Re-evaluation of Bitcoin's Potential as an Inflation Hedge

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Abstract

In this paper, we use the most current data available to re-evaluate the existing evidence surrounding Bitcoin's potential to be used as a hedge against inflation. Using vector autoregressive (VAR) models and GARCH-VAR models to assess correlation and causal relationships, we find evidence that refutes the findings of previous studies that Bitcoin can be used to hedge against inflation [1, 2, 3]. Using a bootstrapping algorithm combined with the GARCH-VAR models, we also establish the presence of a paradigm shift occurring around the start of the COVID pandemic that serves as evidence supporting a pandemic impact on the behaviour of both Bitcoin returns and expected inflation rates. The multi-series VAR models also identify the oil and real estate index to be more strongly correlated with expected inflation rates than Bitcoin returns, suggesting the presence of confounding variables that may be behind any apparent correlation between Bitcoin returns and expected inflation. Based on the results of our study, the existing claims on the relationship between Bitcoin returns and inflation no longer appear to hold true based on more current post-pandemic data.

The complete code used in this study can be found at: https://git.uwaterloo.ca/s362xu/stat974-term-paper

1 Research Question

Inflationary pressures have been high in 2022 due to the combined impact of the ongoing pandemic and the war that broke out unexpectedly in Europe. As a result, research around assets that can be used to hedge against inflation has become an increasingly popular topic. We have chosen to investigate the cryptocurrency Bitcoin as the potential candidate in this paper due to its high popularity and the amount of existing research already available. The objective of this paper is to evaluate the existing evidence supporting Bitcoin's potential to be used as an inflation hedge and investigate whether the claims still hold true based on more current post-pandemic data. We also aim to contribute additional research to address some of the inadequacies that exist in some of the current published works.

2 Selected Literature Survey

2.1 *Inflation and Bitcoin: A descriptive time-series analysis* [1]

The main objective of this study was to investigate the presence of covariation between Bitcoin prices and inflation rates to determine whether Bitcoin can be effectively used as a hedge against inflation [1]. The researchers report to have used Bitcoin price data from January 1, 2019 to December 31, 2020 and forward inflation expectations from the same period to fit a vector autoregressive

(VAR) model. Granger causality tests were then used to test for causation between the two series [1]. Results suggest a one way causal relationship with changes in Bitcoin prices causing changes in expected inflation rate [1]. To investigate possible endogenous variables that may be behind the apparent causal relationship, the team simulated shocks to the system and evaluated the impacts [1]. Results showed that shocks to Bitcoin prices cause an immediate and lasting increase in expected forward inflation rates [1]. No causal relationship in the reverse direction was observed [1]. The study concludes that Bitcoin price increases cause increases in inflation and thus, the evidence supports the idea that Bitcoin can be used as a hedge against inflation [1].

We find that the above study has an important inadequacy that needs to be addressed. The researchers do not consider other confounding variables that can also be contributing to the apparent causal relationship between bitcoin prices and inflation. As Bitcoin makes up for a very minor component of the global economy, it is implausible that it could be the sole cause of changes in inflation. Therefore to fully establish a causal relationship, we believe that additional investigation into factors in the housing market, crude oil prices, and other potential confounding variables is warranted. This point will be one of the areas we will explore in this paper as we evaluate the impact of a number of other variables that may also contribute to changes in inflation expectation.

2.2 *Bitcoin: An inflation hedge but not a safe haven* [2]

The objective of this study was to answer the question of whether Bitcoin can serve as a "safe haven" and an alternative to gold when considering potential hedges for inflation [2]. The study investigates the relationship between changes in Bitcoin price and changes in inflation rates using a Vector Autoregressive (VAR) model [2]. Results indicated a positive correlation between Bitcoin price increases and inflation rate increases, providing evidence for Bitcoin's potential to be used as an inflation hedge [2]. The researchers also reports Bitcoin's response to financial uncertainty shocks and found that Bitcoin prices tend to decline which is in contrast to that of gold. Based on the above results, the study concluded that although there is evidence suggesting that Bitcoin may be used to hedge against inflation, it cannot act as a "safe haven" which is what gold is often deemed as [2].

Similar to the first study, this study also uses a VAR model to model the relationship between Bitcoin prices and expected forward inflation. However, this study performs an additional investigation on the responses of Bitcoin prices to financial uncertainty shocks and compares it to the responses of gold. We found the insights presented to be very interesting. However, we note that a further step can be taken to investigate other assets in addition to gold that may share similar properties with Bitcoin. In evaluating the relationship between expected forward inflation and assets such as real estate, crude oil, and treasury bonds, we can potentially generate more insights into the inflation hedging capabilities of Bitcoin. This will be an area of research our study will investigate.

2.3 *Bitcoin, gold and the dollar – A GARCH volatility analysis* [3]

The objective of the study was to investigate the presence of correlation in price movements between Bitcoin, gold, and the power of the American dollar. The data used in this study was collected from a period prior to the COVID pandemic [3]. Both a GARCH model and an exponential GARCH model were used to model Bitcoin price changes with gold prices, federal funds rate and exchange rates included as predictors [3]. The estimated coefficients suggest that Bitcoin prices are significantly correlated with the federal funds rate and the dollar to pound exchange rate [3]. Interestingly, the study reported that Bitcoin prices seem to be more affected by the dollar to pound exchange rate than the dollar to euro exchange rate. The study cites a similar finding in gold prices. Analysis of the variance equation also show that positive volatility shocks to the predictors cause a decrease in volatility of Bitcoin returns [3]. The exponential GARCH model investigates the effect of positive and negative news on Bitcoin prices [3]. The study reports that analysis indicated a symmetric effect, supporting the idea that Bitcoin can be used to hedge against market risks which often have asymmetric impacts on the prices of other assets. Based on the above results, the study concluded that Bitcoin shares many characteristics with both gold and the American dollar and respond similarly to changes in the explanatory variables used in the GARCH models [3]. Thus, the results of the study support the capabilities of Bitcoin as a hedge against inflation [3].

Contrary to the previous two studies, this study uses a GARCH model to capture the time-varying volatility in the data and uses it to draw inferences on the response of Bitcoin to positive volatility shocks to the explanatory variables [3]. The study also investigates multiple potential explanatory variables compared to a single explanatory variable in the previous two studies [3]. However, the study arrives at a similar conclusion regarding the capabilities of Bitcoin as a hedge against inflation [3].

In summary, all three studies above arrived at similar conclusions and are largely in agreement on Bitcoin's capabilities as an inflation hedge [1, 2, 3]. We will now attempt to reproduce the results on more current post-pandemic data to investigate the below two main questions:

1. Are the claims still supported by more current post-pandemic data?

2. When other potential explanatory variables are added to the equation, is the causal relationship between Bitcoin and expected inflation rate as claimed in the first paper still significant?

