Exercise 1 –mandatory—marked–

Read Chapter 17 (*Karnaugh Maps*) of the textbook.

Consider the following Karnaugh Map of a boolean expression:

		cd			
		00	01	11	10
ab	00	1	0	0	1
	01	1	0	0	1
	11	0	0	1	1
	10	0	0	1	1

(a) Identify (by marking on the map) the blocks of 1s and give the boolean expression in DNF. Disjunctive Normal Form $\vee(\wedge P_{ij})$

(b) Identify (by marking on the map) the blocks of 0s and give the boolean expression in CNF. Conjunctive Normal Form $\wedge(\vee P_{ij})$

(a) $(\neg a \vee \neg c \vee \neg d) \wedge (\neg a \vee c \vee \neg d) \wedge (a \vee c)$ "CNF"

$= \neg(a \wedge c \wedge d) \wedge \neg(a \wedge \neg c \wedge d) \wedge \neg(a \wedge c)$ De Morgan's Law.

$= \neg((a \wedge c \wedge d) \vee (a \wedge \neg c \wedge d) \vee (a \wedge c))$ De Morgan's Law

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(b)

$\neg((\neg a \wedge d) \vee (a \wedge \neg c))$ "DNF".

$= \neg(\neg a \wedge d) \wedge \neg(a \wedge \neg c)$ De Morgan's Law.

$= (a \vee \neg d) \wedge (\neg a \vee c)$ De Morgan's Law.

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Exercise 2 –mandatory—marked–

A three-variable Karnaugh map can be drawn with one variable on the left and two on the top, or vice versa. Putting r on the left (as a row variable), and a, b on top (as column variables), give a Karnaugh map for the expression

$$r \leftrightarrow (a \to b)$$

and produce a DNF equivalent.

Make the DNF *minimal*: with as few occurrences of literals as possible. (Pay attention to the distinction between the number of literals in an expression and the number of occurrences of literals, as the same literal may occur several times.)

a	b	a → b
1	1	1
0	1	1
1	0	0
0	0	1

P	Q	P ↔ Q
1	1	1
0	1	0
1	0	0
0	0	1

K-map

(a → b)

ab

r	11	01	10	00
1	1	1	0	1
0	0	0	1	0

$\neg((\neg r \wedge b) \vee (r \wedge a \wedge \neg b) \vee (\neg r \wedge \neg a \wedge \neg b))$ #

Exercise 3 –optional—marked–

Consider four variables a, b, c, d and the following set of two clauses (a CNF clause is a disjunction of literals):

$$a \vee \neg b, \ \neg a \vee \neg d$$

(∧)

(a) Viewing ‘,’ as conjunction, as we do on the left-hand side of a sequent, draw the (single) 4-variable Karnaugh map for these two clauses. Show on your map the blocks of zeros arising from each of the two clauses. (Because the clauses are being and’ed together, the map has zero wherever either of the clauses has zero.)

(a) $(a \vee \neg b) \wedge (\neg a \vee \neg d)$

ab

cd	00	10	11	01
00	1	1	1	0
10	1	1	1	0
11	1	0	0	0
01	1	0	0	0

(b) total δ = 21

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	0	0	0
01	1	0	0	0

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	0	0	1
01	1	0	0	1

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	0	1	1
01	1	0	1	1

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	1	0
01	1	1	1	0

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	0	1
01	1	1	0	1

	00	10	11	01
00	1	1	1	0
10	1	1	1	0
11	1	1	1	1
01	1	1	1	1

(b) Use the map to find a new clause δ, *different from the given clauses*, such that

$$a \vee \neg b,\ \ \neg a \vee \neg d \vDash \delta$$

(Hint: if $\Gamma \vDash \delta$, what do you know about the cells where δ is 0 or 1 in terms of the cells where Γ is 0 or 1?)

How many different δ can you find?

if Γ is 1. δ is 1.

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	1	0
01	1	1	1	0

	00	10	11	01
00	1	1	1	1
10	1	1	1	0
11	1	1	1	0
01	1	1	1	1

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	0	0
01	1	1	1	1

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	1	1
01	1	1	0	0

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	0	0	1
01	1	1	1	1

	00	10	11	01
00	1	1	1	1
10	1	1	1	1
11	1	1	1	1
01	1	0	0	1

	00	10	11	01
00	1	1	1	0
10	1	1	1	1
11	1	1	1	1
01	1	1	1	0

	00	10	11	01
00	1	1	1	0
10	1	1	1	1
11	1	1	1	1
01	1	1	0	1

...