Heuristic function

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1 Jack Sun's Heuristic Function

Let S be the set of all legal states (which is 5x4 matrix) of Hua Rong Dao. For any $s \in S$, $s_{i,j}$ be the element on ith row, jth column in s, where the index starts at 0. For example, $s_{0,0}$ denotes the element in upper-left corner.
In this paragraph, the elements of the state matrix can be 0 to 7, which is the same as input format. We convert the 0-7 to 0-4 format for my program to print results, but we wont convert in this paragraph.
Let's denote the row and column coordinate of upper-left position of 2x2 piece as $s.bigpiecerow$ and $s.bigpiececol$ respectively.
We say that 2x2 is in left if $s.bigpiececol = 0$, middle if =1, right if =2.
The cost from any state to its successor is 1. Let $g : S \to \{0, 1, ...\}$ be the Manhattan distance between the 2x2 piece and the bottom opening. Mathematically, $g(s) = |s.bigpiecerow - 3| + |s.bigpiececol - 1|$

Construct heuristic function $f : S \to \{0, 1, ...\}$ as following:
(1) If x is a final state, set $f(x) = 0$.
(2) Else, if the 2x2 piece is able to move (in some direction) to decrease the manhattan distance, set $f(x) = g(x)$.
(3) Otherwise, set $f(x) = g(x) + 1$.
Specifically, the condition (2) is:
$(g(x) \neq 0) \wedge ((x.bigpiecerow \leq 2 \wedge x_{x.bigpiecerow+1,x.bigpiececol} = x_{x.bigpiecerow+1,x.bigpiececol+1} = 0) \vee (x.bigpiececol = 0 \wedge x_{x.bigpiecerow,x.bigpiececol+2} = x_{x.bigpiecerow+1,x.bigpiececol+2} = 0) \vee (x.bigpiececol = 2 \wedge x_{x.bigpiecerow,x.bigpiececol-1} = x_{x.bigpiecerow+1,x.bigpiececol-1} = 0))$
Prove that f dominates g: If $g(x) = 0$, then $f(x) = 0$, so $f(x) \geq g(x)$. If $g(x) \neq 0$, $f(x)$ can either be $g(x)+1$ or $g(x)$. So $\forall x \in S, f(x) \geq g(x)$. Also, when $x = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 3 & 1 & 1 & 2 \\ 4 & 4 & 5 & 5 \\ 6 & 6 & 7 & 7 \\ 7 & 7 & 0 & 0 \end{bmatrix}$, $g(x) = 3, f(x) = 4$. So $f(x) > g(x)$. So f dominates g.
Prove f is admissible:
Clearly $f(x)$ is nonnegative.
Let $f^*(x)$ denotes the step of cheapest path from state x to a goal state.
For contradiction, assume $f^*(x) < f(x)$ for some state x. By admissibility of g, $g(x) \leq f^*(x) < f(x)$. By construction of f, $g(x) + 1 = f(x)$, so x must satisfy condition 3 that x is not final state and the 2x2 piece is unable to move to decrease the manhattan distance. In the cheapest path, x first goes to some state y which has room for 2x2 piece to move to decrease the manhattan distance (during which the 2x2 piece doesnt move, so value of function g doesnt change). It takes at least 1 step. At state y, it takes at least $g(y) + 1$ steps to final state. Therefore, $f^*(x) \geq 1 + g(y) = f(x)$.But it contradicts $f^*(x) < f(x)$.
So f is admissible.