Because the system and the environment are mutually constrained
系统和环境相互约束

Adaptive Impedance Based Force and Position Control for Pneumatic Compliant System 气动柔性系统

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柔顺控制

***Abstract*—The compliant control of the robot is widely used in the Human-Machine Interface (HMI) and robot bionics. Because the system and the environment are mutually constrained, when the external environment changes, the control environment of the system will change with it, which deteriorates the control performance of the system. This article solves the problem of how to realize the compliant control with changing environment and proposes the adaptive impedance control and compensation based on a specific compliance system. We first model our compliance system by focusing on the system's internal cylinder, pressure difference transmitter its other components and then acquire its transfer function.**

Then we apply the impedance control and adaptive impedance control to the compliance system. Comparing these two methods, we prove that the adaptive impedance control has better tracking performance and robustness in uncertain environment. Furthermore, the stability of the adaptive impedance compliance system is proved by the Lyapunov function. Finally, we verify this algorithm with a flange experimental platform and design an experiment about contact force between the flange and different objects. The stability and practicability of the experimental algorithm are substantiated.

***Index Terms*—compliant control, adaptive control, impedance control, flange**

I. Introduction

The force control of robots is an important field of robotic research. In general, there are two modes of operation which are successfully used for robots in industrial applications: First is the unconstrained motion of robotic manipulator in space, which can effectively track the space trajectory; the other is that the manipulator of the robot is in contact with the environment and the movement is constrained. When the robot is in contact with external environment, we should consider controlling both the position of the robot and the mutual force between robot and the external environment. In the meanwhile, we also need to guarantee that the robot has a high compliance in order to have the ability to produce any force on it in a specific environment. The contradiction or trade off between flexibility and rigidity is one of the core contradictions in robotics research. In order to solve the contradiction, experts on robotics all over the world have done a lot of research work on compliance control [1].

灵活性和刚度

Hogan proposed impedance control which is one of the basic control methods about compliance control [2]. It has been widely used in the realization of robot compliance control. Impedance control is a kind of method that makes the force and position satisfy certain ideal dynamic relationship by adjusting the impedance parameters of the manipulator. But it is an indirect method to realize the force control. In the application, the environment is often very inaccurate or unknown. Thus, a smaller estimate error will result in a lot of force error, which badly affects the precision. Lasky and Hsia proposed an internal and external loop control strategy including the correction law of the reference position of the outer ring by establishing the quadratic performance index of force error [3]. The uncertainty of dynamic model is compensated for the robust position control in the ring.

In Section II, based on the compliance robot system, both the pneumatic compliant actuator and the gas flow mass are modeled and the open loop transfer function of the system is established by analyzing the force of the compliant system. Then in Section III, we first use impedance control to control the system, which provides the compliant system with a unified control framework scheme. Next, in order to make the compliant system adapt to the environmental change, a more advanced adaptive impedance control algorithm is proposed. By employing the Lyapunov function, the stability of adaptive impedance compliance system is verified. In Section IV, in order to prove the real-time state of robot's compliant system, we build the experimental platform to verify the stability of the system and the feasibility of the control algorithm. The performance of the two control methods is compared in response time and other aspects.

II. The Model of Compliant System

To control the compliant system, it is necessary to analyze the compliant system and then establish the model of pneumatic actuator with lumped parameters. The basic dynamic properties of pneumatic actuator include dynamic characteristics of air cylinder pressure and mass-flow equation of cylinder chambers. Given the fact that the flow of gas is quite difficult to analyze and easily affected by many factors, we ignore some unnecessary parameters as simplification.

A. Force Analysis for Pneumatic Actuator

The sketch and photo of the pneumatic actuator are illustrated in Fig.1. The pneumatic actuator is connected to the mobile platform by flange. The angle between the mobile platform and vertical direction is α. The mass of moving platform is set as

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M, the total mass of the flange as m, the output force of the pneumatic actuator as F_1 and the counterforce as F_2.

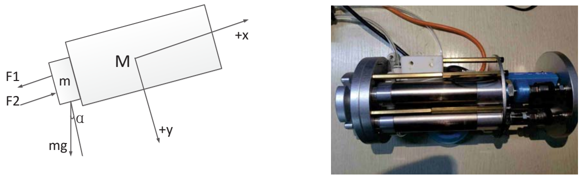

Fig. 1. Sketch and photograph of the pneumatic actuator

According to Newton's Second Law, we can get the force equilibrium equation in the following:

$$F_1 - F_2 + G_\alpha - f = m\ddot{x} + K_v\dot{x} \tag{1}$$

where G_α is the component of the gravity and external load in the α direction and it can be calculated by the equation $G_\alpha = mg\sin\alpha$, K_v is the viscous friction coefficient and f is the friction.

