Lessons 2 and 3: Exploring Quadratic Relations and Properties of Graphs of Quadratics

By the end of this lesson you should be able to:

- Define key terms relating to quadratic functions
- List the main properties of Quadratic relations
- Solve problems using key features of quadratic graphs

Define the following terms:

Second difference:

the values you get when you do the finite differences twice. If they are all equal, the relation is quadratic. If they are not equal, it is not quadratic nor linear.

Quadratic:

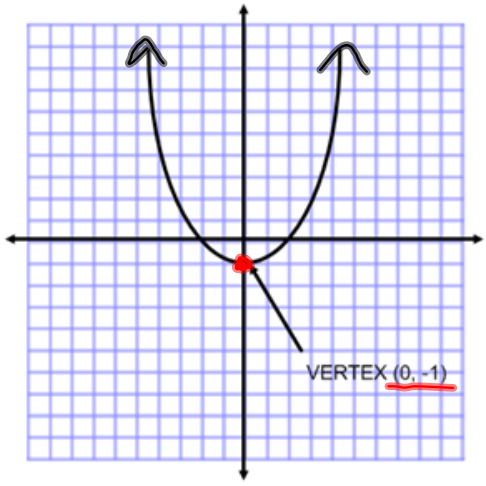
is the relationship you have when the second differences are constant. It is a second degree polynomial.

Degree:

is the highest exponent that appears in any term of the expanded form of a polynomial.

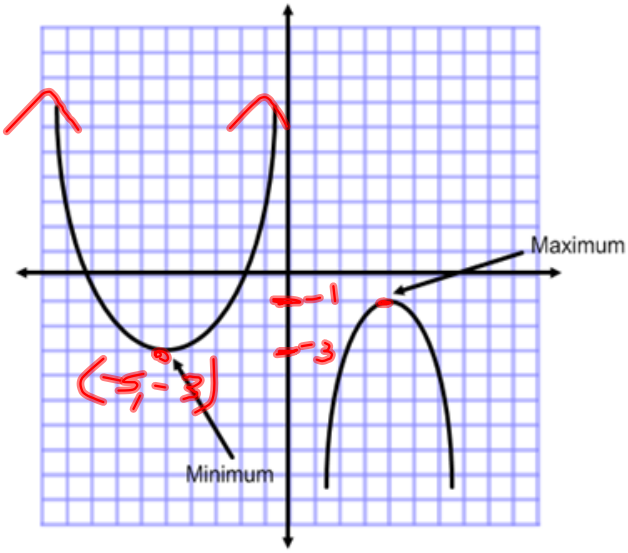
A parabola: is the graph of a quadratic relation in the shape of the letter U.

A vertex: is the point on the graph of a parabola with the greatest y-coordinate (if the graph opens down) or the least y-coordinate (if the graph opens up). It is usually represented by the coordinates (h, k).

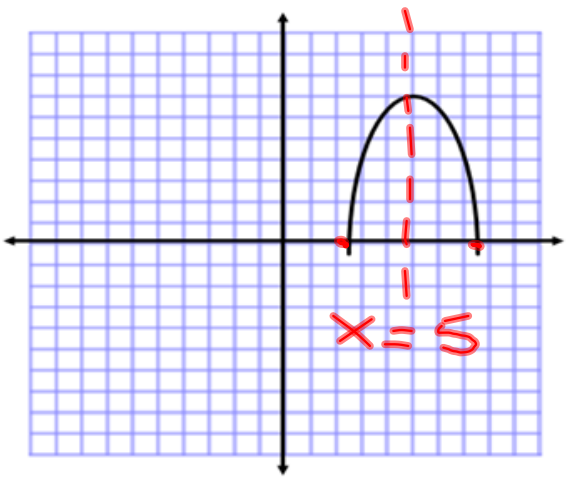


Maximum: is the greatest y value of the quantity being modeled, when the parabola opens down.

Minimum: is the smallest y value of the quantity being modeled, when the parabola opens upwards.



Axis of symmetry: is the vertical line that passes through the vertex. It is the perpendicular bisector of the segments joining any two points on the parabola that have the same y-coordinates.



Zeros or x-intercepts: are the points where the parabola crosses the x axis.

