ECON2271

Business Econometrics

Or: Practical Econometrics for Beginners

Week 4 Revision

Week 1: Introduction to key concepts

Agenda and learning outcomes

- **Topic 1: Data, distributions and comparing means**
  - Data scales and quality
    - ➢ Students able to identify data scales and quality
  - Cross-sectional, time-series and panel
    - ➢ Students able to identify format and implications for analysis
  - Distributions, sampling and distributional characteristics
    - ➢ Students able to understand the concept of a probability and frequency distribution and sampling, key characteristics of the normal distribution, and key measures of central tendency and dispersion.
  - Testing means: hypotheses, tests and confidence intervals
    - ➢ Students able to identify and test hypotheses comparing means “manually”, identifying and interpreting a 95% confidence interval for a mean.

Topic 2:Univariate Regression and OLS

Agenda and learning outcomes

- **Topic 2(i): Univariate Regression:** $Y = b_0 + b_1X + u$

  a) Introduce the classic univariate linear regression model
     - Able to distinguish between model variables and parameters;
     - Interpret meaning of model parameters and error term

  b) Using univariate simple regression with single dummy to test for differences in group means [Continuous Y, single binary X]
     - Students able to test simple hypotheses comparing means using dummy regression

  c) Using OLS to estimating intercept and slope for a linear function [Continuous Y, single continuous X]
     - Understand the basic premise of OLS.
     - Understand how we evaluate goodness of fit (R-squared and F-stat)
     - Students able to estimate simple univariate regression model and interpret key output correctly.

Topic 2(ii): Univariate Regression and OLS

Agenda and learning outcomes

- **Topic 2 (ii): Univariate Regression:** $Y = b_0 + b_1X + e$

  d) The classical assumptions behind OLS:
     - Understand meaning of BLUE estimates;
     - Understand the assumptions about Y and its relationship with X
     - Understand key assumptions about disturbances (*e* or *u*)

  e) Working with nonlinear relationships:
     - Recognise some common nonlinear functional forms, and understand how to specify a function of Y which is linear in the model parameters.
     - Students able to interpret model estimates for simple linear functions, when Y is linear in X, and also some linearized non-linear functions, where Y is not linear in X but is linear in a function of X (i.e. linear in the parameters).
     - Students able to apply techniques required to identify correct functional form and linearized model specification.

Test 1: Information provided

THE UNIVERSITY OF WESTERN AUSTRALIA
Achieve International Excellence

$$\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} \qquad s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} \qquad t = \frac{\bar{x}-\mu}{SE(\bar{x})} = \frac{\bar{x}-\mu}{s/\sqrt{n}} \qquad t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

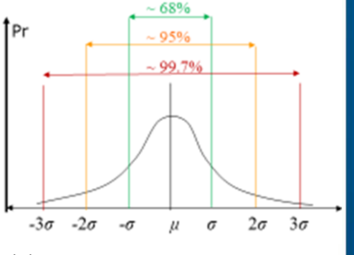
Model	**dy/dx**
$Y = b_0 + b_1X$	b_1
$Y = b_0 + b_1X + b_2X^2$	$b_1 + 2b_2X$
$Y = b_0 + b_1X + b_2X^2 + b_3X^3$	$b_1 + 2b_2X + 3b_3X^2$
$Y = b_0 + b_1 lnX$	b_1/X

+ Table of critical values for t-stat

2. Data distributions
Reminder: *Distributional characteristics*

THE UNIVERSITY OF WESTERN AUSTRALIA
Achieve International Excellence

- **Standard deviations and the probability distribution:**
  - A <u>normal</u> distribution has specific properties:
    - ABOUT 99.7% of observations will lie within 3 standard deviations (σ's)either side of the population mean (μ)
    - ABOUT 95% of observations will lie within 2 standard deviations (σ's)either side of the population mean (μ)
    - ABOUT 68% of observations will lie within 1 standard deviations (σ's)either side of the population mean (μ)



- ❖ The t-distribution is different!! But it approaches the normal dist. as n $\rightarrow\infty$
- ❖ Critical values for *t*, used to calculate CI, vary with n!
  - ➢ When n = 60:
    - 90% CI = $\bar{x} \pm 1.671SE$; 95% CI = $\bar{x} \pm 2.000SE$; 99% CI = $\bar{x} \pm 2.617SE$
  - ➢ When n = ∞:
    - 90% CI = $\bar{x} \pm 1.645SE$; 95% CI = $\bar{x} \pm 1.960SE$; 99% CI = $\bar{x} \pm 2.576SE$

THE UNIVERSITY OF WESTERN AUSTRALIA
Achieve International Excellence

Table T Critical Values of the *t* Distribution

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Source: From *Biometrika Tables for Statisticians*, Vol. 1, Third Edition, edited by E. S. Pearson and H. O. Hartley, 1966, p. 146. Reprinted by permission of the Biometrika Trustees.

Describing nonlinear functions
(and finding maxima and/or minima)

THE UNIVERSITY OF WESTERN AUSTRALIA
Achieve International Excellence

- The linear function ($Y = b_0 + b_1X$) is positive monotonic when b_1 is positive, which means Y is increasing in X. Moreover, the change in Y for every given change in X is constant.
- The logarithmic function ($Y = b_0 + b_1\ln X$) is also positive monotonic (so long as b_1 is positive). However, the change in Y for every given change in X is NOT constant. Rather, the change in Y is constant for a given *proportionate* change in X.
- See the table below. In the left-hand-side table, the X variable increases in 10-fold increments (it grows by a factor of 10, or by 1000%). Here, you can see that Y responds by changing by constant amounts, specifically about 2.3, which is – of course – the same as ln10. On the right-hand-side table, the X variable is doubling (or growing by a factor of 2, or by 200%). Here, you can see that Y responds by changing by constant amounts, of exactly ln2.