B. Mass-flow Equation of Cylinder Chambers

Supposing that the flow of gas is continuous, according to the law of conservation of mass, we can deduce that the change of the mass flow rate is equal to the difference between the mass flow rate flowing in and the mass flow rate flowing out. Thus, we can obtain the equation:

$$\sum \dot{M}_{in} - \sum \dot{M}_{out} = \frac{\mathrm{d}M}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\rho V) = \rho\frac{\mathrm{d}V}{\mathrm{d}t} + V\frac{\mathrm{d}\rho}{\mathrm{d}t} \tag{2}$$

Then we take the ideal gas equation $\rho = \frac{p}{RT}$ into Eq. (2) and the equation below:

$$q_{m_1} = \frac{1}{R}\frac{\mathrm{d}(\frac{p_1V_1}{T_1})}{\mathrm{d}t} = \frac{1}{RT_1}(p_1\frac{\mathrm{d}V_1}{\mathrm{d}t} + V_1\frac{\mathrm{d}p_1}{\mathrm{d}t} - \frac{p_1V_1}{T_1}\frac{\mathrm{d}T_1}{\mathrm{d}t}) \tag{3}$$

$$q_{m_2} = \frac{1}{R}\frac{\mathrm{d}(\frac{p_2V_2}{T_2})}{\mathrm{d}t} = \frac{1}{RT_2}(p_2\frac{\mathrm{d}V_2}{\mathrm{d}t} + V_2\frac{\mathrm{d}p_2}{\mathrm{d}t} - \frac{p_2V_2}{T_2}\frac{\mathrm{d}T_2}{\mathrm{d}t}) \tag{4}$$

Considering that the whole process is adiabatic, we can acquire the relationship between the temperature at any moment T and the initial temperature T_0 in the following:

$$T = T_0\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} \tag{5}$$

Take the derivation of Eq. (5) with respect to time and plug it into Eq. (3) and Eq. (4) ,

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{1}{RT}\left(p\frac{\mathrm{d}V}{\mathrm{d}t} + \frac{V}{k}\frac{\mathrm{d}p}{\mathrm{d}t}\right) \tag{6}$$

From the Eq. (6), we can see the mass flow rate changes along with the volume and pressure. Therefore, for chamber A and chamber B of the cylinder, there exist two equations respectively:

$$\begin{cases} \dot{M}_a = \dfrac{1}{RT_a}\left(p_a\dfrac{\mathrm{d}V_a}{\mathrm{d}t} + \dfrac{V_a}{k}\dfrac{\mathrm{d}p_a}{\mathrm{d}t}\right) \\ \dot{M}_b = \dfrac{1}{RT_b}\left(p_b\dfrac{\mathrm{d}V_b}{\mathrm{d}t} + \dfrac{V_b}{k}\dfrac{\mathrm{d}p_b}{\mathrm{d}t}\right) \end{cases} \tag{7}$$

Assuming that S_a and S_b are the outward and inward strokes separately. We can make assumptions that $S_aP_a = S_bP_b$, $T_a = T_b = T$ in the initial condition and the movement of the piston is close to the middle. Supposing that the effective area ratio of the two chambers is $n = S_b/S_a$ and the initial volume ratio of the two chambers is $r = V_b/V_a$, the mass flow rate in chamber A and chamber B can be expressed as:

$$\begin{cases} \dot{M}_a = \dfrac{1}{RTk}\left(kS_ap_a\dfrac{\mathrm{d}V}{\mathrm{d}t} + V_a\dfrac{\mathrm{d}p_a}{\mathrm{d}t}\right) \\ \dot{M}_b = \dfrac{1}{RTk}\left(kS_bp_b\dfrac{\mathrm{d}V}{\mathrm{d}t} + \gamma V_a\dfrac{\mathrm{d}p_b}{\mathrm{d}t}\right) \end{cases} \tag{8}$$

