Week3_ECON_PS1

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Exercise 1

$$\frac{1}{e^{z_t}K_t^\alpha - K_{t+1}} = \beta E_t\left\{\frac{\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^\alpha - K_{t+2}}\right\}$$

Sub in the guessed policy function and $z_t = 0$ we get

$$\frac{1}{K_t^\alpha - AK_t^\alpha} = \beta\left\{\frac{\alpha(AK_t{}^\alpha)^{\alpha-1}}{(AK_t{}^\alpha)^\alpha - A(AK_t{}^\alpha)^\alpha}\right\}$$

$$1 = \frac{\beta}{A}\left\{\frac{\alpha(AK_t{}^\alpha)^{\alpha-1}}{(AK_t{}^\alpha)^{\alpha-1} - (AK_t{}^\alpha)^{\alpha-1}}\right\}$$

$$1 = \frac{\alpha\beta}{A}$$

$$A = \alpha\beta$$

So $\overline{K}$ is:

$$\overline{K} = (\alpha\beta)^{\frac{1}{1-\alpha}}$$

Check for steady state policy function, if K_t is a steady state, then

$$K_{t+1} = (\alpha\beta)\left[(\alpha\beta)^{\frac{1}{1-\alpha}}\right]^\alpha = (\alpha\beta)^{\frac{1}{1-\alpha}} = K_t$$

Exercise 2

Characterizing functions:

$$c_t = (1-\tau)\left[w_t\ell_t + (r_t - \delta)k_t\right] + k_t + T_t - k_{t+1} \tag{1}$$

$$\frac{1}{c_t} = \beta E_t\left\{\frac{1}{c_{t+1}}\left[(r_{t+1}-\delta)(1-\tau)+1\right]\right\} \tag{2}$$

$$\frac{\alpha}{1-\ell_t} = \frac{1}{c_t}w_t(1-\tau) \tag{3}$$

$$r_t = \alpha e^{z_t}K_t^{\alpha-1}L_t^{1-\alpha} \tag{4}$$

$$w_t = (1-\alpha)e^{z_t}K_t^\alpha L_t^{-\alpha} \tag{5}$$

$$\tau\left[w_t\ell_t + (r_t-\delta)k_t\right] = T_t \tag{6}$$

$$z_t = (1-\rho_z)\,\overline{z} + \rho_z z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \sim \text{ i.i.d. } \left(0, \sigma_z^2\right) \tag{7}$$

Exercise 3
Characterizing functions:

$$c_t = (1-\tau)\left[w_t\ell_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1} \tag{8}$$
$$c_t{}^{-\gamma} = \beta E_t\left\{c_{t+1}{}^{-\gamma}\left[(r_{t+1}-\delta)(1-\tau)+1\right]\right\} \tag{9}$$
$$\frac{\alpha}{1-\ell_t} = c_t{}^{-\gamma}w_t(1-\tau) \tag{10}$$
$$r_t = \alpha e^{z_t}K_t^{\alpha-1}L_t^{1-\alpha} \tag{11}$$
$$w_t = (1-\alpha)e^{z_t}K_t^{\alpha}L_t^{-\alpha} \tag{12}$$
$$\tau\left[w_t\ell_t + (r_t-\delta)\,k_t\right] = T_t \tag{13}$$
$$z_t = (1-\rho_z)\,\overline{z} + \rho_z z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \sim \text{ i.i.d. } \left(0,\sigma_z^2\right) \tag{14}$$

