TITLE: Algebra of Fuzzy Sets **AUTHOR**: Giacomo Garbarino **DATE**: 31/05/2022

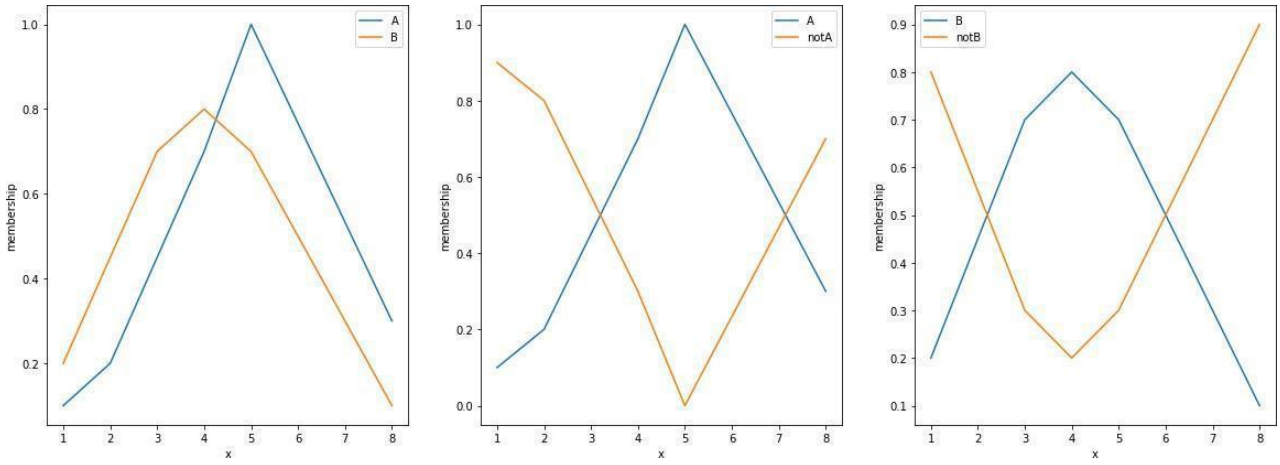
HOMEWORK DESCRIPTION

The homework consist of implementing and testing the following fuzzy set operations: complement, intersection, union, alfa-level set, algebraic sum, bounded sum, bounded difference, bounded product, algebraic product, drastic product, drastic sum and cartesian product.

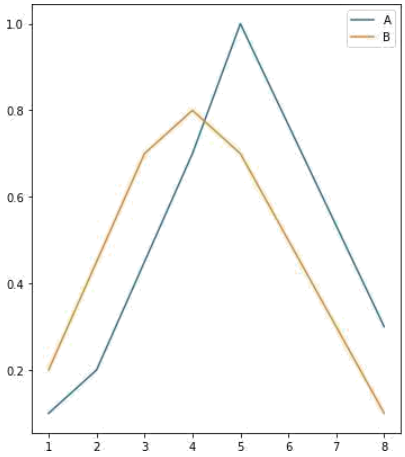
HOMEWORK APPROACH

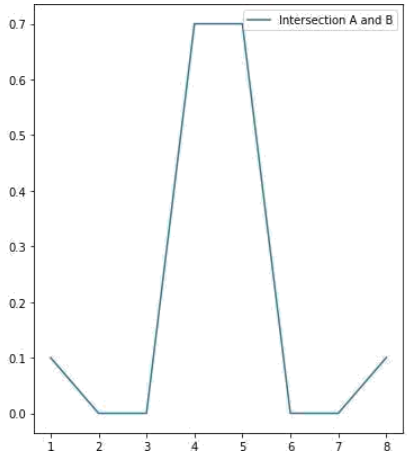
The implementation of each operation consists of creating an initial empty array, comparing the membership degree's value of each element of one input array with the membership degree's value of the corresponding element of the other input array and according to the result of the comparison, each element with its resulting membership degree's value according to the logic of the operation is added as a tuple in the array which is then returned as output. In particular, the implementation of the cartesian product consists of comparing the membership degree's value of each element of an input array with all the membership degree's value of each element of the other input array and the resulting array contains one tuple for each comparison. The tuple contains the two elements compared and the minimum membership degree's value between them. The alfa-level operation, more precisely, consists of comparing the membership degree's value of each element of the input set with a value given as paramether and each element with a membership degree's value equal or greater than that value is added to an initial empty array which is then returned as output. In the test phase of each operation except the the cartesian product, the input sets and the resulting one have been plotted and compared using 2d graphs. In the test phase of the cartesian product, the resulting set has been plotted using a 3d scatterplot in two different point of views. In complement operation, alfa-level set operation and cartesian product operation, the *sets()* operation has not been used inside the implementation because it was not necessary.

