# Piecewise Exponential Distributional Regression Model for Survival Analysis

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## 1. Introduction

- The Piecewise Exponential Model (PEM) can be utilised to approximate many distributions found in a survival setting, allowing for variation of hazard rates between intervals, while keeping hazards constant within intervals.

$$h(t) = \begin{cases} \lambda_1 & 0 \le t < t_{(1)} \\ \lambda_2 & t_{(1)} \le t < t_{(2)} \\ \vdots & \\ \lambda_m & t_{(m-1)} \le t < t_{(m)} \end{cases}$$

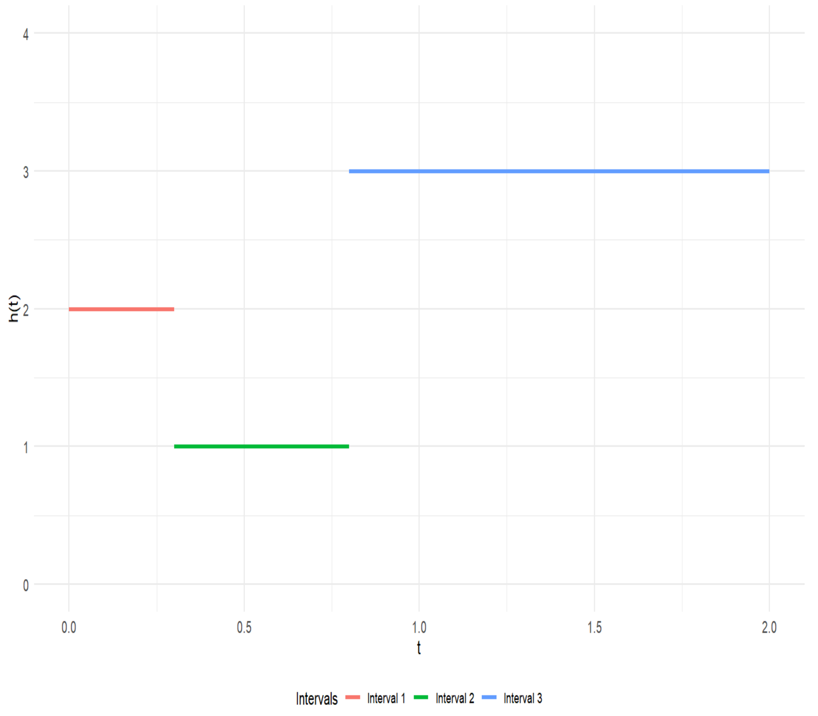

**Figure 1.** Piecewise exponential hazard, where Interval i, $I_i, = [t_{(i-1)}, t_{(i)})$

## 2. Methods

- PEM can be extended to include covariates.

$$\lambda_m = e^{(\beta_{0m} + \beta_1 x_1 + \ldots + \beta_p x_p)}$$

- The above link reflects the variation between intervals via $\beta_{0m}$ while ensuring covariate effects remain constant across intervals.
- The log-likelihood from this extension is as follows:

$$log(L) = \sum_{m=1}^{M} \sum_{i \epsilon I_m} \left\{ \delta_i (B_m X_i) - t_i e^{B_m X_i} - \sum_{j=1}^{m-1} t_{(j)} \left( e^{B_j X_i} - e^{B_{j+1} X_i} \right) \right\}$$

with $B_m = (\beta_{0m}, \beta_1, \ldots, \beta_p)$

- Parameters were optimised using the Newton-Raphson method, implemented through nlm in the R package 'stats'.

## 3. Simulation Study

- A simulation study was carried out to test the performance of the proportional hazard PEM and compare with the cox model.
- The data was simulated using a piecewise distribution with 3 true intervals (see figure 1), and 3 covariate effects.
- The PEM was fitted using 10 intervals, designed to keep the number of events consistent across intervals.

| | True $\beta_j$ | Mean $\hat{\beta}_j$ | True SE($\hat{\beta}_j$) | Est SE($\hat{\beta}_j$) | Coverage |
|---|---|---|---|---|---|
| $\beta_1$ | 0.5 | 0.51 | 0.07 | 0.07 | 0.95 |
| $\beta_2$ | -0.5 | -0.50 | 0.06 | 0.07 | 0.95 |
| $\beta_3$ | 0.01 | 0.01 | 0.01 | 0.01 | 0.91 |

**Table 1.** Mean, Standard error and coverage of covariate effects calculated from 100 simulations

- The covariate estimates were also compared to the estimates obtained from the cox model.

