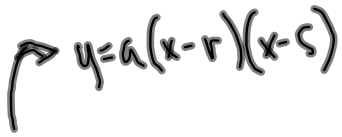
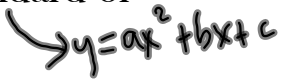
Lesson 7: Solving problems using quadratic equations

By the end of this lesson you should be able to
- Change an equation from standard to vertex form
- Solve word problems using an equation

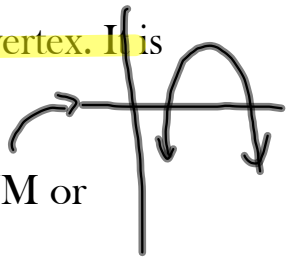


- Quadratic relations can be expressed in factored, standard or vertex form.



- The value of a is the same in all 3 forms.

- In vertex form: $y = a(x-h)^2 + k$, h is the x value at the vertex. It is also the AXIS OF SYMMETRY.



- k is the y value at the vertex. It is also the MAXIMUM or MINIMUM value.

- Depending on what the question asks you to solve for, you may want to put the equation in a different form

Example 1: Finding the vertex form (from standard form)

Change $y = 4x^2 + 20x + 25$ to vertex form

STEP 1: Rewrite the equation in factored form

$y = 4x^2 + 20x + 25$

M: $(a \times c)$ 100
A: 20
N: 10, 10

$= 4x^2 + 10x + 10x + 25$

$= 2x(2x + 5) + 5(2x + 5)$

$= (2x + 5)(2x + 5)$

STEP 2: Find the zeros

$(2x + 5) = 0$

$2x + 5 = 0$

$2x = -5$

$\frac{2x}{2} = \frac{-5}{2}$

$x = -2.5$

$(2x + 5) = 0$

$x = -2.5$

∴ One x-intercept

STEP 3: Find the axis of symmetry

$\text{aofs} = \frac{-2.5 + (-2.5)}{2}$

$= \frac{-5}{2}$

$x = -2.5$

STEP 4: Use the axis of symmetry to find y at the vertex

$y = (2x+5)(2x+5)$

$= (2(-2.5)+5)(2(-2.5)+5)$

$= (-5+5)(-5+5)$

$= 0$

vertex $(-2.5, 0)$

STEP 5: The value of a

$a = 4$ because it is the same in all 3 forms.

STEP 6: Write your equation using a, h and k

$y = a(x-h)^2 + k$

$y = 4(x-(-2.5))^2 + 0$

$y = 4(x+2.5)^2$

$h = -2.5$

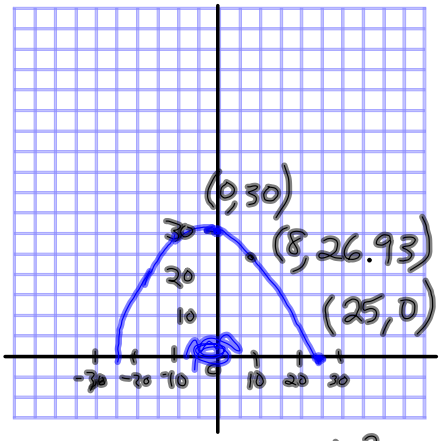
$k = 0$

$a = 4$

Example 2: (from page 295)
The underside of a bridge forms a parabolic arch. The arch has a maximum height of 30 m and a width of 50 m. Can sailboat pass under the bridge, 8 m from the axis of symmetry, if the top of its mast is 27 m above the water? Justify your solution.

Sketch the parabola to help you solve:

(x, y)
(8, y)



$h = 0$
$k = 30$
$a = ?$

other point
(25, 0)

$y = a(x - 0)^2 + 30$

$y = ax^2 + 30$

$0 = a(25)^2 + 30$

$0 = 625a + 30$ -30

$\frac{-30}{625} = \frac{625a}{625}$

$-0.048 = a$

∴ vertex form: $y = -0.048x^2 + 30$

sub 8 in for x: $y = -0.048(8)^2 + 30$

$= -0.048(64) + 30$

$= -3.072 + 30$

$= 26.928$

∴ The sailboat is too high!

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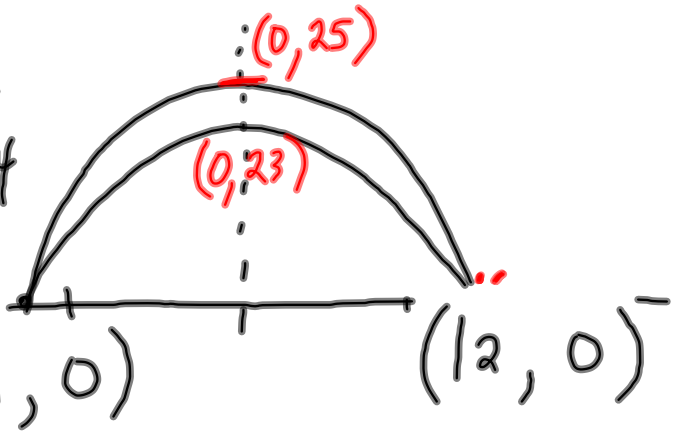

$h = 0$
$K = 25$
$a = ?$
other point $(12,0)$

$y = ax^2 + 25$

$0 = a(12)^2 + 25$

$0 = 144a + 25$

$\frac{-25}{144} = \frac{144a}{144}$

$-0.17 = a$

$y = -0.17x^2 + 25$