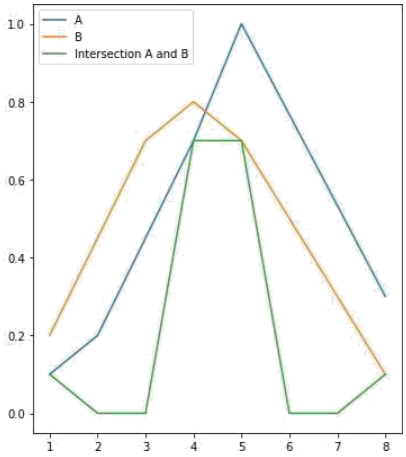
RESULTS



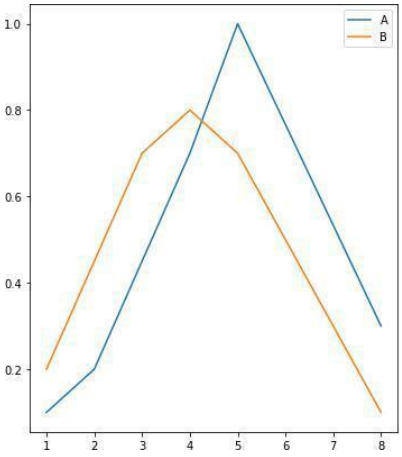
As expected, the plot of the complementary set is exactly the opposite of the original one for both input sets.

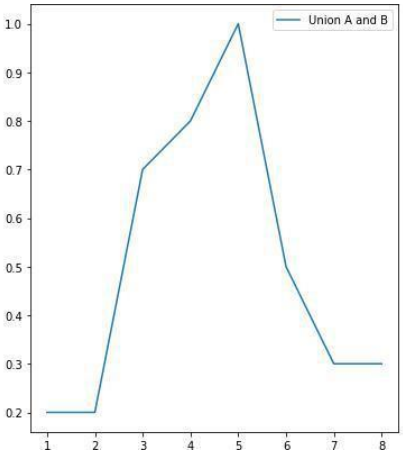


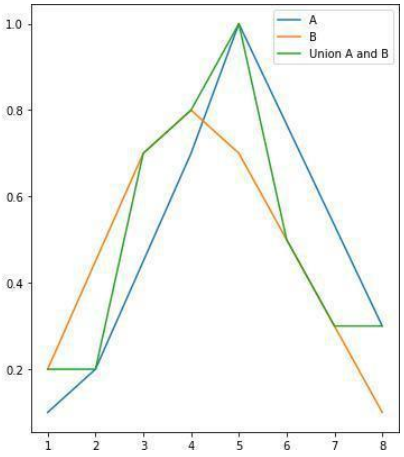




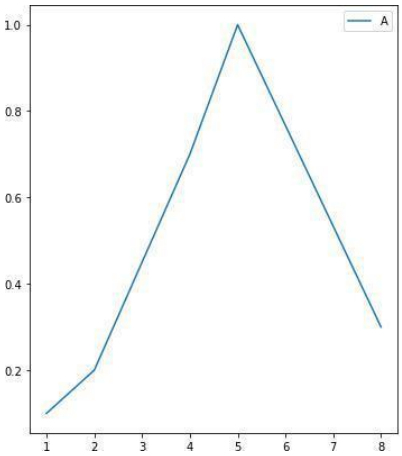
The plot of the intersection set shows that most of elements has a very low membership degree's value respect to those of the original sets which are a little bit higher.