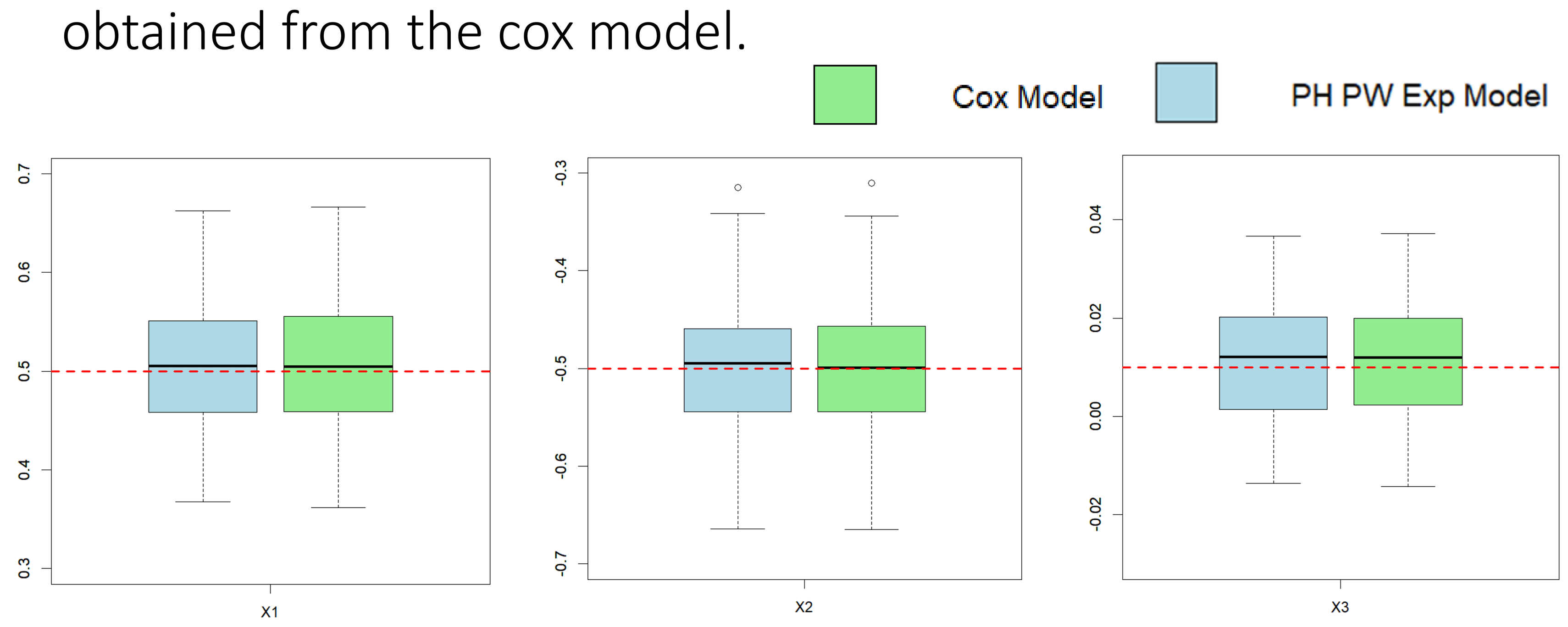


**Figure 2.** Boxplots of Covariate estimates, with true vales shown by - - -

## 4. Case Study

- Next, we compare the performance of both models on a set of lung cancer data from the survival package in R.
- Significant covariates included both sex and ECOG score.

