

## Lesson 7: Solving systems by elimination

By the end of this lesson you should be able to:

- Explain how to use the elimination method to solve a linear system
- Use the elimination method to solve a linear system (find the POI)



To solve systems using substitution, a good first step is to...

- A substitute the expression into the equation.
- B isolate one of the variables.
- C solve for the second variable.
- D substitute the second variable for the first.

2

Solving  $x + 4y = 2$  and  $x - 2y = 8$  by substitution, it is determined that  $x = 6$ . What is the value of  $y$ ?

Substitute  $x = 6$  into either equation

$$x - 2y = 8$$

$$(6) - 2y = 8$$

$$- 2y = 8 - 6$$

$$\underline{-2y = 2}$$

$$\underline{-2} \quad \underline{-2}$$

$$y = -1$$

Solving using the ELIMINATION METHOD allows you to remove a variable from the system completely, making the step to find your first variable quicker than the substitution method.

To use the elimination method:

- 1) The variable that you want to eliminate must be in the same position in both equations
- 2) You must make the coefficient of the variable you want to eliminate the same by multiplying the entire equation by a constant greater than 0.
- 3) You either ADD the equations or SUBTRACT. Add if the coefficients have opposite signs, and subtract if the signs are the same.
- 4) Once you have solved for one variable, plug it into either equation to solve for the other variable.
- 5) Verify your answers. (left side/right side check)

What is the value of  $s$  for  $4r + 2s = 6$  and  $4r - 2s = 18$ , using elimination?

System:

$$\begin{array}{r} 4r + 2s = 6 \\ - \quad 4r - 2s = 18 \\ \hline \end{array}$$

$$\cancel{4r} + 4s = -12$$

$\frac{4}{4} \quad \frac{4}{4}$

$$s = -3$$

### EXAMPLE 2

Use elimination to solve a system where the coefficient is not the same for a given variable in both equations

$$\begin{cases} 13x + 2y = 47 \\ 8x + 29y = 140 \end{cases} \begin{matrix} \times 29 \\ \times 2 \end{matrix} \rightarrow \begin{array}{r} 377x + 58y = 1363 \\ - 16x + 58y = 280 \\ \hline 361x = 1083 \\ \hline x = \frac{1083}{361} \end{array}$$

$$\boxed{x = 3}$$

Find y:  $13x + 2y = 47$   
 $13(3) + 2y = 47$   
 $39 + 2y = 47$

$$\frac{2y}{2} = \frac{8}{2}$$

$\therefore$  The POI is  $(3, 4)$ .

$$\boxed{y = 4}$$

①  $13x + 2y = 47$   
 $13(3) + 2(4) = 47 \checkmark$

②  $8x + 29y = 140$   
 $8(3) + 29(4) = 140 \checkmark$

### Example 3:

Solve a system with rational (fraction) coefficients

This is the same type of question as the previous, it just looks complicated because of the fractions

To remove the denominator, simply multiply the whole equation by a number that both 8 and 11 go into

$$88 \left( \frac{x}{11} - \frac{y}{8} = -2 \right) \longrightarrow 8x - 11y = -176$$

$$8 \left( \frac{x}{2} - \frac{y}{4} = 3 \right) \longrightarrow 4x - 2y = 24$$

To remove the denominator, simply multiply the whole equation by a number that both 2 and 4 go into

$$\begin{array}{rcl} 8x - 11y & = & -176 \\ 2(4x - 2y = 24) & \rightarrow & 8x - 4y = 48 \\ \hline & & -7y = -224 \\ & & \underline{-7} \quad \underline{-7} \\ & & y = 32 \end{array}$$

Find  $x$  by plugging  $y$  into

$$4x - 2y = 24$$

$$4x - 2(32) = 24$$

$$4x - 64 = 24$$

$$4x = 24 + 64$$

$$\frac{4x}{4} = \frac{88}{4}$$

$$x = 22$$

$\therefore$  The POI  
is  $(22, 32)$