Neurocomputing 384 (2020) 46–56

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Lag $\mathcal{H}_\infty$ synchronization and lag synchronization for multiple derivative coupled complex networks

Lin-Hao Zhao[a], Jin-Liang Wang[a,b,*]

[a] Tianjin Key Laboratory of Autonomous Intelligence Technology and Systems, School of Computer Science and Technology, Tiangong University, Tianjin 300387, China

[b] School of Information Science and Technology, Linyi University, Linyi 276005, China

ARTICLE INFO

Article history:
Received 3 June 2019
Revised 8 September 2019
Accepted 29 November 2019
Available online 5 December 2019

Communicated by Prof. S. Arik

Keywords:
Adaptive state feedback controller
Complex networks (CNs)
Lag $\mathcal{H}_\infty$ synchronization
Lag synchronization
Multiple derivative couplings

ABSTRACT

This paper mainly devotes to the study of lag $\mathcal{H}_\infty$ synchronization and lag synchronization issues for the multiple derivative coupled complex networks (MDCCNs) with and without external disturbances, which have never been investigated. On one side, with the help of state feedback controller, adaptive state feedback controller and Lyapunov functionals, two criteria are developed to insure the lag $\mathcal{H}_\infty$ synchronization for the MDCCN with external disturbances. On the other side, we also discuss the lag synchronization in the MDCCN in virtue of choosing appropriate state feedback controller and adaptive state feedback controller. Lastly, two numerical examples are put forward to verify the lag $\mathcal{H}_\infty$ synchronization and lag synchronization criteria.

1. Introduction

It is generally known that there exist many important dynamical behaviors in complex networks (CNs) [1], such as synchronization, passivity, stability, etc. Especially, the synchronization for CNs has became a focus of interest for many researchers [2–15]. In virtue of utilizing inequality techniques, Lyapunov functionals and edge-based adaptive control strategies, Wang et al. [3] discussed the synchronization for two kinds of coupled reaction–diffusion neural networks. In [4], the authors considered an adaptive coupled reaction–diffusion neural networks, procured several passivity criteria by devising suitable adaptive scheme and employing inequality techniques, and developed a criterion of synchronization based on the result of the passivity. Toopchi et al. [7] put forward a fractional Proportional-Integral pinning control strategy to address the synchronization problem of fractional order CNs. Furthermore, in the real-life world, there may exist the external disturbances (EDs) in CNs. As the matter of fact, the synchronization in CNs may be destroyed on account of the existence of EDs. Consequently, how to weaken or avoid the influence of EDs has became the focus of interest for lot of reasearhces from home and abroad [16–20]. Based on inequality techniques, Lyapunov functional approach, and adaptive control strategies, Wang and Wu [16] studied the $\mathcal{H}_\infty$ synchronization and synchronization problems of hybrid coupled reaction–diffusion neural networks. In [18], the authors came up with a discrete stochastic CN, and investigated the bound $\mathcal{H}_\infty$ synchronization issue of such network on the basis of the Kronecker product and time-varying real-valued function.

In the world in which we live today, the time-delay commonly exists due to the finite speed of transmission and network congestion, which is always considered as the impact factor for the instability of system. Therefore, it is very intriguing to research the lag synchronization for the CNs [21–28]. In [21], with the help of devised adaptive controllers, the authors studied the lag synchronization for two types of CNs which are referred to as response and drive systems. Ji et al. [22] considered the lag synchronization between the delayed coupled uncertain CN and a nonidentical reference node based on the adaptive control approach. In [25], through devised discontinuous and continuous feedback controllers, Li et al. both coped with the lag complete synchronization and lag quasi-synchronization issues of coupled memristive neural networks. In this existing literature [21–28], unfortunately, the authors only investigated the lag synchronization for CNs. But, there exists very few results about the lag $\mathcal{H}_\infty$ synchronization issue for CNs [29].

* Corresponding author at: Tianjin Key Laboratory of Autonomous Intelligence Technology and Systems, School of Computer Science and Technology, Tiangong University, Tianjin 300387, China; School of Information Science and Technology, Linyi University, Linyi 276005, China

E-mail address: wangjinliang1984@163.com (J.-L. Wang).

https://doi.org/10.1016/j.neucom.2019.11.100

On the basis of Lyapunov functional method, inequality techniques, and adaptive control approach, Wang et al. [29] studied the lag $\mathcal{H}_\infty$ synchronization in spatial diffusion coupled and state coupled neural networks with reaction–diffusion terms. Consequently, it makes the lag $\mathcal{H}_\infty$ synchronization worthy of being researched further.

Moreover, lots of systems in the real-life world should be depicted as multi-weighted complex networks (MWCNs) [30–36], for instance, social networks, coupled neural networks, etc. More recently, the synchronization issue for the MWCNs has became a focus of interest for some researchers in various fields. In [30], the authors put forward two multi-weighted complex dynamical networks, and established some synchronization criteria for these network models based on Lyapunov functional method, inequality techniques, pinning control approach and the obtained results of the passivity. By selecting suitable Lyapunov functionals and using pinning control approach, Wang et al. [31] investigated the synchronization in undirected and directed MWCNs. Apparently, it is also meaningful to study the $\mathcal{H}_\infty$ synchronization for the MWCNs [33–36]. In [33], the authors discussed the $\mathcal{H}_\infty$ output synchronization issue for the MWCNs on the basis of Barbalat's lemma and pinning control approach. Qin et al. [35] analyzed the robust $\mathcal{H}_\infty$ synchronization of uncertain multiple time-delayed CNs based on inequality techniques, and gave several robust $\mathcal{H}_\infty$ synchronization criteria for the presented network with the help of the devised adaptive state feedback controller. However, there exists few results about lag synchronization in MWCNs [32]. In particular, the lag $\mathcal{H}_\infty$ synchronization for MWCNs has not been considered. Consequently, it makes the lag $\mathcal{H}_\infty$ synchronization and lag synchronization of the MWCNs worthy of being studied further.

As the matter of fact, there exist three kinds of coupling forms in the CNs: state coupling [4–6], output coupling [37–39] and derivative coupling [40–46]. More recently, the synchronization issue of CNs with derivative coupling has caused lots of concerns [40–43]. In [40], the authors coped with the synchronization problem for the derivative coupled complex-valued complex delayed dynamical networks with parameters perturbation on the basis of intermittent pinning control strategy and inequality techniques. Zheng [42] discussed the synchronization issue for the derivative coupled CNs with the help of a pinning controller and a pinning impulsive controller. Especially, some authors have further investigated the synchronization for the multiple derivative coupled CNs [44–46]. In virtue of utilizing inequality techniques and the devised adaptive controllers, Wang et al. [44] investigated the output synchronization of multiple output or output derivative coupled CNs. In [46], the authors studied the $\mathcal{H}_\infty$ output synchronization and output synchronization for the multiple derivative coupled CNs with the help of Lyapunov functional approach, matrix theory, some inequality techniques and the devised adaptive controllers. However, the state synchronization, output synchronization and $\mathcal{H}_\infty$ output synchronization were taken into account in these existing literatures [40–46]. But, there have not been any studies on the lag synchronization and lag $\mathcal{H}_\infty$ synchronization for CNs with multiple derivative couplings.

In this paper, we respectively research the lag $\mathcal{H}_\infty$ synchronization and lag synchronization for multiple derivative coupled complex networks (MDCCNs). The main contributions of this paper are given as follows. Firstly, we put forward two kinds of MDCCNs. Secondly, in virtue of utilizing appropriate state feedback controller, adaptive state feedback controller, and selecting suitable Lyapunov functionals, two criteria of the lag $\mathcal{H}_\infty$ synchronization are developed for MDCCN. Thirdly, we also investigate the lag synchronization issue for MDCCN by employing suitable state feedback controller and adaptive feedback controller.

2. Preliminaries

2.1. Notations

$\mathcal{B}$ denotes the undirected connections set in MDCCN, $\mathcal{N}_i$ denotes the neighbors of the node i, $\lambda_L(\mathcal{Y})$ and $\lambda_H(\mathcal{Y})$ represent the minimum and maximum eigenvalues of the real symmetric matrix $\mathcal{Y}$.

2.2. Lemma

Lemma 2.1. *(see [47]) If the differentiable function $o(t)$ has a finite limit as $t \to +\infty$ and $\dot{o}(t)$ is uniformly continuous, then $\dot{o}(t) \to 0$ as $t \to +\infty$.*

3. Lag $\mathcal{H}_\infty$ synchronization of MDCCN

3.1. MDCCN model

The MDCCN model considered in this section is described by

$$\dot{k}_i(t) = Ak_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m \dot{k}_j(t) + u_i(t) + \zeta_i(t), \tag{1}$$

where $i = 1, 2, \ldots, \mathcal{Q}$; $\mathbb{R}^{n\times n} \ni A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$; $k_i(t) = (k_{i1}(t), k_{i2}(t), \ldots, k_{in}(t))^T \in \mathbb{R}^n$ denotes the state vector of the ith node; $\mathbb{R}^{n\times n} \ni \Psi^m = \mathrm{diag}(\psi_1^m, \psi_2^m, \ldots, \psi_n^m) > 0$ represents the inner coupling matrix; $\mathbb{R}^n \ni u_i(t) = (u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t))^T$ is the control input vector of node i; $b_m (m = 1, 2, \ldots, q)$ is the coupling strength of the mth coupling form; the outer coupling matrix $\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}} \ni F^m = (F_{ij}^m)_{\mathcal{Q}\times\mathcal{Q}} (m = 1, 2, \ldots, q)$ has the following definition: if there exists a connection between nodes j and $i (j \neq i)$, then $\mathbb{R} \ni F_{ji}^m = F_{ij}^m > 0$; elsewise, $\mathbb{R} \ni F_{ji}^m = F_{ij}^m = 0 \ (i \neq j)$; furthermore, $F_{ii}^m = -\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{Q}} F_{ij}^m$; $\mathbb{R}^n \ni \zeta_i(t)$ is external disturbance, and

$$\int_0^{t_s} \zeta^T(t)\zeta(t)dt < +\infty$$

for any $\mathbb{R} \ni t_s \geqslant 0$. In this section, the MDCCN (1) is connected.

Remark 1. As the matter of fact, the change of the node state may be affected by the state derivatives of the neighbor nodes in many real networks [40–43]. For instance, the stock transaction system is a complex dynamical network, in which a node represents a stock and an edge denotes the correlations between different stocks. Apparently, the price of each stock is affected by the price fluctuating rates of other stocks. Consequently, the derivative coupling should be considered in the stock transaction system. Moreover, there exist different influencing factors for the price fluctuating rate of the stock, such as international events, natural disasters and so on. Therefore, the stock transaction system should be modeled by the complex network with multiple derivative couplings. Obviously, it is very meaningful to investigate the complex networks with multiple derivative couplings.