3 Data, Variables, Samples Periods

For this study, we obtained data on Bitcoin prices, SP500 return index, FTSE return index, oil index, real estate index, policy index, US 3-month treasury bond rates, US 1-year treasury bond rates, gold prices, and the 5-year expected forward inflation rate. All prices are in USD. The data on Bitcoin prices, SP500 return index, FTSE return index, and US 3-month treasury bond rates were obtained from Yahoo Finance and all series data were obtained for the period from 2014-09-17 to 2022-11-23. Please refer to Table 1 for specific data sources and data set descriptions.

Data	Source	Additional Information
Bitcoin Daily Return (BTC.Ret)	Yahoo Finance	The daily return calculated using the daily open/close prices of Bitcoin in USD.
SP500 Daily Return (SP500.Ret)	Yahoo Finance	Daily return of the average annual return on stocks of the 500 leading companies in the US by S&P.
Oil index (WTI)	FRED - WTI	Crude Oil Price retrieved from FRED, Federal Reserve Bank of St. Louis
Real estate index (D.JON.RE)	Market Watch	The index tracking the performance of real estate investment trusts (REIT) and other companies that invest directly or indirectly in real estate.
UK Stock Market Daily Return (FTSE.Ret)	Yahoo Finance	Daily Return of the FTSE Index in GBP.
US 3-month treasury bond rates (US13W)	Yahoo Finance	3 month return on US treasury bonds.
US 1-year treasury bond rates (US1Y)	Market Watch	Annual return on US treasury bonds.
5-year expected forward inflation rate (FYFY)	FRED - T5YIFR	Expected inflation rate for the 5-year period starting 5 years from current day.[1]
Gold prices (GLD)	Investing.com	Gold futures price (GCG3)
Volatility Index (VIX)	Yahoo Finance	
Policy index (DPI)	US EPU	Daily economic Policy uncertainty index which is based on newspaper archives from Access World New's NewsBank service.

Table 1: Data sources and explanations

Figure 1 presents a visual summary of all the data series used in this study. It is evident that there exists some degree of auto-correlation and volatility clustering effect in most of series. In addition, some series appears to undergo some form of Brownian motion.

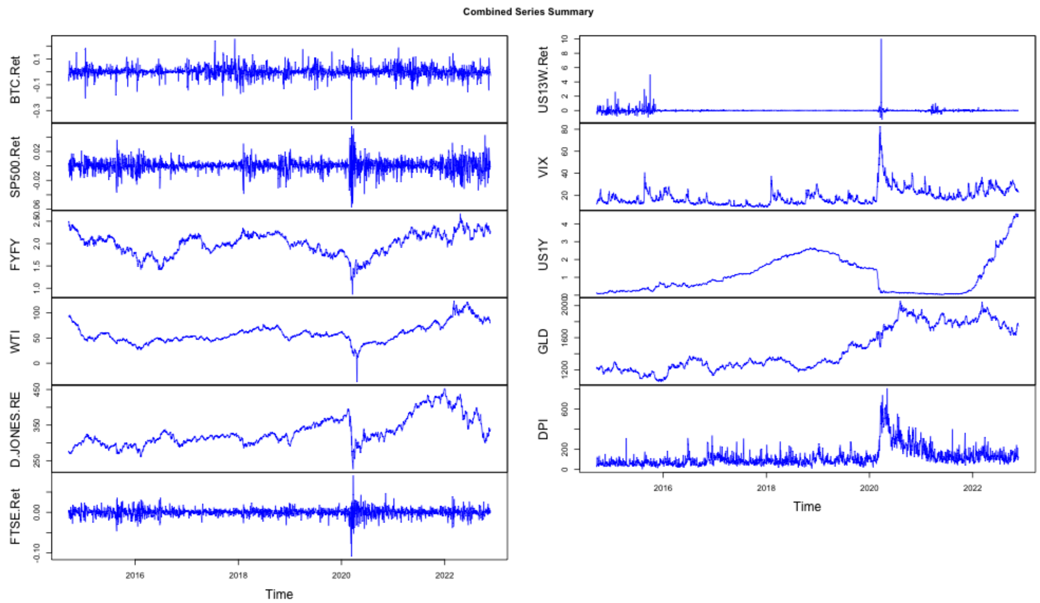

Figure 1: Series Summary

Therefore, we performed the Ljung-Box test (Test for autocorrelation within each series), the Lagrange Multiplier test (Test for ARCH effects) and the Dickey-Fuller test (Test for stationarity) for each series. The results are shown in the table 2 below.

Series	Ljung-Box Test p-value	LM Test p-value	Dickey-Fuller Test p-value
BTC.Ret	0.0	0.0	0.01
SP500.Ret	0.0	0.0	0.01
WTI	0.0	0.0	0.09752
D.JON.RE	0.0	0.0	0.04189
FTSE.Ret	0.0	0.0	0.01
US13W.Ret	0.0	0.0	0.01
VIX	0.0	0.0	0.01
US1Y	0.0	0.0	0.99
FYFY	0.0	0.0	0.2433
GLD	0.0	0.0	0.4264
DPI	0.0	0.0	0.0124

Table 2: Testing on the original data series

From the test results we can see that all series are under the influence of ARMA and ARCH effects. Additionally, the Dickey-Fuller test statistics for most of the series are significant at the 5% confidence level, suggesting that they are not under the influence of the unit disk and hence are stationary.

4 Application

In this study, we re-evaluate Bitcoin's potential to act as an inflation hedge based on a larger data set that includes more current post-pandemic data. Results of the study, if significant, can be used to support informed decision making for potential investors looking for assets to hedge against inflation in the post-pandemic setting.

5 Experimental Methodology

Our study can be divided into the following four main phases:

5.1 *Phase 1: VAR Models*

In this phase, we fit a VAR model on the Bitcoin returns and forward inflation rate series to establish the possible presence of correlation between the two series. The optimal number of lags is selected by minimizing the Akaike information criterion (AIC). Based on the optimal maximum number of lags, we select the below VAR(5) process:

$$FYFY_t = Constant_1 + \alpha_{11} * FYFY_{t-1} + \beta_{11} * BT_{t-1} + ... + \alpha_{15} * FYFY_{t-5} + \beta_{15} * BT_{t-5} + \epsilon_{1,t}$$

$$BT_t = Constant_2 + \alpha_{21} * BT_{t-1} + \beta_{21} * FYFY_{t-1} + ... + \alpha_{25} * BT_{t-5} + \beta_{25} * FYFY_{t-5} + \epsilon_{2,t}$$

We then perform Granger Causality tests on the fitted model to evaluate causal relationships between the series. We also evaluate the models, assess the residuals and test for remaining ARCH effect using the LM test. Any remaining ARCH effect is addressed using a GARCH-VAR model in the next phase.

5.2 *Phase 2: GARCH-VAR Models*

In order to capture the conditional heteroscedasticity in the series, we fit a selection of GARCH models to the residuals of the VAR model fitted in Phase 1.

