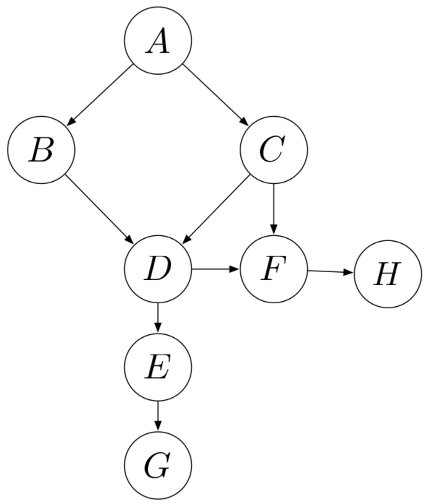
CS 6300 **HW07: Bayes Nets I** **April 4, 2023**

1 D-Separation

For the Bayes net below, determine if each independence assertion is guaranteed to be true.



1. $B \perp\!\!\!\perp C$

   Paths: BAC, BDC, and BDFC

   Triples:
   BAC - Active
   BDC -
   BDF -
   DFC -

   Any active path indicates not guaranteed. BAC is active

2. $B \perp\!\!\!\perp C \mid G$

   Paths: BAC, BDC, and BDFC

   Triples:
   BAC - Active
   BDC -
   BDF -
   DFC -

   Any active path indicates not guaranteed. BAC is active

3. $B \perp\!\!\!\perp C \mid H$

   Paths: BAC, BDC, and BDFC

   Triples:
   BAC - Active

BDC -
BDF -
DFC -

Any active path indicates not guaranteed. BAC is active

4. $A \perp\!\!\!\perp D \mid G$

   Paths: ABD, ACD, and ACFD

   Triples:
   ABD - Active
   ACD -
   ACF -
   CFD -

   Any active path indicates not guaranteed. ABD is active

5. $A \perp\!\!\!\perp D \mid H$

   Paths: ABD, ACD, and ACFD

   Triples:
   ABD - Active
   ACD -
   ACF -
   CFD -

   Any active path indicates not guaranteed. ABD is active

6. $B \perp\!\!\!\perp C \mid A, F$

   Paths: BAC, BDC, and BDFC

   Triples:
   BAC - Inactive
   BDC - Inactive
   BDF - Active
   DFC - Active

   Any active path indicates not guaranteed. BDFC is active

7. $F \perp\!\!\!\perp B \mid D, A$

   Paths: FDB, FCAB, FCDB, and FDCAB

   Triples:
   FDB - Inactive
   FCA -
   CAB - Inactive
   FCD - Active
   CDB - Active
   FDC - Inactive
   DCA -

Any active path indicates not guaranteed. FCDB is active.

8. $F \perp\!\!\!\perp B \mid D, C$

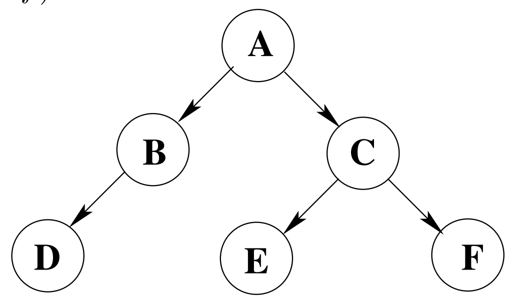
   Paths: FDB, FCAB, FCDB, and FDCAB

   Triples:
   FDB - Inactive
   FCA - Inactive
   CAB -
   FCD - Inactive
   CDB -
   FDC - Inactive
   DCA -

   All paths are have an inactive triple and are thus inactive, therefore we are guaranteed that the independence assertion is true

2 Inference by Enumeration

Consider the following Bayes' net. Derive $P(A|+e,-f)$.



