Lesson 7: Factoring a Perfect Square Trinomial and a Difference of Squares

By the end of today's lesson you should be able to:

- Differentiate between a difference of squares and a perfect square trinomial
- Factor a perfect square trinomial using decomposition and using reasoning
- Factor a difference of squares

When a perfect square trinomial (three terms - $ax^2 + bx + c$) is factored, it has two identical binomial factors.
For example:

$x^2 + 8x + 16 = (x+4)(x+4)$ These two binomials are exactly the same

A perfect square trinomial can be in two forms:

$a^2 + 2ab + b^2$ OR $a^2 - 2ab + b^2$

$=(a+b)(a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

$=(a-b)(a-b)$

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

A difference of squares involves the subtraction of 2 squares (two terms to the power of 2).

$a^2 - b^2$ is a difference of squares

$a^2 - b^2 = (a+b)(a-b)$

The two factors will ALWAYS be the sum of the square roots and the difference of the square roots.

$4x^2 - 9$

$= (2x + 3)(2x - 3)$

Example 1: factoring a perfect square using decomposition method (from last class)

$9x^2 - 24x + 16$

$= 9x^2 - 12x - 12x + 16$

$= 3x(3x-4) - 4(3x-4)$

$= (3x-4)(3x-4)$

$= (3x-4)^2$

* Find two numbers that add up to -24 and multiply to 144.

$(a)(c) = 144$

*Recall that (a)(c) gives you the value that you must factor to find the values that add to b.

144

−12 −12

Recognizing a perfect square trinomial:

What do you notice about the a and c terms in the quadratic expression? Perfect squares

Choose the correct answer:

Which expressions are perfect square trinomials?

A $16a^2 + 24a + 9$ ✓

B $7x^2 + 19x - 6$ ✗

C $4p^2 - 20p + 25$ ✓

D $9n^2 - 16$ ✗

Example 2: Factor

$16x^2 + 40x + 25$

$= (4x + 5)(4x + 5)$

$= (4x + 5)^2$

Use reasoning to answer

Factor by inspection:
If the a and c terms are both perfect squares, this is likely a perfect square trinomial. Using the square roots of the a and the c terms, try to determine what the factored form of $9x^2 - 42x + 49$ will be.

$(3x-7)(3x-7) = (3x-7)^2$

Check to make sure:

Example 3: Factoring a Difference of Squares

A difference of squares is not a trinomial. There are only two terms and they are both perfect squares. Since it is a DIFFERENCE of squares, there will be a minus sign between them.

a) $x^2 - 49$

Both x^2 and 49 are perfect squares

$(x+7)(x-7)$

b) $64x^4 - 16y^2$

$= 16(4x^4 - 1y^2)$

Which of the following are examples of a difference of squares?

A $4x^2 - 16$

B $9p^4 - 16x^2$

C $25x - 9$

D $81x^2 - 25y^2$

Factor the differences of squares:

A $= 4 (x^2 - 4)$
$= 4 (x-2)(x+2)$

B $= (3p^2-4x)(3p^2+4x)$

D $=(9x-5y)(9x+5y)$