C. Equations of The Pressures of Two Cylinder Chambers

In the cylinder, suppose that the intensity of pressure inside chamber A is p_a and the volume is V_a, while the intensity of pressure inside chamber B is p_b and the volume is V_b; L is the length of stroke. Thus, the pressure differential equation of the cylinder can be obtained. According to the law of conservation of energy, the energy equation of two cylinder chambers can be simplified as the following:

$$p_{(a,b)} = \frac{\gamma RT}{V_{(a,b)}}\dot{M}_{(a,b)} - \frac{\gamma \cdot p_{(a,b)}\dot{V}_{(a,b)}}{V_{(a,b)}} \tag{9}$$

Assuming y is the distance between piston and the end of chamber A, Eq. (9) can be also written as:

$$\begin{cases} \dfrac{\mathrm{d}p_a}{\mathrm{d}t} = RkT_s\dfrac{\dot{M}_a}{V_a} - \dfrac{kp_a\dot{y}}{y} \\ \dfrac{\mathrm{d}p_b}{\mathrm{d}t} = RkT_s\dfrac{\dot{M}_b}{V_b} - \dfrac{kp_b\dot{y}}{L-y} \end{cases} \tag{10}$$

D. Transfer Function of Compliant System

The mass flow through the valve is primarily influenced by the opening area and exit pressure of the valve. Thus, the mass flow is the function of pressure and the size of the opening area. The function can be written as:

$$\dot{m}_{(a,b)} = qX_v\psi_{(a,b)}(p_u, p_d) \tag{11}$$

Linearizing above equation with Tylor formula [4],

$$\begin{cases} \dot{m}_a = K_{p_a}X_v - K_{C_a}p_a \\ \dot{m}_b = K_{p_b}X_v - K_{C_b}p_b \end{cases} \tag{12}$$

where $K_{p_a} = \frac{\partial \dot{m}_a}{\partial x}\Big|_{X_V=0}$, $K_{p_b} = \frac{\partial \dot{m}_b}{\partial x}\Big|_{X_V=0}$, $K_{C_a} = \frac{\partial \dot{m}_a}{\partial p_a}\Big|_{p_a=0}$ and $K_{C_b} = \frac{\partial \dot{m}_a}{\partial p_b}\Big|_{p_b=0}$.

However, the dynamic response of the valve is much faster than that of compliant and mechanical systems. Therefore in the following analysis, we concentrate more on the dynamic characteristics of compliant system as well as the load of the cylinder and neglect the dynamic characteristics of the valve. Assuming the pressure of the load is $p_L = p_a - np_b$, the force equilibrium equation analyzed previously can be written in the following form:

$$\begin{aligned} &\dot{m}_a + \frac{n}{r}\dot{m}_b = \left(K_{p_a} + \frac{n}{r}K_{p_b}\right)X_v - \left(\frac{n}{r}K_{C_b} + nK_{Ca}\right)p_b + \\ &K_{C_a}p_L = \frac{1}{RTk}\left[\left(\frac{\mathrm{d}p_a}{\mathrm{d}t} - n\frac{\mathrm{d}p_b}{\mathrm{d}t}\right)V_a + \left(1 + \frac{n}{r}\right)kS_ap_a\frac{\mathrm{d}y}{\mathrm{d}t}\right] \end{aligned} \tag{13}$$

then, take Laplace transformation of Eq. (13),

$$\begin{aligned}&(K_{p_a}+\frac{n}{r}K_{p_b})X_v(s)-(\frac{n}{r}K_{C_b}+nK_{C_a})p_b(s)\\&-\frac{1}{RT}(1+\frac{n}{r})S_a p_a sy(s)=(K_{C_a}+\frac{sV_a}{RTk})p_L(s)\end{aligned} \tag{14}$$

$$S_a p_a - S_b p_b = (ms^2+K_v s)y(s)+f(s)+F_2(s)-G_\alpha(s) \tag{15}$$

To get the transfer function between $Y(s)$ and $P_L(s)$, we can assume the pressure of chamber B and $X_v(s)$ are zero and ignore other external forces. Then the linear transfer function of the system can be obtained:

$$\frac{Y(s)}{P_L(s)}=\frac{S_a RTk(K_{c_b}+K_{c_a}r)}{mV_a rs^3+(K_{c_a}RTkmr+K_v V_a r)s^2+\beta_n s} \tag{16}$$

where $\beta_n = S_a^2 P_a kr + S_a^2 P_a kn + K_{c_a} K_v RTkr$. For ease of analysis, Eq. (16) can be simplified as:

$$\frac{Y(s)}{P_L(s)}=\frac{K_n}{s(\frac{s^2}{w_n{}^2}+2\frac{\xi_n}{w_n}s+1)} \tag{17}$$

where $\xi_n = \frac{1}{2}(K_{c_a}RTkmr+K_v V_a r)\sqrt{\frac{\beta_n}{mV_a r}}$, $\omega_n = \sqrt{\frac{\beta_n}{mV_a r}}$, $K_n = \frac{S_a RTk(K_{c_b}+K_{c_a}r)}{\beta_n}$.

