# QUEEN'S UNIVERSITY, Smith Engineering APSC 174 - All Sections - Introduction to Linear Algebra Final Exam $^1$ - April 2025

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HAND IN answers recorded on exam paper

Sample Multiple-Choice Problems

- Write your name and student number on this page before you begin.
- This examination is 3 HOURS in length.
- Answer all questions in the space provided. If you need more room, continue your answer on one of the blank pages at the back, providing clear directions to the marker, e.g., Continued on Page 15.
- Question 1 contains 16 multiple-choice problems. A Bubble Sheet for multiple-choice problems is provided at the end of this package. Please use the Bubble Sheet to record your responses. Note that answers written on the question pages (pages 2-7) will not be graded.
- Questions 2-5 are long answer questions. For full marks, you must show all your work and explain how you arrived at your answers. Correct answers with incorrect support may be given no credit.
- Only answers written in this booklet will be graded; all scrap paper from the test will be discarded.

**PLEASE NOTE:** Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.

Q 1	Q 2	Q 3	Q 4	Q 5	Total
16	10	7	10	5	48

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## Question 1. Multiple-Choice Problems [1 mark each]

Each problem has several possible answers, labeled (A), (B), (C), and/or (D).

Choose the **most appropriate answer** and completely fill in the rectangle associated with your response for each problem on the attached **Bubble Sheet** (last page of this exam package).

Note the Bubble Sheet is not replaceable, so please follow these instructions carefully:

- mark only one answer per question;
- use a pencil (not pen) to allow erasing if you change your answer;
- fill in the bubbles completely;
- erase any mistakes completely;
- do not fold, tear, or write outside the bubble area.
- (1) Which of the following are linear combinations of  $\mathbf{v}_1 = (0, -1, 1)$  and  $\mathbf{v}_2 = (1, 2, 1)$ ?
  - (A) (0,0,0)
  - (B) (2,2,4)
  - (C) Both A and B
  - (D) Neither A nor B
- (2) Are the vectors  $\mathbf{v}_1 = (2, 0, -1)$ ,  $\mathbf{v}_2 = (40, 0, 7)$ , and  $\mathbf{v}_3 = (-1, 100, 4)$  linearly dependent in  $\mathbb{R}^3$ ?
  - (A) Yes
  - (B) No
  - (C) Not enough information to determine
  - (D) None of the above
- (3) Which of the following is true?
  - (A) Every linearly independent set of seven vectors in  $\mathbb{R}^7$  forms a basis for  $\mathbb{R}^7$ .
  - (B) Every set of seven vectors that spans  $\mathbb{R}^7$  forms a basis for  $\mathbb{R}^7$ .
  - (C) Every set of vectors that spans  $\mathbb{R}^7$  contains a basis for  $\mathbb{R}^7$ .
  - (D) All are true.

(4)	Consider the vector space $\mathbb{R}^2$ and $\mathbf{v} = (2,0) \in \mathbb{R}^2$ . Then the linear span of $\mathbf{v}$ is
	(A) $\{\mathbf{v}\}$ (B) $\{(x,y): x \in \mathbb{R}, y = 0\}$ (C) $\{(0,y): y \in \mathbb{R}\}$ (D) $\mathbb{R}^2$
(5)	If A is a $6 \times 4$ matrix, then the number of leading ones in the reduced row echelon form of A is at most
	(A) 3
	(B) 4
	(C) 5
	(D) 6
(6)	To determine whether a set of $n$ vectors from $\mathbb{R}^n$ is linearly independent, we can form a matrix $A$ whose columns are the vectors in the set and then put $A$ into its reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?
	(A) A row of all zeros
	(B) A column of all zeros
	(C) An identity matrix
	(D) A row that has all zeros except in the last position
(7)	A square matrix $A$ is invertible if and only if
	(A) $\lambda = 0$ is not an eigenvalue of $A$
	(B) $\lambda = 0$ is an eigenvalue of $A$
	(C) $\lambda = 1$ is not an eigenvalue of $A$
	(D) $\lambda = 1$ is an eigenvalue of A

(8) If A and B are  $n \times n$  real matrices, A is invertible, and  $AB = \mathbf{0}$ , then B =

- (A) Zero matrix
- (B) Identity matrix
- (C) Invertible matrix
- (D) The inverse of A

(9) If A is an  $n \times n$  matrix, and columns of A are linearly dependent, then

(A) 
$$\det(A) = 0$$

(B) 
$$\det(A) \neq 0$$

(C) 
$$\det(A) < 0$$

(D) 
$$\det(A) > 0$$

(A) 
$$2^4$$

$$(C)$$
 1

(D) 
$$0$$

(11) The characteristic polynomial of the matrix  $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$  is

(A) 
$$\lambda(\lambda-2)$$

(B) 
$$(\lambda - 2)(\lambda + 1)$$

(C) 
$$\lambda(\lambda+2)$$

(D) 
$$(\lambda - 1)(\lambda - 2)$$

(12) Suppose the matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  has an eigenvalue 1 with associated eigenvector  $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ . What is  $A^{27}\mathbf{v}$ ?

