<u>Lesson 4:Completing the square!</u>

By the end of this lesson you should be able to

- write the vertex form of a parabola from the standard form by completing the square
- use this form to determine the vertex

What is 'completing the square'

- This is the process of re-writing a quadratic equation (process of going from standard to vertex form)

- $y=ax^2 + bx + c \longrightarrow y=a(x-h)^2 + k$

$a(x-h)(x-h) + k$

This is accomplished by creating a perfect square within the expression and then factoring it.

To do this you MUST factor out only the coefficient of x2 from the terms a and b.

Then b is divided by 2 and squared.

This value of c must then be added and subtracted to create an equivalent expression

EXAMPLE 1: Completing the square algebraically

- To begin we always factor a out of the expression.
- Then using the "new" value of b, we divide the b term by 2 and square it.

$y = -2x^2 + 12x - 11$

$y = -2(x^2 - 6x) - 11$

Partial factor the first two terms (factor "a" only)

$y = -2(x^2 - 6x + 9 - 9) - 11$ $\quad -9 \times -2$

b term is: $\left(-\frac{b}{2}\right)^2 = 9$
add and subtract this term.

$y = -2(x^2 - 6x + 9) + 18 - 11$

Remove the negative value from the brackets and remember to multiply by the a value.

$y = -2(x-3)(x-3) + 7$

M: 9
A: -6
N: -3, -3

Factor the trinomial inside the brackets.

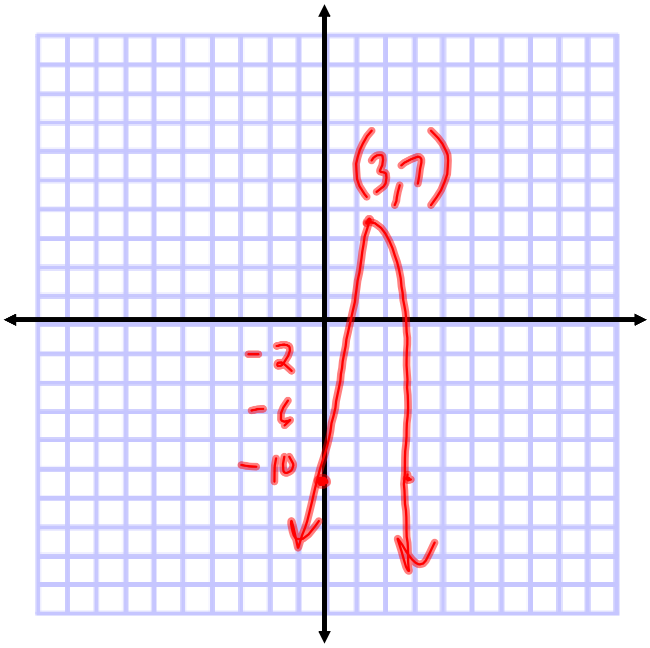
$y = -2(x-3)^2 + 7$

Combine the constants outside the brackets.

vertex (3, 7)

∴ the vertex form is:

Graph the quadratic equation from the previous slide



Vertex: (3, 7)

Y-intercept: (0, -11)

Example 2: finding the maximum or minimum value

If a question asks you to find the maximum or minimum, it is looking for the y value at the vertex. If you are given standard form, the quickest way to find your answer is by completing the square.

Does the following quadratic equation have a minimum or maximum value, and what is it?

$y = 8x^2 - 96x + 15$

min value: find vertex

$y = 8(x^2 - 12x) + 15$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-12}{2}\right)^2 = 36$

$y = 8(x^2 - 12x + 36 - 36) + 15$

$y = 8(x^2 - 12x + 36) - 288 + 15$

$y = 8(x-6)(x-6) - 273$

$(6, -273)$

$\therefore y = 8(x-6)^2 - 273$

$\therefore$ The min value -273

EXAMPLE 3:

Water sprayed from a garden hose follows a path modelled by $h= -4.9t^2 - 19.6t + 0.5$, where h is the height in metres and t is the time is seconds that the water is in the air. What is the maximum height of the water?

$y= -4.9(t^2+4t)+0.5$

$\left(\frac{4}{2}\right)^2 = (2)^2$

$= 4$

$y=-4.9(t^2+4t+4-4)+0.5$

$y=-4.9(t^2+4t+4)+0.5+19.6$

$y=-4.9(t+2)(t+2)+20.1$

vertex form: $y=-4.9(t+2)^2+20.1$

$\therefore$ The max height is 20.1m