So, this is the Eq.(17) of position and output power of the system.

III. Control Algorithm of The Compliant System

A. Impedance Control

In this paper, our objective is to control the force and position accurately. If the end of the flange is not in contact with the external environment, it can be seen as a relatively isolated system, so the interaction force can be ignored. In this case, we should only control the end position of the flange [5]. But in general, when flange contacts with the external environment, the force with external environment of the flange cannot be ignored. The controller needs to adjust its dynamic behavior by controlling the position and force of the flange end.

According to the definition of Hogan, the compliant system has both impedance and admittance properties. In other words, if the deviation exists in the compliant system, then the system will produce resistance force, this is the impedance property; if there is force applied at the end of the system, then it will produce displacement, this is the admittance property. However, the external environment has only the admittance property. Then, second-order linear impedance can be expressed as [6]:

$$\mathbf{M}_d(\ddot{\vec{X}}-\ddot{\vec{X}}_d)+\mathbf{B}_d(\dot{\vec{X}}-\dot{\vec{X}}_d)+\mathbf{K}_d(\vec{X}-\vec{X}_d)=\vec{E} \tag{18}$$

where $\mathbf{M}_d, \mathbf{B}_d, \mathbf{K}_d$ are called target inertia matrix, damping matrix and stiffness matrix respectively. $\vec{X}, \dot{\vec{X}}, \ddot{\vec{X}}$ are the position, velocity, acceleration vectors of the end of the flange. $\vec{X}_d, \dot{\vec{X}}_d, \ddot{\vec{X}}_d$ are the excepted position, velocity and acceleration vectors. $\vec{E}=\vec{F}_r-\vec{F}_e$ is the force error signal, $\vec{F}_r$ is the reference force vector. $\vec{F}_e$ is the force vector which is applied to the end of the flange.

Since the flange is the single-degree-of-freedom system, we can get:

$$m_d(\ddot{x}-\ddot{x}_d)+b_d(\dot{x}-\dot{x}_d)+k_d(x-x_d)=e \tag{19}$$

The compliant system can work in two movement space modes: free space and contact space. In the free space, the contact force $f_e=0$, thus,

$$m_d(\ddot{x}-\dot{x}_d)+b_d(\dot{x}-\dot{x}_d)+k_d(x-x_d)=f_r \tag{20}$$

When the flange contacts with the environment, the flange is not an isolated system any more. Instead, it is an integrated dynamic system which consists of flange and the environment. Then the mode of the flange with environment can be simplified to a linear spring model [7]. The model can be obtained as:

$$f_e = k_e(x-x_e) \tag{21}$$

where k_e is the environment stiffness and x_e is the position of pneumatic actuator.

In frequency domain, the following impedance function can be written as Eq. (22)

$$x_f(s)=\frac{e(s)}{m_d s^2+b_d s+k_d} \tag{22}$$

$$x_r = x_d + x_f \tag{23}$$

where x_f is the output position of impedance control and x_r is the reference position.

When the flange is in the free space, there is no contact with the environment. So $f_e=0$. If $f_r=0$, we will get the following result.

$$x_r = x_d \tag{24}$$

Eq. (24) means that the output position of flange tracks the input of the desired position. When the flange contacts with the environment, assuming that the controller is accurate enough, we can get $x=x_r$. Hence, $x_f=x+x_d$. As a result, impedance control can be used to solve the free movement and the restricted movement with an unified method. That is one of the advantages of impedance control.

B. Simulation Results of Impedance Control

Reference force as a given signal is directly taken into the force control method, which is used to feedback the contact force signal between the end of the flange and environment [8].

In the simulation, the target mass m_d is 2kg and the environment stiffness k_e is 700N/m. We use the fourth - order Runge - Kutta method in the simulation and set up a fixed step length. The simulation time is 10s, and the sampling time is 0.001s.

