# Analysing a Slot Machine Bonus Game

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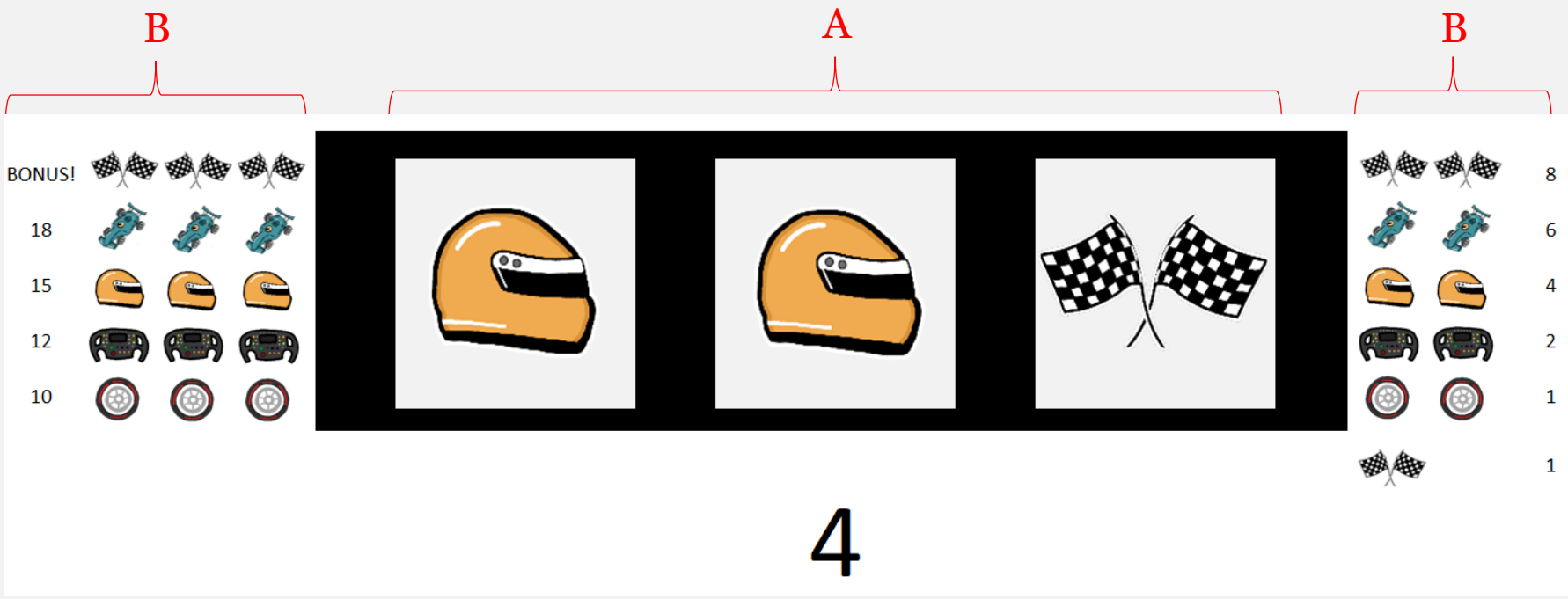



