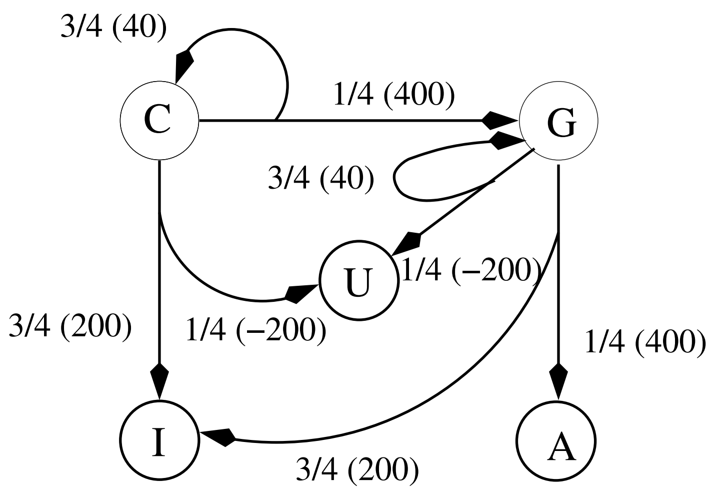
CS 6300 **HW05: Policy Iteration and TD Learning** **Due Feb 28, 2023**

Please use the LaTeX template to produce your writeups. See the Homework Assignments page on the class website for details. Hand in via gradescope.

1 Temporal Difference Learning



For the MDP above, you decide to use experience and TD learning to find the values. You experience the following 3 episodes.

S	A	R	S	A	R	S	A	R
C	s	40	C	s	40	C	s	400
C	s	40	C	g	200	G	s	40
C	s	400	I			G	g	400
G	s	40				A		
G	s	-200						
U								

The learning rate is $\alpha = (1/2)^n$, where n is the episode number. Initialize all values to 0 and perform TD learning to estimate the state values $V^\pi(S)$ using the above three episodes.
Equation for TD learning: $V^\pi(S) = (1-\alpha)V^\pi(S) + \alpha(R(S,A,S') + \gamma V^\pi(S'))$
Episode 1:
1: $V^\pi(C) = (1-\frac{1}{2}) * 0 + \frac{1}{2}(40 + 1 * 0) = 20$
2: $V^\pi(C) = (1-\frac{1}{2}) * 20 + \frac{1}{2}(40 + 1 * 20) = 40$
3: $V^\pi(C) = (1-\frac{1}{2}) * 30 + \frac{1}{2}(400 + 1 * 0) = 215$
4: $V^\pi(G) = (1-\frac{1}{2}) * 0 + \frac{1}{2}(40 + 1 * 0) = 20$
5: $V^\pi(G) = (1-\frac{1}{2}) * 20 + \frac{1}{2}(-200 + 1 * 20) = -80$

Episode 2:
1: $V^\pi(C) = (1-\frac{1}{4}) * 215 + \frac{1}{4}(40 + 1 * 215) = 225$

2: $V^{\pi}(C) = (1 - \frac{1}{4}) * 225 + \frac{1}{4}(200 + 1 * 0) = 218.75$

Episode 3:
1: $V^{\pi}(C) = (1 - \frac{1}{8}) * 218.75 + \frac{1}{8}(400 + 1 * -80) = 231.40625$
2: $V^{\pi}(G) = (1 - \frac{1}{8}) * -80 + \frac{1}{8}(40 + 1 * -80) = -75$
3: $V^{\pi}(G) = (1 - \frac{1}{8}) * -75 + \frac{1}{8}(400 + 1 * 0) = -15.625$

2 Q-learning

In this simplied version of blackjack, the deck is infinite and the dealer always has a fixed count of 15. The deck contains cards 2 through 10, J, Q, K, and A, each of which is equally likely to appear when a card is drawn. Each number card is worth the number of points shown on it, the cards J, Q, and K are worth 10 points, and A is worth 11. At each turn, you may either *hit* or *stay*.

- **If you choose to *hit*, you receive no immediate reward and are dealt an additional card.**
- **If you stay, you receive a reward of 0 if your current point total is exactly 15, +10 if it is higher than 15 but not higher than 21, and -10 otherwise (i.e., lower than 15 or larger than 21).**
- **After taking the *stay* action, the game enters a terminal state *end* and ends.**
- **A total of 22 or higher is refered to as a *bust*; from a *bust*, you can only choose the action *stay*.**

As your state space you take the set $\{0, 2, \ldots, 21, bust, end\}$ **indicating point totals.**

Given the partial table of initial Q-values below left, fill in the partial table of Q-values on the right after the episode center below occurs. Assume $\alpha = 0.5$ **and** $\gamma = 1$**. The initial portion of the episode has been omitted. Show the derivation of the Q values that are updated.**

s	a	$Q(s,a)$
19	**hit**	**-2**
19	**stay**	**5**
20	**hit**	**-4**
20	**stay**	**7**
21	**hit**	**-6**
21	**stay**	**8**
bust	**stay**	**-8**

s	a	r
19	**hit**	**0**
21	**hit**	**0**
bust	**stay**	**-10**

s	a	$Q(s,a)$
19	**hit**	**3**
19	**stay**	**5**
20	**hit**	**-4**
20	**stay**	**7**
21	**hit**	**-7**
21	**stay**	**8**
bust	**stay**	**-9**

Equation: $Q(s_t, a_t) = (1-\alpha)Q(s_t, a_t) + \alpha(R(s_t, a_t, s_{t+1}) + \gamma max_{a'} Q(s_{t+1}, a'))$
Q value updates:
$Q(19, hit) = (1 - \frac{1}{2}) * -2 + \frac{1}{2}(0 + 1 * 8) = 3$
$Q(21, hit) = (1 - \frac{1}{2}) * -6 + \frac{1}{2}(0 + 1 * -8) = -7$
$Q(bust, stay) = (1 - \frac{1}{2}) * -8 + \frac{1}{2}(-10 + 1 * 0) = -9$