Scaling by Two Standard Deviations

POSC 3410 – Quantitative Methods in Political Science

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Goal for Today

Make the most of regression by making coefficients directly interpretable.

Introduction

You all should be familiar with regression by now.

Introduction

Regression coefficients communicate:

- Estimated change in y for one-unit change in x.
    - This is in linear regression.
- Estimated change in *logged odds* of y for one-unit change in x.
    - This is the interpretation for logistic regression.

These communicate some quantities of interest.

- After all, you want to know the effect of x on y!

Introduction

However, it's easy (and tempting) to provide misleading quantities of interest.

- Our variables are seldom (if ever) on the same scale.
    - e.g. age can be anywhere from 18 to 100+, but years of education are typically bound between 0 and 25 (or so).
- Worse yet, zero may not occur in any variable.
    - We would have an uninterpretable *y*-intercept.
- From my experience, this can lead to false convergence of the model itself.

Your goal: regression results should be as easily interpretable as possible.

Interpreting by Standardizing the Input

Gelman (2008) offers a technique for interpreting regression results: scale the non-binary input data by two standard deviations.

- This makes continuous inputs (roughly) on same scale as binary inputs.
- It allows a preliminary evaluation of relative effect of predictors otherwise on different scales.

Standardization

Standardization follows z transformations, which you should know.

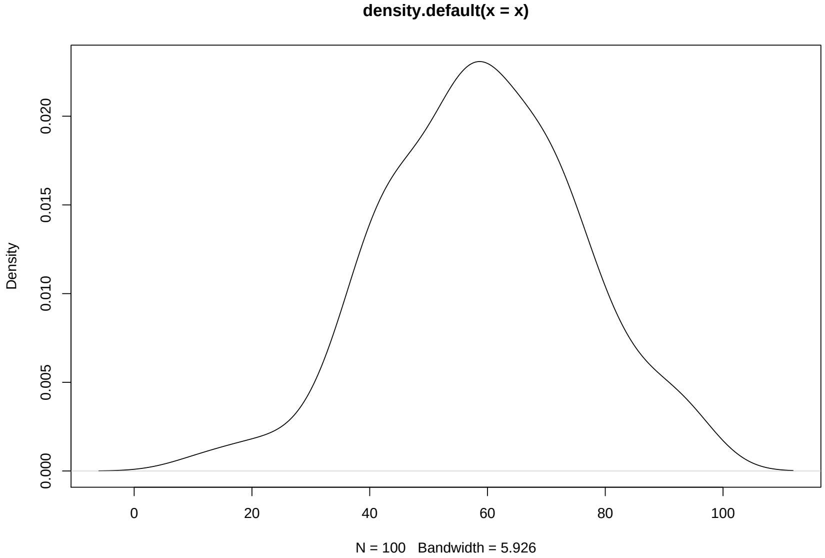
- $z = (\bar{x} - \mu)/\sigma$

This transforms any variable to have a mean of zero and a standard deviation of one.

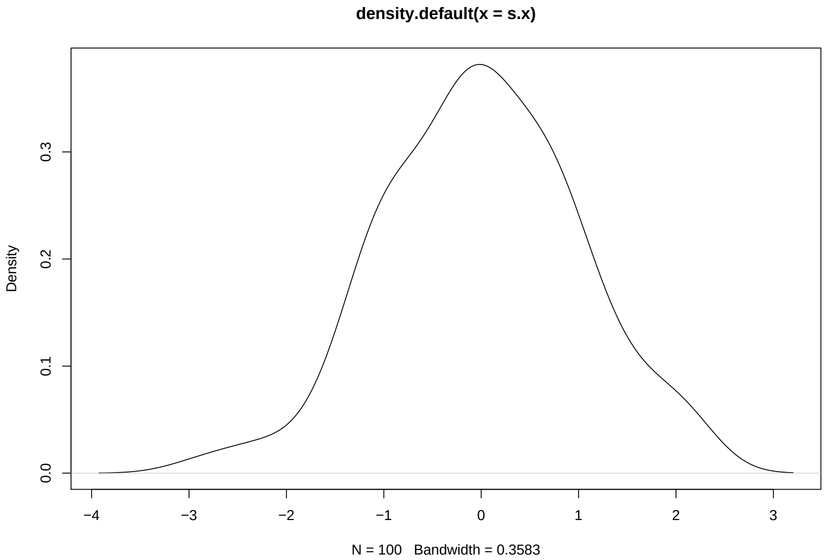
Observe for Normally Distributed Data

```
> set.seed(8675309) ### for reproducibility
> x <- rnorm(100, 58, 17.8)
> mean(x)
[1] 58.93099
> sd(x)
[1] 16.53876
> s.x <- (x - mean(x))/(sd(x))
> mean(s.x)
[1] 1.998618e-16
> sd(s.x)
[1] 1
```