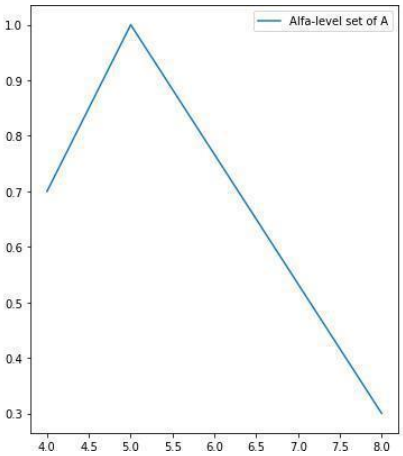


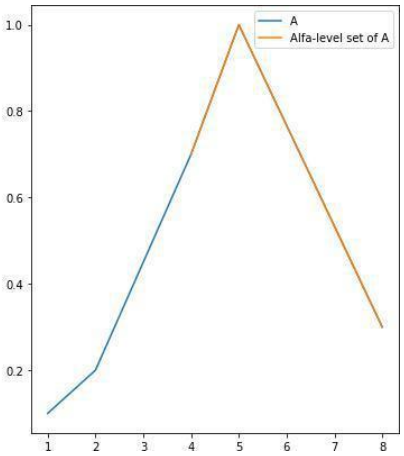




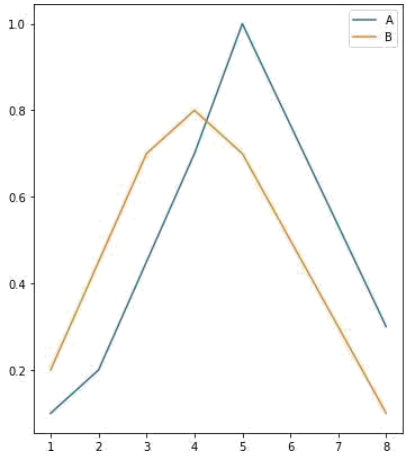
As expected, the plot of the union set shows that most of elements' membership degrees have values quite similar to the original ones. In the third plot, the green line is very close to the orange and blue line and it sometimes overlaps on them.

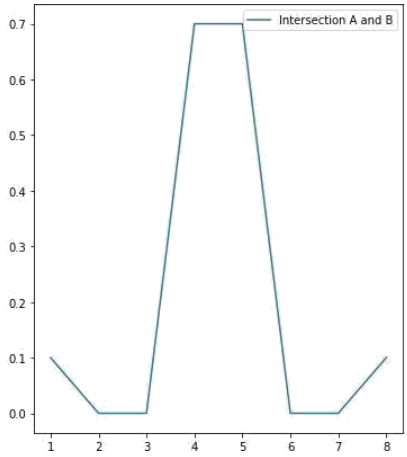


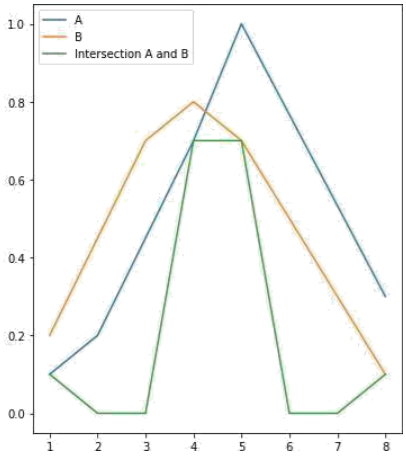




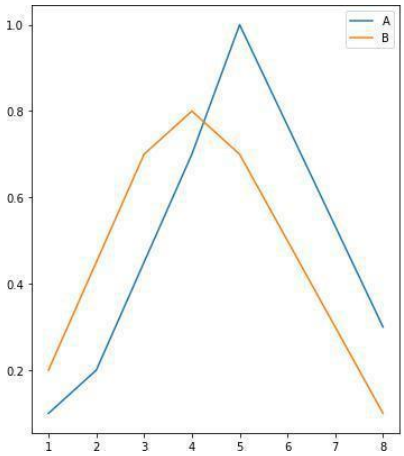
As expected, since the alfa-level set operation is a sort of filter operation, the plot from a certain point (0.7) overlaps the plot of the original one.