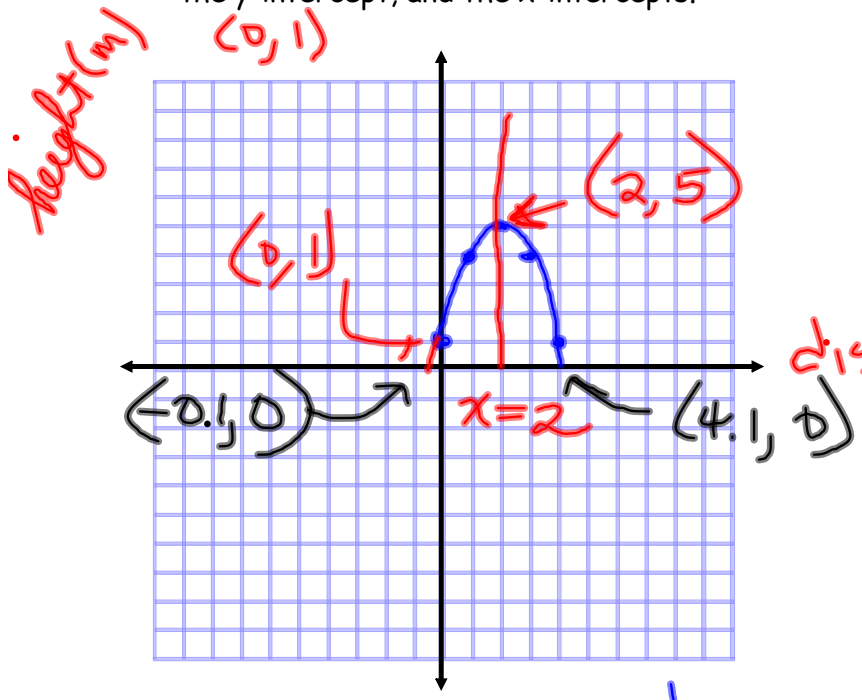
Example 1: Holly hits a baseball and the path of the ball is approximated by the equation $y = -x^2 + 4x + 1$, where x represents the horizontal distance travelled by the ball in metres and y represents the height of the ball in metres.

$y = -(1)^2 + 4(1) + 1$
$= -1 + 4 + 1$

Using a table of values, sketch this graph and then determine the following:

equation of the axis of symmetry, $x = 2$
the coordinates of the vertex, $(2, 5)$
the y-intercept, and the x-intercepts.

x	y
0	1
1	4
2	5
3	4
4	1



$y = -(2)^2 + 4(2) + 1$
$= -4 + 8 + 1$
$= 5$

$y = -(3)^2 + 4(3) + 1$
$= -9 + 12 + 1$
$= 4$

$y = (5)^2 + 4(5) + 1$
$= -25 + 20 + 1$
$=$

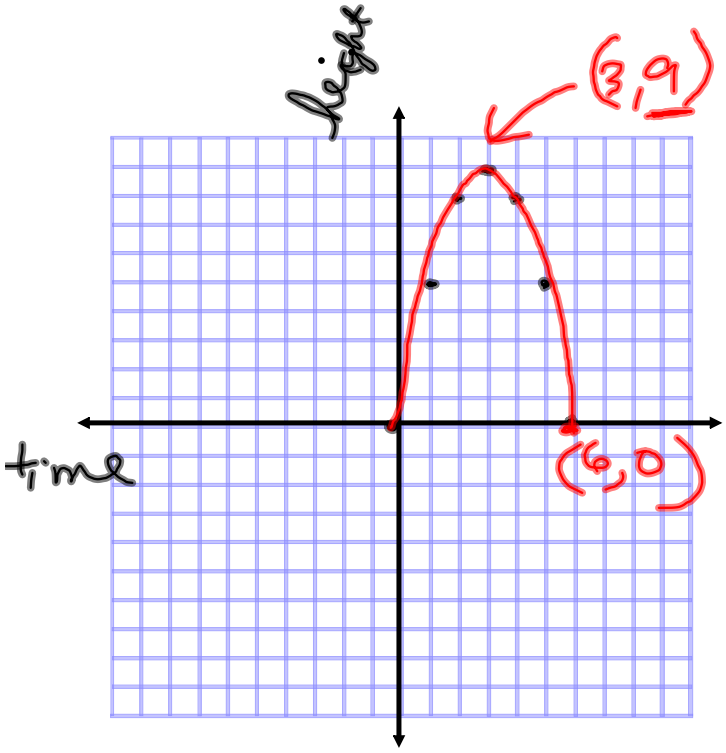
Example 2:

Bonnie kicks a soccer ball that follows the path $y = -x^2+6x$, where y is the height of the ball in metres and x is the time in seconds.

Find the maximum height in metres that the ball reaches and how long the ball in the air before it lands.

$y = -x^2 + 6x + 0$

x	y
0	0
1	5
2	8
3	9
4	8
5	5
6	0



∴ The max. height is 9m, and the ball is in the air for 6 s.