Standard GARCH model with normal / skewed student-t innovations:

$$\epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,t} + \sigma_{1,t} * \eta_{1,t} \\ \mu_{2,t} + \sigma_{2,t} * \eta_{2,t} \end{bmatrix} \quad \text{Where, } \eta_{\{1,2\},t} \sim i.i.d \begin{cases} \text{Normal}(0,1) \\ \text{Skewed Student-t}(\sigma, \lambda) \end{cases}$$

$$\mu_t = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 + \varphi_1 \sigma_{1,t} * \eta_{1,t} + \phi_1(\epsilon_{1,t-1} - c_1) \\ c_2 + \varphi_2 \sigma_{2,t} * \eta_{2,t} + \phi_2(\epsilon_{2,t-1} - c_2) \end{bmatrix}$$

$$\sigma_t^2 = \begin{bmatrix} \sigma_{1,t}^2 \\ \sigma_{2,t}^2 \end{bmatrix} = \begin{bmatrix} \omega_1 + \alpha_1(\epsilon_{1,t-1} - \mu_{1,t})^2 + \beta_1 \sigma_{1,t-1}^2 \\ \omega_2 + \alpha_2(\epsilon_{2,t-1} - \mu_{2,t})^2 + \beta_2 \sigma_{2,t-1}^2 \end{bmatrix}$$

GJR-GARCH model with normal and skewed student-t innovations:

$$\epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_{1,t} + \sigma_{1,t} * \eta_{1,t} \\ \mu_{2,t} + \sigma_{2,t} * \eta_{2,t} \end{bmatrix} \quad \text{Where, } \eta_{\{1,2\},t} \sim i.i.d \begin{cases} \text{Normal}(0,1) \\ \text{Skewed Student-t}(\sigma, \lambda) \end{cases}$$

$$\mu_t = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 + \varphi_1 \sigma_{1,t} * \eta_{1,t} + \phi_1(\epsilon_{1,t-1} - c_1) \\ c_2 + \varphi_2 \sigma_{2,t} * \eta_{2,t} + \phi_2(\epsilon_{2,t-1} - c_2) \end{bmatrix}$$

$$\sigma_t^2 = \begin{bmatrix} \sigma_{1,t}^2 \\ \sigma_{2,t}^2 \end{bmatrix} = \begin{bmatrix} \omega_1 + \alpha_1 * (|\epsilon_{1,t-1} - \mu_{1,t}| + \gamma_1 * (\epsilon_{1,t-1} - \mu_{1,t}))^2 + \beta_1 * \sigma_{1,t-1}^2 \\ \omega_2 + \alpha_2 * (|\epsilon_{2,t-1} - \mu_{2,t}| + \gamma_2 * (\epsilon_{2,t-1} - \mu_{2,t}))^2 + \beta_2 * \sigma_{2,t-1}^2 \end{bmatrix}$$

We then use the AIC values for each GARCH model to select the optimal model. Again, we perform the usual LM and non-normality tests on the selected model to assess model performance.

5.3 *Phase 3: GARCH-VAR Models on Separated Data*

Algorithm 1: Bootstrapping Algorithm for finding the optimal split point

Input: X: input data, a N × K matrix; n: Number of splits; M: The minimum number of data required to be in each splits; lag: lag variable used in the VAR model estimation

for $t = 1, \ldots, n$ **do**
- Choose sample index $i \in (M, N - M)$ without replacement `// bootstrapping`
- $\text{preModel}_t \leftarrow \text{VAR}(X_{:i,:}, \text{lag})$
- $\text{postModel}_t \leftarrow \text{VAR}(X_{i+1:,:}, \text{lag})$
- $\text{MSE}_t \leftarrow \frac{\sum_{k=1}^{K} \text{MSE}(\text{preModel}_t)_k}{2 \times i} + \frac{\sum_{k=1}^{K} \text{MSE}(\text{postModel}_t)_k}{2 \times (n-i)}$ `// Weighted MSE`

In this phase, we use the above bootstrapping algorithm to investigate the presence of a a paradigm shift. If such a shift in fact exists, there should exist a split point in the series that would result in a significant decrease in the combined mean squared errors when two individual VAR models are fitted on the two reduced series resulting from the split. In the algorithm, we randomly vary the split point, repeatedly fit a pair of models on the split data series, and assess the combined MSE of each pair of models. If a paradigm shift as a result of the COVID pandemic in fact exists, we should see a significant drop in the combined MSE when the split point is chosen around early 2020 marking the start of the pandemic.

If the presence of such a split point is detected around the start of the COVID pandemic, we will split the expected inflation and Bitcoin return series in two at the split point 2020-02-03 chosen based on the date COVID was declared an international health emergency [8]. We then fit the GARCH-VAR model selected in phase 3 separately on the pre-pandemic and post-pandemic series. We then perform ARCH tests and Ljung-box tests on the pre-pandemic and post-pandemic models' residuals and compare them to the same test results of the model fitted with the entire series. If the results of the reduced models indicate a significantly stronger model performance, we will have strong evidence to suggest a paradigm shift occurring as a result of the pandemic onset.

5.4 *Phase 4: VAR Model (with regularization) Incorporating All Series*

In this phase, we investigate possible correlations between expected inflation and the full set of explanatory variables in our data set. We first fit the multi-series regularized VAR(p) model shown below with the $\lambda \cdot \sum_{i=1}^{p} \sum_{k \in K} |\beta_{i,k}|$ penalty term to do a round of variable selection. Where $\beta_{i,k}$ represents coefficient corresponding to the variable k on the i th lag.

$$FYFY_t = Constant + \sum_{i=1}^{p} \beta_{i,FYFY} \cdot FYFY_{t-i} + \beta_{i,BT} \cdot BT_{t-i} + \beta_{i,DPI} \cdot DPI_{t-i} + ... + \epsilon_{1,t} \quad (1)$$

We then fit another VAR(p) model on the series corresponding to the selected variables. Similar to phase 1, the optimal number of lags is selected by minimizing the Akaike information criterion. We again perform Granger Causality tests on the model to evaluate causal relationships between the series to find potential variables that may have a causal relationship with expected forward inflation.

Based on the results of the four phases, we will draw a conclusion on whether the results of the papers in our literature survey still hold true based on the more current data set used in our study. We will also attempt to establish causal relationships between expected inflation and a selection of other explanatory variables.

6 Results and Discussion

We divide the results and discussion sections into phases similar to the methodology section. Here, we will discuss and compare the results of different models in detail.

6.1 *Phase 1: VAR Models*

The fitted VAR model with estimated coefficients for the 5-year expected forward inflation rate and Bitcoin return series are presented in Table 3 and Table 4 respectively. Please note that $FYFY.lx$ and $BTC.Ret.lx$ in table represent the model coefficient associated with the x th lag of 5-year expected forward inflation rate and Bitcoin return series. We highlight the coefficient p-value if they are significant at the 5% confidence level.