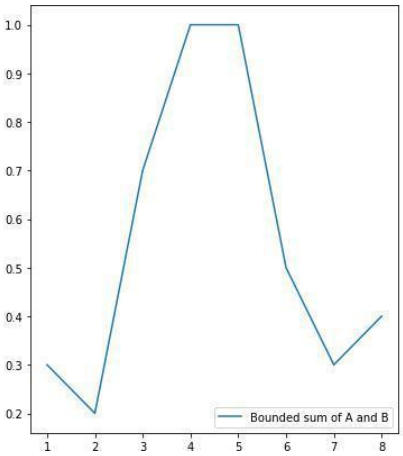


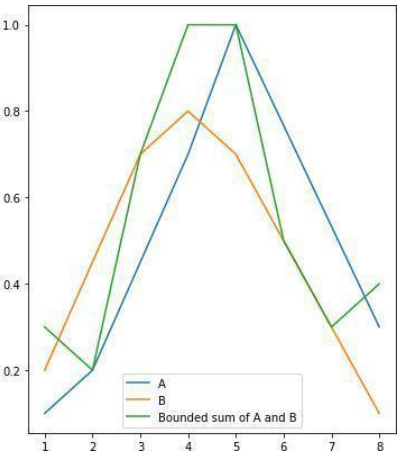




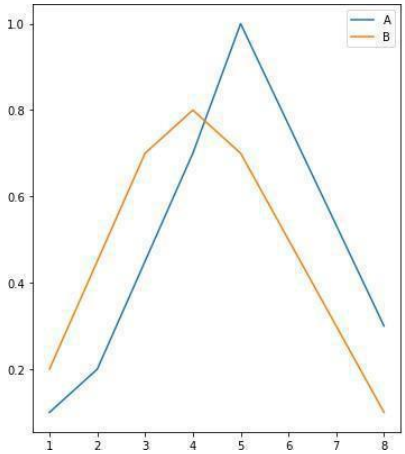
The plot of the algebraic sum set shows that most of elements' membership degrees have values quite similar to the original ones and the green line sometimes overlaps the other two ones.

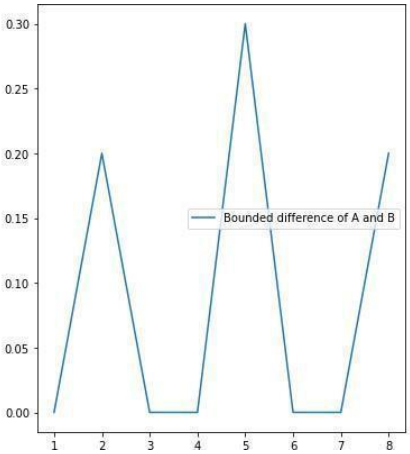


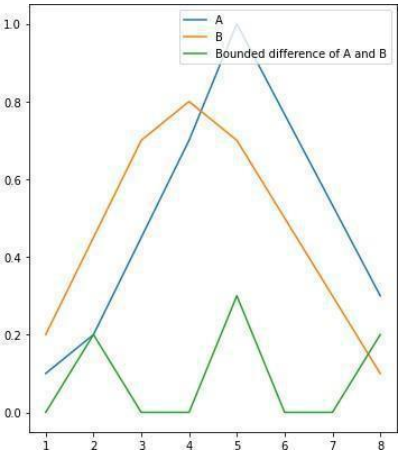




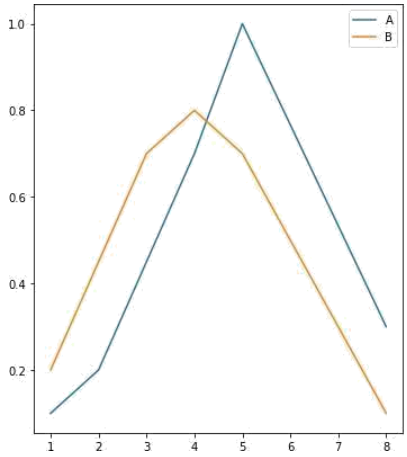
As in the algebraic sum, the plot of the bounded sum set shows a similar behaviour.