We set the sine input bias as 10N, the amplitude as 2N and the frequency as 0.8Hz. The simulation below shows that the system has better tracking performance in position, but there is a certain lag in the response of force. The system tracking performance is poor with a sine signal. So we propose a new method to improve impedance control. The results of simulation are shown below.

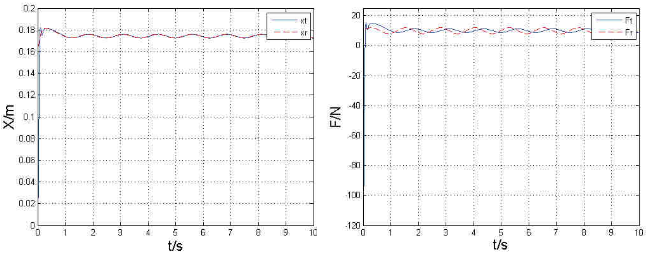

Fig. 2. Force response and position response

C. Adaptive Controller Design And Analysis

If the parameters of controlled objects are fixed or there are relatively slight changes in the parameters which can be ignored during the control process, then the conventional control methods will achieve the expected effects. In this paper the control system is composed of flange and environment, however, the changes of environmental factors are variable, so the system is an integrated dynamic system which is quite difficult to be controlled satisfactorily by utilizing the conventional impedance control with fixed parameters. Based on the situation above, a new control method is proposed by designing an adaptive controller to optimize the impedance controller. It can adjust the target parameters of the impedance controller in real time to make the control performance achieve the best state when environmental factors and parameters of controlled objects change [9].

In the process of compliant force control, the adaptive controller creates a new impedance equation and obtains a new adjustment amount by changing the target impedance relation. In this process, a small adjustment amount Δx_f is input on the basis of the original impedance control and a new target impedance relationship is obtained by adjusting Δx_f according to the adjustment rule of the adaptive controller to acquire the expression of x_f. Then the new reference position can be written as:

$$x_r = x_f + \Delta x_f \tag{25}$$

where Δx_f is the position correction amount generated by the adaptive controller. In this paper, we take the position correction as the following expression:

$$\Delta x_f = g(t) + p(t)e(t) + d(t)\dot{e}(t) \tag{26}$$

where $e(t)$ is the force control error; $g(t)$ is the auxiliary term used to compensate for the static error; $p(t)$ and $d(t)$ are the proportional factor and the differential factor respectively. According to the assumption in the previous chapter, the model of the flange contacting with the environment is a linear spring model, substituting Eq. (25) and Eq. (26) into the position-based impedance control equation:

$$\begin{aligned}\ddot{e} + \frac{b_d + k_d k_e d(t)}{m_d}\dot{e} + \frac{k_d + k_e + k_d k_e p(t)}{m_d}e \\ = \frac{k_d k_e}{m_d}[x_e - x_d - g(t)]\end{aligned} \tag{27}$$

After simplification, we can assume that

$$\begin{cases} a_p(t) = \frac{b_d + k_d k_e d(t)}{m_d} \\ b_p(t) = \frac{k_d + k_e + k_d k_e p(t)}{m_d} \\ w_p(t) = \frac{k_d k_e}{m_d}[\frac{f_r}{k_e} + x_e - g(t)] \end{cases} \tag{28}$$

Hence, the force error equation can be written as:

$$\ddot{e} + a_p(t)\dot{e} + b_p(t)e = w_p(t) \tag{29}$$

Supposing that $\mathbf{E}_p = \begin{bmatrix} e & \dot{e} \end{bmatrix}^\mathsf{T}$, we can transfer Eq. (29) into the following form:

$$\dot{\mathbf{E}}_p = \mathbf{A}_p(t)\mathbf{E}_p + \begin{bmatrix} 0 \\ w_p(t) \end{bmatrix} \tag{30}$$

where $\mathbf{A}_p(t) = \begin{bmatrix} 0 & 1 \\ -b_p(t) & -a_p(t) \end{bmatrix}$, Eq. (27) is the adjustable system in Model Reference Adaptive Control (MRAC). The purpose of adjusting the coefficients $a_p(t)$, $b_p(t)$, $w_p(t)$ is to reduce the difference between the actual force error and the expected force error so that the response of the actual system follows the response of the reference model. The reference model can be taken as the following ideal second-order system model:

