OSE2019 Week4 Firm Dynamics
Firm entry and exit

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Description of model

- incumbent firm:
  - production function: $y_{jt} = e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$
  - idiosyncratic productivity follows a Markov chain: ε_{jt}
  - capital accumulation: $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$
  - convex adjustment cost (units of output): $-\frac{\varphi}{2}\left(\frac{i_{jt}}{k_{jt}}\right)^2 k_{jt}$
  - fixed cost (units of output) to remain in operation each period: c_f
  - if does not pay the fixed cost, the firm sells its capital with value: $(1-\delta)k$
- potential entrants:
  - continuum of potential entrants
  - ex-ante identical
  - at the beginning of each period, each firm decides whether to pay a fixed cost c_e and enter the economy.
  - if enters, then draw a value from idiosyncratic productivity ε_{jt} from some distribution ν and begins an an incumbent firm with $k_{jt} = 0$.
  - no adjustment costs at $k_{jt} = 0$.
  - free entry among potential entrants, implies the expected value from entering is less than or equal to the entry cost c_e, so $c_e \leq \int v(\varepsilon, 0)\nu(d\varepsilon)$, with equality if entry actually takes place, i.e. $m^* > 0$.
- representative household:
  - preference over consumption C_t and labour supply N_t.
  - expected utility function: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - aN_t)$

- Steady state recursive competitive equilibrium:
  - set of incumbent value functions $v(\varepsilon, k)$
  - policy rules $k'(\varepsilon, k)$ and $n(\varepsilon, k)$
  - a mass of entrants per period m^*
  - a measure of active firms at the beginning of the period $g^*(\varepsilon, k)$
  - real wage w^* such that :
    1. incumbent firms maximize their firm value
    2. the free entry condition holds
    3. the labor market clears
    4. the measure of active firms $g^*(\varepsilon, k)$ is stationary
- Calibration:

$$
\begin{aligned}
\theta &= 0.21 \\
\nu &= 0.64 \\
\delta &= 0.1 \\
\beta &= 0.96 \\
\varphi &= 0.5 \\
\varepsilon_{jt+1} &= \rho \varepsilon_{jt} + \omega_{jt+1} \\
\omega_{jt+1} &\sim N\left(0, \sigma^2\right) \\
\rho &= 0.9 \\
\sigma &= 0.02
\end{aligned}
$$

1: define the recursive competitive equilibrium

- Firm optimization: taking w^* as given, $v(\varepsilon, k)$ solves Bellman equation.

$$v(\varepsilon, k) = \max \left\{(1-\delta)k, v^1(\varepsilon, k) - c_f\right\}$$

$$v^1(\varepsilon, k) = \max_{k', n} \varepsilon k^\theta n^\nu - w^* n - (k' - (1-\delta)k) - \frac{\varphi}{2}\left(\frac{k'}{k} - (1-\delta)\right)^2 k + \beta \mathbb{E}\left[v\left(\varepsilon', k'\right)\right]$$

  Solve for policy rules $k'(\varepsilon, k)$ and $n(\varepsilon, k)$

- Free entry condition holds with market clearing:

$$c_e \leq \int v(\varepsilon, 0) \nu(d\varepsilon)$$

  Solve for w^* together with firm's optimization Problem

- Solve for stationary measure of incumbent firms $g^*(\varepsilon, k)$

2: solve representative firm steady state equilibrium

Suppose that there are no entry and exits, all firms parcitipate in the market in all periods, and $k' = k = \bar{k}$, $\varepsilon = \varepsilon' = 0$, then we have

- Firm's FOC:

$$-1 - \varphi\delta + \beta V_k(k') = 0$$

- Firm's envelope:

$$\begin{aligned} V_k(k) &= \pi_k(k) + (1-\delta) - \frac{\varphi}{2}\left[-2\frac{i}{k} + \left(\frac{i}{k}\right)^2\right] \\ &= \pi_k(k) + (1-\delta) - \frac{\varphi}{2}\left\{\left[\frac{k'(k)}{k} - (1-\delta)\right]^2 - 2\left[\frac{k'(k)}{k} - (1-\delta)\right]\right\} \end{aligned}$$

Hence in steady state,

$$\begin{aligned} \tilde{V}_k\left(k^*\right) &= \pi_k\left(k^*\right) + (1-\delta) - \frac{\varphi}{2}\left(\delta^2 - 2\delta\right) \\ &= \pi_k\left(k^*\right) - \frac{\varphi}{2}\delta^2 + (\varphi - 1)\delta + 1 \end{aligned}$$

- By firm's profit maximization:

$$\pi(k) = \max_n e^{\epsilon} k^{\theta} n^{\nu} - wn$$

take derivative w.r.t n,

$$\begin{aligned} &\implies \nu e^{\epsilon} k^{\theta} n^{\nu-1} - w = 0 \\ &\qquad \implies w = \nu e^{\epsilon} k^{\theta} n^{\nu-1} \end{aligned}$$

- Combining everying above to get Euler equation:

$$-1 - \varphi\delta + \beta\left[\theta\bar{k}^{\theta-1} n^{\nu} - \frac{\varphi}{2}\delta^2 + (\varphi - 1)\delta + 1\right] = 0$$