A: Reels

B: Winning Combinations

C: Pay-out Amount

Slot Machines

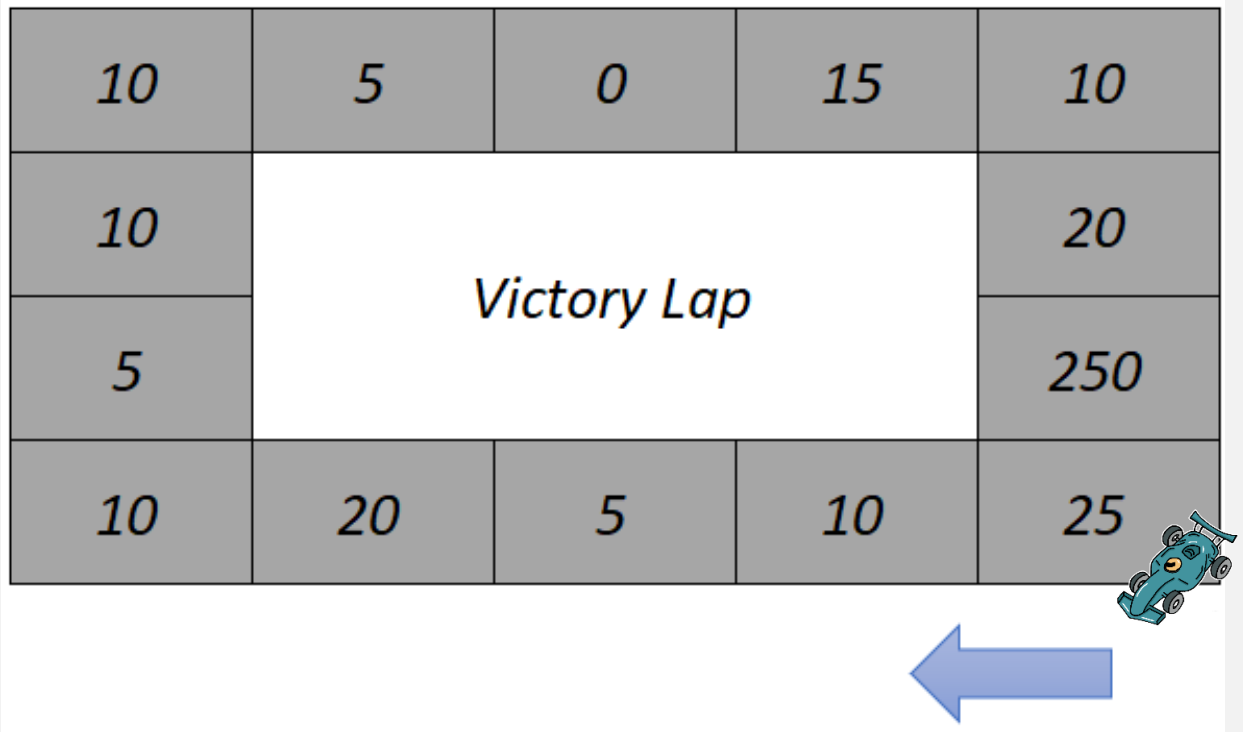
**Problem:**

- Create a bonus game
- Desired expected pay-out: 100

| COMBOS | PAYS | HITS | CONTRIBUTION |
|---|---|---|---|
| 5 5 5 | 100 | 4 | 14.58% |
| 4 4 4 | 18 | 6 | 3.94% |
| 3 3 3 | 15 | 12 | 6.56% |
| 2 2 2 | 12 | 18 | 7.87% |
| 1 1 1 | 10 | 60 | 21.87% |
| 5 5 | 8 | 24 | 7.00% |
| 4 4 | 6 | 22 | 4.81% |
| 3 3 | 4 | 72 | 10.50% |
| 2 2 | 2 | 66 | 4.81% |
| 1 1 | 1 | 220 | 8.02% |
| 5 | 1 | 168 | 6.12% |
| | Totals: | 672 | 96.06% |

**Rules:**

- Start on the 25 tile
- Roll a 6-sided dice to move
- You win the amount shown on every square you land on
- Game ends after 4 rolls
- Or, game ends if you land on 0




Bonus Game

Dice Roll 1: 

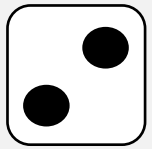
Winnings: 25 + 20 = 45

Dice Roll 2: 

