# Political Uncertainty

Kristy Buzard
March 23, 2015

On Feb. 12, Sebastian and I agreed to focus efforts on finding a base model to facilitate empirical identification. I am pursuing Groseclose & Snyder (1996), "Buying Supermajorities," APSR

- For each legislator $i$, $v(i) = u_i(x) - u_i(s)$, measured in money; this is the reservation price of $i$
  - $x$ is an alternative policy proposal; $s$ is the status quo
  - WLOG, label legislators so that $v(i)$ is a non-increasing function
  - Note legislators only have preferences over how they vote, not over which alternative wins
- There are two vote buyers; each prefers to minimize total bribes paid while passing his preferred policy, but each would prefer to concede the issue rather than pay more than his WTP
  - $A$ prefers $x$; $W_A$ is $A$'s willingness to pay (WTP) for $x$ measured in money
  - $B$ prefers $s$; $W_B$ is $B$'s WTP for $s$
- Bribe offer functions: $a(i)$ and $b(i)$ are $A$ and $B$'s offers to $i$. Legislators take these bribe offers as given and then vote for the alternative that maximizes their payoff
- A moves first; $a(i)$ is perfectly observable to $B$ when he moves
- Goal: characterize SPNE in pure strategies
  - Assume unbribed legislators who are indifferent vote for $s$; all bribed legislators who are indifferent vote for whoever bribed them last
- Assume continuum of legislators on $\left[-\frac{1}{2}, \frac{1}{2}\right]$
- Assume $W_A$ large enough that $x$ wins in equilibrium (no uncertainty case)
- $m + \frac{1}{2}$ is fraction of legislators who vote for $x$ as opposed to the status quo, $s$
- Results
  - Prop 1: three types of equilibria in which $x$ wins; depend on size of $W_B$
  - Prop 2: $m^*$ (the optimal coalition size) is unique, and provides three cases parameterizing its size in terms of $W_B$, $v(-\frac{1}{2})$ and $v(m^*)$
  - Prop 3/4: special case where $v(z) = \alpha - \beta z$

General thoughts on extension to uncertainty

- I think, without uncertainty, you would estimate $m^*$ as a function of the parameters of $v$ and WTP
  - It's useful that $m^*$ is unique. Not clear it would extend to case of uncertainty, but I think it's likely so I'm going to assume it for now
- I'm pretty sure this predicts that $B$ should never pay anything when there is no uncertainty, but I don't see where they say it explicitly (I should read more carefully to verify)
  - Uncertainty should reverse this, right?
  - What is uncertainty? Make $v(z)$ stochastic is most natural
    * I'm going to start with linear parameterization of $v(z)$ and add uncertainty. For now, take same form for $v(z)$
    $$v(z) = \alpha - \beta \cdot z$$
    and take $z$ as a random variable distributed normally with mean $z$ and standard deviation $\sigma_z$. This is unfortunate nomenclature, since later we'll probably want to take this to a standard normal "Z"
      · This means that the legislators are ordered on the interval according to their ideal points, but there is acknowledged uncertainty surrounding their preferences
      · What is the source of this uncertainty? We take it to vary by legislator and also possibly by issue area when we extend the model to account for that
      · Uncertainty comes from: **log-rolling and cross-pressuring; length of voting record; electoral incentives that we can't control for**

Backward induction (legislature moves last; B makes last bribe; A makes first bribe)

1. Legislature
   - Each legislator $z$ will decide whether to vote for $x$ or $s$ given $z$, $a(z)$ and $b(z)$. Votes for $x$ if
   $$v(z) = \alpha - \beta \cdot z + a(z) - b(z) > 0$$
   (whether the inequality is weak or strict depends on tie-breaking rules set out in the paper)
     - Let's start out by thinking of $z$ as being a random variable distributed $N(z, \sigma_z)$
     - Notice that a bribe from $B$ is subtracted because it makes the alternative *less* attractive
   - Payoff for vote buyer A if $x$ wins is $U_A(x) - \int_{-\frac{1}{2}}^{\frac{1}{2}} a(j)dj$
   - Payoff for vote buyer A if $s$ wins is $U_A(s) - \int_{-\frac{1}{2}}^{\frac{1}{2}} a(j)dj$

- I think the cleanest way to write the condition for whether $x$ wins is

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathbb{1}\left[v(j) \geq 0\right] dj \geq \frac{1}{2}$$