Density Plot



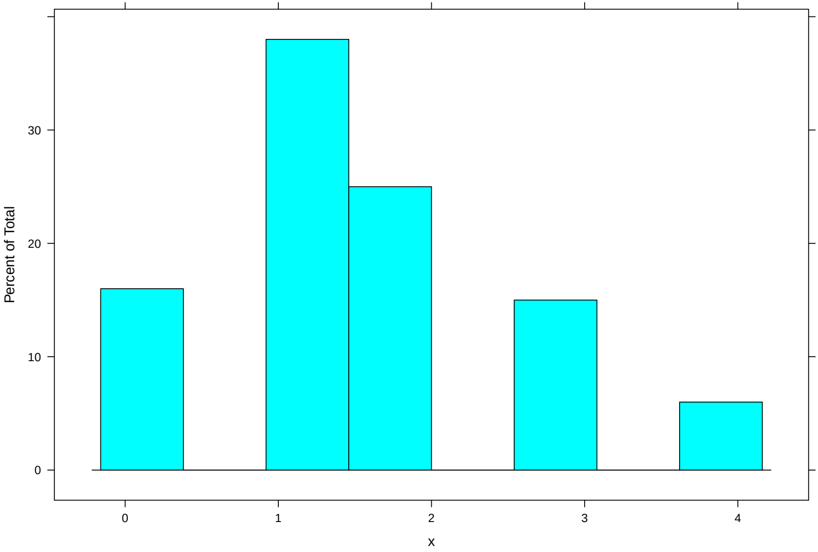
Density Plot (Standardized)



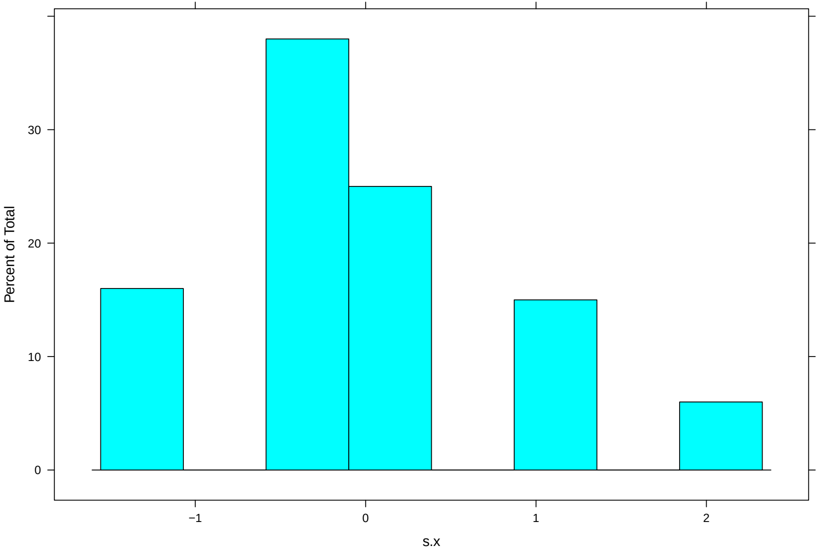
Works with Non-Normally Distributed Data Too

```
> set.seed(8675309)
> x <- rpois(100, 1.5)
> mean(x)
[1] 1.57
> sd(x)
[1] 1.112418
> s.x <- (x - mean(x))/(sd(x))
> mean(s.x)
[1] -7.327933e-17
> sd(s.x)
[1] 1
```

Density Plot



Density Plot (Standardized)



Don't Let the Scale Fool You

```r
unique(x)
```

```
## [1] 0 1 2 4 3
```

```r
unique(s.x)
```

```
## [1] -1.4113394 -0.5123971  0.3865452  2.1844298  1.2854875
```

What Standardization Does to Regression Coefficients

Recall what standardization does when overlaying standardized x-axis with normal x-axis.