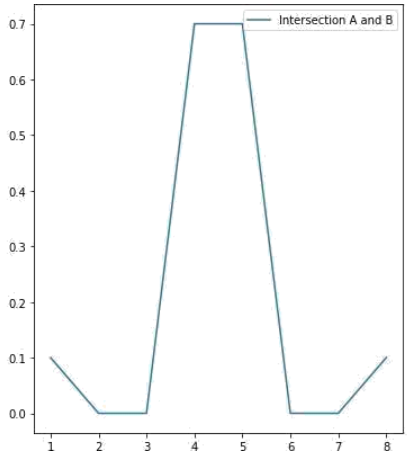


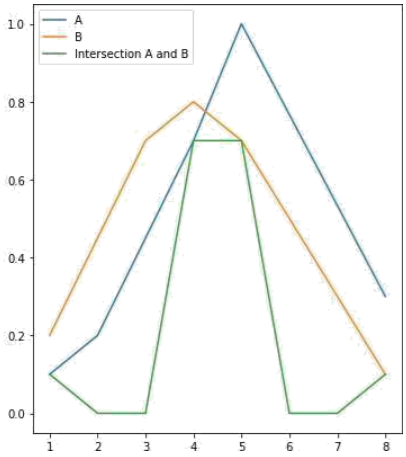




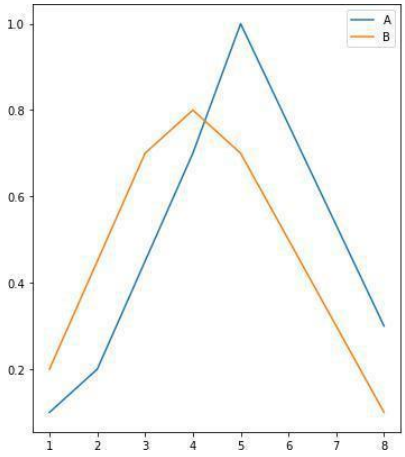
As expected, the plot of the bounded difference set shows that most elements' membership degrees have values very close to 0, in fact the membership values of the original sets are very close each other.

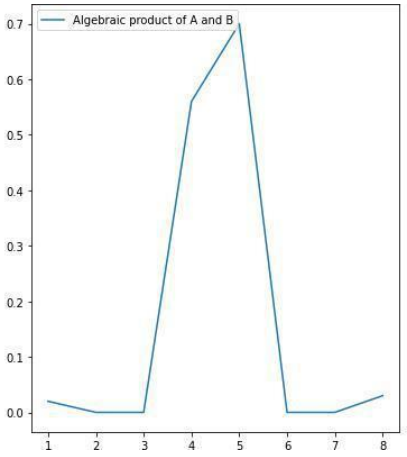


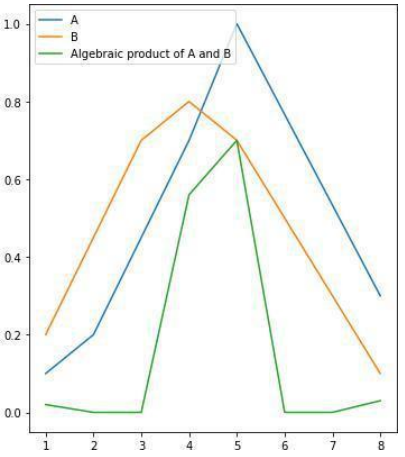




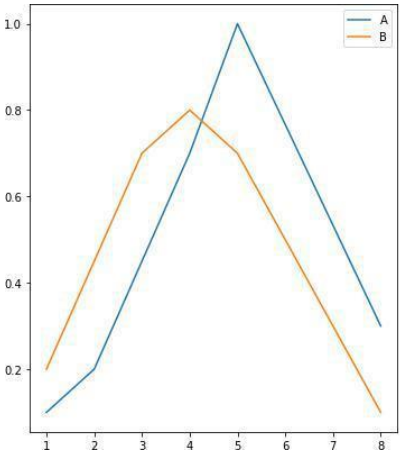
The plot of the bounded product set shows that the value of membership degree of six elements on eight is equal to 0, so only the elements which have high membership degree values in both original sets show a value higher than 0. It can be said this operation is quite drastic.

