Vote Buying, Supermajorities, and Flooded Coalitions

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In a recent paper, Banks (2000), adopting the framework of our model (Groseclose and Snyder 1996), derives several new and noteworthy results. In addition, he provides a counterexample to our proposition 4. Here we explain the error in our proposition but note that we can correct it easily if we invoke an additional assumption: In equilibrium the winning vote buyer constructs a nonflooded coalition, that is, she does not bribe every member of her coalition. We conclude with a brief discussion of the substantive implications of Banks's proposition 1; we note that it provides additional support for our general claim that minimal winning coalitions should be rare in a vote-buying game.

HOW PROPOSITION 4 CAN BE RESTATED CORRECTLY

Our mistake in proposition 4 was to assume that the solution to a step in the proof occurred at an interior point when, in fact, the solution might occur at a boundary point.

In our proof we define m as the surplus of A's coalition. That is, the size of A's coalition is $m + (n + 1)/2$, although only A needs $(n + 1)/2$ legislators (a bare majority) to pass the bill. Vote buyer B is willing to pay W_B to defeat A's bill. Since he must buy $m + 1$ legislators to invade A's coalition successfully, A can prevent B from invading if each of her legislators costs at least $W_B/(m + 1)$ for B to buy. We show that A will adopt a leveling strategy. That is, each member of her paid coalition will cost exactly $W_B/(m + 1)$ for B to buy.

Legislators are labeled $i = 1, 2, \ldots, n$, and we assume that the legislators have linear preferences. That is, the utility that i receives for voting for A's bill is $v(i) = \alpha - \beta[i - (n + 1)/2]$. Define i_0 as the left-most legislator whose initial preferences are less than $W_B/(m + 1)$. This legislator will be the left-most member of A's *bribed* coalition. Let s be the (hypothetical, possibly noninteger-valued) legislator whose preferences for A's bill equals $W_B/(m + 1)$. That is, $W_B/(m + 1) = v(s) = \alpha - \beta[s - (n + 1)/2]$, or $s = [\alpha - W_B/(m + 1)]/\beta + (n + 1)/2$. Define $\delta(m)$ as the distance between i_0 and s.

If A has not bribed all the members of her coalition, we call this a nonflooded coalition. In this case there exists a legislator to the left of i_0, labeled $i_0 - 1$. Furthermore, s must lie between these two legislators. Therefore, $\delta(m) \in [0, 1]$.

If A has constructed a flooded coalition, however, that is, she has bribed all members of her coalition, then there is no legislator to the left of i_0, and it is possible for $\delta(m)$ to be greater than one.

Our mistake is on line 20 of the proof: "Since $\delta(m) \in [0, 1], \ldots$" Of course, this is not necessarily true when A has constructed a flooded coalition. If we assume, however, that A constructs a nonflooded coalition, then the proof follows. Thus, the following is a corrected restatement of proposition 4: Suppose $v(i) = \alpha - \beta[i - (n + 1)/2]$, with $\beta \geq 0$ and $\alpha \leq 0$. If $a^*()$ and $b^*()$ constitute an equilibrium in which A *constructs a nonflooded coalition and* x wins, then $m^* = 0$ only if $W_B \leq [1/3 + (28/9)^{1/2}]\beta < (2.1)\beta$. (The original version does not contain the phrase in italics, but it is otherwise identical.)[1]

While the nonflooded coalition assumption is crucial for our result, the opposite assumption is crucial for Banks's results. As he shows, his A2 assumption implies that the winning vote buyer bribes each member of her coalition in equilibrium, that is, she constructs a flooded coalition.

Substantively, this condition is fairly restrictive. It requires that all members of a coalition, even the most ardent supporters, receive bribes in equilibrium. In contexts in which legislators themselves are the vote buyers, this is impossible. For a genuine flooded coalition to occur, the legislators must receive side payments, not pay them. An example of legislators who are vote buyers is Groseclose's (1996) case study of the Byrd Amendment to the Clean Air Act. Byrd was one vote buyer, and the opposing vote buyer was the team of President George Bush and Majority Leader George Mitchell.[2]

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[1] We should also note that line two of the proof contains a typo. The minus sign in "$b(i) > a(i) - v(i)$" should be a plus sign.

[2] We should note, however, that a minor change in A2 *does* allow Banks's results to apply to contexts where legislators are vote buyers. For instance, suppose legislator 1 is the vote buyer, and she has such intense preferences for x that A2 is violated. As long as the other legislators' preferences satisfy A2, Banks's results do not change qualitatively. Nevertheless, if one restricts the size of the legislature so that it is not too small, and if one restricts preferences of the vote-buying legislators so that they do not differ too drastically from the preferences of the other legislators, then the results do change qualitatively. For instance, suppose (i) preferences take a linear form (as in our proposition 4); (ii) $n \geq 5$; (iii) legislator 1 is vote buyer A (thus, $v_1 = W_A$); and (iv) the preferences of the other legislators satisfy A2. Then there does not exist an equilibrium in which legislator 1 buys votes. (Basically, A2 places a minimum bound on W_B, which makes it too expensive for 1 to buy votes successfully. A proof is available from the authors.) That is, if conditions i–iv are satisfied, then Banks's proposition 1 does not hold in the context where legislators are vote buyers, even with the change in A2.

SUBSTANTIVE IMPLICATIONS OF BANKS'S PROPOSITION 1

We conclude with two brief thoughts on Banks's proposition 1. First, it is a very interesting and impressive result. It provides a necessary *and* sufficient condition for a minimal winning coalition to occur (unlike our proposition 4, which only provides a necessary condition). Furthermore, unlike our proposition 4, the result does not need to assume that preferences are linear. It does require, however, that preferences satisfy assumption A2.

Second, the proposition shows that our original conclusion—that minimal winning coalitions should be rare—is even more general than our original statement.[3] The condition in the proposition requires that $v_{(n+3)/2} \leq -W_B(n - 1)/4$. That is, for a minimal winning coalition to occur, the legislator just to the right of the median voter must strongly favor the right-wing vote buyer's alternative, relative to the size of the legislature (n) and the right-wing vote buyer's intensity of preference (W_B). When the size of the legislature is at least five members, the condition requires that the legislator favor the alternative *more* strongly than the right-wing vote buyer (even though the legislator is fairly "centrist"). In our view, this should occur very rarely in practice; thus it supports our claim that minimal winning coalitions should be rare.

[3] Many of the ideas in this paragraph developed from conversations with Ken Shotts, to whom we are very grateful.

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