the fact that the two plants are physically separated, there will be inherent time delays in this system. It is well known that time delays can destabilize a closed loop system, but it is important to analyze the magnitude of the delay that could be tolerated by such a system. In this section, using the Retarded Functional Differential Equation (RFDE) formulation of the problem, the question of stability in the presence of time delays for this system will be examined.

In this work the state of the quadrotor is transmitted across a communications link to the ground vehicle, and the state of the ground vehicle is transmitted across the communications link to the quadrotor. Since both plants make use of their own state and the state of the other vehicle, the closed loop system expressed in RFDE form will contain both the state of the system at the current time and the state of the system delayed by the time delay in the communications.

Delays in a system are *commensurate* if each delay is an integer multiple of a base delay. Here it is assumed that the time delay between vehicles is the same in each direction. Since a private network is used, and the only signals transmitted on the network are between the vehicles, this is reasonable. We make the additional assumption, for the purpose of analysis, that the time delay is also constant. It is always possible to ensure that this assumption is held in practice by putting buffers into the system to deliver the information at a constant rate.

When the assumption of commensurate delays can be made about the system, a useful body of time delay stability results [10] can be applied to the problem. We define the time delay in this system as τ seconds. In order to use the time delay stability results presented in [10], the closed loop system must be put in the following linear RFDE form,

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau), \qquad \tau \geq 0 \tag{11}$$

where $x \in R^n$ is the state vector, and $A_0 \in R^{n\times n}$ and $A_1 \in R^{n\times n}$ describe the dynamics of the system.

The difficulty with the formulation of this controller when delays are present in the system comes in with the first state in the joint system (9). In the presence of time delays, only one term or the other that makes up e_1 will be delayed. As a result, there is no clear way to express the dynamics (9) with control law (10) in RFDE form.

To overcome this obstacle, we propose the following modification to the coordinated controller to properly take into account time delays. Noting that $e_1(t-\tau) = y_1(t-\tau) - X(t-\tau)$, the following control law is introduced:

$$\begin{aligned}\bar{v}_x(t) &= -k_{11}\left(y_1(t-\tau) - X(t-\tau)\right) - k_{12}\dot{y}_1(t-\tau) \\ &\quad - k_{13}\ddot{y}_1(t-\tau) - k_{14}y_1^{(3)}(t-\tau) \qquad (12)\\ v_1(t) &= -k_{21}\left(y_1(t-\tau) - X(t-\tau)\right) - k_{22}\dot{y}_1(t) \\ &\quad - k_{23}\ddot{y}_1(t) - k_{24}y_1^{(3)}(t) \qquad (13)\end{aligned}$$

Using this formulation of the control law, each side intentionally delays the term in e_1 that comes from itself. As a result, the closed loop system will consist only of terms containing the system state and the system state delayed by τ. It may then be put into RFDE form. Then, it will be proved that this formulation results in a stable closed loop system with a particular delay margin.

The main tool for studying the stability of RFDE systems of a single delay comes from [10, p. 48]. This theorem relies on the computation of generalized eigenvalues of two frequency-dependent matrices. The notation $\lambda_i(A, B)$ represents the i-th generalized eigenvalue of two square matrices A and B. As well,

$$\underline{\rho}(A, B) := \min\{|\lambda| \,|\, \det(A - \lambda B) = 0\}$$

In this theorem, θ_k^i represents the phase angle of the complex generalized eigenvalue at frequency ω_k^i.

Theorem 3.1: Suppose that the system (11) is stable at $\tau = 0$, and let $q = \mathrm{rank}(A_1)$. Furthermore, define

$$\bar{\tau}_i := \begin{cases} \min_{1\leq k\leq n} \frac{\theta_k^i}{\omega_k^i} & \text{if } \lambda_i(j\omega_k^i I - A_0, A_1) = \\ & e^{-j\theta_k^i} \text{ for some } \omega_k^i \in (0,\infty), \\ & \theta_k^i \in [0, 2\pi] \\ \infty & \text{if } \underline{\rho}(j\omega I - A_0, A_1) > 1, \\ & \forall \omega \in (0, \infty) \end{cases}$$

Then,

$$\bar{\tau} := \min_{1\leq i\leq q} \bar{\tau}_i$$

The system (11) is stable for all $\tau \in [0, \bar{\tau})$, but becomes unstable at $\tau = \bar{\tau}$.