(A) 
$$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(B) 
$$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

(C) 
$$\begin{bmatrix} -1 \\ 2^{27} \end{bmatrix}$$

(D) 
$$\left[ \begin{array}{cc} a^{27} & b^{27} \\ c^{27} & d^{27} \end{array} \right]$$

### STUDENT NUMBER:

Questions (13) to (16) all refer to the following information. A linear transformation L has the standard matrix A shown below. Also shown is the reduced row echelon form (RREF) of A.

$$A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 3 & -9 & 4 \\ 0 & 2 & -6 & 2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (13) Which of the following describes the input and output spaces for L?
  - (A)  $L: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$
  - (B)  $L: \mathbb{R}^4 \longrightarrow \mathbb{R}^3$
  - (C)  $L: \mathbb{R}^3 \longrightarrow \mathbb{R}^4$
  - (D)  $L: \mathbb{R}^4 \longrightarrow \mathbb{R}^4$
- (14) What is  $\dim(\operatorname{Im}(L))$ ?
  - (A)  $\dim(\operatorname{Im}(L)) = 1$
  - (B)  $\dim(\operatorname{Im}(L)) = 2$
  - (C)  $\dim(\operatorname{Im}(L)) = 3$
  - (D)  $\dim(\operatorname{Im}(L)) = 4$
- (15) What is  $\dim(\text{Ker}(L))$ ?
  - (A)  $\dim(\operatorname{Ker}(L)) = 0$
  - (B)  $\dim(\operatorname{Ker}(L)) = 1$
  - (C)  $\dim(\operatorname{Ker}(L)) = 2$
  - (D)  $\dim(\operatorname{Ker}(L)) = 3$
- (16) Which of the following vectors are in the kernel of L?
  - $(A) \quad \begin{bmatrix} 0 \\ 6 \\ 2 \\ 0 \end{bmatrix}$

 $(B) \quad \begin{bmatrix} 0 \\ -2 \\ 6 \\ 0 \end{bmatrix}$ 

 $(C) \quad \begin{bmatrix} -2\\1\\-3\\1 \end{bmatrix}$ 

(D)

#### Note for students

These are a sample of possible questions; the actual exam questions may cover other material from the course.

#### Correct answers are:

- 1-(C)
- 2-(B)
- 3-(D)
- 4-(B)
- 5-(B)
- 6-(C)
- 7-(A)
- 8-(A)
- 9-(A)
- 10-(D)
- 11-(A)
- 12-(A)
- 13-(B)
- 14-(B)
- 15-(C)
- 16-(A)

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7	CAJ	EBJ	EC3	EDJ	C83	32		E83	E03			57	EAD	EB3	EC3			82	CAI	E83	C03	EDJ	CEI
8	CAJ					33	CAJ	CSI	CCI			58	CAJ		COI			83	CAD				CEI
9	CAI	EB3	E03		CED	34	CAD	E83	E03		CED	59	CAI	CBJ	E03	C03	CED	84	CAJ	EBO	C03		CEI
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12	CAJ	CBJ	E¢3		CED	37		EBI	ECI	EDJ	CEJ	62	CAJ	EB3	E03	E03	CEI	87	CAJ	CB3	C03	E03	CED
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16	CAJ	CBD	CCI		CED	41	CAJ		ECI	CDD	CED	66	CAD	EB3	COD	E03	CED	91	CAD	CBJ	E03		CEI
17	CAI	E83	□¢⊒			42			EC]		CED	67	EAJ		E¢⊒		CED	92	CAJ				CED
18	CAJ	EB3	E¢J		CED	43		E83	ECI	EDJ	CED	68		CBJ	EQI		CED	93	CAJ	EB3	CCI		CEI
19	CAI	CBJ	CCI		CEI	44	□A□			EDJ	CEJ	69	CAI	EBI	ECI		CED	94	CAI	EB3			CED
20	CAJ	E83	ECI	EDD	CED	45	CAJ	CB3	Eca	EDE	CED	70	EAD	EB3	E03	EDD	CED	95	EAD	EBO	ECI	E03	DED