Defining $k^*(t) = \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} k_\rho(t)$, one has

$$\begin{aligned}
\dot{k}^*(t) &= \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}\left(Ak_\rho(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{\rho j}^m \Psi^m \dot{k}_j(t) + u_\rho(t) + \zeta_\rho(t)\right)\\
&= \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} Ak_\rho(t) + \frac{1}{\mathcal{Q}}\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m\left(\sum_{\rho=1}^{\mathcal{Q}} F_{\rho j}^m\right)\Psi^m \dot{k}_j(t)\\
&\quad + \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t)\\
&= Ak^*(t) + \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t).
\end{aligned}$$

Letting $\check{k}_i(t) = k_i(t) - k^*(t)$, one gets

$$\begin{aligned}\dot{\check{k}}_i(t) &= \dot{k}_i(t) - \dot{k}^*(t)\\ &= Ak_i(t) - Ak^*(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m (\dot{\check{k}}_j(t) + \dot{k}^*(t)) + u_i(t)\\ &\quad - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \zeta_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t)\\ &= A\check{k}_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m \dot{\check{k}}_j(t) + u_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t)\\ &\quad + \zeta_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t). \end{aligned} \tag{2}$$

Definition 3.1. The lag $\mathcal{H}_\infty$ synchronization for MDCCN (1) is achieved if

$$\int_0^{t_s} E(t)dt \leqslant V(0) + \eta^2 \sum_{i=1}^{\mathcal{Q}} \int_0^{t_s} \zeta_i^T(t)\zeta_i(t)dt, \tag{3}$$

in which $E(t) = \sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}(\check{k}_i(t) - \check{k}_j(t-\tau_{ji}))^T \Upsilon (\check{k}_i(t) - \check{k}_j(t-\tau_{ji}))$, $\mathbb{R}^{n\times n} \ni \Upsilon = \mathrm{diag}(\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_n) > 0$, $\mathbb{R} \ni \tau_{ji} = \tau_{ij} > 0$ are the time delay between nodes $i \in \{1, 2, \ldots, \mathcal{Q}\}$ and $j \in \mathcal{N}_i$, the function $V(\cdot) \geq 0$, $\mathbb{R} \ni \eta > 0$, $\mathbb{R} \ni t_s \geqslant 0$.

Denote

$$\zeta(t) = (\zeta_1(t), \zeta_2(t), \ldots, \zeta_{\mathcal{Q}}(t)),$$

$$\check{k}(t) = (\check{k}_1(t), \check{k}_2(t), \ldots, \check{k}_{\mathcal{Q}}(t)).$$

3.2. State feedback controller

For the purpose of ensuring the lag $\mathcal{H}_\infty$ synchronization for MDCCN (1), an appropriate state feedback controller is devised as follows:

$$u_i(t) = -d\check{k}_i(t) + d\sum_{j\in\mathcal{N}_i}\left(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)\right), \tag{4}$$

where $\mathbb{R} \ni d > 0$.

By (2) and (4), one has

$$\begin{aligned}\dot{\check{k}}_i(t) &= A\check{k}_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m \dot{\check{k}}_j(t)\\ &\quad - d\check{k}_i(t) + d\sum_{j\in\mathcal{N}_i}(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t)\\ &\quad + \zeta_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t), \end{aligned} \tag{5}$$

where $i = 1, 2, \ldots, \mathcal{Q}$.

Theorem 3.1. *On the basis of the state feedback controller (4), the MDCCN (1) is lag $\mathcal{H}_\infty$ synchronized if the following condition is met:*

$$\Upsilon A - d\Upsilon + \frac{\Upsilon^2}{2d\eta^2} \leqslant 0.$$

Proof. Consider the following Lyapunov functional for network (2):

$$\begin{aligned}V_1(t) &= \sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\check{k}_i(t) - \sum_{m=1}^{q} b_m \check{k}^T(t)\left[F^m \otimes (\Psi^m\Upsilon)\right]\check{k}(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\int_{t-\tau_{ji}}^{t} \check{k}_j^T(s)\Upsilon\check{k}_j(s)ds. \end{aligned} \tag{6}$$

In what follows, one gets

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\dot{\check{k}}_i(t) - 2\sum_{m=1}^{q} b_m \check{k}^T(t)\left[F^m \otimes (\Psi^m\Upsilon)\right]\dot{\check{k}}(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Upsilon\check{k}_j(t) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\ &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\Bigg[A\check{k}_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m \dot{\check{k}}_j(t) - d\check{k}_i(t)\\ &\quad + d\sum_{j\in\mathcal{N}_i}(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \zeta_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t)\Bigg]\\ &\quad - 2\sum_{m=1}^{q} b_m \check{k}^T(t)[F^m \otimes (\Psi^m\Upsilon)]\dot{\check{k}}(t) + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Upsilon\check{k}_j(t)\\ &\quad - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\ &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\left(A\check{k}_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \zeta_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t)\right)\\ &\quad - 2d\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\check{k}_i(t) + 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\left(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)\right)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Upsilon\check{k}_j(t) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji}). \end{aligned} \tag{7}$$

On account of

$$\begin{aligned}\sum_{i=1}^{\mathcal{Q}} \check{k}_i(t) &= \sum_{i=1}^{\mathcal{Q}} (k_i(t) - k^*(t))\\ &= \sum_{i=1}^{\mathcal{Q}} \left(k_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} k_\rho(t)\right)\\ &= \sum_{i=1}^{\mathcal{Q}} k_i(t) - \sum_{\rho=1}^{\mathcal{Q}} k_\rho(t)\\ &= 0. \end{aligned}$$

Apparently, one has

$$\sum_{i=1}^{\mathcal{Q}} \check{k}_i(t)\left(\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t) + \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} \zeta_\rho(t)\right) = 0. \tag{8}$$

By (7) and (8), one obtains

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon A\check{k}_i(t) + 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\zeta_i(t) - 2d\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\check{k}_i(t)\\ &\quad + 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\check{k}_j(t-\tau_{ji}) - 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\check{k}_i(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Upsilon\check{k}_j(t) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji}). \end{aligned} \tag{9}$$

Since the MDCCN (1) is undirected, one gets

$$\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\check{k}_i(t) = \sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Upsilon\check{k}_j(t). \tag{10}$$

Accordingly, one has

$$\begin{aligned}\dot{V}_1(t) &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon(A - dI_n)\check{k}_i(t) + 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Upsilon\zeta_i(t)\\ &\quad + 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\check{k}_j(t-\tau_{ji}) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Upsilon\check{k}_i(t) \end{aligned}$$

$$
\begin{aligned}
&- d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\
&= 2\check{k}^T(t)[I_{\mathcal{Q}}\otimes(\Upsilon A-d\Upsilon)]\check{k}(t)+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t).
\end{aligned}
\tag{11}
$$

By (11), one gets

$$
\begin{aligned}
&d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt\\
&=d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt+\int_0^{t_s}\dot{V}_1(t)+V_1(0)-V_1(t_s)\\
&\leqslant d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt+2\int_0^{t_s}\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)dt\\
&\quad-d\int_0^{t_s}E(t)dt+2\int_0^{t_s}\check{k}^T(t)[I_{\mathcal{Q}}\otimes(\Upsilon A-d\Upsilon)]\check{k}(t)dt+V_1(0)\\
&=V_1(0)-\int_0^{t_s}\left[\left(I_{\mathcal{Q}}\otimes\frac{\Upsilon}{\sqrt{d}\eta}\right)\check{k}(t)-\sqrt{d}\eta\zeta(t)\right]^T\left[\left(I_{\mathcal{Q}}\otimes\frac{\Upsilon}{\sqrt{d}\eta}\right)\check{k}(t)\right.\\
&\quad\left.-\sqrt{d}\eta\zeta(t)\right]dt+2\int_0^{t_s}\check{k}^T(t)\left[I_{\mathcal{Q}}\otimes\left(\Upsilon A-d\Upsilon+\frac{\Upsilon^2}{2d\eta^2}\right)\right]\check{k}(t)\\
&\leqslant V_1(0).
\end{aligned}
\tag{12}
$$

Obviously, we can conclude that

$$
\int_0^{t_s}E(t)dt\leqslant V(0)+\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt,
$$

where $V(t)=\frac{V_1(t)}{d}$.

Consequently, on the basis of the controller (4), the MDCCN (1) is lag $\mathcal{H}_\infty$ synchronized. □

3.3. Adaptive state feedback controller

$\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}}\ni\mathcal{Y}^m(t)=(\mathcal{Y}_{ij}^m(t))_{\mathcal{Q}\times\mathcal{Q}}$ represents a matrix varied with time, and it has the following definition:

$$
\mathcal{Y}_{ij}^m(t)=\begin{cases}\mathcal{Y}_{ji}^m(t)>0, & \text{if } (i,j)\in\mathcal{B},\\ -\sum_{\substack{\rho=1\\ \rho\neq i}}^{\mathcal{Q}}\mathcal{Y}_{i\rho}^m(t), & \text{if } i=j,\\ 0, & \text{otherwise,}\end{cases}
$$

in which $i=1,2,\dots,\mathcal{Q}$.

For MDCCN (1), devising the adaptive controller as follows:

$$
\begin{aligned}
&u_i(t)=\sum_{j\in\mathcal{N}_i}d\big(\check{k}_j(t-\tau_{ji})-\check{k}_i(t)\big)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\mathcal{Y}_{ij}^m(t)\Psi^m\check{k}_j(t),\\
&\dot{\mathcal{Y}}_{ij}^m(t)=\begin{cases}\iota_{ij}^m\big(k_i(t)-k_j(t)\big)^T\Psi^m\Upsilon\big(k_i(t)-k_j(t)\big), & \text{if } (i,j)\in\mathcal{B},\\ -\sum_{\substack{\rho=1\\ \rho\neq i}}^{\mathcal{Q}}\dot{\mathcal{Y}}_{i\rho}^m(t), & \text{if } i=j,\\ 0, & \text{otherwise,}\end{cases}
\end{aligned}
\tag{13}
$$

in which $\mathbb{R}\ni\iota_{ij}^m=\iota_{ji}^m>0$, $\mathbb{R}\ni d>0$, and $\mathbb{R}^{n\times n}\ni\Upsilon=\mathrm{diag}(\Upsilon_1,\Upsilon_2,\dots,\Upsilon_n)>0$.

By (2) and (13), one gets

$$
\begin{aligned}
\dot{\check{k}}_i(t)&=A\check{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\dot{\check{k}}_j(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\mathcal{Y}_{ij}^m(t)\Psi^m\check{k}_j(t)\\
&\quad+d\sum_{j\in\mathcal{N}_i}\big(\check{k}_j(t-\tau_{ji})-\check{k}_i(t)\big)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\
&\quad+\zeta_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}\zeta_\rho(t),
\end{aligned}
\tag{14}
$$

in which $i=1,2,\dots,\mathcal{Q}$.

