

The project of Computational Methods in Physics

Xu Jue

March 6, 2016

1 Calculate the magnetic field

We can get the magnetic field which is generated by a coil by integrating

$$\begin{cases} B_{x0}(x, y, z) = \int_0^{2\pi} \frac{Rz \cos \phi}{(x^2 + y^2 + z^2 + R^2 - 2xR \cos \phi - 2yR \sin \phi)^{3/2}} d\phi \\ B_{y0}(x, y, z) = \int_0^{2\pi} \frac{Rz \sin \phi}{(x^2 + y^2 + z^2 + R^2 - 2xR \cos \phi - 2yR \sin \phi)^{3/2}} d\phi \\ B_{z0}(x, y, z) = \int_0^{2\pi} \frac{R(R - x \cos \phi - y \sin \phi)}{(x^2 + y^2 + z^2 + R^2 - 2xR \cos \phi - 2yR \sin \phi)^{3/2}} d\phi \end{cases} \quad (1)$$

Then we can get the magnetic field by a transformation

$$\begin{cases} B_x(x, y, z) = B_{x0}(x, y, z - d/2) + B_{x0}(x, y, z + d/2) \\ B_y(x, y, z) = B_{y0}(x, y, z - d/2) + B_{y0}(x, y, z + d/2) \\ B_z(x, y, z) = B_{z0}(x, y, z - d/2) + B_{z0}(x, y, z + d/2) \end{cases} \quad (2)$$

Then we need to solve the second order Ordinary Differential Equations,

$$\begin{cases} x'' = -sc(y'Bz(x, y, z) - z'By(x, y, z)) \\ y'' = -sc(z'Bx(x, y, z) - x'Bz(x, y, z)) \\ z'' = -sc(x'By(x, y, z) - y'Bx(x, y, z)) \end{cases} \quad (3)$$

We transfer the them into first order Ordinary Differential Equations,

$$\begin{cases} vx' = -sc(vy * Bz(x, y, z) - vz * By(x, y, z)) \\ vy' = -sc(vz * Bx(x, y, z) - vx * Bz(x, y, z)) \\ vz' = -sc(vx * By(x, y, z) - vy * Bx(x, y, z)) \\ x' = vx \\ y' = vy \\ z' = vz \end{cases} \quad (4)$$

Its initial values are

$$\begin{cases} vx(0) = 0 \\ vy(0) = 0 \\ vz(0) = 0.15e6 \\ x(0) = 0 \\ y(0) = 0.78R \\ z(0) = -0.75d \end{cases} \quad (5)$$

1.1 Solution

1.1.1 Equation to be solved

1.1.2 Numerical method used

Gaussian quadrature method is applied.

1.1.3 Results

The value of the triple integral is 0.09603.

1.1.4 Discussions