	Estimate	Std. Error	t value	Pr(>|t|)
$FYFY.l_1$	0.85	0.02	37.81	0.00
$BTC.Ret.l_1$	0.02	0.02	1.14	0.26
$FYFY.l_2$	0.10	0.03	3.38	0.00
$BTC.Ret.l_2$	0.04	0.02	1.95	0.05
$FYFY.l_3$	0.02	0.03	0.74	0.46
$BTC.Ret.l_3$	0.02	0.02	0.96	0.34
$FYFY.l_4$	-0.04	0.03	-1.47	0.14
$BTC.Ret.l_4$	0.01	0.02	0.68	0.49
$FYFY.l_5$	0.07	0.02	2.90	0.00
$BTC.Ret.l_5$	0.01	0.02	0.35	0.73
$const$	0.02	0.01	2.43	0.02

Table 3: VAR model fitted for the 5-year expected forward inflation rate

	Estimate	Std. Error	t value	Pr(>|t|)
$FYFY.l_1$	0.00	0.02	0.03	0.97
$BTC.Ret.l_1$	0.01	0.02	0.22	0.82
$FYFY.l_2$	0.02	0.03	0.77	0.44
$BTC.Ret.l_2$	0.00	0.02	0.17	0.86
$FYFY.l_3$	-0.03	0.03	-1.05	0.29
$BTC.Ret.l_3$	0.03	0.02	1.26	0.21
$FYFY.l_4$	0.01	0.03	0.33	0.74
$BTC.Ret.l_4$	0.03	0.02	1.24	0.21
$FYFY.l_5$	-0.01	0.02	-0.56	0.58
$BTC.Ret.l_5$	-0.06	0.02	-2.57	0.01
$const$	0.02	0.01	3.25	0.00

Table 4: VAR model fitted for the Bitcoin daily return

In the FYFY model, the reported p-values suggest an overall insignificant correlation between 5-year expected forward inflation rate and the Bitcoin return series. The p-values reported in the BTC.Ret model suggest a similar finding. To assess the validity of the results. we performed multiple tests on the model residuals with results presented in Table 5. The Portmanteau Test on the fitted VAR model and corresponding plots of model residuals are shown in figures 2 and 3 below.

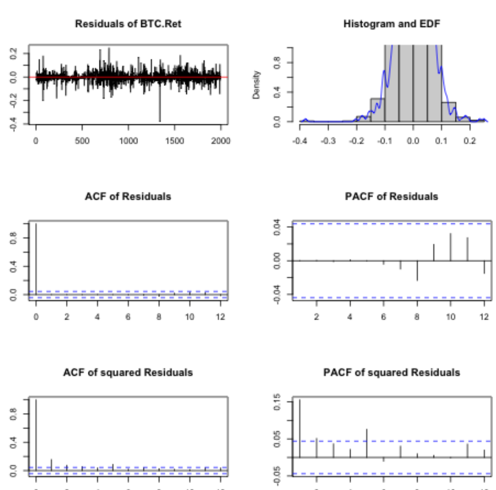

Figure 2: Serial test results on the FYFY

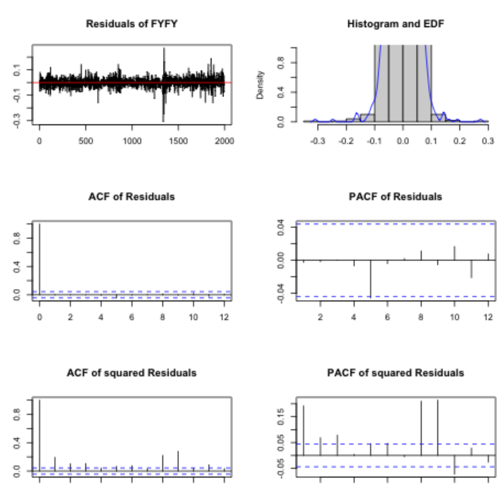

Figure 3: Serial test results on the BTC.Ret

The residual plots suggest the presence of autocorrelation among the squared residuals suggesting that the model is unlikely to be an adequately fitted model on the data series. The ARCH and normality tests on the residuals series also indicate strong evidence for remaining ARCH effect and non-normality among the residuals. It is evident that there is still significant correlation to be explained in the residuals.

Test	Series	Test Statistic	Pr(>|t|)
ARCH LM-Test	FYFY	278.1	0.0
	BTC.Ret	72.926	0.0
Box-Ljung Test	FYFY	6.3003	0.9002
	BTC.Ret	6.11	0.9104
JB-Test		6909.8	0.0
Skewness	multivariate	27.881	0.0
Kurtosis		6881.9	0.0

Table 5: ARCH/Normality test on the residual series

Additionally, we observe that the 5-year expected forward inflation rate is significantly better explained by the VAR model compared to the Bitcoin return series. The models' adjusted R^2 values are 0.9733 and 0.005516 respectively. To further investigate this asymmetrical effect, we performed the Granger causality test. The results of this test are shown in table 11.

The p-values for the test are 0.2407 and 0.05746 corresponding to the null hypothesis that Bitcoin return series do not granger-cause the 5-year expected forward inflation rate and vice versa. The p-value for null hypothesis that there is no instantaneous causality between the two series is 0.0486. Based on the p-values, We conclude that there is no significant evidence for a Granger-cause relationship from BTC.Ret to FYFY and vice versa. However, there is evidence to support instantaneous causality between the two series, suggesting that future values for Bitcoin returns may have significant correlation with current 5-year expected inflation rates. Nonetheless, the lack of Granger causation contradicts with the findings of first study in our literature survery that Bitcoin has a significant granger-causality effect on the FYFY but not the other way around[1].

To further investigate the causality relationships, we generate impulse response functions to describe the response of BTC.Ret to an FYFY shock and vice versa. The results of the shock effect are shown in the figures 4 and 5 below.

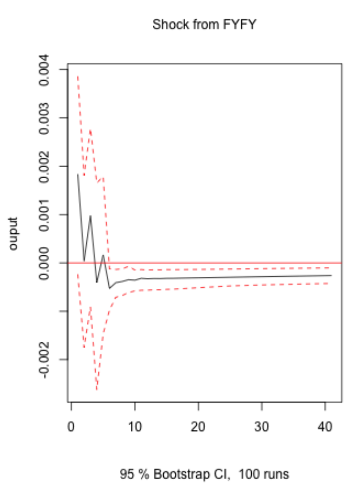

Figure 4: ISF simulated from FYFY

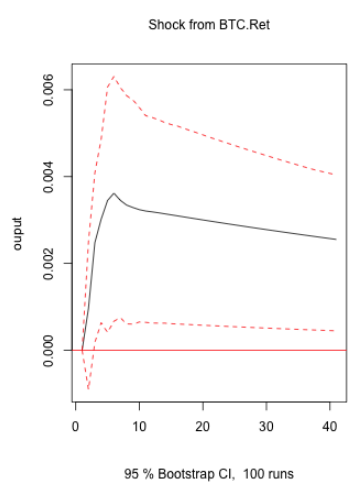

Figure 5: ISF simulated from BTC.Ret

Figures 4 and 5 show that the shock from BTC.Ret creates a positive and persistent shock to the FYFY series and FYFY creates a weak and negative shock to the BTC.Ret. This again seconds our conclusion from the R^2 test that the model on the 5-year expected forward inflation rate better explains the variation than the model for Bitcoin returns. we also observe that the resulting plots are very similar to those presented in the first paper in our literature review which also show that a shock to the Bitcoin returns cause a strong and persistent shock to the FYFY series [1]. This portion of our findings are in support of the aforementioned study's findings [1].

