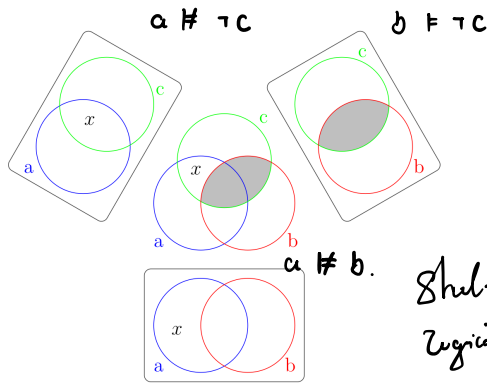## Question 1 –mandatory–

The following diagram provides a Venn diagram check of the soundness of a sound syllogism. Which syllogism does it check? (N.B. the order of premises or variables may differ from the tables of sound syllogisms.)

$$a \not\models \neg c \qquad b \models \neg c$$

$$a \not\models b.$$

Shading Area representing the logical relationship between c & b. Once we took off c. b has no longer intersect with c.

Explain why there is no shading in the bottom diagram although corresponding portions of the central diagram are shaded.

$$\frac{a \not\models \neg c \qquad b \models \neg c}{a \not\models b}$$

Question 2 –mandatory–

Derive

$$\frac{a \models \neg b \qquad a \not\models \neg c}{c \not\models b}$$

from *barbara*, using contraposition of sequents and rules, replacement of predicates, and double negation, as required.

Contreposition

$$\frac{a \models \neg b \qquad a \not\models \neg c}{c \not\models b}$$

$$\frac{a \models b \qquad b \models c}{a \models c}$$

↓ Substitution

$$\frac{a \models \neg b \qquad \neg b \models \neg c}{a \models \neg c}$$

↓ Contreposition / Double negation cancellation

$$\frac{a \models \neg b \qquad c \models b}{a \models \neg c}$$

Question 3 –mandatory–

Prove using the sequent calculus that

$$(p \wedge q) \vee \neg p \vee \neg q$$

is a tautology.

$$\cfrac{\cfrac{\cfrac{\cfrac{\overline{p, q \models p}^{I} \qquad \overline{p, q \models q}^{I}}{p, q \models p \wedge q} \wedge R}{(p \wedge q), \neg p, \neg q} \neg R, \neg R}{(p \wedge q) \vee \neg p \vee \neg q} \vee R, \vee R}$$

Since it can reduced to identity, hence no further assumption is needed. So we can conclude that its a tautology

Question 4 –optional–

Logical implication $p \to q$ is defined to be $q \vee \neg p$.

Use the definition to create new sequent calculus rules $\to R$ and $\to L$ that work directly on implication formulae. That is, fill in the top lines in the rules

$$\frac{\cdots}{\Gamma, p \to q \vDash \Delta}\ (\to L) \quad \frac{\cdots}{\Gamma \vDash p \to q, \Delta}\ (\to R)$$

$$p \to q$$

$$\frac{\Gamma,\ q \vee \neg p \vDash \Delta}{\Gamma,\ p \to q \vDash \Delta}\ \to L \qquad \frac{\Gamma \vDash q \vee \neg p,\ \Delta}{\Gamma \vDash p \to q,\ \Delta}\ \to R$$