$$\ddot{e} + a_m\dot{e} + b_m e = 0 \tag{31}$$

Assuming that $\mathbf{E}_m = \begin{bmatrix} e_m & \dot{e}_m \end{bmatrix}^\mathsf{T}$ is the state variable matrix of the reference model. Similarly, according to Eq. (31), we can get the following equation about reference model:

$$\dot{\mathbf{E}}_m = \mathbf{A}_m(t)\mathbf{E}_m \tag{32}$$

where $\mathbf{A}_m(t) = \begin{bmatrix} 0 & 1 \\ -b_m & -a_m \end{bmatrix}$. Subtract Eq. (30) from Eq. (32) to obtain the error equation of the reference model and the actual system response:

$$\dot{\mathbf{E}}_e = \mathbf{A}_m\mathbf{E}_e + (\mathbf{A}_m - \mathbf{A}_p(t))\mathbf{E}_p - \begin{bmatrix} 0 \\ w_p(t) \end{bmatrix} \tag{33}$$

where $\mathbf{E}_e = \begin{bmatrix} e_m - e & \dot{e}_m - \dot{e} \end{bmatrix}^\mathsf{T}$ is the state variable of the state equation about the total error. Assume that $\dot{a}_p(t) = -\frac{\chi}{\beta_1}\dot{e}$, $\dot{b}_p(t) = -\frac{\chi}{\beta_0}e$, $\dot{w}_p(t) = \frac{\chi}{\beta_2}$, where $\chi = p_2(e_m - e) + p_3(\dot{e}_m - \dot{e})$ So we can acquire the adjustment rules of $p(t)$, $d(t)$, $g(t)$ as follows:

$$\begin{cases} \chi(t) = -(\lambda_p e(t) + \lambda_v \dot{e}(t)) \\ d(t) = d_0 - \eta \int_0^t \chi \dot{e} dt \\ p(t) = p_0 - \mu_2 \int_0^t \chi e dt \\ g(t) = g_0 - \mu_1 \int_0^t \chi dt + \left(\frac{m_d}{k_d}\ddot{x}_e + \frac{b_d}{k_d}\dot{x}_e \right) \end{cases} \tag{34}$$

where λ_p, λ_v, η, μ_1, μ_2 are small positive numbers; d_0, p_0 and g_0 correspond to the initial values of three time - varying coefficients. Since the impedance control has ensured the continuity of the reference position, they can be taken as zero. According to the relationship between the end of flange and the environmental position :$e = f_r - f_e = -k_e(x - x_e)$, we will know that $\dot{e} = -k_e\dot{x}$. And then we can replace it with the differential signal of position and put the result into Eq. (34). In order to enhance the robustness of the system, the adaptive control rate above can be corrected by σ - correction method, the new control rate is:

$$\begin{cases} \chi(t) = -(\lambda_p e(t) + \lambda_d \dot{x}(t)) \\ d(t) = d_0 - \mu_3 \int_0^t \chi \dot{x}(t)dt - \sigma_3 \int_0^t d(t)dt \\ p(t) = p_0 - \mu_2 \int_0^t \chi e(t)dt - \sigma_2 \int_0^t p(t)dt \\ g(t) = g_0 - \mu_1 \int_0^t \chi dt + \sigma_1 \int_0^t g(t)dt \end{cases} \tag{35}$$

Substituting the controlled variables in Eq.(26) with Eq.(35), we can obtain the adaptive tracking control item:

$$\Delta x_f = g(t) + p(t)e(t) + d(t)\dot{x}(t) \tag{36}$$

We can insert the small correction amount of pose Δx_f to the adjustment item of the classical impedance controller so as to indirectly adjust the impedance parameter in real time.

D. *Lyapunov Stability Verification*

Construct quadratic energy function $V(\mathbf{E}, t)$ based on Lyapunov stability theorem:

$$\begin{aligned} V(\mathbf{E}_e, t) =& \frac{1}{2}\mathbf{E}_e^{\mathsf{T}}\mathbf{P}\mathbf{E}_e + \frac{1}{2}\mathbf{Z}^{\mathsf{T}}\mathbf{R}\mathbf{Z} \\ =& \frac{1}{2}\mathbf{E}_e^{\mathsf{T}}\mathbf{P}\mathbf{E}_e + \frac{1}{2}\beta_0(b_p(t) - b_m)^2 \\ &+ \frac{1}{2}\beta_1(a_p(t) - a_m)^2 + \frac{1}{2}\beta_2 w_p(t)^2 \end{aligned} \tag{37}$$