In summary, the results of this phase suggest that the VAR model cannot adequately capture variation in the FYFY series used in our study. We note three possible contributing reasons that will be investigated in later phases:

- There exists conditional heteroscedasticity in both series which is not captured by the VAR model.
- The data sample period used in the selected literature does not contain current post-pandemic data and there exists a paradigm shift at some time point covered by the newly added data.
- There exists potential confounding variables in the relationship between the 5-year expected forward inflation rate and Bitcoin returns

The first point will be investigated using a GARCH-VAR model in phase 2, the second using a bootstrapping technique in phase 3, and the third using a multi-series VAR model in phase 4.

6.2 *Phase 2: GARCH-VAR Models*

In this phase, we fit a selection of GARCH-VAR models on the FYFY and BTC.Ret series. To capture the conditional heteroscedasticity in the VAR models, we fit a multivariate GARCH model on the residuals[2] For the GARCH part, we fitted an ARMA(1,1)-GARCH(1,1) model based on obervations from the ACF of the residuals/squared-residuals shown in figure 3. We test and compare different GARCH(1,1) models including the standard GARCH and GJR-GARCH under both normal and skewed student-t innovations. The motivation for the skewed student-t distributions is the residuals histogram and the result of the multivariate skewness test as shown in table 5.

The resulting model statistics are shown in the table 6. We highlight the statistics of the model with the highest average likelihood and lowest AIC and BIC scores.

Model (Distribution)	Average Log-Likelihood	Akaike	Bayes
SGARCH (normal)	3.73	-7.4470	-7.4133
SGARCH (sstd)	3.898	-7.7795	-7.7346
GJR-GARCH (normal)	3.731	-7.4482	-7.4089
GJR-GARCH (sstd)	3.898	-7.7786	-7.7281

Table 6: Information Criteria for GARCH-VAR Models

Based on the above results, the preferred model in our case is the standard GARCH (S-GARCH) model with skewed student-t innovations. The model's estimated coefficients are presented in table 7. Again, we highlight the p-values corresponding to the coefficients that are significant at a confidence level of 5%. From the results, it is evident that the the S-GARCH model with skewed student-t innovations is a good fit for the data as almost all coefficients are correlated with a significant p-value. Note that the skewed parameter for BTC.Ret and FYFY are both significant, symbolizing the the data is highly likely to follow a skewed distribution.

We continue to examine the model residuals using the ARCH LM-test and the Box-Ljung test. The results are shown in table 10. Comparing the current test statistics for the ARCH test and the values in table 5, it is evident that there is a significant drop suggesting that the GARCH-VAR model is capturing substantially more ARCH effect in both the BTC.Ret and the FYFY series.

[2] According to Professor Tony Wirjanto's email on Dec. 9 2022

	Estimate	Std. Error	t value	Pr(>|t|)
$FYFY.\mu$	2.484747	0.006170	402.74331	0.000000
$FYFY.ar_1$	0.998589	0.001157	862.80009	0.000000
$FYFY.ma_1$	-0.152258	0.024625	-6.18304	0.000000
$FYFY.\omega$	0.000036	0.000012	2.88821	0.003874
$FYFY.\alpha_1$	0.066086	0.012947	5.10428	0.000000
$FYFY.\beta_1$	0.908899	0.016742	54.28878	0.000000
$FYFY.skew$	0.984600	0.030071	32.74237	0.000000
$FYFY.shape$	6.858945	1.047528	6.54775	0.000000
$BTC.Ret.\mu$	0.002049	0.000889	2.30651	0.021082
$BTC.Ret.ar_1$	0.972813	0.005715	170.21669	0.000000
$BTC.Ret.ma_1$	-0.964070	0.000675	-1428.69855	0.000000
$BTC.Ret.\omega$	0.000026	0.000029	0.88622	0.375497
$BTC.Ret.\alpha_1$	0.125985	0.025342	4.97137	0.000001
$BTC.Ret.\beta_1$	0.873015	0.042657	20.46614	0.000000
$BTC.Ret.skew$	1.012112	0.026325	38.44752	0.000000
$BTC.Ret.shape$	3.258060	0.196410	16.58808	0.000000

Table 7: VAR-SGARCH model fitted

In summary, we have produced in this phase a GARCH-VAR model that captures the remaining ARCH effect and is adequately fitted to the data. In the next phase, we will address the question of whether there exists a paradigm shift at some point in the sample period that may also be contributing to the differing results seen in phase 1 between our study and the first study in our selected literature survey.

6.3 *Phase 3: VAR Models on Separated Data*

In this phase, we aim to establish the existence of a change in paradigm in the sample period and aim to pinpoint a rough location of this shift. With this objective, we implement bootstrapping algorithm to detect candidate time points. We randomly choose time points covered by the sample period and split the series in two at the chosen time point. We then fit a pair of individual GARCH-VAR models on the resulting two series and assess the average mean squared error of the model pair. We repeated this process 300 times and present our results in figure 6. Note that the vertical line in the figure indicates the rough time point of when the COVID virus outbreak began in the United States. The solid/dashed horizontal lines represent the estimated mean and 95% confidence interval of the averaged mean squared error series.