A	$P(A)$
+a	0.3
-a	0.7

B	A	$P(B\|A)$
+b	+a	0.7
+b	-a	0.6

C	A	$P(C\|A)$
+c	+a	0.2
+c	-a	0.9

D	B	$P(D\|B)$
+d	+b	0.3
+d	-b	0.4

E	C	$P(E\|C)$
+e	+c	0.4
+e	-c	0.5

F	C	$P(F\|C)$
+f	+c	0.1
+f	-c	0.8

From the Bayes' net we have

$$P(A, B, C, D, +e, -f) = P(A)P(B|A)P(C|A)P(D|B)P(+e|C)P(-f|C)$$

The order in which to joint factors is arbitrary, and we wil use B, C, A resulting in the factors below.

$$\begin{aligned} f_1(A, B, D) &= P(B|A)P(D|B) \\ f_2(A, C, +e, -f) &= P(C|A)P(+e|C)P(-f|C) \\ f_3(A, B, C, D, +e, -f) &= P(A)f_1(A, B, D)f_2(A, C, +e, -f) \end{aligned}$$

We need this for factor 2.

$$P(C) = P(C|A) * P(A)$$

A	C	$P(C\|A)$	P(A)	P(C)
+a	+c	0.2	0.3	0.06
+a	-c	0.8	0.3	0.24
-a	+c	0.9	0.7	0.63
-a	-c	0.1	0.7	0.07

C	P(C)
+c	0.69
-c	0.31

Fill in the following factor tables:

A	B	D	$f_1(A, B, D)$
+a	+b	+d	0.7 * 0.3 = 0.21
+a	+b	-d	0.7 * 0.7 = 0.49
+a	-b	+d	0.3 * 0.4 = 0.12
+a	-b	-d	0.3 * 0.6 = 0.18
-a	+b	+d	0.6 * 0.3 = 0.18
-a	+b	-d	0.6 * 0.7 = 0.42
-a	-b	+d	0.4 * 0.4 = 0.16
-a	-b	-d	0.4 * 0.6 = 0.24

A	C	$f_2(A, C, +e, -f)$
+a	+c	0.2 * (0.4 / 0.69) * (0.9 / 0.69) = 0.151
+a	-c	0.8 * (0.5 / 0.31) * (0.2 / 0.31) = 0.832
-a	+c	0.9 * (0.4 / 0.69) * (0.9 / 0.69) = 0.681
-a	-c	0.1 * (0.5 / 0.31) * (0.2 / 0.31) = 0.104

Using f_1 and f_2, compute f_3:

A	B	C	D	$f_3(A, B, C, D, +e, -f)$
+a	+b	+c	+d	0.3 * 0.21 * 0.151 = 0.0095
+a	+b	+c	-d	0.3 * 0.49 * 0.151 = 0.0221
+a	+b	-c	+d	0.3 * 0.21 * 0.832 = 0.0524
+a	+b	-c	-d	0.3 * 0.49 * 0.832 = 0.1123
+a	-b	+c	+d	0.3 * 0.12 * 0.151 = 0.0054
+a	-b	+c	-d	0.3 * 0.18 * 0.151 = 0.0081
+a	-b	-c	+d	0.3 * 0.12 * 0.832 = 0.0299
+a	-b	-c	-d	0.3 * 0.18 * 0.832 = 0.0449
-a	+b	+c	+d	0.7 * 0.18 * 0.681 = 0.0858
-a	+b	+c	-d	0.7 * 0.42 * 0.681 = 0.2002
-a	+b	-c	+d	0.7 * 0.18 * 0.104 = 0.0131
-a	+b	-c	-d	0.7 * 0.42 * 0.104 = 0.0305
-a	-b	+c	+d	0.7 * 0.16 * 0.681 = 0.0762
-a	-b	+c	-d	0.7 * 0.24 * 0.681 = 0.1144
-a	-b	-c	+d	0.7 * 0.16 * 0.104 = 0.0116
-a	-b	-c	-d	0.7 * 0.24 * 0.104 = 0.0174

Then we marginalize over B,C, and D

A	$\sum_{B,C,D} f_3(A, B, C, D, +e, -f)$
+a	0.0095 + 0.0221 + 0.0524 + 0.1123 + 0.0054 + 0.0081 + 0.0299 + 0.0449 = 0.2846
-a	0.0858 + 0.2002 + 0.0131 + 0.0305 + 0.0762 + 0.1144 + 0.0116 + 0.0174 = 0.5492

Finally, we normalize to get our answer

A	$P(A\| + e, -f)$
+a	0.2846 / 0.8338 = 0.3413
-a	0.5492 / 0.8338 = 0.6587