CS 6300 **HW04: Value and Policy Iteration** **Due Feb 21, 2023**

1 Value Iteration

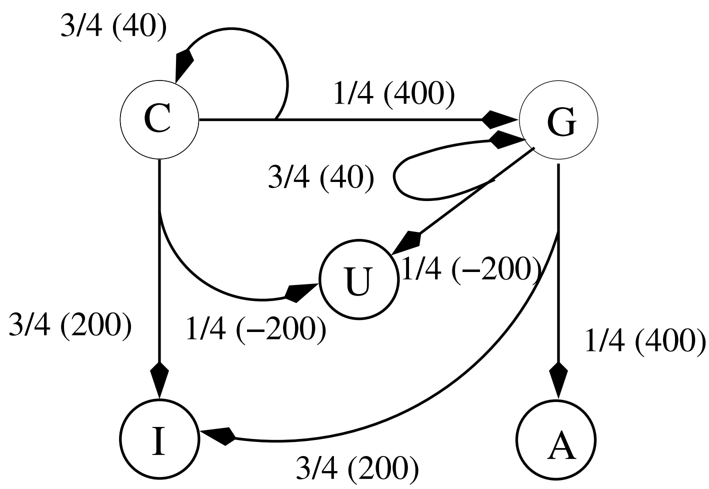
In the MDP below, there are 5 states: C(ollege), G(rad school), I(ndustry), A(cademia), and U(nemployed). States I, A and U are terminal states. Probabilities of transitions are either 1/4 or 3/4, and the values in parentheses are the rewards for that transition. The possible actions from states C and G are:

- State C: You may choose to stay in C, but with a probability of 1/4 you may end up going to state G.

  You may also choose to go to state I, but with probability 1/4 you end up in state U.

- State G: You may choose to stay in state G, but with probability 1/4 you end up in state U.

  You may also choose to go to state A, but with probability 3/4 you end up in state I.



1. You start in state C. Perform two iterations of value iteration, where you first compute the Q values and then take the maximum of the Q values. The discount is $\gamma = 1$.

2. Perform policy extraction after these two iterations to find $\pi^*(s)$. Please show all work.

Round 1:
$Q_1(C, C) = T(C, C, C)[(R(C, C, C) + V_0^*(C)] + T(C, C, G)[(R(C, C, G) + V_0^*(G)]$
$Q_1(C, C) = \frac{3}{4}[40 + 0] + \frac{1}{4}[400 + 0] = 130$

$Q_1(C, I) = T(C, I, I)[(R(C, I, I) + V_0^*(I)] + T(C, I, U)[(R(C, I, U) + V_0^*(U)]$
$Q_1(C, I) = \frac{3}{4}[200 + 0] + \frac{1}{4}[-200 + 0] = 100$

$Q_1(G, G) = T(G, G, G)[(R(G, G, G) + V_0^*(G)] + T(G, G, U)[(R(G, G, U) + V_0^*(U)]$
$Q_1(G, G) = \frac{3}{4}[40 + 0] + \frac{1}{4}[-200 + 0] = -20$

$Q_1(G,A) = T(G,A,A)[(R(G,A,A) + V_0^*(A)] + T(G,A,I)[(R(G,A,I) + V_0^*(I)]$
$Q_1(G,A) = \frac{1}{4}[400+0] + \frac{3}{4}[200+0] = 250$

Round 1 value update:
$V_1(C) = \max_{a\in\{C,I\}} Q_1^*(C,a) = 130$
$V_1(G) = \max_{a\in\{G,A\}} Q_1^*(G,a) = 250$
$V_1(I) = 0$
$V_1(U) = 0$
$V_1(A) = 0$

Round 2:
$Q_1(C,C) = T(C,C,C)[(R(C,C,C) + V_1^*(C)] + T(C,C,G)[(R(C,C,G) + V_1^*(G)]$
$Q_1(C,C) = \frac{3}{4}[40+130] + \frac{1}{4}[400+250] = 290$

$Q_1(C,I) = T(C,I,I)[(R(C,I,I) + V_1^*(I)] + T(C,I,U)[(R(C,I,U) + V_1^*(U)]$
$Q_1(C,I) = \frac{3}{4}[200+0] + \frac{1}{4}[-200+0] = 100$

$Q_1(G,G) = T(G,G,G)[(R(G,G,G) + V_1^*(G)] + T(G,G,U)[(R(G,G,U) + V_1^*(U)]$
$Q_1(G,G) = \frac{3}{4}[40+250] + \frac{1}{4}[-200+0] = 167.5$

$Q_1(G,A) = T(G,A,A)[(R(G,A,A) + V_1^*(A)] + T(G,A,I)[(R(G,A,I) + V_1^*(I)]$
$Q_1(G,A) = \frac{1}{4}[400+0] + \frac{3}{4}[200+0] = 250$

Round 2 value update:
$V_2(C) = \max_{a\in\{C,I\}} Q_2^*(C,a) = 290$
$V_2(G) = \max_{a\in\{G,A\}} Q_2^*(G,a) = 250$
$V_2(I) = 0$
$V_2(U) = 0$
$V_2(A) = 0$

Policy extraction after 2 rounds:
$\pi_2(s) = arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + V_i^*(s')]$

$$\pi_2(C) = \begin{cases} T(C,C,C)[(R(C,C,C) + V_2^*(C)] + T(C,C,G)[(R(C,C,G) + V_2^*(G)] \\ T(C,I,I)[(R(C,I,I) + V_2^*(I)] + T(C,I,U)[(R(C,I,U) + V_2^*(U)] \end{cases}$$

