The project of Computational Methods in Physics

Xu Jue

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1 Principle of the magnetic bottle

1.1 Calculate the magnetic field

We can get the magnetic field which is generared by a coil by integrating

$$\begin{cases} B_{x0}(x,y,z) = \int_0^{2\pi} \frac{Rzcos\phi}{(x^2+y^2+z^2+R^2-2xRcos\phi-2yRsin\phi)^{3/2}} d\phi \\ B_{y0}(x,y,z) = \int_0^{2\pi} \frac{Rzsin\phi}{(x^2+y^2+z^2+R^2-2xRcos\phi-2yRsin\phi)^{3/2}} d\phi \\ B_{z0}(x,y,z) = \int_0^{2\pi} \frac{R(R-xcos\phi-ysin\phi)}{(x^2+y^2+z^2+R^2-2xRcos\phi-2yRsin\phi)^{3/2}} d\phi \end{cases} \tag{1}$$

Then we can get the magnetic field of a magnetic bottle (two coils) by a transformation

$$\begin{cases} B_x(x,y,z) = B_{x0}(x,y,z-d/2) + B_{x0}(x,y,z+d/2) \\ B_y(x,y,z) = B_{y0}(x,y,z-d/2) + B_{y0}(x,y,z+d/2) \\ B_z(x,y,z) = B_{z0}(x,y,z-d/2) + B_{z0}(x,y,z+d/2) \end{cases} \tag{2}$$

1.2 Calculate the movement of the electron

According to the Newton equation and Lorentz force equation

$$\begin{cases} F = ma \\ F_{Lorentz} = qv \times x \end{cases} \tag{3}$$

Then we can get the equation,

$$qv \times x = ma \tag{4}$$

Then we need to solve the second order Ordinary Dierential Equations, because we study this problem in 3D.

$$\begin{cases} x'' = -\frac{m}{q}(y'Bz(x,y,z) - z'By(x,y,z)) \\ y'' = -\frac{m}{q}(z'Bx(x,y,z) - x'Bz(x,y,z)) \\ z'' = -\frac{m}{q}(x'By(x,y,z) - y'Bx(x,y,z)) \end{cases} \tag{5}$$

We transfer the them into first order Ordinary Dierential Equations,

$$
\begin{cases}
vx' = -sc(vy * Bz(x,y,z) - vz * By(x,y,z)) \\
vy' = -sc(vz * Bx(x,y,z) - vx * Bz(x,y,z)) \\
vz' = -sc(vx * By(x,y,z) - vy * Bx(x,y,z)) \\
x' = vx \\
y' = vy \\
z' = vz
\end{cases}
\tag{6}
$$

Its intial values are

$$
\begin{cases}
vx(0) = 0 \\
vy(0) = 0 \\
vz(0) = 0.15e6 \\
x(0) = 0 \\
y(0) = 0.78R \\
z(0) = -0.75d
\end{cases}
\tag{7}
$$

1.3 Solution

1.3.1 Equation to be solved

1.3.2 Numerical method used

1.3.3 Results

1.3.4 Discussions