- Distance between 0 and $1 = 34\%$ of data.

Thus, a regression coefficient for standardized variable estimates the effect of x on y for a one-standard deviation change from the mean.

- i.e. effect of a change across 34% of the data of x.

Benefits/Limitations of Standardization

Standardizing by one standard deviation is helpful for a couple reasons.

- It creates a meaningful zero (i.e. the mean) for the y-intercept.
- Regression coefficient captures a magnitude change.

However, it won't help us make preliminary comparisons with dummy variables.

The Problem of Dummy Variables

Dummy variables are special class of nominal variables.

- An indicator is either “there” or “not there”.

In regression, this has an important effect.

- Coefficient goes up, all else equal.
    - So does standard error.

It may be misleading to think that binary variables have the largest effect on an outcome, but a regression coefficient may suggest this.

Scaling by Two Standard Deviations

Take a continuous (non-binary) input variable and divide it by two standard deviations instead of one.

- This will transform the data to have a mean of zero and standard deviation of .5.
- Regression coefficient would communicate estimated change in y for change across 47.7% of data in x.

Observe with Normally Distributed Data

```
> set.seed(8675309)
> x <- rnorm(100, 58, 17.8)
> mean(x)
[1] 58.93099
> sd(x)
[1] 16.53876
> s.x <- (x - mean(x))/(2*sd(x)) ### two SDs instead of one
> mean(s.x)
[1] 9.993091e-17
> sd(s.x)
[1] 0.5
```

Same Thing with Non-Normally Distributed Data

```
> set.seed(8675309)
> x <- rpois(100, 1.5)
> mean(x)
[1] 1.57
> sd(x)
[1] 1.112418
> s.x <- (x - mean(x))/(2*sd(x)) ### two SDs instead of one
> mean(s.x)
[1] -3.663966e-17
> sd(s.x)
[1] 0.5
```

Comparison with Binary Independent Variables

Why do this? Consider a dummy IV with 50/50 split between 0s and 1s.

- $p(dummy = 1) = .5$
- Then, standard deviation equals .5 ($\sqrt{.5 * .5} = \sqrt{.25} = .5$)
- We can directly compare this dummy variable with our new standardized input variable!

This works well in most cases, except when $p(dummy = 1)$ is really small.

- e.g. $p(dummy = 1) = .25$, then $\sqrt{.25 * .75} = .4330127$

An Application with State Education-Turnout

I revisit the state education-turnout example from the Pollock book.

I do use newer data.

- **state education**: % of state having HS diploma (2009).
- **turnout**: state-level turnout of VEP in 2012 general election.
- **region**: factor/"fixed effects"
  - 0 = West, 1 = Northeast, 2 = Midwest, 3 = South.

A Simple Regression

```
> M1 <- lm(turnout ~ perhsdiploma +
           factor(regioncondensed), data=Data)
> summary(M1)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -33.7839    27.3483  -1.235  0.22312
perhsdiploma   1.0464     0.3105   3.370  0.00155 **
Northeast      3.4049     2.4388   1.396  0.16952
Midwest        4.1645     2.2680   1.836  0.07294 .
South          3.7147     2.4732   1.502  0.14009
```

Some Confusion (with the Results)

- We see that more educated states have higher turnout.
- Midwestern states have higher turnout in comparison to states in West.

However, are we to believe that the Midwest is the largest predictor?

- and what about that uninterpretable y-intercept?

Let's standardize the education variable by two standard deviations.

A More Readable Regression

```
> M2 <- lm(turnout ~ z.perhsdiploma +
    factor(regioncondensed), data=Data)
    > summary(M2)

    Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      57.133      1.592  35.881  < 2e-16 ***
z.perhsdiploma    7.128      2.115   3.370  0.00155 **
Northeast         3.405      2.439   1.396  0.16952
Midwest           4.164      2.268   1.836  0.07294 .
South             3.715      2.473   1.502  0.14009
```

Interpretation

Notice that the effects ultimately didn't change for the region fixed effects.

- t value for education variable is unchanged too.

However, this regression table is much more readable.

- y-intercept is much more meaningful.
- We also see that education does appear to have the largest effect.

Table of Contents