Theorem 3.2. *With the help of the adaptive controller (13), the MDCCN (1) can realizes the lag $\mathcal{H}_\infty$ synchronization.*

Proof. Select the following Lyapunov functional for network (14):

$$
\begin{aligned}
V_2(t)&=\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\check{k}_i(t)-\sum_{m=1}^{q}b_m\check{k}^T(t)\big[F^m\otimes(\Psi^m\Upsilon)\big]\check{k}(t)\\
&\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\frac{b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)^2}{2\iota_{ij}^m}\\
&\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\int_{t-\tau_{ji}}^{t}\check{k}_j^T(s)\Upsilon\check{k}_j(s)ds,
\end{aligned}
\tag{15}
$$

in which $\mathbb{R}\ni\hat{\mathcal{Y}}_{ij}^m=\hat{\mathcal{Y}}_{ji}^m\geqslant 0$ $(i\neq j)$, $\hat{\mathcal{Y}}_{ij}^m=0$ when and only when $\mathcal{Y}_{ij}^m(t)=0$, $\check{k}(t)=(\check{k}_1^T(t),\check{k}_2^T(t),\dots,\check{k}_{\mathcal{Q}}^T(t))^T$.

In what follows, one obtains

$$
\begin{aligned}
\dot{V}_2(t)&=2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\dot{\check{k}}_i(t)-2\sum_{m=1}^{q}b_m\check{k}^T(t)\big[F^m\otimes(\Psi^m\Upsilon)\big]\dot{\check{k}}(t)\\
&\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\frac{b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)}{\iota_{ij}^m}\dot{\mathcal{Y}}_{ij}^m(t)+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t)\Upsilon\check{k}_j(t)\\
&\quad-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\
&=2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\Bigg[A\check{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\dot{\check{k}}_j(t)\\
&\quad+d\sum_{j\in\mathcal{N}_i}(\check{k}_j(t-\tau_{ji})-\check{k}_i(t))+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\mathcal{Y}_{ij}^m(t)\Psi^m\check{k}_j(t)\\
&\quad-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)+\zeta_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}\zeta_\rho(t)\Bigg]\\
&\quad-2\sum_{m=1}^{q}b_m\check{k}^T(t)\big[F^m\otimes(\Psi^m\Upsilon)\big]\dot{\check{k}}(t)\\
&\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)\big(\check{k}_i(t)-\check{k}_j(t)\big)^T\Psi^m\Upsilon\big(\check{k}_i(t)-\check{k}_j(t)\big)\\
&\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t)\Upsilon\check{k}_j(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\
&=2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\left(A\check{k}_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)+\zeta_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}\zeta_\rho(t)\right)\\
&\quad+2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_i^T(t)\Upsilon\big(\check{k}_j(t-\tau_{ji})-\check{k}_i(t)\big)\\
&\quad+2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m\mathcal{Y}_{ij}^m(t)\check{k}_i^T(t)\Psi^m\Upsilon\check{k}_j(t)\\
&\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)\big(\check{k}_i(t)-\check{k}_j(t)\big)^T\Psi^m\Upsilon\big(\check{k}_i(t)-\check{k}_j(t)\big)\\
&\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t)\Upsilon\check{k}_j(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji}).
\end{aligned}
\tag{16}
$$

Furthermore, one derives

$$
\begin{aligned}
&\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)\big(\check{k}_i(t)-\check{k}_j(t)\big)^T\Psi^m\Upsilon\big(\check{k}_i(t)-\check{k}_j(t)\big)\\
&=-2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m(\mathcal{Y}_{ij}^m(t)-\hat{\mathcal{Y}}_{ij}^m)\check{k}_i^T(t)\Psi^m\Upsilon\check{k}_j(t),
\end{aligned}
\tag{17}
$$

in which $\hat{\mathcal{Y}}_{ii}^m=-\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{Q}}\hat{\mathcal{Y}}_{ij}^m$.

By (16) and (17), one has

$$\begin{aligned}\dot{V}_2(t) &= 2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon A\check{k}_i(t)+2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\zeta_i(t)\\
&\quad+2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_i^T(t)\Upsilon\check{k}_j(t-\tau_{ji})-2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_i^T(t)\Upsilon\check{k}_i(t)\\
&\quad+2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m\hat{\mathcal{Y}}_{ij}^m\check{k}_i^T(t)\Psi^m\Upsilon\check{k}_j(t)\\
&\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t)\Upsilon\check{k}_j(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\check{k}_j^T(t-\tau_{ji})\Upsilon\check{k}_j(t-\tau_{ji})\\
&= 2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon A\check{k}_i(t)+2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\zeta_i(t)\\
&\quad+2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m\hat{\mathcal{Y}}_{ij}^m\check{k}_i^T(t)\Psi^m\Upsilon\check{k}_j(t)-dE(t)\\
&\quad+2\sum_{i=1}^{\mathcal{Q}}\check{k}_i^T(t)\Upsilon\zeta_i(t)-dE(t)\\
&= 2\check{k}^T(t)\left[I_{\mathcal{Q}}\otimes(\Upsilon A)+\sum_{\substack{m=1\\m\neq r}}^{q}b_m\hat{\mathcal{Y}}^m\otimes(\Psi^m\Upsilon)+b_r\hat{\mathcal{Y}}^r\otimes(\Psi^r\Upsilon)\right]\check{k}(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t)\\
&\leqslant 2\check{k}^T(t)\big[I_{\mathcal{Q}}\otimes(\Upsilon A)+b_r\hat{\mathcal{Y}}^r\otimes(\Psi^r\Upsilon)\big]\check{k}(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t),\end{aligned}\tag{18}$$

in which $\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}}\ni\hat{\mathcal{Y}}^m=(\hat{\mathcal{Y}}_{ij}^m)_{\mathcal{Q}\times\mathcal{Q}}$, $\check{k}(t)=(\check{k}_1^T(t),\check{k}_2^T(t),\dots,\check{k}_{\mathcal{Q}}^T(t))^T$, and $E(t)=\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}(\check{k}_i(t)-\check{k}_j(t-\tau_{ji}))^T\Upsilon(\check{k}_i(t)-\check{k}_j(t-\tau_{ji}))$.

Clearly one acquires an orthogonal matrix $\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}}\ni\Gamma=(\gamma_1,\gamma_2,\dots,\gamma_{\mathcal{Q}})$ such that

$$\Gamma^T\hat{\mathcal{Y}}^r\Gamma=\mathcal{Y}=\mathrm{diag}(\mathcal{Y}_1,\mathcal{Y}_2,\dots,\mathcal{Y}_{\mathcal{Q}})\in\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}},$$

in which $0=\mathcal{Y}_1>\mathcal{Y}_2\geqslant\mathcal{Y}_3\geqslant\cdots\geqslant\mathcal{Y}_{\mathcal{Q}}$. Let $\omega(t)=(\omega_1^T(t),\omega_2^T(t),\dots,\omega_{\mathcal{Q}}^T(t))^T=(\Gamma^T\otimes I_n)\check{k}(t)$. By reason of $\gamma_1=\frac{1}{\sqrt{\mathcal{Q}}}(1,1,\dots,1)^T$, it is easily to see that $\omega_1(t)=(\gamma_1^T\otimes I_n)\check{k}(t)=0$. Consequently, one gets

$$\begin{aligned}\dot{V}_2(t) &\leqslant 2\check{k}^T(t)\Big\{I_{\mathcal{Q}}\otimes(\Upsilon A)+b_r(\Gamma\otimes I_n)[\mathcal{Y}\otimes(\Psi^r\Upsilon)](\Gamma^T\otimes I_n)\Big\}\check{k}(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t)\\
&= 2\check{k}^T(t)\big[I_{\mathcal{Q}}\otimes(\Upsilon A)\big]\check{k}(t)+2b_r\omega^T(t)[\mathcal{Y}\otimes(\Psi^r\Upsilon)]\omega(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t)\\
&\leqslant 2\check{k}^T(t)\big[I_{\mathcal{Q}}\otimes(\Upsilon A)\big]\check{k}(t)+2b_r\mathcal{Y}_2\omega^T(t)[I_{\mathcal{Q}}\otimes(\Psi^r\Upsilon)]\omega(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t)\\
&= 2\check{k}^T(t)\Big\{I_{\mathcal{Q}}\otimes(\Upsilon A)+b_r\mathcal{Y}_2[I_{\mathcal{Q}}\otimes(\Psi^r\Upsilon)]\Big\}\check{k}(t)\\
&\quad+2\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)-dE(t).\end{aligned}\tag{19}$$

Then, one has

$$\begin{aligned}&d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt\\
&=d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt+\int_0^{t_s}\dot{V}_2(t)dt+V_2(0)-V_2(t_s)\\
&\leqslant d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt+V_2(0)+2\int_0^{t_s}\check{k}^T(t)(I_{\mathcal{Q}}\otimes\Upsilon)\zeta(t)dt\\
&\quad-d\int_0^{t_s}E(t)dt+2\int_0^{t_s}\check{k}^T(t)\Big\{I_{\mathcal{Q}}\otimes(\Upsilon A)+b_r\mathcal{Y}_2[I_{\mathcal{Q}}\otimes(\Psi^r\Upsilon)]\Big\}\check{k}(t)dt\\
&=V_2(0)-\int_0^{t_s}\left[\left(I_{\mathcal{Q}}\otimes\frac{\Upsilon}{\sqrt{d}\eta}\right)\check{k}(t)-\sqrt{d}\eta\zeta(t)\right]^T\left[\left(I_{\mathcal{Q}}\otimes\frac{\Upsilon}{\sqrt{d}\eta}\right)\check{k}(t)\right.\\
&\quad\left.-\sqrt{d}\eta\zeta(t)\right]dt+2\int_0^{t_s}\check{k}^T(t)\Big\{I_{\mathcal{Q}}\otimes(\Upsilon A)+b_r\mathcal{Y}_2[I_{\mathcal{Q}}\otimes(\Psi^r\Upsilon)]\\
&\quad+\left(I_{\mathcal{Q}}\otimes\frac{\Upsilon^2}{2d\eta^2}\right)\Big\}\check{k}(t).\end{aligned}\tag{20}$$

By selecting $\hat{\mathcal{Y}}_{ij}^r$ sufficiently large such that

$$\lambda_H(\Upsilon A)+b_r\mathcal{Y}_2\lambda_L(\Psi^r\Upsilon)+\lambda_H\left(\frac{\Upsilon^2}{2d\eta^2}\right)\leqslant 0.\tag{21}$$

By (20) and (21), one obtains

$$d\int_0^{t_s}E(t)dt-d\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt\leqslant V_2(0).\tag{22}$$

Obviously, we can conclude that

$$\int_0^{t_s}E(t)dt\leqslant V(0)+\eta^2\int_0^{t_s}\zeta^T(t)\zeta(t)dt,$$

where $V(t)=\frac{V_2(t)}{d}$.