where $\mathbf{Z} = \begin{bmatrix} b_p(t) - b_m & a_p(t) - b_m & w_p(t) \end{bmatrix}^{\mathsf{T}}$,$\mathbf{R} = \mathrm{diag}(\beta_0, \beta_1, \beta_2)$, $\mathbf{P} = \left[\begin{smallmatrix} p_1 & p_2 \\ p_2 & p_3 \end{smallmatrix}\right]$. β_0, β_1, β_2 are positive constants; $\mathbf{P}$ is the singular positive definite symmetric matrix. It is obvious that the energy is positive. According to Lyapunov stability theorem, we can know the stability condition of the system: for a given real positive definite symmetric matrix $\mathbf{Q}$, there exists the matrix $\mathbf{P}$ discussed above which satisfies the expression $\mathbf{A}_m^{\mathsf{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_m = -\mathbf{Q}$. If we take the derivation of the energy function, we will get the following equation:

$$\begin{aligned} \dot{V}(\mathbf{E}_e, t) = \frac{1}{2}\mathbf{E}_e^{\mathsf{T}}\mathbf{Q}\mathbf{E}_e + (b_p(t) - b_m)(\chi e + \beta_0 \dot{b}_p(t)) \\ +(a_p(t) - a_m)(\chi \dot{e} + \beta_1 \dot{a}_p(t)) + w_p(t)(\beta_2 \dot{w}_p(t) - \chi) \end{aligned} \tag{38}$$

where $\chi = \mathbf{P}_2(e_m - e) + \mathbf{P}_3(\dot{e}_m - \dot{e})$, $\mathbf{P}_2$, $\mathbf{P}_3$ are the elements of the matrix $\mathbf{P}$.

Taking $\dot{a_p}(t) = -\frac{\chi}{\beta_1}\dot{e}$, $\dot{b}_p(t) = -\frac{\chi}{\beta_0}e$,$\dot{w}_p(t) = \frac{\chi}{\beta_2}$ into Eq.(38), we can attain the equation $\dot{V}(\mathbf{E}_e, t) = -\frac{1}{2}\mathbf{E}_e^{\mathsf{T}}\mathbf{Q}\mathbf{E}_e$.

It is apparent that $\dot{V}(\mathbf{E}_e, t) \leq 0$ and thus the condition is satisfied. Hence, the system based on Model Reference Adaptive Control (MRAC) is stable.

E. *The Simulation of Adaptive Impedance Control*

In MATLAB/SIMULINK, a fixed step length is used for simulation, and the sampling period is 0.01s. We use the Shikerongkuta method to simulate the step signal, sine signal and slope signal. By trying multiple sets of data, we find out a set of proper parameters of adaptive impedance controller. The parameters λ_p, λ_d, μ_1, μ_2, μ_3, σ_1, σ_2, σ_3 are taken as 1, 8, 0.01, 0.0001, 0.01, 0.1, 0.1, 0.1.

We do some special experiments. At beginning, the input signal of the system is 10N step signal. After 3 seconds, we change the input signal as sine whose amplitude is 2N, bias is 10N, and the frequency is 1Hz. After 6 seconds, the environmental stiffness changed to two times (1400N/m) as beginning. From the simulation results in Fig.3 and Fig.4, the conventional impedance control position tracking effect is better, but the force tracking effect is not good. During the tracking of sine signal, there is a relatively serious lag. When the environment stiffness changes, the response is slower and the overshoot is much larger. Compared with the impedance control, the adaptive impedance control system not only has less overshoot than conventional impedance control, but also fast response. In other words, the compliant system can be adjusted faster, when the environmental stiffness changes. As a result, adaptive impedance controller enhances robustness and reduces the dependence on the environment. Adaptive impedance controller makes the whole system more robust. Thus, it is an effective method to solve the problem of the flange uncertain environment.

