Exercise 1 –mandatory—marked–

Read Chapter 14 (*Sequent Calculus*) of the textbook.

Derive the second of de Morgan's laws

$$\neg(a \wedge b) = \neg a \vee \neg b$$

using a similar argument to the one presented in the textbook for the first law on page 122.

$$\dfrac{\dfrac{\neg c \models a \qquad \neg c \models b}{\neg c \models a \wedge b}\ \wedge}{\neg(a \wedge b) \models c}\ \text{Contraposition}$$

$$\dfrac{\dfrac{\neg c \models a \qquad \neg c \models b}{\neg a \models c \qquad \neg b \models c}\ \text{Contraposition}}{\neg a \vee \neg b \models c}\ \vee$$

As both start from the same permises.
They provide equivalence for result.

Exercise 2 –mandatory—marked–

Write a proof which reduces the conclusion

$$(x \wedge y) \vee (x \wedge z) \models x \wedge (y \vee z)$$

to premises that can't be reduced further.

Is it universally valid? If not, give a counterexample.

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x, y \models x}\ \mathrm{I}}{(x \wedge y) \models x}\ \wedge L \qquad \dfrac{x, z \models x}{x \wedge z \models x}\ \wedge L}{(x \wedge y) \vee (x \wedge z) \models x}\ \vee L \qquad \dfrac{\dfrac{\dfrac{}{x, y \models y, z}\ \mathrm{I}}{(x \wedge y) \models (y \wedge z)} \qquad \dfrac{\dfrac{}{x, z \models y, z}\ \mathrm{I}}{(x \wedge z) \models (y \wedge z)}}{(x \wedge y) \vee (x \wedge z) \models y \wedge z}\ \vee L}{(x \wedge y) \vee (x \wedge z) \models x \wedge (y \vee z)}\ \wedge R$$

It's universally valid. because all permises can be obtained by apply calulus immediate rule.

Exercise 3 –optional—marked–

Write a proof which reduces the conclusion

$$\models (x \wedge \neg y) \vee (\neg(x \vee z) \vee (y \vee z))$$

to premises that can't be reduced further.

Expressions φ like $(x \wedge \neg y) \vee (\neg(x \vee z) \vee (y \vee z))$ used in the antecedents and succedents of sequents are called:

- *tautologies* when $\models \varphi$ is valid (the antecedent is empty);
- *contradictions* when $\varphi \models$ is valid (the succedent is empty);

Is $(x \wedge \neg y) \vee (\neg(x \vee z) \vee (y \vee z))$ a tautology, a contradiction, or neither?

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{x, y, z \vDash x}\;\mathrm{I}}{\vDash x, (\neg(x, z), (y, z))}\;\neg R}{\vDash x, (\neg(x \vee z)(y \vee z))}\;\vee R \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{y, x \vdash y, z}\;\mathrm{I} \quad \dfrac{}{y, z \vDash y, z}\;\mathrm{I}}{y, (x \vee z) \vDash y, z}\;\vee L}{y, (x \vee z) \vDash y \vee z}\;\vee R}{y \vDash \neg(x \vee z), (y \vee z)}\;\neg R}{\vDash \neg y, (\neg(x \vee z)(y \vee z))}\;\neg R}{\dfrac{\vDash (x \wedge \neg y), (\neg(x \vee z), (y \vee z))}{\dfrac{\vDash (x \wedge \neg y), (\neg(x \vee z) \vee (y \vee z))}{\vDash (x \wedge \neg y) \vee (\neg(x \vee z) \vee (y \vee z))}\;\vee R}\;\vee R}\;\wedge R
$$

Since it can reduce to identity, no extra assumption required.
Therefore. it is a toutology