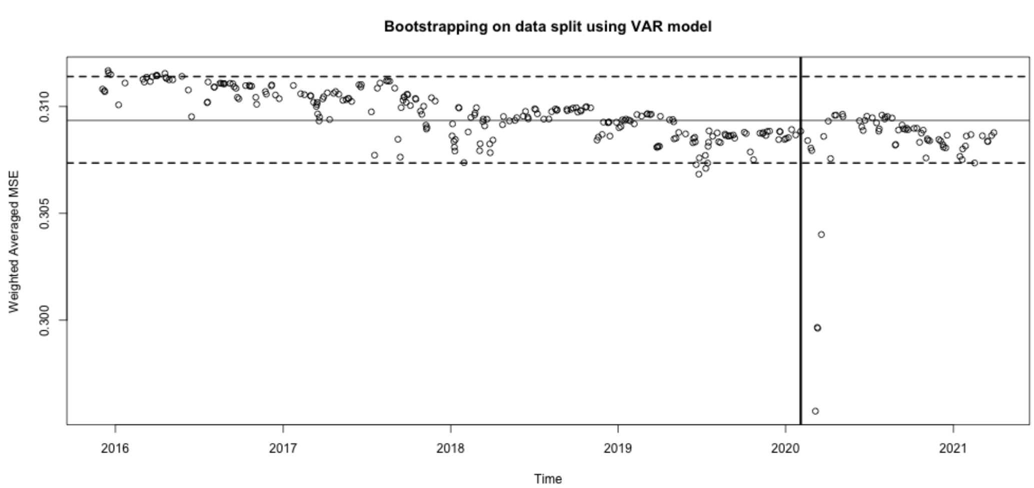

Figure 6: Boostrapping MSE from VAR model

As seen in the plot, there is a significant drop in average mean squared error of the reduced models when the split is done around April - May of 2020 that is far outside the 95% confidence interval. This is strong evidence in support of a paradigm shift occurring around that time point as the result of the split models perform significantly better compared to the split models at any other split point. This is also a strong indicator that the COVID virus, which occurred at roughly the same time, is the cause of this paradigm shift in the relationship between the 5-year expected forward inflation rate and Bitcoin return series.

We then proceed to split the series in two at the time point February 3 2020 which marks the declaration of a COVID health emergency in the US[8]. We will now refer to the resulting reduced series as the pre-covid series and the post-covid series. We then fit the model on the two separated series and present the results in tables 8 and 9.

model	coefficient	Estimate	Std. Error	t value	Pr(>|t|)
SGARCH for FYFY	$FYFY.\mu$	2.486082	0.004448	558.88431	0.000000
	$FYFY.ar_1$	0.999873	0.001282	780.11907	0.000000
	$FYFY.ma_1$	-0.200768	0.030825	-6.51306	0.000000
	$FYFY.\omega$	0.000027	0.000013	2.14047	0.032317
	$FYFY.\alpha_1$	0.055455	0.013331	4.15978	0.000032
	$FYFY.\beta_1$	0.919846	0.019745	46.58621	0.000000
	FYFY.skew	0.966393	0.035200	27.45435	0.000000
	FYFY.shape	8.982772	2.095210	4.28729	0.000018
SGARCH for BTC.Ret	$BTC.Ret.\mu$	0.001977	0.000703	2.81335	0.004903
	$BTC.Ret.ar_1$	-0.310888	0.338768	-0.91770	0.358775
	$BTC.Ret.ma_1$	0.342423	0.330317	1.03665	0.299899
	$BTC.Ret.\omega$	0.000028	0.000032	0.86606	0.386458
	$BTC.Ret.\alpha_1$	0.165554	0.045008	3.67836	0.000235
	$BTC.Ret.\beta_1$	0.833446	0.068423	12.18078	0.000000
	$BTC.Ret.skew$	1.007822	0.032319	31.18340	0.000000
	$BTC.Ret.shape$	3.220900	0.229555	14.03106	0.000000
VAR for FYFY	$FYFY.l_1$	0.776100	0.027621	28.0980	0.000000
	$BTC.Ret.l_1$	0.008123	0.022312	0.3640	0.71586
	$FYFY.l_2$	0.156740	0.034742	4.5120	0.000000
	$BTC.Ret.l_2$	0.010820	0.022308	0.4850	0.62773
	$FYFY.l_3$	0.055130	0.027592	1.9980	0.04592
	$BTC.Ret.l_3$	-0.001372	0.022269	-0.0620	0.95087
	$const$	0.022853	0.008647	2.6430	0.00832
VAR for BTC.Ret	$FYFY.l_1$	-0.03544	0.03413	-1.0380	0.2992
	$BTC.Ret.l_1$	0.052430	0.02757	1.9020	0.0574
	$FYFY.l_2$	0.076440	0.04293	1.7810	0.0752
	$BTC.Ret.l_2$	-0.028040	0.02756	-1.0170	0.3092
	$FYFY.l_3$	-0.046570	0.03409	-1.3660	0.1722
	$BTC.Ret.l_3$	0.061970	0.02752	2.2520	0.0245
	$const$	0.013150	0.01069	1.2310	0.2187

Table 8: VAR-SGARCH(SSTD) model fitted pre-COVID outbreak Period

model	coefficient	Estimate	Std. Error	t value	Pr(>|t|)
SGARCH for FYFY	$FYFY.\mu$	1.702527	0.005764	295.38797	0.000000
	$FYFY.ar_1$	0.997402	0.003466	287.75164	0.000000
	$FYFY.ma_1$	-0.060535	0.043539	-1.39038	0.164412
	$FYFY.\omega$	0.000096	0.000044	2.17695	0.029485
	$FYFY.\alpha_1$	0.074791	0.027465	2.72316	0.006466
	$FYFY.\beta_1$	0.880239	0.036681	23.99684	0.000000
	$FYFY.skew$	0.979498	0.052681	18.59305	0.000000
	$FYFY.shape$	5.242760	1.099814	4.76695	0.000002
SGARCH for BTC.Ret	$BTC.Ret.\mu$	0.001356	0.001509	0.89827	0.369044
	$BTC.Ret.ar_1$	-0.528883	0.709922	-0.74499	0.456279
	$BTC.Ret.ma_1$	0.487911	0.734206	0.66454	0.506343
	$BTC.Ret.\omega$	0.000062	0.000044	1.40101	0.161211
	$BTC.Ret.\alpha_1$	0.078028	0.027849	2.80184	0.005081
	$BTC.Ret.\beta_1$	0.909350	0.025803	35.24180	0.000000
	$BTC.Ret.skew$	1.015068	0.048484	20.93633	0.000000
	$BTC.Ret.shape$	3.191463	0.444681	7.17697	0.000000
VAR for FYFY	$FYFY.l_1$	0.91737	0.03839	23.898	0.000000
	$BTC.Ret.l_1$	0.05787	0.04101	1.411	0.1587
	$FYFY.l_2$	0.07398	0.03843	1.925	0.0547
	$BTC.Ret.l_2$	0.08830	0.04094	2.157	0.0314
	$const$	0.01819	0.01172	1.552	0.1211
VAR for BTC.Ret	$FYFY.l_1$	0.03395	0.03601	0.943	0.3462
	$BTC.Ret.l_1$	-0.07100	0.03847	-1.846	0.0654
	$FYFY.l_2$	-0.04914	0.03605	-1.363	0.1733
	$BTC.Ret.l_2$	0.03559	0.03840	0.927	0.3544
	$const$	0.03275	0.01099	2.978	0.0030

Table 9: VAR-SGARCH(SSTD) model fitted post-COVID outbreak Period

In evaluating the p-values associated with each of the estimated coefficients, it is evident that the model identifies a significant portion of the variables to be significant. This observation is in support of a likely good fitting model for the separated data. By comparing the two models, we can make the following observations:

- The auto-correlation between the 5-year expected forward inflation rate and Bitcoin return series is not significant in both the pre-pandemic and post-pandemic periods. The p-value associated with BTC.Ret in VAR model for the FYFY and the vice versa almost all have p-values above the significance level of 5%. This is very different from the initial VAR model fitted on the full series.
- The GARCH model fitted on both the pre-pandemic and post-pandemic series suggests that the BTC.Ret series is under the influence of a strong GARCH effect. However, the alpha term seems to be significantly smaller in the post-COVID model compared to the pre-COVID model. This suggests that the variance model of the Bitcoin series is more related to past random effects other than the past innovation terms.
- The best number of lags selected by minimizing the AIC scores are different with the pre-COVID model selecting three lags while two lags is selected for the post-COVID model.
- The skew and shape parameters estimated for both 5-year expected forward inflation series and Bitcoin return series are roughly the same, suggesting that the underlying distribution that generates the random innovation did not change in the pre-COVID and post-COVID periods.