Exercise 4
Characterizing functions:

$$c_t = (1-\tau)\left[w_t\ell_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1} \tag{15}$$
$$c_t{}^{-\gamma} = \beta E_t\left\{c_{t+1}{}^{-\gamma}\left[(r_{t+1}-\delta)(1-\tau)+1\right]\right\} \tag{16}$$
$$-\alpha(1-\ell_t)^{-\epsilon} = c_t{}^{-\gamma}w_t(1-\tau) \tag{17}$$
$$r_t = \alpha e^{z_t}K_t^{\eta-1}\left[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}\right]^{\frac{1-\eta}{\eta}} \tag{18}$$
$$w_t = (1-\alpha)e^{z_t}L_t^{\eta-1}\left[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}\right]^{\frac{1-\eta}{\eta}} \tag{19}$$
$$\tau\left[w_t\ell_t + (r_t-\delta)\,k_t\right] = T_t \tag{20}$$
$$z_t = (1-\rho_z)\,\overline{z} + \rho_z z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \sim \text{ i.i.d. } \left(0,\sigma_z^2\right) \tag{21}$$

Exercise 5
Characterizing functions:

$$c_t = (1-\tau)\left[w_t + (r_t-\delta)\,k_t\right] + k_t + T_t - k_{t+1} \tag{22}$$
$$c_t{}^{-\gamma} = \beta E_t\left\{c_{t+1}{}^{-\gamma}\left[(r_{t+1}-\delta)(1-\tau)+1\right]\right\} \tag{23}$$
$$0 = c_t{}^{-\gamma}w_t(1-\tau) \tag{24}$$
$$r_t = \alpha e^{z_t}K_t^{\alpha-1}L_t^{1-\alpha} \tag{25}$$
$$w_t = (1-\alpha)e^{z_t(1-\alpha)}K_t^{\alpha}L_t^{-\alpha} \tag{26}$$
$$\tau\left[w_t + (r_t-\delta)\,k_t\right] = T_t \tag{27}$$
$$z_t = (1-\rho_z)\,\overline{z} + \rho_z z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \sim \text{ i.i.d. } \left(0,\sigma_z^2\right) \tag{28}$$

Steady state versions:

$$c = (1-\tau)\left[w + (r-\delta)\,k\right] + T \tag{29}$$
$$c^{-\gamma} = \beta\left\{c^{-\gamma}\left[(r-\delta)(1-\tau)+1\right]\right\} \tag{30}$$
$$0 = c^{-\gamma}w(1-\tau) \tag{31}$$
$$r = \alpha k^{\alpha-1} \tag{32}$$
$$w = (1-\alpha)k^{\alpha} \tag{33}$$
$$\tau\left[w + (r-\delta)\,k\right] = T \tag{34}$$

Algebraic solusion:

Exercise 6

Characterizing functions:

$$c_t = (1-\tau)\left[w_t \ell_t + (r_t - \delta)\, k_t\right] + k_t + T_t - k_{t+1} \tag{35}$$

$$c_t{}^{-\gamma} = \beta E_t \left\{ c_{t+1}{}^{-\gamma} \left[(r_{t+1} - \delta)(1-\tau) + 1 \right] \right\} \tag{36}$$

$$-a(1-\ell_t)^{-\epsilon} = c_t{}^{-\gamma} w_t (1-\tau) \tag{37}$$

$$r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \tag{38}$$

$$w_t = (1-\alpha) e^{z_t(1-\alpha)} K_t^{\alpha} L_t^{-\alpha} \tag{39}$$

$$\tau\left[w_t + (r_t - \delta)\, k_t\right] = T_t \tag{40}$$

$$z_t = (1-\rho_z)\,\overline{z} + \rho_z z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \sim \text{ i.i.d. } \left(0, \sigma_z^2\right) \tag{41}$$

Steady state versions:

$$c = (1-\tau)\left[w\ell + (r-\delta)\, k\right] + T \tag{42}$$

$$c^{-\gamma} = \beta c^{-\gamma}\left[(r-\delta)(1-\tau) + 1\right] \tag{43}$$

$$-a(1-\ell_t)^{-\epsilon} = c^{-\gamma} w (1-\tau) \tag{44}$$

$$r = \alpha k^{\alpha-1} \ell^{1-\alpha} \tag{45}$$

$$w = (1-\alpha) k^{\alpha} \ell^{-\alpha} \tag{46}$$

$$\tau\left[w + (r-\delta)\, k\right] = T \tag{47}$$