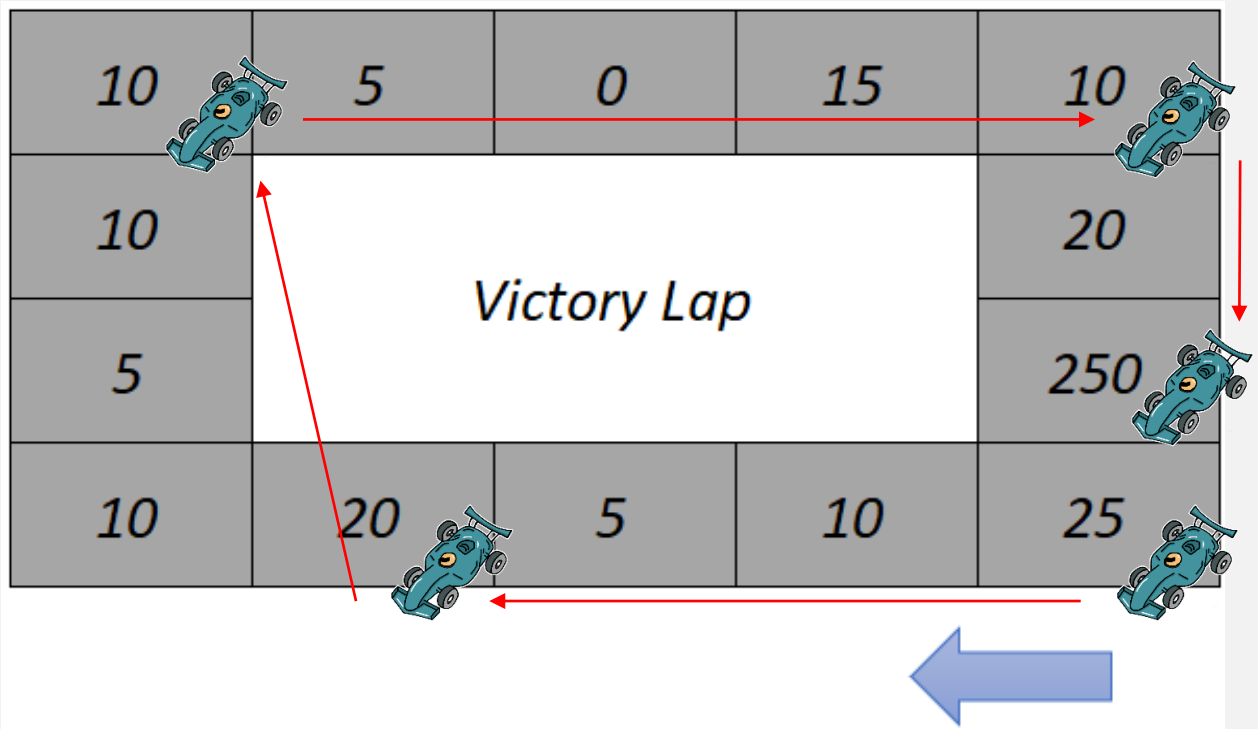
Winnings: 45 + 10 = 55

Dice Roll 3: 

Winnings: 55 + 10 = 65

Dice Roll 4: 

