1. In (a), (b), and (c) below, the matrix on the right hand side is obtained from the one on the left by an elementary row operation. Identify the operation in each case.

(a)

$$\begin{bmatrix} 4 & 2 & 1 & 5 \\ -1 & 3 & 1 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & 2 & 1 & 5 \\ 3 & -9 & -3 & 0 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -5 & 6 \\ 6 & 3 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 6 & 3 & 1 \\ 2 & -5 & 6 \\ 1 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ -3 & 1 & 8 \end{bmatrix} \longrightarrow \begin{bmatrix} -3 & 0 & -5 \\ 2 & -1 & 4 \\ -3 & 1 & 8 \end{bmatrix}$$

(d) Apply the three row operations you have identified, successively, in the same order, to the matrix below and write down the resulting matrix.

$$\begin{bmatrix} 1 & -2 & 0 & 3 & 0 & 4 \\ -3 & 0 & 1 & 4 & 0 & 6 \\ 0 & 7 & 0 & 0 & 1 & -9 \end{bmatrix} \longrightarrow \begin{bmatrix} & & & ? & & \end{bmatrix}$$

2. Use row operations to put the following matrices into RREF.

(a) $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 6 & 9 \\ 15 & 9 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -6 & -1 & 6 \\ 4 & -12 & 2 & 20 \\ 3 & -9 & 0 & 12 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 6 & -1 & -1 \\ 3 & -18 & 1 & 0 \\ 2 & -12 & 3 & 0 \end{bmatrix}$

3. For each of the following matrices, already in RREF, parameterize all the solutions to the corresponding system of equations. Write your solution in vector form (in $\mathbb{R}^n$ where n is the number of unknown variables).

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 8 \\ 0 & 1 & 2 & 2 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & -4 & 0 & -\frac{3}{2} & 5 \\ 0 & 0 & 1 & 6 & \frac{1}{9} \end{array}\right]$

(c) $\left[\begin{array}{ccccc|c} 1 & 0 & 3 & 0 & 1 & -5 \\ 0 & 1 & 2 & 0 & 8 & 9 \\ 0 & 0 & 0 & 1 & 7 & 4 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right]$

(e) $\left[\begin{array}{cccccc|c} 1 & -2 & 0 & 3 & 0 & 11 & -7 \\ 0 & 0 & 1 & 4 & 0 & 6 & 21 \\ 0 & 0 & 0 & 0 & 1 & -9 & 14 \end{array}\right]$

4. Parameterize all solutions to each of the systems of linear equations below, writing the parameterization in vector form. (That is, for each of the systems below, go through the steps of encoding the system in a matrix, using row operations to put each matrix in RREF, and writing down the general solution from the RREF form of the matrix.)

(a)
$$\begin{cases} 4x & + & 3y & + & 6z & + & 5w & = & -7 \\ & - & y & + & 2z & + & 2w & = & -9 \\ x & + & 4y & - & 5z & - & 5w & = & 27 \end{cases}$$

(b)
$$\begin{cases} 2x & + & 2y & + & 3z & + & 4w & = & 2 \\ -x & + & y & + & 2z & + & 7w & = & 1 \\ -3x & & & + & z & + & 5w & = & 1 \\ 6x & + & 2y & + & 1z & + & 3w & = & -1 \end{cases}$$

(c)
$$\begin{cases} 3x_1 & & & + & 9x_3 & + & 6x_4 & = & 3 \\ 2x_1 & + & x_2 & + & 5x_3 & + & 2x_4 & = & 1 \\ 5x_1 & + & 8x_2 & + & 7x_3 & - & 6x_4 & = & -3 \end{cases}$$