This theorem requires a frequency sweeping test to evaluate the generalized eigenvalues continuously between 0 and ∞ rad/sec. In practice, a numerical approach is used [10]. The frequency axis is broken up into a grid, and at each grid point the generalized eigenvalues of $j\omega_k^i I - A_0$ and A_1 are computed. If $\underline{\rho}(j\omega I - A_0, A_1) > 1$ for all ω, the system has a delay margin of ∞. If not, the computation results in pairs (ω_k^i, θ_k^i) that provide estimates of the delay margin $\bar{\tau}_i$. The smallest $\bar{\tau}_i$ will be the delay margin for the system. The pairs are found in the cases where the generalized eigenvalues have magnitude one. At these points, the phase θ_k^i is found, since the generalized eigenvalues are complex in general.

For the application of Theorem 3.1 the RFDE system must be stable for no delays. From the design of the state feedback controller (10), this is assured. In order to determine the delay margin for the closed loop quadrotor-ground vehicle system, one must design the state feedback controller, and then numerically apply the results of Theorem 3.1.

The delay margin of the closed loop was determined for several sets of closed loop poles. For the system (9) with a controller giving closed loop poles at $\{-1, -2, -3, -4\}$, the delay margin is determined as $\tau = 0.52327$ seconds. If each closed loop pole is made more negative by an order of magnitude, giving closed loop poles at $\{-10, -20, -30, -40\}$, the delay margin is computed as $\tau = 0.050108$ seconds. If the closed loop poles of the system are chosen to be $\{-0.1, -0.2, -0.3, -0.4\}$, then the delay margin is found to be $\tau = 6.1237$ seconds. The trend observed here is that, as the closed loop system becomes more sluggish, the delay margin increases. Intuitively this stands to reason, as a slow system has a lower bandwidth than a fast system and reacts less aggressively to the delayed signals.

IV. Simulation Results

The coordinated rendezvous approach previously presented is now verified through a simulation study. The joint decentralized controller is applied to both the x and y states in the inertial frame, while the altitude of the quadrotor is controlled using the feedback linearizing controller that provides trajectory tracking in z.

The effects of wind gusts and sensor noise are included in the simulations. Wind is simulated using the Dryden wind gust model, and affects the quadrotor through the resulting aerodynamic drag. Zero mean Gaussian noise is added to all measurements. The standard deviation of the noise added to position measurements is 10 cm, while the standard deviation of the noise added to the velocity measurements is 10 cm/s. The orientation measurements are corrupted with noise having a standard deviation of 5 degrees, and the time derivatives of the orientation states are corrupted with noise having a standard deviation of 0.1 degrees/s. No noise is applied to the motion models of the vehicles, just their measurements. Each vehicle receives noisy measurements from the other without any filtering. The coefficient of drag for the quadrotor was chosen as $C_d = 0.5$, and the area of the vehicle was computed as 0.2027 m^2.

In the UGV model, x_0 is set to 3.7 cm, and the same value is used in the UGV controller. The initial state is $[X, Y, \theta]^T = [0, 1, 45]^T$, where the linear units are in metres and the angular units in degrees. For the quadrotor l was set to 1 m. The gains for the ψ controller were chosen as $k_{\psi 1} = 24$ and $k_{\psi 2} = 144$. The gains for the z portion of the tracking controller (4) were chosen as $K_1 = 8$, $K_2 = 24$, $K_3 = 32$, and $K_4 = 16$. To ensure that the reference trajectory for z was kept within the bandwidth of the system, the desired z was passed through a 5-th order low-pass filter with poles at $s = -2$. This additionally provides up to the fourth derivative of the filtered reference trajectory. The initial pose of the quadrotor was set to $[x, y, z, \phi, \theta, \psi]^T = [5, 4, 5, 10, 10, 10]^T$, where the linear units are in metres and the angular units in degrees. The quadrotor was initialized with linear and angular rates set to zero. The simulations were implemented using a first order Runge-Kutta solver with a fixed sample period of 0.001 seconds.