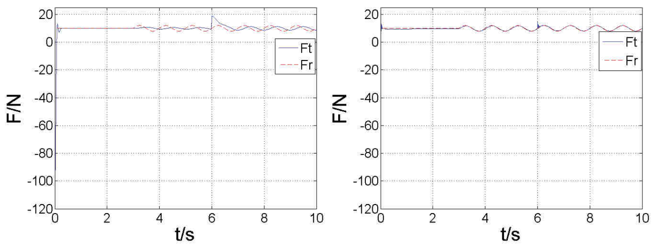

Fig. 3. Force response with impedance control and adaptive control

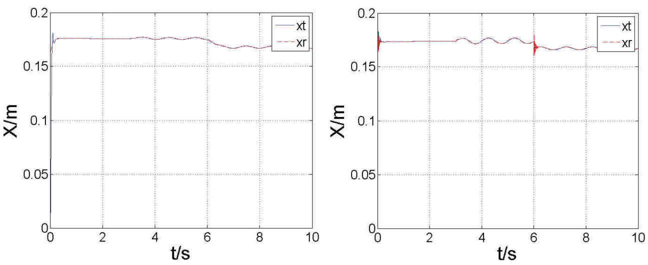

Fig. 4. Position response with impedance control and adaptive control

IV. Experiment on Pneumatic Compliant System

In this section, we perform an experiment on the pneumatic compliant system to verify the accuracy of our previous analysis. The experiment can be divided into the following two parts: one is force response experiment with impedance control and the other is with adaptive impedance control. By analyzing the results of the experiment, we can compare the control performance of two methods.

A. *Brief Introduction about The Experimental Platform*

Flange is the core of our experimental platform, since by controlling the reciprocating motion of the cylinder through two-position and five-way electronic magnetic valve, we can control output force and position of the end of the flange to make the compliant robot accomplish the designed task.

The flange can be separated into two parts. First is the cylinder which provides power for the whole system and the other are spline rails of the flange used for transmission. We use MQM series cylinder produced by SMC company. The platform consist of two-position and five-way electronic magnetic valve (A05PS25X-1P), differential pressure transmitter (PRE-U PS12001-080-010), and position sensor (MPS-0327STPO). The photograph and structure sketch of experimental platform are shown in Fig.5 and Fig.6.

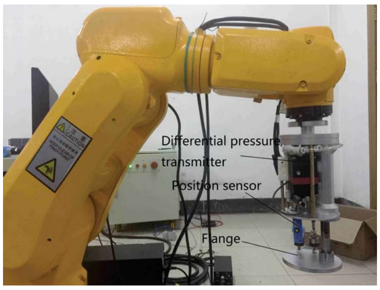

Fig. 5. Photograph of experimental platform

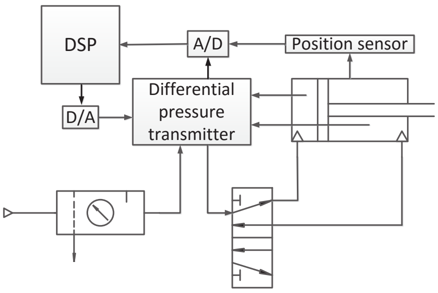

Fig. 6. Structure sketch of experimental platform

B. Force Response Experiment with Impedance Control And Adaptive Control

We control the compliant system by using two methods: impedance control and adaptive control. When the system is in a steady state, we change the external environment from wood to iron and observe the force response of the system with the two methods respectively. The results are shown in Fig. 7. By analysing the results of the experiment, we can know the fact that the overshoot and setting time of the adaptive control is less than that of impedance control. It means that adaptive control in compliant flange increase the robustness of the system, which make the compliant system adapt to the changing environment better. Therefore, we can draw the conclusion that the control performance of the adaptive control is better than that of the impedance control.

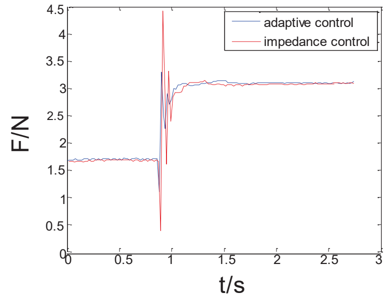

Fig. 7. Force response of the system in changing environment

V. Conclusion

This paper discusses about establishing the model and transfer function of the system. To control the system, two different kinds of control methods are proposed, impedance control and adaptive control. Next, we compare the control performance of these two methods by analyzing the results of simulation with MATLAB/SIMULINK and find out that adaptive control has better robustness. For further comparison and verification of the results of simulation, we build the experiment platform and do the environment variation experiment with two methods. By analysing the results of the experiment, we draw the conclusion that the two methods can both accomplish the task, however, adaptive control has better performance in uncertain environment, which accords with our judgement in pervious sections. In the near future, we will optimize our adaptive control algorithm for better system performance by referring to latest theories.

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