Consequently, on the basis of the controller (13), the MDCCN (1) is lag $\mathcal{H}_\infty$ synchronized. □

4. Lag synchronization for MDCCN

4.1. MDCCN model

The MDCCN model considered in this section is described by

$$\dot{k}_i(t)=Ak_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\dot{k}_j(t)+u_i(t),\tag{23}$$

where $i=1,2,\dots,\mathcal{Q}$; $A, b_m, F_{ij}^m, \Psi^m, k_i(t), u_i(t)$ have the same definitions as these in the Section 3. In this section, the MDCCN (23) is also connected.

Taking $k^*(t)=\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}k_\rho(t)$, one has

$$\begin{aligned}\dot{k}^*(t) &= \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}\left(Ak_\rho(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{\rho j}^m\Psi^m\dot{k}_j(t)+u_\rho(t)\right)\\
&= \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}Ak_\rho(t)+\frac{1}{\mathcal{Q}}\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\left(\sum_{\rho=1}^{\mathcal{Q}}F_{\rho j}^m\right)\Psi^m\dot{k}_j(t)+\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\
&= Ak^*(t)+\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t).\end{aligned}\tag{24}$$

Defining $\check{k}_i(t)=k_i(t)-k^*(t)$, one obtains

$$\begin{aligned}\dot{\check{k}}_i(t) &= \dot{k}_i(t)-\dot{k}^*(t)\\
&= Ak_i(t)-Ak^*(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\left(\dot{\check{k}}_j(t)+\dot{k}^*(t)\right)\\
&\quad+u_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\
&= A\check{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\dot{\check{k}}_j(t)+u_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t),\end{aligned}\tag{25}$$

where $i=1,2,\dots,\mathcal{Q}$.

In what follows, the lag synchronization for the MDCCN (23) is defined:

Definition 4.1 [32]**.** The MDCCN (23) is lag synchronized if

$$\lim_{t\to+\infty}\left\|\check{k}_i(t)-\check{k}_j(t-\tau_{ji})\right\|=0,\quad\text{for all } i\neq j,$$

where $\mathbb{R}\ni\tau_{ji}=\tau_{ij}>0$ are the time delay between nodes $i\in\{1,2,\dots,\mathcal{Q}\}$ and $j\in\mathcal{N}_i$.

4.2. State feedback controller

In order to ensure the lag synchronization of MDCCN (23), devising a suitable controller has following the form:

$$u_i(t) = -d\check{k}_i(t) + d\sum_{j\in\mathcal{N}_i}\left(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)\right), \tag{26}$$

where $\mathbb{R} \ni d > 0$.

By (25) and (26), one has

$$\begin{aligned}\dot{\check{k}}_i(t) &= A\check{k}_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m \Psi^m \dot{\check{k}}_j(t)\\ &\quad - d\check{k}_i(t) + d\sum_{j\in\mathcal{N}_i}\left(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)\right) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t),\end{aligned} \tag{27}$$

where $i = 1, 2, \ldots, \mathcal{Q}$.

Theorem 4.1. *On the basis of the state feedback controller (26), the MDCCN (23) is lag synchronized if there exists a matrix $\mathbb{R}^{n\times n} \ni \Theta = \mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_n) > 0$ such that*

$$\Theta A - d\Theta \leqslant 0.$$

Proof. Consider the following Lyapunov functional for MDCCN (27):

$$\begin{aligned}V_3(t) &= \sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\check{k}_i(t) - \sum_{m=1}^{q} b_m \check{k}^T(t)\left[F^m \otimes (\Psi^m\Theta)\right]\check{k}(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\int_{t-\tau_{ji}}^{t} \check{k}_j^T(s)\Theta\check{k}_j(s)ds.\end{aligned} \tag{28}$$

In what follows, one obtains

$$\begin{aligned}\dot{V}_3(t) &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\dot{\check{k}}_i(t) - 2\sum_{m=1}^{q} b_m \check{k}^T(t)[F^m \otimes (\Psi^m\Theta)]\dot{\check{k}}(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Theta\check{k}_j(t) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Theta\check{k}_j(t-\tau_{ji})\\ &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\Bigg[A\check{k}_i(t) + \sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m F_{ij}^m\Psi^m\dot{\check{k}}_j(t)\\ &\quad - d\check{k}_i(t) + d\sum_{j\in\mathcal{N}_i}(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t)) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t)\Bigg]\\ &\quad - 2\sum_{m=1}^{q} b_m\check{k}^T(t)[F^m\otimes(\Psi^m\Theta)]\dot{\check{k}}(t)\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Theta\check{k}_j(t) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Theta\check{k}_j(t-\tau_{ji})\\ &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\left(A\check{k}_i(t) - \frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t)\right) - 2d\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\check{k}_i(t)\\ &\quad + 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Theta(\check{k}_j(t-\tau_{ji}) - \check{k}_i(t))\\ &\quad + d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t)\Theta\check{k}_j(t)\\ &\quad - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Theta\check{k}_j(t-\tau_{ji})\\ &= 2\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta A\check{k}_i(t) - 2d\sum_{i=1}^{\mathcal{Q}} \check{k}_i^T(t)\Theta\check{k}_i(t)\\ &\quad + 2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Theta\check{k}_j(t-\tau_{ji}) - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_i^T(t)\Theta\check{k}_i(t)\\ &\quad - d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i} \check{k}_j^T(t-\tau_{ji})\Theta\check{k}_j(t-\tau_{ji}),\\ &= 2\check{k}^T(t)\left[I_{\mathcal{Q}}\otimes(\Theta A - d\Theta)\right] - d\hat{E}(t)\\ &\leqslant -d\hat{E}(t),\end{aligned} \tag{29}$$

where $\hat{E}(t) = \sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\left(\check{k}_i(t) - \check{k}_j(t-\tau_{ji})\right)^T\Theta\left(\check{k}_i(t) - \check{k}_j(t-\tau_{ji})\right)$, $\check{k}(t) = (\check{k}_1^T(t), \check{k}_2^T(t), \ldots, \check{k}_{\mathcal{Q}}^T(t))^T$.

In light of (29), one gets $\lim_{t\to+\infty} V_3(t)$ exists and

$$\hat{E}(t) \leqslant -\frac{\dot{V}_3(t)}{d}.$$

Accordingly, one has

$$\begin{aligned}\int_0^t \hat{E}(q)dq &\leqslant -\int_0^t \frac{\dot{V}_3(q)}{d}dq\\ &= \frac{V_3(0)}{d} - \frac{V_3(t)}{d}\\ &\leqslant \frac{V_3(0)}{d}.\end{aligned}$$

Therefore, $\lim_{t\to+\infty}\int_0^t \hat{E}(q)dq$ exist. Furthermore,

$$\dot{\hat{E}}(t) = 2\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\left(\check{k}_i(t) - \check{k}_j(t-\tau_{ji})\right)^T\Theta\left(\dot{\check{k}}_i(t) - \dot{\check{k}}_j(t-\tau_{ji})\right).$$

Based on (29) and the definition of $V_3(t)$, one gets $\check{k}_i(t)$ is bounded for any $t \in [0, +\infty)$. Then, one has $\check{k}_j(t-\tau_{ji})$ is bounded for any $t \in [0, +\infty)$. Therefore, $u_i(t)$ is bounded.

From (27), one obtains

$$\mathcal{M}\dot{\check{k}}(t) = (I_{\mathcal{Q}}\otimes A)\check{k}(t) + u(t) - 1_{\mathcal{Q}}\otimes\left(\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}} u_\rho(t)\right),$$

where $\mathcal{M} = I_{\mathcal{Q}n} - \sum_{m=1}^{q} b_m F^m\otimes\Psi^m$, $u(t) = (u_1(t), u_2(t), \ldots, u_{\mathcal{Q}}(t))$, $1_{\mathcal{Q}} = (1, 1, \ldots, 1)^T \in \mathbb{R}^{\mathcal{Q}}$.

On account of $\mathcal{M} > 0$, one obtains $\dot{\check{k}}_i(t), \dot{\check{k}}_j(t-\tau_{ji}), i = 1, 2, \ldots, \mathcal{Q}$ are also bounded. Apparently, it is easy to infer that $\left|\dot{\hat{E}}(t)\right|$ is bounded. Therefore, $\hat{E}(t)$ is uniformly continuous.

With the help of Lemma 2.2, one concludes that

$$\lim_{t\to+\infty}\hat{E}(t) = 0,$$

Scilicet,

$$\lim_{t\to+\infty}\left\|\check{k}_i(t) - \check{k}_j(t-\tau_{ji})\right\| = 0,\ \ i = 1, 2, \ldots, \mathcal{Q},\ \ j\in\mathcal{N}_i.$$

Therefore, by virtue of the controller (26), the MDCCN (23) is lag output synchronized. □

4.3. Adaptive state feedback controller

$\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}} \ni \tilde{\mathcal{Y}}^m(t) = (\tilde{\mathcal{Y}}_{ij}^m(t))_{\mathcal{Q}\times\mathcal{Q}}$ represents a matrix varied with time, and it has the following definition:

$$\tilde{\mathcal{Y}}_{ij}^m(t) = \begin{cases}\tilde{\mathcal{Y}}_{ji}^m(t) > 0, & \text{if } (i,j)\in\mathcal{B},\\ -\sum_{\substack{\rho=1\\ \rho\neq i}}^{\mathcal{Q}}\tilde{\mathcal{Y}}_{i\rho}^m(t), & \text{if } \ i = j,\\ 0, & \text{otherwise},\end{cases}$$

in which $i = 1, 2, \ldots, \mathcal{Q}$.

For MDCCN (23), devising the adaptive controller as follows:

$$u_i(t)=\sum_{j\in\mathcal{N}_i} d\big(\breve{k}_j(t-\tau_{ji})-\breve{k}_i(t)\big)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_m\mathfrak{Y}_{ij}^m(t)\Psi^m\breve{k}_j(t),$$

$$\dot{\mathfrak{Y}}_{ij}^m(t)=\begin{cases}\iota_{ij}^m\big(k_i(t)-k_j(t)\big)^T\Psi^m\Theta\big(k_i(t)-k_j(t)\big), & \text{if } (i,j)\in\mathcal{B},\\ -\sum_{\substack{\rho=1\\ \rho\neq i}}^{\mathcal{Q}}\dot{\mathfrak{Y}}_{i\rho}^m(t), & \text{if } \ i=j,\\ 0, & \text{otherwise},\end{cases} \tag{30}$$

in which $\mathbb{R}\ni\iota_{ij}^m=\iota_{ji}^m>0$, $\mathbb{R}\ni d>0$, and $\mathbb{R}^{n\times n}\ni\Theta=\mathrm{diag}(\theta_1,\theta_2,\dots,\theta_n)>0$.