We then proceed to investigate model performance using tests on the standarized residuals from the pre-COVID, post-COVID, and full period models. The results are shown in Table 10.

Model	Test	Series	Test Statistic	Pr(>|t|)
Full period model	ARCH LM-Test	FYFY	35.452	0.0003966
		BTC.Ret	2.2516	0.9989
	Box-Ljung Test	FYFY	5.6552	0.9325
		BTC.Ret	19.633	0.07434
Pre-COVID outbreak model	ARCH LM-Test	FYFY	7.8121	0.7996
		BTC.Ret	10.351	0.5852
	Box-Ljung Test	FYFY	8.2173	0.7679
		BTC.Ret	21.861	0.03911
Post-COVID outbreak model	ARCH LM-Test	FYFY	30.601	0.002266
		BTC.Ret	1.111	1
	Box-Ljung Test	FYFY	6.7662	0.8727
		BTC.Ret	14	0.3007

Table 10: Test statistic for VAR-SGARCH Models' residuals

It is evident from the results that splitting the data into two periods at the split point of the start of the COVID pandemic allowed the models to capture more ARCH effect in the data series. The test statistic for ARCH LM-test drop significantly for both the pre-COVID and post-COVID models compared to that of the full model. However, we are also seeing significant p-value for the Box-Ljung Test on the BTC.Ret residuals for the pre-COVID model, and the ARCH Test on FYFY residuals for the pre-COVID model. The opposite finding is seen in the post-COVID model. This inconsistency again confirms that the split data series are following two different mechanisms with the COVID outbreak being a significant driver for this paradigm shift.

we then proceed to perform the Granger causality test for the two series and compare them against the VAR model fitted in phase 1. The combined results are presented in table 11.

Model	Granger-Cause Direction	Test Statistic	Pr(>|t|)
Full period model	FYFY $\rightarrow$ BTC.Ret	3.8926	0.0485
	BTC.Ret $\rightarrow$ FYFY	1.3488	0.2407
Pre-COVID outbreak model	FYFY $\rightarrow$ BTC.Ret	6.11	0.9104
	BTC.Ret $\rightarrow$ FYFY	0.12971	0.9425
Post-COVID outbreak model	FYFY $\rightarrow$ BTC.Ret	3.1159	0.04466
	BTC.Ret $\rightarrow$ FYFY	4.6911	0.009327

Table 11: Granger Causality test on the VAR models

From the table, we can easily see that the resulting Granger-causality in the full period model is mostly explained by the post-COVID data. In the pre-COVID model, the Granger causality from FYFY to BTC.Ret and vice versa is quite insignificant. This echoes the finding in the fitted model in table 8 for the pre-COVID outbreak that the variables associates with BTC.Ret in the VAR model fitted for FYFY are all above significance level.

To summarize, the result of phase 3 indicate the presence of a paradigm shift caused by the outbreak of COVID. Interestingly, the post-COVID outbreak data is accountable for most of the Granger causality observed from 5-year expected forward inflation rate to Bitcoin return series and vice versa. This finding contradicts the findings in the first study in our literature survey that the Bitcoin price Granger causes changes in the 5-year expected forward inflation rate in a sample period that covers little of the post-pandemic period.

In the next phase, we will explore some of the other data series included in our data set that may act as potential confounding variables in the relationship between the 5-year expected forward inflation rate and Bitcoin returns.

6.4 *Phase 4: VAR Model (with regularization) Incorporating All Series*

The regularized VAR model on the FYFY series selected non-zero coefficients for the following 3 series: DPI, WTI, D.JONES.RE (see Table 12). A lag of 8 was chosen again by minimizing the Akaike information criterion.

The results of the variable selection is as expected. Oil is both an essential source of energy and hence the fluctuation in oil prices is likely to have influences on the inflation expectation. Additionally, housing costs have also been a major component of the consumer price index (CPI) and hence a surge in rent is likely to lead to growths in inflation expectation. Policy uncertainty can lead to economic uncertainty which is also likely to be correlated with expected inflation. However, by examining the estimated coefficients in Table 12, we note that the crude oil price and the real estate index both express negative correlations with expected inflation. This may indicate the presence of significant noises in the model or hidden correlation between some of the explanatory series that is causing the somewhat counter-intuitive results.

	Estimate
$DPI.l_7$	0.33796050
$WTI.l_7$	-0.02099501
$D.JONES.RE.l_7$	-0.01626382
$WTI.l_8$	0.008482229

Table 12: Non-zero coefficients for VAR model with regularization fitted on the FYFY series

Figure 7 shows the cross-validation process for tuning the lambda parameter. Under l1 regularization using the optimal lambda, the estimated coefficients for many lags and variables are regularized (set to zero). We examine the variables and lags with non-zero coefficients.

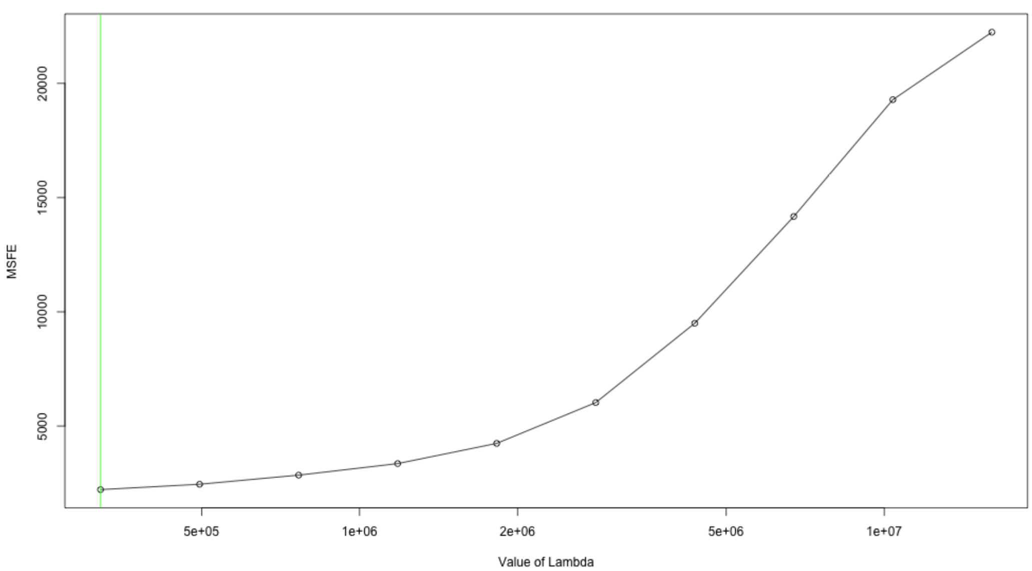

Figure 7: Cross validation loss over lambda

The estimates of the un-regularized VAR model fitted on the chosen variables are shown in Table 13. Results indicate a statistically significant correlation between the FYFY series and the WTI and D.JONES.RE series.