In all simulations the joint decentralized controller (10) is used to draw the x and y positions of each vehicle together. The gain matrix K for both the x and y dynamics is computed using an LQR approach with diagonal Q and R matrices having diagonals $\{0.1, 1, 1, 1\}$ and $\{10, 10\}$ respectively. Using Theorem 3.1, the delay margin for this controller was determined to be 6.8984 seconds. One would not expect to have delays of that magnitude when two vehicles are relatively close to each other and operating over a private radio network. Nonetheless, it is valuable to be able to quantify the delay margin, and encouraging that for some choices of the closed loop poles it can be quite large.

Initially a time delay of 1 second in each direction is used. Fig. 1 shows the difference between the quadrotor and ground vehicle positions. It is the objective of the joint decentralized controller to drive these signals to zero. From Fig. 1 it is clear that even in the presence of a non-negligible time delay the closed loop system remains asymptotically stable. The presence of noise on the states as well as the aerodynamic drag including Dryden wind gusts has a clear effect on the simulation in terms of driving the dynamics of both vehicles. As a result, one would not expect to see ideal asymptotic convergence. However, the trend is that the difference in position tends to zero. The trajectory of each of the vehicles in space is shown in Fig. 2. They converge to the same area in x and y and remain in that area. The noise

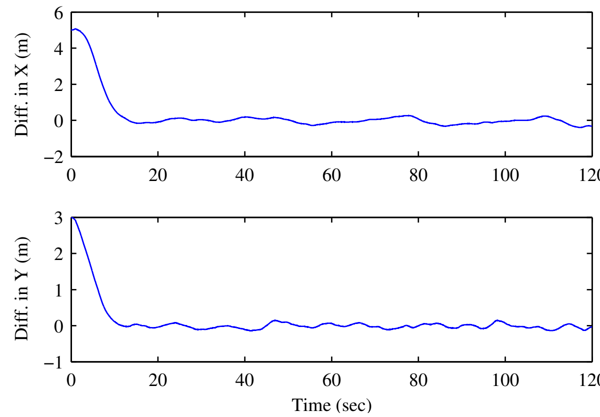

Fig. 1. Difference between the quadrotor and ground vehicle trajectories in simulation with a time delay of 1 second in each direction.

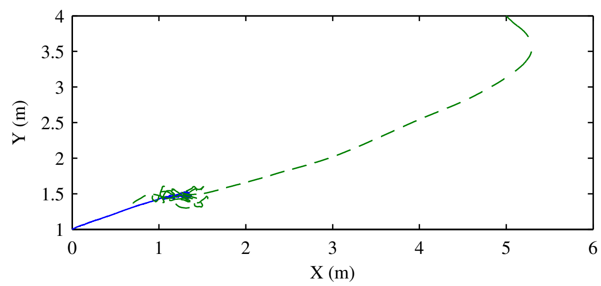

Fig. 2. Trajectories taken by the quadrotor (dashed) and ground vehicle (solid) with a time delay of 1 second in each direction.

and wind gusts continue to drive the vehicles around in that area. In the ideal case one would observe that both vehicles meet at the same location and execute a landing procedure.

The position states of the quadrotor are shown for this simulation in Fig. 3. After 120 seconds the x and y states have converged. The z state remains stable and executes the commanded landing trajectory from 5 m to 1 m 25 seconds in. This shows the effectiveness of the feedback linearizing controller (3) at decoupling the quadrotor states, even in the presence of noise and unmodeled forces such as drag, and executing a manoeuvre to land the quadrotor on the UGV.

A second simulation was performed with a time delay of 6.2 seconds in each direction. It was determined through simulation that with the sensor noise and wind gusts the simulation did not remain stable for time delays past 6.2 seconds. However, in the ideal case the system can be simulated with a time delay right up to the delay margin. Fig. 4 shows the difference in positions for the quadrotor and ground vehicle for this simulation. The solid lines show the simulation with sensor noise and wind gusts, while the dashed lines show the corresponding simulation when noise, drag, and wind gusts are not included. It is clear from this plot that the increased time delay has brought the system closer to the stability margin. The oscillations in Fig. 4 are much more sustained with the longer time delay. Even in the presence of significant time delay, sensor noise, and unmodeled wind disturbances the joint decentralized