By (25) and (30), one gets

$$\begin{aligned}\dot{\breve{k}}_i(t)&=A\breve{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_mF_{ij}^m\Psi^m\dot{\breve{k}}_j(t)+u_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\ &=A\breve{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}} b_mF_{ij}^m\Psi^m\dot{\breve{k}}_j(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\ &\quad+\sum_{j\in\mathcal{N}_i}d(\breve{k}_j(t-\tau_{ji})-\breve{k}_i(t))\\ &\quad+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\mathfrak{Y}_{ij}^m(t)\Psi^m\breve{k}_j(t),\end{aligned} \tag{31}$$

in which $i=1,2,\dots,\mathcal{Q}$.

Theorem 4.2. *With the help of the adaptive controller (30), the MDCCN (23) realizes the lag synchronization.*

Proof. Select the following Lyapunov functional for network (31):

$$\begin{aligned}V_4(t)&=\sum_{i=1}^{\mathcal{Q}}\breve{k}_i^T(t)\Theta\breve{k}_i(t)-\sum_{m=1}^{q}b_m\breve{k}^T(t)\big[F^m\otimes(\Psi^m\Theta)\big]\breve{k}(t)\\ &\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\frac{b_m(\mathfrak{Y}_{ij}^m(t)-\check{\mathfrak{Y}}_{ij}^m)^2}{2\iota_{ij}^m}\\ &\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\int_{t-\tau_{ji}}^{t}\breve{k}_j^T(s)\Theta\breve{k}_j(s)ds,\end{aligned} \tag{32}$$

in which $\mathbb{R}\ni\check{\mathfrak{Y}}_{ij}^m=\check{\mathfrak{Y}}_{ji}^m\geqslant 0\ (i\neq j)$, $\check{\mathfrak{Y}}_{ij}^m=0$ when and only when $\mathfrak{Y}_{ij}^m(t)=0$, $\breve{k}(t)=(\breve{k}_1^T(t),\breve{k}_2^T(t),\dots,\breve{k}_{\mathcal{Q}}^T(t))^T$.

In what follows, one obtains

$$\begin{aligned}\dot{V}_4(t)&=2\sum_{i=1}^{\mathcal{Q}}\breve{k}_i^T(t)\Theta\dot{\breve{k}}_i(t)-2\sum_{m=1}^{q}b_m\breve{k}^T(t)[F^m\otimes(\Psi^m\Theta)]\dot{\breve{k}}(t)\\ &\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\frac{b_m(\mathfrak{Y}_{ij}^m(t)-\check{\mathfrak{Y}}_{ij}^m)}{\iota_{ij}^m}\dot{\mathfrak{Y}}_{ij}^m(t)\\ &\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t)\Theta\breve{k}_j(t)-\ d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t-\tau_{ji})\Theta\breve{k}_j(t-\tau_{ji})\\ &=2\sum_{i=1}^{\mathcal{Q}}\breve{k}_i^T(t)\Theta\Bigg[A\breve{k}_i(t)+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_mF_{ij}^m\Psi^m\dot{\breve{k}}_j(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\\ &\quad+\sum_{j\in\mathcal{N}_i}d(\breve{k}_j(t-\tau_{ji})-\breve{k}_i(t))+\sum_{m=1}^{q}\sum_{j=1}^{\mathcal{Q}}b_m\mathfrak{Y}_{ij}^m(t)\Psi^m\breve{k}_j(t)\Bigg]\\ &\quad-2\sum_{m=1}^{q}b_m\breve{k}^T(t)[F^m\otimes(\Psi^m\Theta)]\dot{\breve{k}}(t)+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathfrak{Y}_{ij}^m(t)\\ &\quad-\check{\mathfrak{Y}}_{ij}^m)\big(\breve{k}_i(t)-\breve{k}_j(t)\big)^T\Psi^m\Theta\big(\breve{k}_i(t)-\breve{k}_j(t)\big)\\ &\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t)\Theta\breve{k}_j(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t-\tau_{ji})\Theta\breve{k}_j(t-\tau_{ji})\\ &=2\sum_{i=1}^{\mathcal{Q}}\breve{k}_i^T(t)\Theta\left(A\breve{k}_i(t)-\frac{1}{\mathcal{Q}}\sum_{\rho=1}^{\mathcal{Q}}u_\rho(t)\right)\\ &\quad-2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_i^T(t)\Theta\breve{k}_i(t)+2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_i^T(t)\Theta\breve{k}_j(t-\tau_{ji})\\ &\quad+2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m\mathfrak{Y}_{ij}^m(t)\breve{k}_i^T(t)\Psi^m\Theta\breve{k}_j(t)\\ &\quad+\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathfrak{Y}_{ij}^m(t)-\check{\mathfrak{Y}}_{ij}^m)\big(\breve{k}_i(t)\\ &\quad-\breve{k}_j(t)\big)^T\Psi^m\Theta\big(\breve{k}_i(t)-\breve{k}_j(t)\big)\\ &\quad+d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t)\Theta\breve{k}_j(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t-\tau_{ji})\Theta\breve{k}_j(t-\tau_{ji}).\end{aligned} \tag{33}$$

Furthermore, one derives

$$\begin{aligned}&\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}b_m(\mathfrak{Y}_{ij}^m(t)-\check{\mathfrak{Y}}_{ij}^m)\big(\breve{k}_i(t)-\breve{k}_j(t)\big)^T\Psi^m\Theta\big(\breve{k}_i(t)-\breve{k}_j(t)\big)\\ &\quad=-2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m(\mathfrak{Y}_{ij}^m(t)-\check{\mathfrak{Y}}_{ij}^m)\breve{k}_i^T(t)\Psi^m\Theta\breve{k}_j(t),\end{aligned} \tag{34}$$

in which $\check{\mathfrak{Y}}_{ii}^m=-\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{Q}}\check{\mathfrak{Y}}_{ij}^m$.

By (33) and (34), one has

$$\begin{aligned}\dot{V}_4(t)&=2\sum_{i=1}^{\mathcal{Q}}\breve{k}_i^T(t)\Theta A\breve{k}_i(t)-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_i^T(t)\Theta\breve{k}_i(t)\\ &\quad+2d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_i^T(t)\Theta\breve{k}_j(t-\tau_{ji})\\ &\quad+2\sum_{m=1}^{q}\sum_{i=1}^{\mathcal{Q}}\sum_{j=1}^{\mathcal{Q}}b_m\check{\mathfrak{Y}}_{ij}^m\breve{k}_i^T(t)\Psi^m\Theta\breve{k}_j(t)\\ &\quad-d\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\breve{k}_j^T(t-\tau_{ji})\Theta\breve{k}_j(t-\tau_{ji})\\ &=2\breve{k}(t)\left[I_{\mathcal{Q}}\otimes(\Theta A)+\sum_{m=1}^{q}b_m\check{\mathfrak{Y}}^m\otimes(\Psi^m\Theta)\right]\breve{k}(t)-d\hat{E}(t)\\ &=2\breve{k}(t)\Bigg[I_{\mathcal{Q}}\otimes(\Theta A)+\sum_{\substack{m=1\\ m\neq r}}^{q}b_m\check{\mathfrak{Y}}^m\otimes(\Psi^m\Theta)\\ &\quad+b_r\check{\mathfrak{Y}}^r\otimes(\Psi^r\Theta)\Bigg]\breve{k}(t)-d\hat{E}(t)\\ &\leqslant 2\breve{k}(t)\big[I_{\mathcal{Q}}\otimes(\Theta A)+b_r\check{\mathfrak{Y}}^r\otimes(\Psi^r\Theta)\big]\breve{k}(t)-d\hat{E}(t),\end{aligned} \tag{35}$$

in which $\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}}\ni\check{\mathfrak{Y}}^m=(\check{\mathfrak{Y}}_{ij}^m)_{\mathcal{Q}\times\mathcal{Q}}$, $\breve{k}(t)=(\breve{k}_1^T(t),\breve{k}_2^T(t),\dots,\breve{k}_{\mathcal{Q}}^T(t))^T$, and $\hat{E}(t)=\sum_{i=1}^{\mathcal{Q}}\sum_{j\in\mathcal{N}_i}\big(\breve{k}_i(t)-\breve{k}_j(t-\tau_{ji})\big)^T\Theta\big(\breve{k}_i(t)-\breve{k}_j(t-\tau_{ji})\big)$.

Clearly, one acquires an orthogonal matrix $\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}}\ni\hat{\Gamma}=(\hat{\gamma}_1,\hat{\gamma}_2,\dots,\hat{\gamma}_{\mathcal{Q}})$ such that

$$\hat{\Gamma}^T\check{\mathfrak{Y}}^r\hat{\Gamma}=\bar{\mathfrak{Y}}=\mathrm{diag}(\bar{\mathfrak{Y}}_1,\bar{\mathfrak{Y}}_2,\dots,\bar{\mathfrak{Y}}_{\mathcal{Q}})\in\mathbb{R}^{\mathcal{Q}\times\mathcal{Q}},$$

in which $0=\bar{\mathfrak{Y}}_1>\bar{\mathfrak{Y}}_2\geqslant\bar{\mathfrak{Y}}_3\geqslant\dots\geqslant\bar{\mathfrak{Y}}_{\mathcal{Q}}$. Let $\hat{\omega}(t)=(\hat{\omega}_1^T(t),\hat{\omega}_2^T(t),\dots,\hat{\omega}_{\mathcal{Q}}^T(t))^T=(\hat{\Gamma}^T\otimes I_n)\breve{k}(t)$. By reason of $\hat{\gamma}_1=\frac{1}{\sqrt{\mathcal{Q}}}(1,1,\dots,1)^T$, it is easily to see that $\hat{\omega}_1(t)=(\hat{\gamma}_1^T\otimes I_n)\breve{k}(t)=0$. Consequently, one gets

$$\begin{aligned}
\dot{V}_4(t) &\leqslant 2\check{k}^T(t)\big\{I_Q\otimes(\Theta A)+b_r(\hat{\Gamma}\otimes I_n)\big[\bar{\mathcal{Y}}\otimes(\Psi^r\Theta)\big](\hat{\Gamma}^T\otimes I_n)\big\}\check{k}(t)-d\hat{E}(t)\\
&= 2\check{k}^T(t)[I_Q\otimes(\Theta A)]\check{k}(t)+2b_r\hat{\omega}^T(t)\big[\bar{\mathcal{Y}}\otimes(\Psi^r\Theta)\big]\hat{\omega}(t)-d\hat{E}(t)\\
&\leqslant 2\check{k}^T(t)[I_Q\otimes(\Theta A)]\check{k}(t)+2b_r\bar{\mathcal{Y}}_2\hat{\omega}^T(t)[I_Q\otimes(\Psi^r\Theta)]\hat{\omega}(t)-d\hat{E}(t)\\
&= 2\check{k}^T(t)\big[I_Q\otimes(\Theta A)+b_r\bar{\mathcal{Y}}_2 I_Q\otimes(\Psi^r\Theta)\big]\check{k}(t)-d\hat{E}(t). \qquad (36)
\end{aligned}$$