	Y = lnX	
X	Y:	change Y:
1	0	
10	2.302585	2.302585
100	4.60517	2.302585
1000	6.907755	2.302585
10000	9.21034	2.302585

	Y = lnX	
X:	Y:	change Y:
1	0	
2	0.693147	0.693147
4	1.386294	0.693147
8	2.079442	0.693147
16	2.772589	0.693147

CRICOS Provider Code: 00126G

9

Describing nonlinear functions
(and finding maxima and/or minima)

- Say you estimate the function $LS = 5.65 + 0.21\ln(INC) + u$
  - Below, you see a table of predicted LS (pred LS) for different income levels.
  - Here, the change in LS for a 10-fold change in INC is always 0.21*ln(10) = 0.48.
  - The relationship between INC and LS is logarithmic. Specifically, LS increases at a decreasing rate and INC increases.
  - The slope dLS/dINC is equal to slope in the lin-log specification divided by INC – so you can see that it will always diminish as INC increases.
    - When INC = 10, each extra dollar is associated with a 0.21/10 = 0.021 unit increase in LS.
    - When INC = 1000, each extra dollar is associated with a 0.21/1000 = 0.00021 unit increase in LS.
    - And so on…

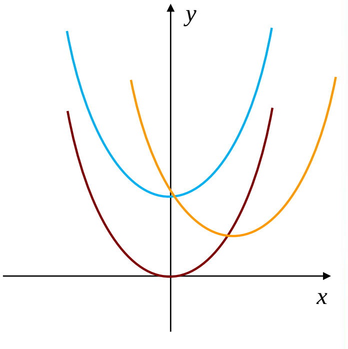
PS: Why do we often add 1 to income before taking the log (e.g. $LS = b_0 + b_1\ln(INC+1)$? Often we want to include individuals with zero income. Because we can't take the log of zero, these people will automatically be excluded. If we add $1, they will be included, with ln(INC+1) = 0.

INC	lnINC	pred LS	change
1	0	5.65	
10	2.302585	6.133543	0.483543
100	4.60517	6.617086	0.483543
1000	6.907755	7.100629	0.483543
10000	9.21034	7.584171	0.483543
100000	11.51293	8.067714	0.483543
1000000	13.81551	8.551257	0.483543

10

Describing nonlinear functions
(and finding maxima and/or minima)

- The quadratic function ($Y = b_0 + b_1X + b_2X^2$) can have both upward and downward-sloping sections. It depends what values X can have.
- For example, if there is constant or no linear component in the model ($Y = b_2X^2$), and $b_2 > 0$, then the function will look like the red function below. If X can only be positive, this means Y is always increasing in X in the domain for which the function is defined.
- If we add a constant ($Y = b_0 + b_2X^2$), then the function in the vertical plane. E.g. if the constant is 2, the function moves up by 2 (blue curve).
- If we add a linear term to this ($Y = b_0 + b_1X + b_2X^2$), then the function moves in both planes. E.g. if the linear term is -2X, the function moves to the right and down. This means that this function will have both downward and upward-sloping sections when X is positive.



11

Describing nonlinear functions
(and finding maxima and/or minima)

- The quadratic function ($Y = b_0 + b_1X + b_2X^2$) can therefore have a minimum or maximum turning point.
- We find the turning point by finding the value for X for which Y is stationary: that is, where the slope dy/dx = 0. For the general quadratic function above:

$$\frac{dy}{dx} = b_1 + 2b_2\mathrm{X} = 0 \Rightarrow \mathrm{X} = -\frac{b_1}{2b_2}$$

- To find out if it's a minimum or a maximum, we have to find out whether the function is concave up (like a smiley face) or down (like a sad face). To do this, we evaluate the sign of the second derivative at the turning point. For a quadratic function, the second derivative is a constant, and equal to $2b_2$, so all we need to do is to look at the sign of b_2. If it's positive, the function is concave up, and any turning point we find will be a minimum. If it's negative, the function is concave down, and any turning point we find will be a maximum.

12

Describing nonlinear functions
(and finding maxima and/or minima)

THE UNIVERSITY OF WESTERN AUSTRALIA
Achieve International Excellence

- The cubic function ($Y = b_0 + b_1X + b_2X^2 + b_3X^3$) can have both a minimum and a maximum turning point. It depends, and especially on the values of X for which the model is defined.
- We can identify the values of X for which the function is increasing or decreasing by evaluating the sign of the slope (dy/dx).
- We can identify turning points by finding the value for X for dy/dx = 0. For the general cubic function above:

$$\frac{dy}{dx} = b_1 + 2b_2\mathrm{X} + 3b_3X^2 = 0 \Rightarrow \quad X = \frac{-2b_2 \pm \sqrt{(2b_2)^2 - 4(3b_3)(b_1)}}{2(3b_3)}$$

To find out if it's a minimum or a maximum, we have to find out whether the function is concave up (like a smiley face) or down (like a sad face). To do this, we evaluate the sign of the second derivative at the turning point. For this general cubic function, the second derivative is a function of X: $\frac{d^2y}{dx^2} = 2b_2 + 6b_3X$

If the value of this derivative is positive at a given point, the function is concave up at this point, and any turning point we find will be a minimum. If it's negative, the function is concave down, and any turning point we find will be a maximum.

CRICOS Provider Code: 00126G