

Tutorial 10

---

1. Compute these matrix multiplications:

$$(a) \begin{bmatrix} 2 & -1 \\ 5 & 4 \\ 7 & -4 \end{bmatrix} \begin{bmatrix} 2 & -1 & -2 \\ 3 & 5 & 1 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & -1 & -2 \\ 3 & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & 4 \\ 7 & -4 \end{bmatrix}$$

$$(c) \begin{bmatrix} -3 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix}$$

$$(d) \begin{bmatrix} 3 & 1 & 2 \\ -2 & -1 & 3 \\ 7 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 8 & 3 & 8 \\ 2 & 1 & 0 \end{bmatrix}$$

2. Suppose we have two linear transformations  $L_1 : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  and  $L_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  given by these formulas:

$$L_1(x, y, z) = (7x + 3z, 2x + y + 8z) \quad \text{and} \quad L_2(u, v) = (4u + v, 2u + 3v, -u + 5v).$$

(a) Give the formulas for the composite function  $L = L_2 \circ L_1$ .

(b) Using these formulas, find the standard matrix  $C$  for  $L$ .

(c) Find the standard matrix  $A$  for  $L_1$  and  $B$  for  $L_2$ .

(d) Compute the matrix product  $BA$  showing the details of how you computed the entries. (You should, of course, get the matrix  $C$  as an answer.)

3. Let  $L : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be a linear transformation. Prove that  $L$  is surjective if and only if it is injective.