Winnings: 65 + 250 = 315

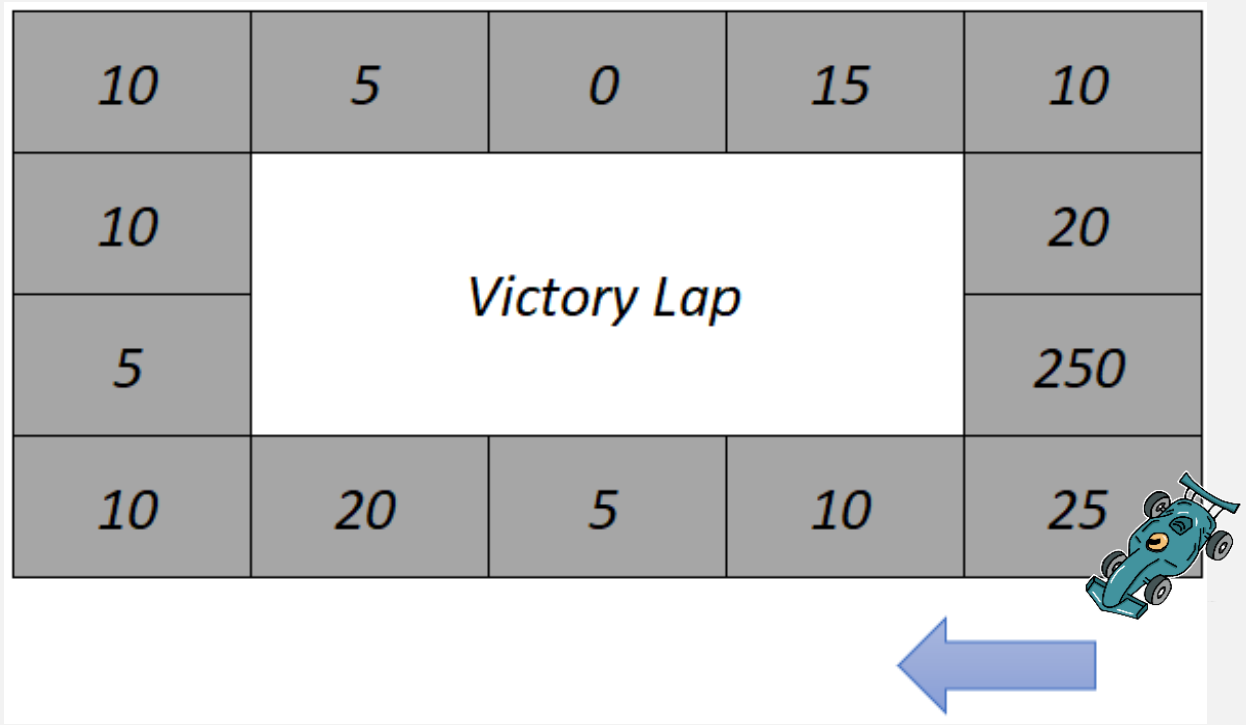



Example Game

# Markov Chains

- Used to find probabilities of sequences of events

- The probabilities of the next event depend <u>only</u> on the current event.

