

Lesson 4: Relating a Quadratic Relation in factored form to its graph

By the end of this lesson you should be able to:

- Relate the factored form of an equation to its zeros (x-intercepts)
- Use the factored form to graph an equation
- Use a graph to find the factored form of an equation

The factored form of a quadratic is written as: $y = a(x-r)(x-s)$, where $a \neq 0$

This form can be useful when graphing the relation because we can easily find the zeroes. To find the zeroes we set the brackets equal to zero and solve for x.

Example 1:

find the y-intercept, zeroes, axis of symmetry and vertex from the equation

$$y = 2(x-4)(x+2)$$

to find the y-intercept, just set $x=0$

y-int:

$$y = 2(0-4)(0+2)$$

$$y = 2(-4)(2)$$

$$y = -16$$

$$(0, -16)$$

x-int

$$y = (x-4)$$

$$0 = x-4$$

$$4 = x$$

$$(4, 0)$$

$$y = (x+2)$$

$$0 = x+2$$

$$-2 = x$$

$$(-2, 0)$$

$$a \text{ of } s: \frac{x_1 + x_2}{2}$$

$$= \frac{4 + (-2)}{2}$$

$$= \frac{2}{2} = 1$$

$$a \text{ of } s: \boxed{x=1}$$

vertex: find y when $x=1$

$$y = 2(x-4)(x+2)$$

$$= 2(1-4)(1+2)$$

$$= 2(-3)(3)$$

$$y = -18 \text{ @ } x=1$$

$$\therefore \text{vertex } (1, -18)$$

Find the x-intercepts or "zeros":

Set each factor equal to zero and then solve for x . This will give you two values of x .

Axis of Symmetry:

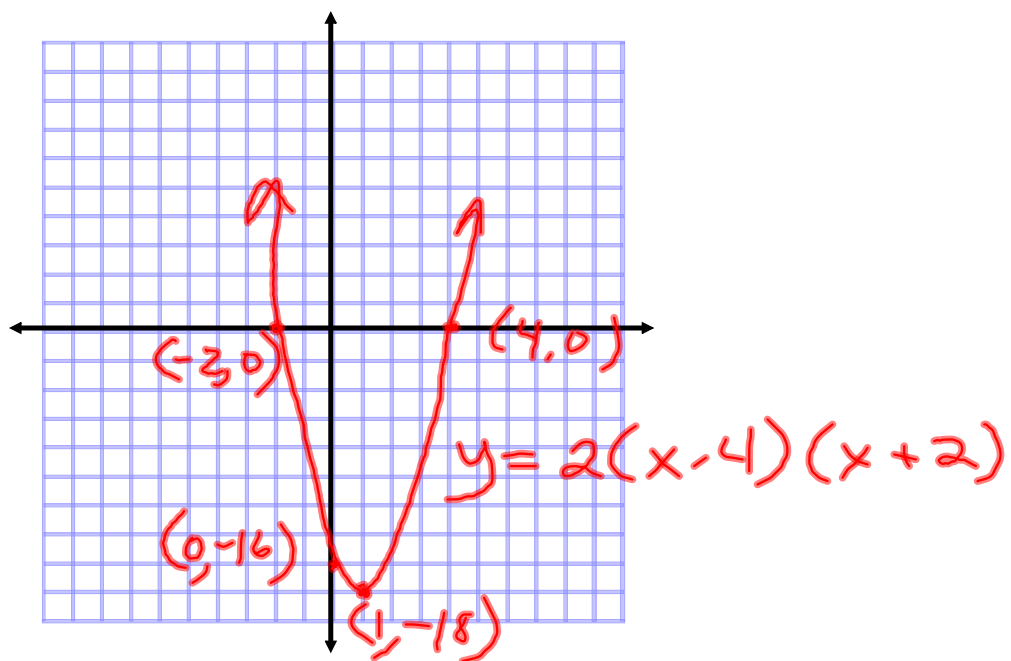
To find take the average of the two zeros added together

The Vertex:

The vertex lies on the axis of symmetry. This means that it will be the y -value that occurs when x equals the axis of symmetry.

To find, just plug $x = 1$ into the equation.

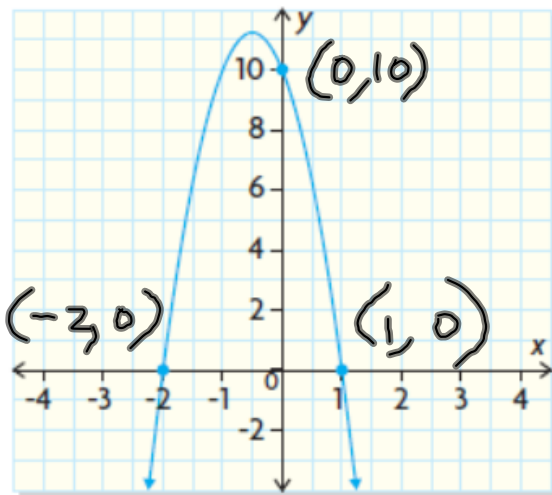
Use the y-intercept, zeros, and vertex to graph the function:



Example 2:

use a graph to find the equation

Find the zeros by looking at the graph. The zeros occur at:



Plug these zeros into the factored form: $y = a(x-r)(x-s)$

$$y = a(x - (-2))(x - 1)$$

$$y = a(x + 2)(x - 1)$$

Now, find the value of a . To do this we use the y -intercept.

Substitute (0, 10) into the above equation:

$$y = a(x + 2)(x - 1)$$

$$10 = a(0 + 2)(0 - 1)$$

$$10 = a(2)(-1)$$

$$10 = -2a$$

Therefore, the equation is: $y = -5(x + 2)(x - 1)$

$$\rightarrow \frac{10}{-2} = \frac{-2a}{-2}$$

$$\boxed{-5 = a}$$