2. Vote buyer B

- GS assumption on vote buyers' objective is "each prefers to minimize total bribes paid while passing his preferred policy, but each would prefer to concede the issue rather than pay more than his WTP"
- This has to be adapted to our situation with uncertainty
  - Groseclose and Snyder formulation would suggest something like

$$\min_{b(z)} \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \quad \text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq U_B(s) - U_B(x)$$

    and however we write the "while passing his preferred policy" constraint
  - With asymmetric $v(z)$ or WTP parameters, it would be easy to get equilibria where only $A$ or $B$ buys votes. But we also have lots of outcomes where both buy votes.
  - They can easily both have positive probability of winning. But what do we need for this to be an equilibrium in this three stage game?
  - Uncertainty buys us a lot: no longer this knife edge condition of $A$ pushing to the point that $B$ buys no votes
- Let's start with the simple maximize expected value of winning (WTP times probability of winning) net of bribes:

$$\max_{b(j)} W_B \left\{\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathbb{1}\left[\alpha - \beta \cdot j + a(j) - b(j) \leq 0\right] dj\right] \geq \frac{1}{2}\right\} - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj$$
$$\text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq W_B \quad \text{and} \quad b(j) \geq 0\ \forall j \tag{1}$$

  - Problem is that the indicator function will never equal 1 when $J \sim N(j, \sigma_j)$
  - So what's a reasonable objective function?
    * Maximizing the total probability mass where $v(j) \geq 0$?

$$\max_{b(j)} W_B \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr\left[\alpha - \beta \cdot j + a(j) - b(j) \leq 0\right] dj\right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj$$
$$\text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq W_B \quad \text{and} \quad b(j) \geq 0\ \forall j \tag{2}$$

      · Note the non-negativity constraint means you can't extract money from a legislator who is past the FOC point in order to reallocate to another legislator

- · Some legislators who are past FOC point will not be lobbied
- ∗ Suppressing the constraint for now and using the fact that $J \sim N(j, \sigma_j)$, this can be re-written as

$$\max_{b(j)} W_B \left[ \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr \left[ \frac{\alpha + a(j) - b(j)}{\beta} \le j \right] dj \right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j) dj \tag{3}$$

or

$$\max_{b(j)} W_B \left[ \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr \left[ \frac{\frac{\alpha + a(j) - b(j)}{\beta} - j}{\sigma_j} \le z \right] dj \right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j) dj \tag{4}$$

where $Z \sim N(0, 1)$.

- ∗ So, this can be written as

$$\max_{b(j)} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[ W_B \left( 1 - \Phi \left( \frac{\frac{\alpha + a(j) - b(j)}{\beta} - j}{\sigma_j} \right) \right) - b(j) \right] dj$$

$$\text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j) dj \le W_B \quad \text{and} \quad b(j) \ge 0 \; \forall j \tag{5}$$

- ∗ If the realizations of $Z$ are i.i.d. and the bribes are independent across legislators, (I believe) this can be maximized pointwise–except for the constraint
- ∗ The FOC for each $j$ is then

$$-W_B \cdot \frac{\partial \Phi \left( \frac{\frac{\alpha + a(j) - b(j)}{\beta} - j}{\sigma_j} \right)}{\partial b(j)} = 1 \tag{6}$$

as long as the constraint is satisfied.

- · I don't know how decisions are made if the constraint binds—how the rationing happens
- · If the constraint is not binding, vote buyer B will pay every single legislator (we could put in some kind of fixed cost of dealing with each legislator to short circuit this outcome)
- · Perhaps there's something in equilibrium I'm not seeing yet that will reduce the number of legislators that are paid

3. Vote buyer A

- Whatever we decide for vote buyer B will be the same for vote buyer A

Notes from 3/16 Skype chat (Kristy and Sebastian)

- Seems like FOC for lobby will boil down to each one buying votes until marginal benefit = marginal cost
- May want to use some simplified rule / heuristic for lobby's decision: perhaps they reorder the legislators by their ±2 std. dev. and make some decision based on that ordering who to lobby
- Will some kind of rule that looks like RMSE come out of the math? Is it possible to get anything closed-form at all?

Notes from 3/19 Skype chat (Kristy and Sebastian)

- We can use the data we have to horse-race this model against other hypotheses about how lobbyists distribute bribes
  - Allocate equally among legislators
  - Groseclose Snyder with full information: only one side pays
  - Our model with one dimension
    - Could we use the econometric model to benchmark to one where uncertainty disappears? Or, as it does theoretically, does the model have to change completely?
  - Our model with multiple dimensions
- We can think of this as looking for the effect of uncertainty on prices—we'd be pricing uncertainty relative to a model with uncertainty
  - "a metric in dollars of uncertainty"
  - this is a model of vote buying under uncertainty
- Given how different the environment with uncertainty is, Kristy should explore both the sequential model that parrots GS96 and a simultaneous model that is more like GH94 (menu auction)
- When mapping to the data, we're going to want to know from the model whether/when total contributions represent WTP.
  - It clearly doesn't in the case of certainty. One side pays nothing; the other pays either as much as is necessary to shut down its opponent, or nothing at all the necessary amount exceeds WTP
  - May be able to show it doesn't matter, that the proportion of total expenditures is a sufficient statistic
- Finish writing the model, and then see if we can use the estimates we already have to write a first, very rough draft