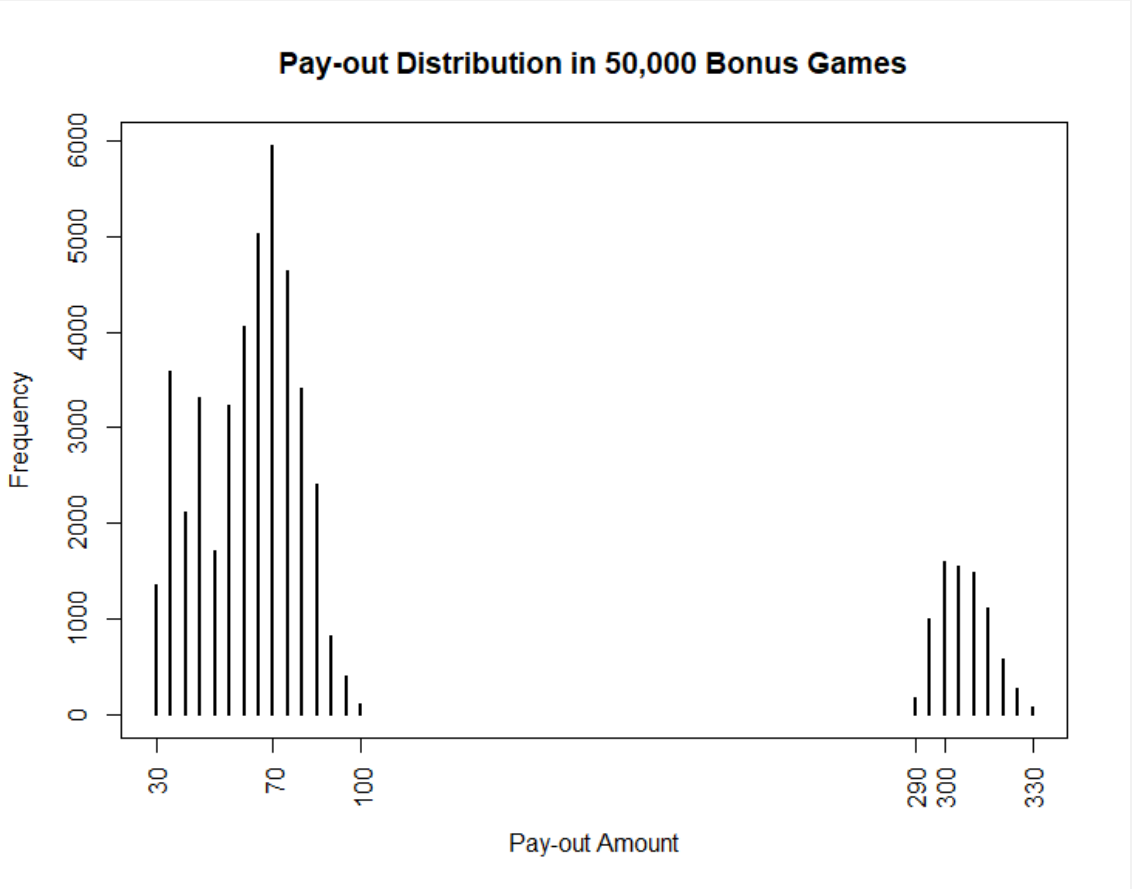


## Model Making

- For each tile, found the probabilities of landing on every other tile
- Found the probability of being on each tile after 1 step
- Iterated over 4 steps
- Tweaked model to achieve desired pay-out

- Built bonus game in R

- Performed Monte Carlo Simulation to check expected value of pay-outs




Check in R

# Appendix

## Bonus Game Code:

```
13  # Creating Bonus Game and Checking its Expected Value
14
15  tilePayouts <- c(25,10,5,20,10,5,10,10,5,0,15,10,20,250)
16  dice <- c(1:6)
17  totalBonusWinnings <- c()
18  for (sim in 1:50000) {
19    bonusWinnings <- 25
20    currentTile <- 0
21    for(turn in 1:4){
22      diceRoll <- sample(dice,1,TRUE)
23      currentTile <- (currentTile + diceRoll) %% 14
24      bonusWinnings <- bonusWinnings + tilePayouts[currentTile+1]
25      if(tilePayouts[currentTile+1] == 0) break
26    }
27    totalBonusWinnings <- c(totalBonusWinnings,bonusWinnings)
28  }
29  expectedBonusPayout <- sum(totalBonusWinnings)/50000
30  expectedBonusPayout
```

# Appendix

## Markov Chain Transition Matrix:

**Transition Matrix**

| | | Next Tile | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Current Tile | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 |
| | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 |
| | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 |
| | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |
| | 8 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |
| | 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 10 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 | 1/6 |
| | 11 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 | 1/6 |
| | 12 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1/6 |
| | 13 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## Pay-out Calculation:

| Tile | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial State | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Turn 1 | 0 | 0.16667 | 0.16667 | 0.16667 | 0.16667 | 0.16667 | 0.16667 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Turn 2 | 0 | 0 | 0.02778 | 0.05556 | 0.08333 | 0.11111 | 0.13889 | 0.16667 | 0.13889 | 0.11111 | 0.08333 | 0.05556 | 0.02778 | 0 |
| Turn 3 | 0.05093 | 0.02778 | 0.02778 | 0.01852 | 0.01852 | 0.02778 | 0.0463 | 0.06944 | 0.09722 | 0.22685 | 0.10648 | 0.10648 | 0.09722 | 0.0787037 |
| Turn 4 | 0.08102 | 0.0733 | 0.07793 | 0.06481 | 0.05015 | 0.03704 | 0.02855 | 0.02778 | 0.03472 | 0.27315 | 0.04321 | 0.05787 | 0.07099 | 0.07947531 |
| Sum of Probabilities | 1.13194 | 0.26775 | 0.30015 | 0.30556 | 0.31867 | 0.34259 | 0.3804 | 0.26389 | 0.27083 | 0.61111 | 0.23302 | 0.21991 | 0.19599 | 0.15817901 |
| Tile Payout | 25 | 10 | 5 | 20 | 10 | 5 | 10 | 10 | 5 | 0 | 15 | 10 | 20 | 250 |
| Tile Contribution | 28.2986 | 2.67747 | 1.50077 | 6.11111 | 3.18673 | 1.71296 | 3.80401 | 2.63889 | 1.35417 | 0 | 3.49537 | 2.19907 | 3.91975 | 39.5447531 |
| Expected Payout | 100.444 | | | | | | | | | | | | | |