By selecting $\check{\mathcal{Y}}^r_{ij}$ sufficiently large such that

$$\lambda_H(\Theta A)+b_r\bar{\mathcal{Y}}_2\lambda_L(\Psi^r\Theta)\leqslant 0. \qquad (37)$$

By (36) and (37), one has

$$\dot{V}_4(t)\leqslant -d\hat{E}(t). \qquad (38)$$

From (38) and the definition of $V_4(t)$, one gets $V_4(t)$ is bounded. Then, one obtains $\tilde{\mathcal{Y}}^m_{ij}(t)$ is also bounded. The following proof is similar as Theorem 4.1, thus we omit its proof to save space. □

Remark 2. In recent years, many authors have been investigated the dynamical behaviors (e.g. synchronization, passivity, etc) of the multi-weighted complex networks, and lots of meaningful results have been obtained [2–15]. Unfortunately, in these existing works [2–15], the nodes in network models are coupled through their states. As the matter of fact, the derivative of node state may affect other nodes in networks. Consequently, it is very intriguing to investigate the dynamical behaviors of complex networks with derivative coupling [40–43]. More recently, some authors have further studied the dynamical behaviors (e.g. synchronization, passivity, etc) of the complex networks with multiple derivative couplings [44–46]. But, the lag synchronization and the lag $\mathcal{H}_\infty$ synchronization for the complex networks with multiple derivative couplings have not yet been discussed.

Remark 3. In this paper, several lag synchronization and lag $\mathcal{H}_\infty$ synchronization conditions are obtained based on the Lyapunov stability theory [see Theorems 3.1, 3.2, 4.1 and 4.2], which are dependent on the dimension and the number of nodes. Apparently, it could be difficult to solve these conditions when node number is very huge. In future, we will adopt some new approaches to obtain lower dimension and easier to solve the lag synchronization and lag $\mathcal{H}_\infty$ synchronization criteria.

5. Numerical example

Example 5.1. Take the following MDCCN into consideration:

$$\begin{aligned}
\dot{k}_i(t) &= Ak_i(t)+0.4\sum_{j=1}^{6}F^1_{ij}\Psi^1\dot{k}_j(t)+0.5\sum_{j=1}^{6}F^2_{ij}\Psi^2\dot{k}_j(t)\\
&\quad+0.6\sum_{j=1}^{6}F^3_{ij}\Psi^3\dot{k}_j(t)+u_i(t)+\zeta_i(t),
\end{aligned}$$

where $i=1,2,\ldots,6, k_i(t)=(k_{i1}(t),k_{i2}(t),k_{i3}(t))^T\in\mathbb{R}^3, u_i(t)=(u_{i1}(t),u_{i2}(t),u_{i3}(t))^T\in\mathbb{R}^3, A=\mathrm{diag}(1,2,3)$,

$$F^1=\begin{pmatrix}-0.4&0.1&0.2&0&0&0.1\\0.1&-0.6&0&0.3&0.2&0\\0.2&0&-0.5&0&0&0.3\\0&0.3&0&-0.7&0.2&0.2\\0&0.2&0&0.2&-0.4&0\\0.1&0&0.3&0.2&0&-0.6\end{pmatrix},$$

$$F^2=\begin{pmatrix}-0.6&0.3&0.1&0&0&0.2\\0.3&-0.5&0&0.1&0.1&0\\0.1&0&-0.6&0&0&0.5\\0&0.1&0&-0.3&0.1&0.1\\0&0.1&0&0.1&-0.2&0\\0.2&0&0.5&0.1&0&-0.8\end{pmatrix},$$

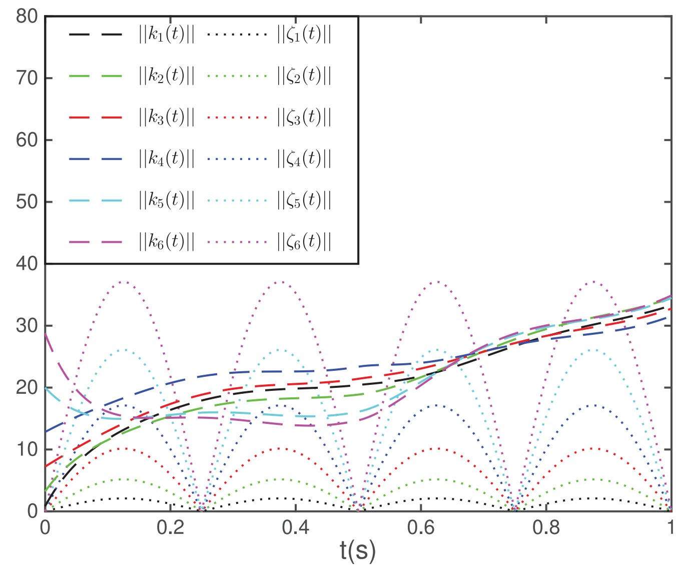

Fig. 1. Change processes of $\|k_i(t)\|$ and $\|\zeta_i(t)\|$ for the MDCCN (1) under the state feedback controller (4), where $i=1,2,\ldots,6$.

$$F^3=\begin{pmatrix}-0.7&0.2&0.3&0&0&0.2\\0.2&-0.6&0&0.2&0.2&0\\0.3&0&-0.5&0&0&0.2\\0&0.2&0&-0.4&0.1&0.1\\0&0.2&0&0.1&-0.3&0\\0.2&0&0.2&0.1&0&-0.5\end{pmatrix}.$$

Case 1: Choosing $\eta=2, \Upsilon=I_3, d=5, \Psi^1=\mathrm{diag}(0.3,0.2,0.4), \Psi^2=\mathrm{diag}(0.4,0.1,0.2), \Psi^3=\mathrm{diag}(0.3,0.5,0.1)$, it is easily to calculate the condition of the Theorem 3.1 is met. On the basis of the Theorem 3.1, the MDCCN (1) is lag $\mathcal{H}_\infty$ synchronized via controller (4). Taking $\zeta_i(t)=(i^2*\sin(4\pi t), 1.2\sqrt{i}*\sin(4\pi t), 1.4i*\sin(4\pi t))^T$, the change processes of the node state $\|k_i(t)\|(i=1,2,\ldots,6)$, and the external distance $\|\zeta_i(t)\|(i=1,2,\ldots,6)$ are shown in Fig. 1.

Case 2: Let $\eta=3, \Upsilon=I_3, d=3, \Psi^1=\mathrm{diag}(0.4,0.5,0.2), \Psi^2=\mathrm{diag}(0.3,0.4,0.6), \Psi^3=\mathrm{diag}(0.2,0.4,0.3)$. In virtue of the controller (13), the MDCCN (1) is lag $\mathcal{H}_\infty$ synchronized. Selecting $\zeta_i(t)=(0.6i^2*\sin(4\pi t), 0.5\sqrt{i}*\sin(4\pi t), 0.4i*\sin(4\pi t))^T$,

$$\mathcal{Y}^1(0)=\begin{pmatrix}-0.05&0.01&0.02&0&0&0.02\\0.01&-0.07&0&0.02&0.04&0\\0.02&0&-0.04&0&0&0.02\\0&0.02&0&-0.04&0.01&0.01\\0&0.04&0&0.01&-0.05&0\\0.02&0&0.02&0.01&0&-0.05\end{pmatrix},$$

$$\mathcal{Y}^2(0)=\begin{pmatrix}-0.06&0.02&0.01&0&0&0.03\\0.02&-0.04&0&0.01&0.01&0\\0.01&0&-0.03&0&0&0.02\\0&0.01&0&-0.03&0.01&0.01\\0&0.01&0&0.01&-0.02&0\\0.03&0&0.02&0.01&0&-0.06\end{pmatrix},$$

$$\mathcal{Y}^3(0)=\begin{pmatrix}-0.04&0.01&0.02&0&0&0.01\\0.01&-0.05&0&0.03&0.01&0\\0.02&0&-0.07&0&0&0.05\\0&0.03&0&-0.05&0.01&0.01\\0&0.01&0&0.01&-0.2&0\\0.01&0&0.05&0.01&0&-0.07\end{pmatrix},$$

the change processes of the node state $\|k_i(t)\|(i=1,2,\ldots,6)$, the external distance $\|\zeta_i(t)\|(i=1,2,\ldots,6)$, and the adaptive feedback gains are displayed in Fig. 2 and Fig. 3, respectively.

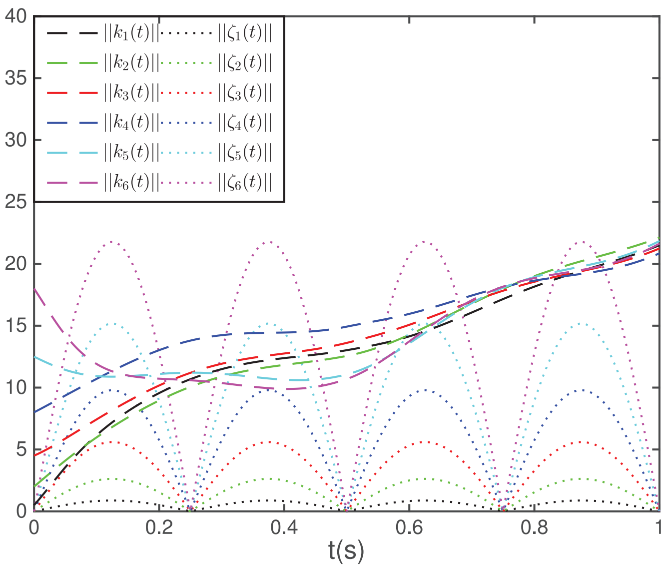

Fig. 2. Change processes of $\|k_i(t)\|$ and $\|\zeta_i(t)\|$ for the MDCCN (1) under the adaptive state feedback controller (13), where $i = 1, 2, \ldots, 6$.