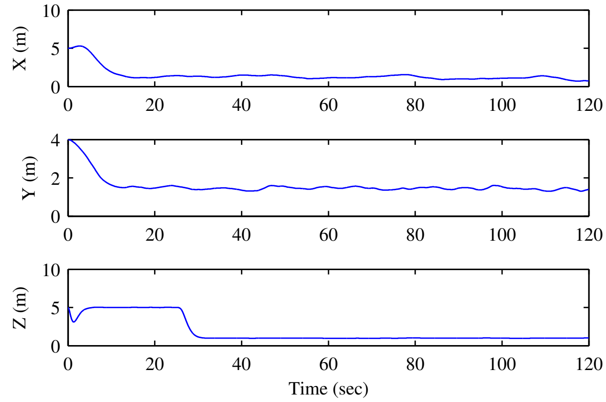

Fig. 3. Position of the quadrotor for the simulation with a time delay of 1 second.

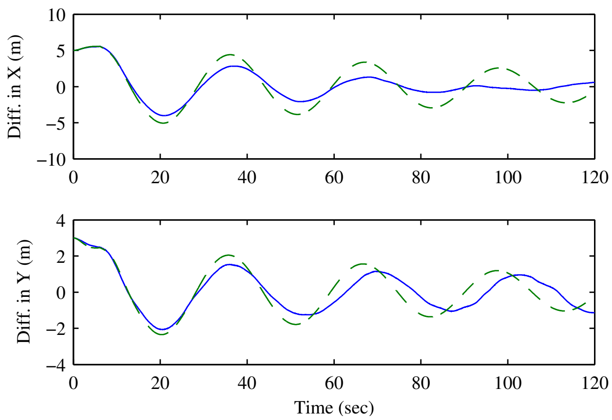

Fig. 4. Difference between the quadrotor and ground vehicle trajectories in simulation with a time delay of 6.2 seconds in each direction.

controller continues to perform as expected. The difference in position between the quadrotor and ground vehicle continues to decay toward zero in both the ideal case and the more realistic case. Interestingly, the case with noise and wind gusts appears to be more damped in the oscillations than the ideal case. This effect will require further study. The trajectories for each vehicle are shown in Fig. 5. In this plot the oscillatory behaviour of each vehicle is evident.

V. Conclusion

This work has presented a new approach for the autonomous landing of a quadrotor on a UGV. By designing local nonlinear controllers for each vehicle that render each vehicle linear through feedback, it is possible to design a joint decentralized controller that draws the two vehicles together. In this work, only the x and y positions of the vehicles are drawn together. Once the vehicles are close enough to each other, an automated landing of the quadrotor is performed using a tracking controller on the quadrotor altitude state. For this strategy each vehicle must have access to the other vehicle's state. Due to the communications involved in a system like this, there will be time delays inherent in the transmission of the state. To address this, the

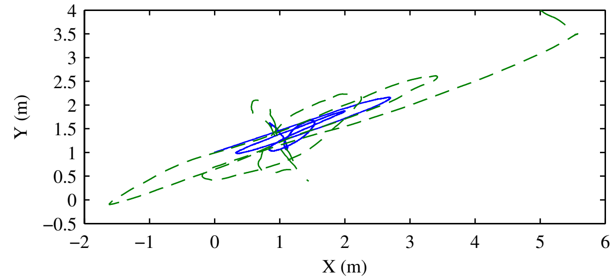

Fig. 5. Trajectories taken by the quadrotor (dashed) and ground vehicle (solid) with a time delay of 6.2 seconds in each direction.

problem was formulated using RFDEs in such a way that the closed loop stability in the presence of time delays could be analyzed. Simulation results were presented to confirm the feasibility of the approach. The addition of sensor noise and wind disturbances to the simulations has shown that the approach is robust to noise and modeling error, even in the presence of significant time delays.

An experimental implementation of this approach is underway in the WAVELab at the University of Waterloo. Initial experiments have begun using GPS and vision systems for state measurements. Future work will examine the use of more complex plant models for each vehicle, the effects of using estimators, and the impact of bounded but time varying communication delays. Additionally, a more in depth study of the effects of modeling error and noise on the delay margin for closed loop stability will be carried out.

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