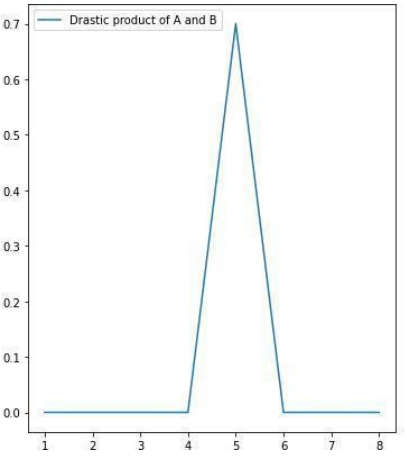


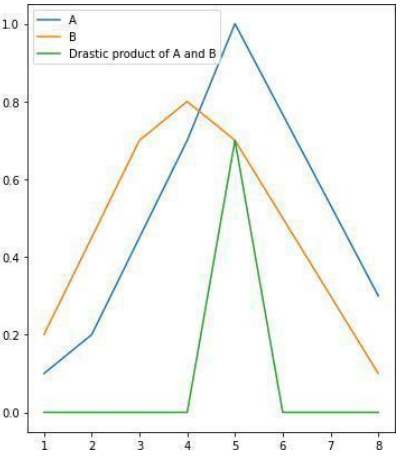




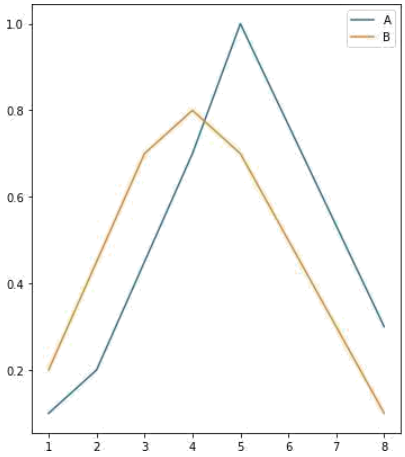
This plot shows that algebraic product and bounded product have a very similar behaviour.

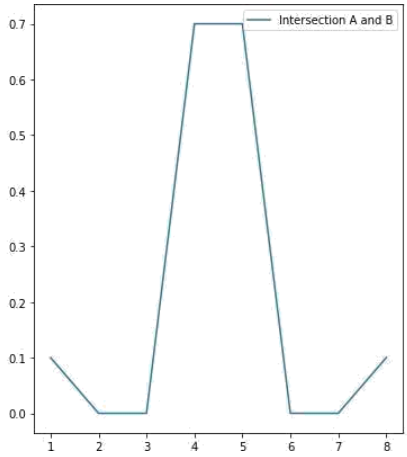


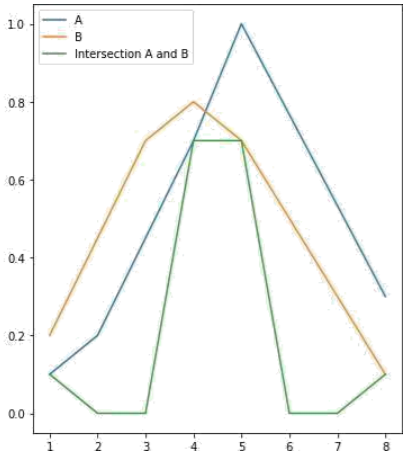




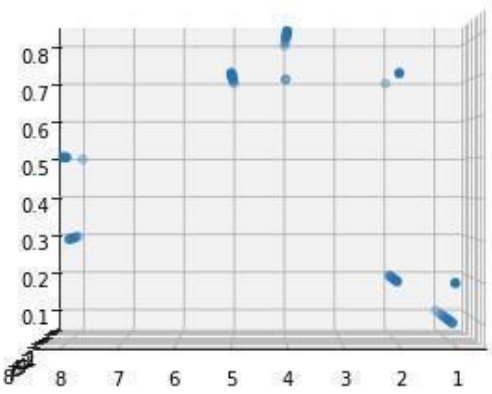
As the name says, the drastic product is very drastic, even more than the bounded product; only one element has a membership degree value higher than 0 (element 5).

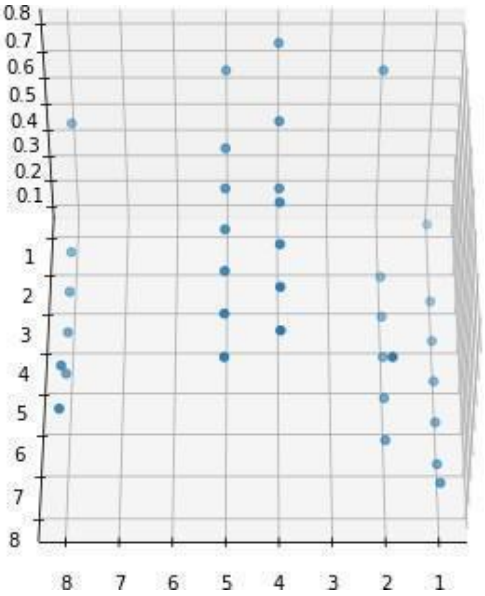






As expected, the plot of the drastic sum set shows that this operation tends to increase the membership degree values of the majority of the elements of the original sets.





The plot of the cartesian product shows that most of the tuples of the form *[[element_A, element_B], degree_value]* have membership degree values higher than 0.3.