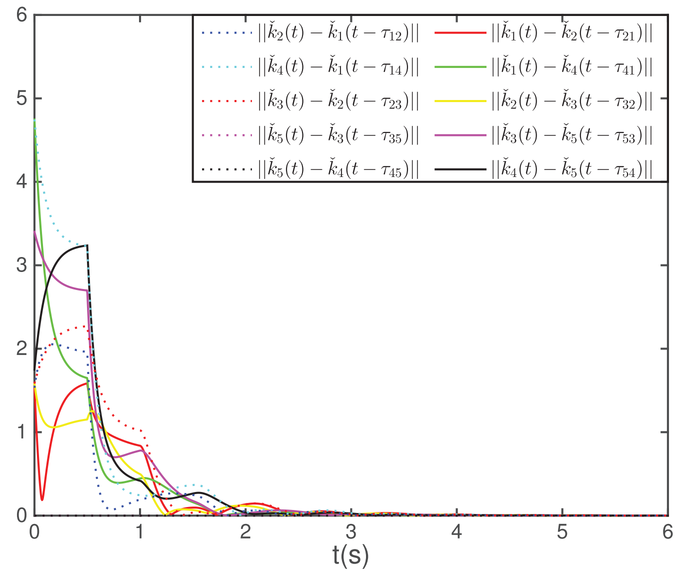

Fig. 4. Change processes of $\|\check{k}_i(t) - \check{k}_j(t - \tau_{ji})\|$ for the MDCCN (23) under the state feedback controller (26), where $i = 1, 2, \ldots, 5,\ j \in \mathcal{N}_i$.

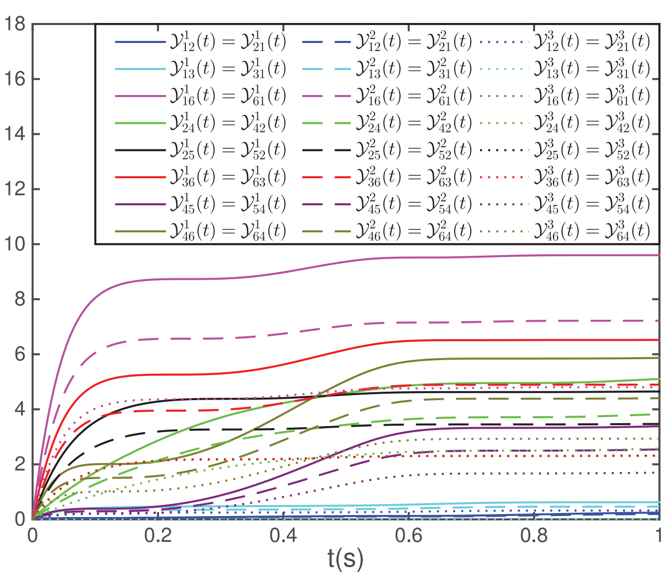

Fig. 3. Change processes of the adaptive feedback gains in the controller (13).

Example 5.2. Take the following MDCCN into consideration:

$$\dot{k}_i(t) = Ak_i(t) + 0.8\sum_{j=1}^{5} F^1_{ij}\Psi^1\dot{k}_j(t) + 0.7\sum_{j=1}^{5} F^2_{ij}\Psi^2\dot{k}_j(t) + 0.6\sum_{j=1}^{5} F^3_{ij}\Psi^3\dot{k}_j(t) + u_i(t),$$

where $i = 1, 2, \ldots, 5, k_i(t) = (k_{i1}(t), k_{i2}(t), k_{i3}(t))^T \in \mathbb{R}^3, u_i(t) = (u_{i1}(t), u_{i2}(t), u_{i3}(t))^T \in \mathbb{R}^3, A = \mathrm{diag}(1, 2, 3)$,

$$F^1 = \begin{pmatrix} -0.6 & 0.4 & 0 & 0.2 & 0 \\ 0.4 & -0.5 & 0.1 & 0 & 0 \\ 0 & 0.1 & -0.3 & 0 & 0.2 \\ 0.2 & 0 & 0 & -0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.2 & -0.4 \end{pmatrix},$$

$$F^2 = \begin{pmatrix} -0.5 & 0.3 & 0 & 0.2 & 0 \\ 0.3 & -0.6 & 0.3 & 0 & 0 \\ 0 & 0.3 & -0.4 & 0 & 0.1 \\ 0.2 & 0 & 0 & -0.3 & 0.1 \\ 0 & 0 & 0.1 & 0.1 & -0.2 \end{pmatrix},$$

$$F^3 = \begin{pmatrix} -0.7 & 0.4 & 0 & 0.3 & 0 \\ 0.4 & -0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & -0.5 & 0 & 0.3 \\ 0.3 & 0 & 0 & -0.4 & 0.1 \\ 0 & 0 & 0.3 & 0.1 & -0.4 \end{pmatrix}.$$

Case 1: Choosing $\Theta = I_3, d = 5, \Psi^1 = \mathrm{diag}(0.5, 0.2, 0.3), \Psi^2 = \mathrm{diag}(0.4, 0.3, 0.1), \Psi^3 = \mathrm{diag}(0.2, 0.5, 0.4)$, it is easily to calculate the condition of Theorem 4.1 is met. On the basis of Theorem 4.1, the MDCCN (23) is lag synchronized via controller (26). The change processes of $\|\check{k}_i(t) - \check{k}_j(t - \tau_{ji})\|$ $(i = 1, 2, \ldots, 5,\ j \in \mathcal{N}_i)$ are shown in Fig. 4.

Case 2: Let $\Theta = I_3,\ d = 3,\ \Psi^1 = \mathrm{diag}(0.5, 0.2, 0.3),\ \Psi^2 = \mathrm{diag}(0.4, 0.3, 0.1), \Psi^3 = \mathrm{diag}(0.2, 0.5, 0.4)$. In virtue of the controller (30), the MDCCN (23) is lag synchronized. Choosing

$$\tilde{\mathcal{Y}}^1(0) = \begin{pmatrix} -0.04 & 0.03 & 0 & 0.01 & 0 \\ 0.03 & -0.05 & 0.02 & 0 & 0 \\ 0 & 0.02 & -0.03 & 0 & 0.01 \\ 0.01 & 0 & 0 & -0.03 & 0.02 \\ 0 & 0 & 0.01 & 0.02 & -0.03 \end{pmatrix},$$

$$\tilde{\mathcal{Y}}^2(0) = \begin{pmatrix} -0.05 & 0.02 & 0 & 0.03 & 0 \\ 0.02 & -0.04 & 0.02 & 0 & 0 \\ 0 & 0.02 & -0.05 & 0 & 0.03 \\ 0.03 & 0 & 0 & -0.04 & 0.01 \\ 0 & 0 & 0.03 & 0.01 & -0.04 \end{pmatrix},$$

$$\tilde{\mathcal{Y}}^3(0) = \begin{pmatrix} -0.07 & 0.04 & 0 & 0.03 & 0 \\ 0.04 & -0.05 & 0.01 & 0 & 0 \\ 0 & 0.01 & -0.03 & 0 & 0.02 \\ 0.03 & 0 & 0 & -0.05 & 0.02 \\ 0 & 0 & 0.02 & 0.02 & -0.04 \end{pmatrix},$$

the change processes of $\|\check{k}_i(t) - \check{k}_j(t - \tau_{ji})\|$ $(i = 1, 2, \ldots, 5,\ j \in \mathcal{N}_i)$ and the adaptive feedback gains are displayed in Fig. 5 and Fig. 6, respectively.

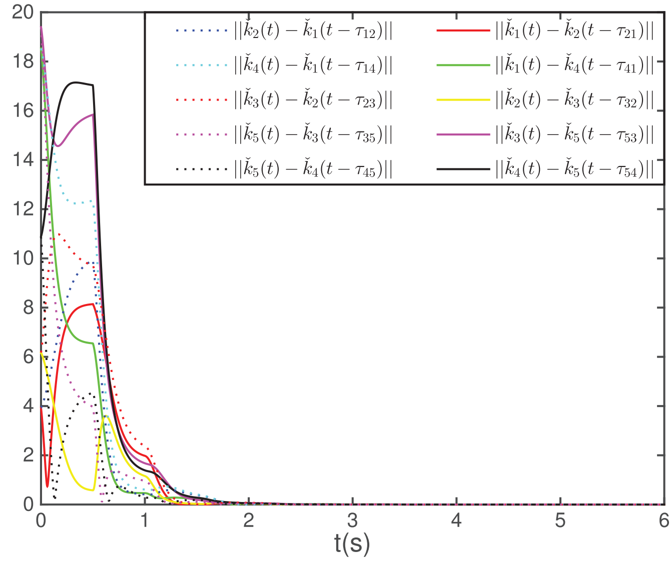

Fig. 5. Change processes of $\|\check{k}_i(t)-\check{k}_j(t-\tau_{ji})\|$ for the MDCCN (23) under the adaptive state feedback controller (30), where $i=1,2,\ldots,5,\ j\in\mathcal{N}_i$.

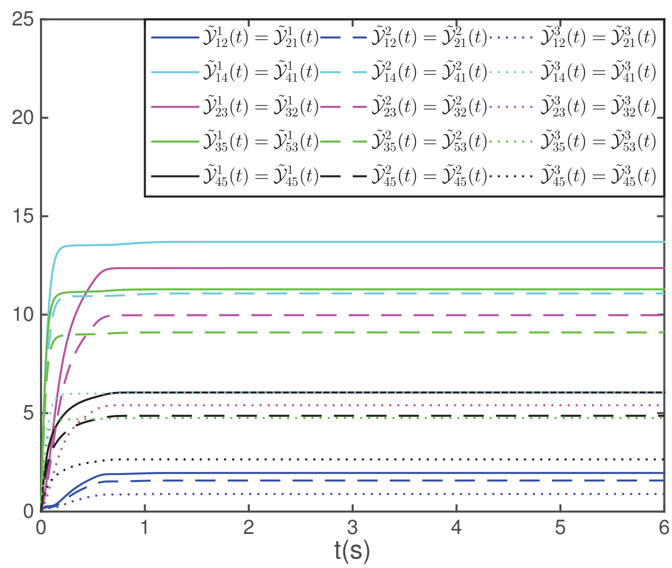

Fig. 6. Change processes of the adaptive feedback gains in the controller (30).

6. Conclusion

In this paper, we have taken the lag $\mathcal{H}_\infty$ synchronization and lag synchronization problems for the MDCCNs into consideration. In virtue of devising appropriate state feedback controller and adaptive state feedback controller, and selecting suitable Lyapunov functionals, two lag $\mathcal{H}_\infty$ synchronization criteria for the MDCCN have been developed. Moreover, by utilizing Barbalat's Lemma, two criteria of the lag synchronization for the MDCCN have been acquired on the basis of the state feedback controller and the adaptive state feedback controller. At last, we have came up with two examples to verify the effectiveness of the obtain consequences.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61773285, in part by the Tianjin Talent Development Special Support Program for Young Top-Notch Talent, in part by the Natural Science Foundation of Tianjin, China, under Grant 19JCYBJC18700, and in part by the Program for Innovative Research Team in University of Tianjin (No. TD13-5032).