$\pi_2(C) = \begin{cases} \frac{3}{4}[40+290] + \frac{1}{4}[400+250] = 410 \\ \frac{3}{4}[200+0] + \frac{1}{4}[-200+0] = 100 \end{cases}$
$\pi_2(C) = C$

$$\pi_2(G) = \begin{cases} T(G,G,G)[(R(G,G,G) + V_2^*(G)] + T(G,G,U)[(R(G,G,U) + V_2^*(U)] \\ T(G,A,A)[(R(G,A,A) + V_2^*(A)] + T(G,A,I)[(R(G,A,I) + V_2^*(I)] \end{cases}$$

$\pi_2(G) = \begin{cases} \frac{3}{4}[40+250] + \frac{1}{4}[-200+0] = 167.5 \\ \frac{1}{4}[400+0] + \frac{3}{4}[200+0] = 250 \end{cases}$
$\pi_2(G) = A$

2 Policy Iteration

Consider again the following MDP. There are 5 states: C(ollege), G(rad school), I(ndustry), A(cademia), and U(nemployed). States I, A and U are terminal states. Probabilities of transitions are either 1/4 or 3/4, and the values in parentheses are the rewards for that transition. The possible actions from states C and G are:

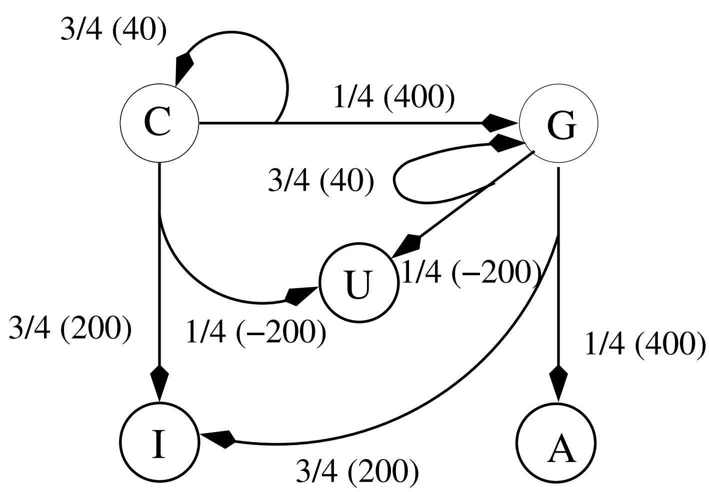
- State C: You may choose to stay s in C, but with a probability of 1/4 you may end up going g to state G.

  You may also choose to go g to state I, but with probability 1/4 you end up in state U.

- State G: You may choose to stay s in state G, but with probability 1/4 you end up in state U.

  You may also choose to go g to state A, but with probability 3/4 you end up in state I.

You start in state C. Assume your initial policy is $\pi_0(s) = s$, i.e., you wish to stay in the current state you're in. Also the discount is $\gamma = 1$.



For this problem you will perform one step of Policy Iteration:

1. Perform policy evaluation to solve for the utility values $V^{\pi_0}(C)$ and $V^{\pi_0}(G)$. Remember that the utility values can be solved for analytically.

   The analytical equation to use is:

$$V^{\pi_i}(s) = \sum_{s'} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi_i}(s')\right]$$

2. Perform policy improvement to find $\pi_1(s)$. Please show all work.

$V^{\pi_0}(C) = \sum_{s'} T(C, \pi(C), s')[R(C, \pi(C), s') + V^{\pi_0}(s')]$
$V^{\pi_0}(C) = T(C, C, C)[(R(C, C, C) + V^{\pi_0}(C)] + T(C, C, G)[(R(C, C, G) + V^{\pi_0}(G)]$
$V^{\pi_0}(C) = \frac{3}{4}[40 + 0] + \frac{1}{4}[400 + 0] = 130$

$V^{\pi_0}(G) = \sum_{s'} T(G, \pi(G), s')[R(C, \pi(G), s') + V^{\pi_0}(s')]$
$V^{\pi_0}(G) = T(G, G, G)[(R(G, G, G) + V_0^{\pi_0}(G)] + T(G, G, U)[(R(G, G, U) + V_0^{\pi_0}(U)]$
$V^{\pi_0}(G) = \frac{3}{4}[40 + 0] + \frac{1}{4}[-200 + 0] = -20$

Policy improvement:
$\pi_1(s) = arg\max_a \sum_{s'} T(s, a, s')[R(s, a, s') + V^{\pi_0}(s')]$

$$\pi_2(C) = \begin{cases} T(C, C, C)[(R(C, C, C) + V^{\pi_0}(C)] + T(C, C, G)[(R(C, C, G) + V^{\pi_0}(G)] \\ T(C, I, I)[(R(C, I, I) + V^{\pi_0}(I)] + T(C, I, U)[(R(C, I, U) + V^{\pi_0}(U)] \end{cases}$$

$\pi_2(C) = \begin{cases} \frac{3}{4}[40 + 0] + \frac{1}{4}[400 + 0] = 130 \\ \frac{3}{4}[200 + 0] + \frac{1}{4}[-200 + 0] = 100 \end{cases}$
$\pi_2(C) = C$

$$\pi_2(G) = \begin{cases} T(G, G, G)[(R(G, G, G) + V^{\pi_0}(G)] + T(G, G, U)[(R(G, G, U) + V^{\pi_0}(U)] \\ T(G, A, A)[(R(G, A, A) + V^{\pi_0}(A)] + T(G, A, I)[(R(G, A, I) + V^{\pi_0}(I)] \end{cases}$$

$\pi_2(G) = \begin{cases} \frac{3}{4}[40 + 0] + \frac{1}{4}[-200 + 0] = -20 \\ \frac{1}{4}[400 + 0] + \frac{3}{4}[200 + 0] = 250 \end{cases}$
$\pi_2(G) = A$