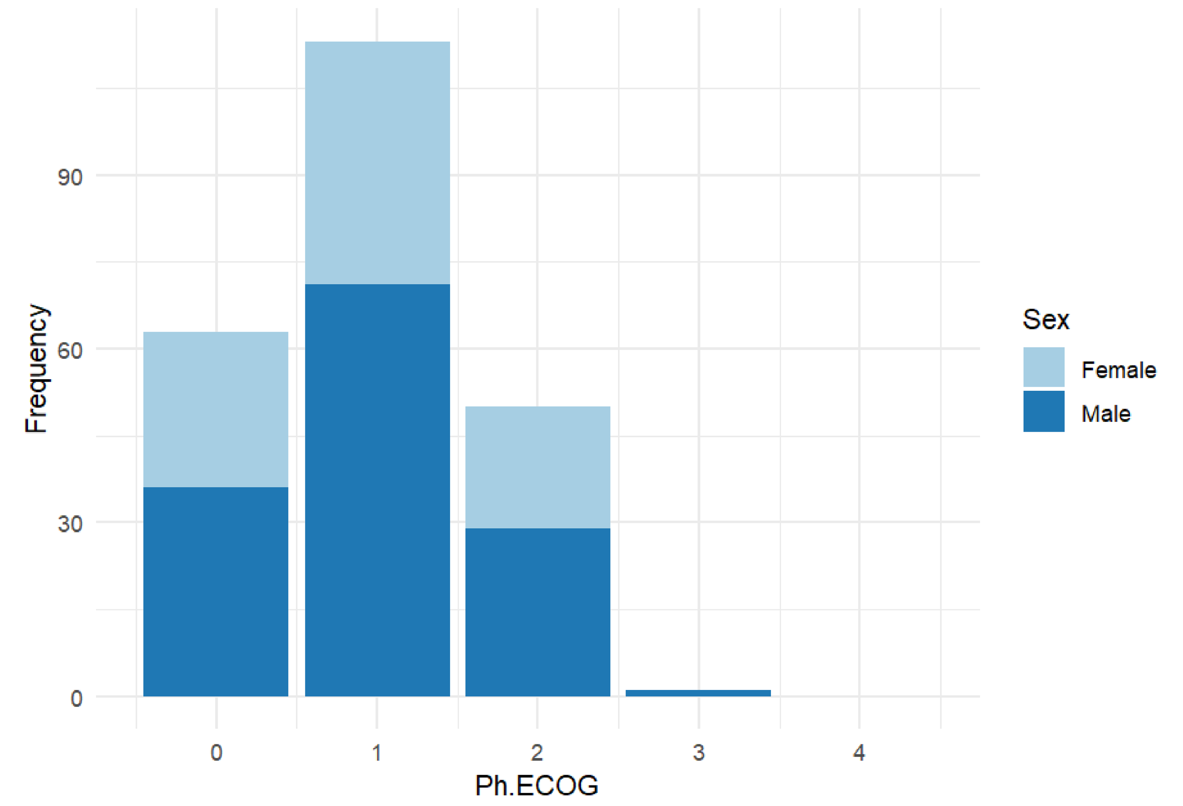


**Figure 3.** Cancer Ph.ECOG and Sex Frequency

- The resulting Cox Model and PEM were compared to the Kaplan-Meier curve for various subsets of the data.

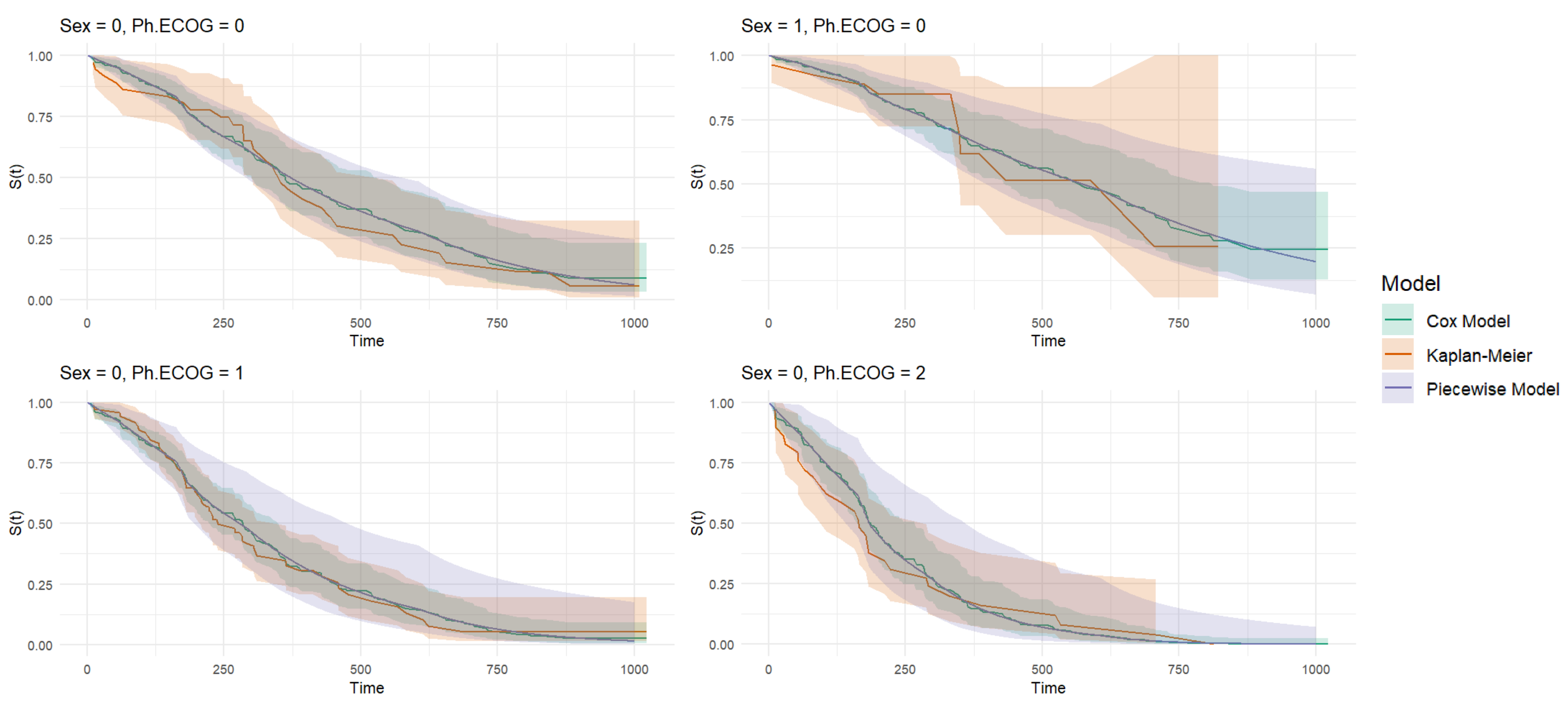


**Figure 4.** Comparison of Survival Curves

## 5. Future Work

- We can further extend PEM by allowing our covariate effects to change between intervals. This can be achieved by redefining $B_m = (\beta_{0m}, \beta_{1m}, \ldots, \beta_{pm})$.
- Further exploration on how the number of intervals affect this model.

## 6. Acknowledgements

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## 7. References

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