	Estimate	Std. Error	t value	Pr(>|t|)
$FYFY.l_1$	0.82	0.02	35.63	0.00
$DPI.l_1$	0.00	0.00	0.51	0.61
$WTI.l_1$	0.00	0.00	4.51	0.00
$D.JONES.RE.l_1$	0.00	0.00	3.26	0.00
$FYFY.l_2$	0.11	0.03	3.67	0.00
$DPI.l_2$	0.00	0.00	1.03	0.30
$WTI.l_2$	-0.00	0.00	-2.07	0.04
$D.JONES.RE.l_2$	-0.00	0.00	-2.92	0.00
$FYFY.l_3$	0.02	0.03	0.70	0.48
$DPI.l_3$	0.00	0.00	1.96	0.05
$WTI.l_3$	0.00	0.00	0.32	0.75
$D.JONES.RE.l_3$	0.00	0.00	0.76	0.45
$FYFY.l_4$	-0.03	0.03	-1.16	0.24
$DPI.l_4$	-0.00	0.00	-0.43	0.67
$WTI.l_4$	0.00	0.00	0.29	0.77
$D.JONES.RE.l_4$	-0.00	0.00	-2.87	0.00
$FYFY.l_5$	0.03	0.03	1.13	0.26
$DPI.l_5$	-0.00	0.00	-0.44	0.66
$WTI.l_5$	-0.00	0.00	-1.99	0.05
$D.JONES.RE.l_5$	0.00	0.00	3.02	0.00
$FYFY.l_6$	0.03	0.03	1.00	0.32
$DPI.l_6$	0.00	0.00	0.38	0.71
$WTI.l_6$	-0.00	0.00	-0.64	0.52
$D.JONES.RE.l_6$	-0.00	0.00	-0.63	0.53
$FYFY.l_7$	0.01	0.03	0.24	0.81
$DPI.l_7$	-0.00	0.00	-0.79	0.43
$WTI.l_7$	0.00	0.00	0.73	0.46
$D.JONES.RE.l_7$	0.00	0.00	1.59	0.11
$FYFY.l_8$	-0.00	0.02	-0.19	0.85
$DPI.l_8$	-0.00	0.00	-1.23	0.22
$WTI.l_8$	0.00	0.00	0.25	0.80
$D.JONES.RE.l_8$	-0.00	0.00	-1.98	0.05
$const$	0.03	0.01	2.48	0.01

Table 13: VAR model fitted on the FYFY series

Results of the ARCH and normality tests on the model residuals are presented in Table 14. The results suggest an inadequate model as there is strong evidence for remaining ARCH effect and non-normality in the residuals. Consequently, we are not able to draw a strong conclusion on the correlation between the series. Nonetheless, we have evidence against the hypothesis that Bitcoin returns are significantly correlated with expected forward inflation as the regularized VAR model had dropped the Bitcoin returns series entirely.

Test	Test Statistic	Pr(>|t|)
ARCH LM-Test	7614	0.00
Box-Ljung Test	0.022849	0.8799
JB-Test	3242193	0.0
Skewness	13996	0.0
Kurtosis	3228197	0.0

Table 14: ARCH/Normality test on the residual series

We then perform Granger causality tests to look for possible causation. The results are presented in Table 15. The results suggest a causal relationship from WTI to expected inflation and from D.JONES.RE to expected inflation. This finding supports the significant correlation findings in the fitted VAR model. As a result, we have evidence to suggest a causal relationship between the oil index and expected inflation and the real estate index and expected inflation. As a result, these variables may act as potential confounding variables in any observed causal relationship between Bitcoin returns and expected forward inflation.

Granger-Cause	Test Statistic	Pr(>|t|)
DPI $\rightarrow$ FYFY	1.3344	0.1268
WTI $\rightarrow$ FYFY	2.1024	0.001272
D.JONES.RE $\rightarrow$ FYFY	5.147	0.00

Table 15: Granger Causality test on the multi-series VAR models

7 Areas Requiring Further Research

We recognize that there exists limitations in this study that will require additional research in order to address. The first being that the GARCH-VAR models produced in Phase 2 do not look for variation in volatility that can be explained by inter-series variables. The models are simply separate GARCH models fitted independently on the residuals of each VAR model produced in phase 1. The models therefore fail to capture potential inter-series dependencies in the time varying volatility. This area was beyond the scope of the study and will require significant additional time to investigate. However, to further improve this study in the future, we intend to consult the GO-GARCH Model section of work 5 in the works consulted list to gain a better understanding of how to implement a true multi-variate GARCH-VAR model that attempts to capture inter-series relationships.

8 Conclusion

In this study, we find that the existing evidence supporting a causal relationship from Bitcoin returns to the expected forward inflation rate as reported by the studies in our selected literature survey no longer hold true based on more current post-pandemic data. The results of phase 1 also failed to find a significant correlation between the two series, contradicting the idea that Bitcoin can be used as an inflation hedge. We have also established the presence of a paradigm shift in the volatility of both series occurring at the start of the COVID pandemic as evidenced through model residual comparisons of different split points used to separate the series. Interestingly, the causal relationship from Bitcoin returns to expected inflation is supported by post-pandemic data but not pre-pandemic data. This observation is evidence against the possibility that the contradicting results to existing research can be attributed to the newly added post-pandemic data. As a result, additional research in this area will be required to fully explain the contradictions. Lastly, we also investigated the possibility of other confounding variables that may be behind the observed positive relationship between inflation and Bitcoin as reported in the studies reviewed. In phase 4 of our study, we find the oil index and real estate index to be potential candidates that also show a possible causal relationship with expected inflation. When these additional variables were added to the regularized VAR model for the inflation series, the correlation between inflation and Bitcoin returns became insignificant. In conclusion, our study finds evidence to refute the claims made in the papers selected in the literature survey section and does not support Bitcoin's potential to be used as an inflation hedge.

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