Political Uncertainty

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On Feb. 12, Sebastian and I agreed to focus efforts on finding a base model to facilitate empirical identification. I am pursuing Groseclose & Snyder (1996), "Buying Supermajorities," APSR

- For each legislator i, $v(i) = u_i(x) - u_i(s)$, measured in money; this is the reservation price of i
  - x is an alternative policy proposal; s is the status quo
  - WLOG, label legislators so that $v(i)$ is a non-increasing function
  - Note legislators only have preferences over how they vote, not over which alternative wins
- There are two vote buyers; each prefers to minimize total bribes paid while passing his preferred policy, but each would prefer to concede the issue rather than pay more than his WTP
  - A prefers x; W_A is A's willingness to pay (WTP) for x measured in money
  - B prefers s; W_B is B's WTP for s
- Bribe offer functions: $a(i)$ and $b(i)$ are A and B's offers to i. Legislators take these bribe offers as given and then vote for the alternative that maximizes their payoff
- A moves first; $a(i)$ is perfectly observable to B when he moves
- Goal: characterize SPNE in pure strategies
  - Assume unbribed legislators who are indifferent vote for s; all bribed legislators who are indifferent vote for whoever bribed them last
- Assume continuum of legislators on $\left[-\frac{1}{2}, \frac{1}{2}\right]$
- Assume W_A large enough that x wins in equilibrium (no uncertainty case)
- $m + \frac{1}{2}$ is fraction of legislators who vote for x as opposed to the status quo, s
- Results
  - Prop 1: three types of equilibria in which x wins; depend on size of W_B
  - Prop 2: m^* (the optimal coalition size) is unique, and provides three cases parameterizing its size in terms of W_B, $v(-\frac{1}{2})$ and $v(m^*)$
  - Prop 3/4: special case where $v(z) = \alpha - \beta z$

General thoughts on extension to uncertainty

- I think, without uncertainty, you would estimate m^* as a function of the parameters of v and WTP
  - It's useful that m^* is unique. Not clear it would extend to case of uncertainty, but I think it's likely so I'm going to assume it for now
- I'm pretty sure this predicts that B should never pay anything when there is no uncertainty, but I don't see where they say it explicitly (I should read more carefully to verify)
  - Uncertainty should reverse this, right?
  - What is uncertainty? Make $v(z)$ stochastic is most natural
    * I'm going to start with linear parameterization of $v(z)$ and add uncertainty:

      $$v(z) = \alpha - \beta \cdot z + \varepsilon_z$$

      we can decide later on the distributional assumptions for ε_z, and whether / when we want to make the ε vary by legislator (z)

Backward induction (legislature moves last; B makes last bribe; A makes first bribe)

1. Legislature
   - Each legislator z will decide whether to vote for x or s given z, $a(z)$ and $b(z)$. Votes for x if

     $$v(z) = \alpha - \beta \cdot z + a(z) + b(z) > 0$$

     (whether the inequality is weak or strict depends on tie-breaking rules set out in the paper)
     - Let's start out by thinking of z as being a random variable distributed $N(z, \sigma_z)$
   - Payoff for vote buyer A if x wins is $U_A(x) - \int_{-\frac{1}{2}}^{\frac{1}{2}} a(j)dj$
   - Payoff for vote buyer A if s wins is $U_A(s) - \int_{-\frac{1}{2}}^{\frac{1}{2}} a(j)dj$
   - I think the cleanest way to write the condition for whether x wins is

     $$\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathbb{1}\left[v(j) \geq 0\right] dj \geq \frac{1}{2}$$

2. Vote buyer B
   - GS assumption on vote buyers' objective is "each prefers to minimize total bribes paid while passing his preferred policy, but each would prefer to concede the issue rather than pay more than his WTP"

- This has to be adapted to our situation with uncertainty
  - Groseclose and Snyder formulation would suggest something like

$$\min_{b(z)} \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \quad \text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq U_B(s) - U_B(x)$$

    and however we write the "while passing his preferred policy" constraint
  - With asymmetric $v(z)$ or WTP parameters, it would be easy to get equilibria where only A or B buys votes. But we also have lots of outcomes where both buy votes.
  - They can easily both have positive probability of winning. But what do we need for this to be an equilibrium in this three stage game?
  - Uncertainty buys us a lot: no longer this knife edge condition of A pushing to the point that B buys no votes
- Let's start with the simple maximize expected value of winning (WTP times probability of winning) net of bribes:

$$\max_{b(j)} W_B \left\{ \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathbb{1}\left[\alpha - \beta \cdot j + a(j) + b(j) \leq 0\right] dj \right] \geq \frac{1}{2} \right\} - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj$$
$$\text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq W_B \quad (1)$$

  - Problem is that the indicator function will never equal 1 when $J \sim N(j, \sigma_j)$
  - So what's a reasonable objective function?
    * Maximizing the total probability mass where $v(j) \geq 0$?

$$\max_{b(j)} W_B \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr\left[\alpha - \beta \cdot j + a(j) + b(j) \leq 0\right] dj \right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj$$
$$\text{subject to} \quad \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \leq W_B \quad (2)$$

    * Suppressing the constraint for now and using the fact that $J \sim N(j, \sigma_j)$, this can be re-written as

$$\max_{b(j)} W_B \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr\left[\frac{\alpha + a(j) + b(j)}{\beta} \leq j\right] dj \right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \quad (3)$$

    or

$$\max_{b(j)} W_B \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr\left[\frac{\frac{\alpha + a(j) + b(j)}{\beta} - j}{\sigma_j} \leq z\right] dj \right] - \int_{-\frac{1}{2}}^{\frac{1}{2}} b(j)dj \quad (4)$$

    where $Z \sim N(0, 1)$.

- * So, this can be written as

$$\max_{b(j)} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left[W_B \left(1 - \Phi \left(\frac{\frac{\alpha + a(j) + b(j)}{\beta} - j}{\sigma_j} \right) \right) - b(j) \right] dj \tag{5}$$

- * If the realizations of Z are i.i.d. and the bribes are independent across legislators, (I believe) this can be maximized pointwise.
- * The FOC for each j is then

$$-W_B \cdot \frac{\partial \Phi \left(\frac{\frac{\alpha + a(j) + b(j)}{\beta} - j}{\sigma_j} \right)}{\partial b(j)} = 1 \tag{6}$$

Don't forget the constraint!

3. Vote buyer A

- Whatever we decide for vote buyer B will be the same for vote buyer A