References

[1] R. Albert, A.L. Barablasi, Statistical mechanics of complex networks, Rev. Modern Phys. 74 (2002) 47–97.
[2] J.L. Wang, H.N. Wu, T. Huang, Passivity-based synchronization of a class of complex dynamical networks with time-varying delay, Automatica 56 (2015) 105–112.
[3] J.L. Wang, H.N. Wu, L. Guo, Novel adaptive strategies for synchronization oflinearly coupled neural networks with reaction–diffusion terms, IEEE Trans. Neural Netw. Learn. Syst. 25 (2) (2014) 429–440.
[4] J.L. Wang, H.N. Wu, T. Huang, S.Y. Ren, Passivity and synchronization of linearly coupled reaction–diffusion neural networks with adaptive coupling, IEEE Trans. Cybern. 45 (9) (2015) 1942–1952.
[5] M. Han, M. Zhang, T. Qiu, M. Xu, UCFTS: a unilateral coupling finite-time synchronization scheme for complex networks, IEEE Trans. Neural Netw. Learn. Systems 30 (1) (2019) 255–268.
[6] J.L. Wang, H.N. Wu, T. Huang, S.Y. Ren, J. Wu, Pinning control for synchronization of coupled reacting-diffusion neural networks with directed topologies, IEEE Trans. Syst. Man Cybern. Systems 46 (8) (2016) 1109–1120.
[7] Y. Toopchi, M. Jalili, J. Sadati, J. Wang, Fractional PI pinning synchronization of fractional complex dynamical networks, J. Comput. Appl. Math. 347 (2019) 357–368.
[8] R. Yu, H. Zhang, Z. Wang, Y. Liu, Synchronization criterion of complex networks with time-delay under mixed topologies, Neurocomputing 295 (2018) 8–16.
[9] F. Parastesh, H. Azarnoush, S. Jafari, B. Hatef, M. Prec, R. Repnik, Synchronization of two neurons with switching in the coupling, Appl. Math. Comput. 350 (2019) 217–223.
[10] M. Shafiei, F. Parastesh, M. Jalili, S. Jafari, M. Perc, M. Slavinec, Effects of partial time delays on synchronization patterns in izhikevich neuronal networks, The European Physical Journal B 92 (2) (2019) 1–7. Article ID 36
[11] Y. Wang, J. Lu, J. Liang, J. Cao, M. Perc, Pinning synchronization of nonlinear coupled lur'e network under hybrid impulses, IEEE Transactions on Circuits and Systems–II: Express Briefs 66 (3) (2019) 432–436.
[12] S. Majhi, B.K. Bera, D. Ghosh, M. Perc, Chimera states in neuronal networks: a review, Physics of Life Reviews 28 (2019) 100–121.
[13] R. Sakthivel, R. Sakthivel, B. Kaviarasan, C. Wang, Y.K. Ma, Finite-time nonfragile synchronization of stochastic complex dynamical networks with semi–markov switching outer coupling, Complexity 2018 (2018) 1–13. Article ID 8546304
[14] B. Kaviarasan, R. Sakthivel, Y. Lim, Synchronization of complex dynamical networks with uncertain inner coupling and successive delays based on passivity theory, Neurocomputing 186 (2016) 127–138.
[15] R. Sakthivel, M. Sathishkumar, B. Kaviarasan, S.M. Anthoni, Synchronization and state estimation for stochastic complex networks with uncertain inner coupling, Neurocomputing 238 (2017) 44–55.
[16] J.L. Wang, H.N. Wu, Synchronization and adaptive control of an array of linearly coupled reaction–diffusion neural networks with hybrid coupling, IEEE Transactions on Cybernetics 44 (8) (2014) 1350–1361.
[17] Z. Xu, P. Shi, H. Su, Z.G. Wu, T. Huang, Global $\mathcal{H}_\infty$ pinning synchronization of complex networks with sampled-data communications, IEEE Trans. Neural Netw. Learn. Systems 29 (5) (2018) 1467–1476.
[18] B. Shen, Z. Wang, X. Liu, Bounded $\mathcal{H}_\infty$ synchronization and state estimation for discrete time-varying stochastic complex networks over a finite horizon, IEEE Trans. Neural Netw.s 22 (1) (2011) 145–157.
[19] Y. Luo, F. Deng, Z. Ling, Z. Cheng, Local $\mathcal{H}_\infty$ synchronization of uncertain complex networks via non-fragile state feedback control, Math. Comput. Simul. 155 (2019) 335–346.
[20] L. Liu, W.H. Chen, X. Lu, Impulsive $\mathcal{H}_\infty$ synchronization for reaction–diffusion neural networks with mixed delays, Neurocomputing 272 (2018) 481–494.
[21] M. Zhao, H.G. Zhang, Z.L. Wang, H.J. Liang, Observer-based lag synchronization between two different complex networks, Communi. Nonlinear Sci. Numer. Simul. 19 (2014) 2048–2059.
[22] D.H. Ji, S.C. Jeong, J.H. Park, S.M. Lee, S.C. Won, Adaptive lag synchronization for uncertain complex dynamical network with delayed coupling, Appl. Math. Comput. 218 (2012) 4872–4880.
[23] G. Al-mahbashi, M.S.M. Noorani, S.A. Bakar, S. Vahedi, Adaptive projective lag synchronization of uncertain complex dynamical networks with disturbance, Neurocomputing 207 (2016) 645–652.
[24] N. Li, J. Cao, A. Alsaedi, F. Alsaadi, Lag synchronization criteria for memristor-based coupled neural networks via parameter mismatches analysis approach, Neural Comput. 29 (6) (2017) 1721–1744.

[25] S. Wen, Z. Zeng, T. Huang, Q. Meng, W. Yao, Lag synchronization of switched neural networks via neural activation function and applications in image encryption, IEEE Trans. Neural Netw. Learn. Syst. 26 (7) (2015) 1493–1502.
[26] Y. Xia, Z. Yang, M. Han, Lag synchronization of unknown chaotic delayed yang-yang-type fuzzy neural networks with noise perturbation based on adaptive control and parameter identification, IEEE Trans. Neural Netw. 20 (7) (2009) 1165–1180.
[27] G. Al-mahbashi, M.S.M. Noorani, Finite-time lag synchronization of uncertain complex dynamical networks with disturbances via sliding mode control, IEEE Access 7 (2019) 7082–7092.
[28] N. Li, J. Cao, Lag synchronization of memristor-based coupled neural networks via ω-measure, IEEE Trans. Neural Netw. Learn. Syst. 27 (3) (2016) 686–697.
[29] Q. Wang, J.L. Wang, S.Y. Ren, Y.L. Huang, Analysis and adaptive control for lag $\mathcal{H}_\infty$ synchronization of coupled reaction–diffusion neural networks, Neurocomputing 319 (2018) 144–154.
[30] J.L. Wang, M. Xu, H.N. Wu, T. Huang, Passivity analysis and pinning control of multi-weighted complex dynamical networks, IEEE Trans. Netw. Sci. Eng. 6 (1) (2018) 60–73.
[31] J.L. Wang, P.C. Wei, H.N. Wu, T. Huang, M. Xu, Pinning synchronization of complex dynamical networks with multiweights, IEEE Trans. Syst. Man Cybern. Syst. 49 (7) (2019) 1357–1370.
[32] Q. Wang, J.L. Wang, Y.L. Huang, S.Y. Ren, Generalized lag synchronization of multiple weighted complex networks with and without time delay, J. Frankl. Inst. 355 (2018) 6597–6616.
[33] J.L. Wang, Z. Qin, H.N. Wu, T. Huang, P.C. Wei, Analysis and pinning control for output synchronization and $\mathcal{H}_\infty$ output synchronization of multiweighted complex networks, IEEE Transactions on Cybernetics 49 (4) (2019) 1314–1326.
[34] J.L. Wang, Z. Qin, H.N. Wu, T. Huang, Finite-time synchronization and $\mathcal{H}_\infty$ synchronization of multiweighted complex networks with adaptive state couplings, IEEE Transactions on Cybernetics (2018), doi:10.1109/TCYB.2018.2870133. Article in Press
[35] Z. Qin, J.L. Wang, Y.L. Huang, S.Y. Ren, Analysis and adaptive control for robust synchronization and $\mathcal{H}_\infty$ synchronization of complex dynamical networks with multiple time-delays, Neurocomputing 289 (2018) 241–251.
[36] Z. Qin, J.L. Wang, Y.L. Huang, S.Y. Ren, Synchronization and $\mathcal{H}_\infty$ synchronization of multi-weighted complex delayed dynamical networks with fixed and switching topologies, J. Frankl. Inst. 354 (2017) 7119–7138.
[37] J.L. Wang, H.N. Wu, Adaptive output synchronization of complex delayed dynamical networks with output coupling, Neurocomputing 142 (2014) 174–181.
[38] G.P. Jiang, W.K.S. Tang, G. Chen, A state-observer-based approach for synchronization in complex dynamical networks, IEEE Transactions on Circuits and Systems-I: Regular Papers 53 (12) (2006) 2739–2745.
[39] Q. Li, J. Guo, C.Y. Sun, Y. Wu, Adaptive synchronisation for a class of output-coupling complex networks with output feedback nodes, IET Control Theory and Applications 11 (18) (2017) 3372–3380.
[40] S. Zheng, L. Yuan, Nonperiodically intermittent pinning synchronization of complex-valued complex networks with non-derivative and derivative coupling, Physica A 525 (2019) 587–605.
[41] Y. Xu, W. Zhou, J. Fang, C. Xie, D. Tong, Finite-time synchronization of the complex dynamical network with non-derivative and derivative coupling, Neurocomputing 173 (2016) 1356–1361.
[42] S. Zheng, Pinning and impulsive synchronization control of complex dynamical networks with non-derivative and derivative coupling, J. Frankl. Inst. 354 (14) (2017) 6341–6363.
[43] Y. Xu, W. Zhou, J.A. Fang, W. Sun, Adaptive synchronization of the complex dynamical network with non-derivative and derivative coupling, Phys. Lett. A 374 (2010) 1673–1677.
[44] J.L. Wang, D.Y. Wang, H.N. Wu, T. Huang, Output synchronization of complex dynamical networks with multiple output or output derivative couplings, IEEE Trans. Cybern. (2019), doi:10.1109/TCYB.2019.2912336. Article in Press
[45] D.Y. Wang, J.L. Wang, S.Y. Ren, Y.L. Huang, Passivity and synchronization of complex dynamical networks with multiple derivative couplings, Int. J. Control (2018), doi:10.1080/00207179.2018.1528387. Article in Press
[46] D.Y. Wang, J.L. Wang, S.Y. Ren, Y.L. Huang, Output synchronization and $\mathcal{H}_\infty$ output synchronization of complex dynamical networks with multiple derivative couplings, J. Frankl. Inst. 356 (1) (2019) 407–440.
[47] J.J. Slotine, W. Li, Applied Nonlinear Control, Prentice-Hall, 1991.

Lin-Hao Zhao received the B.E. degree in Internet of Things Engineering from Hebei University of Architecture, Zhangjiakou, China, in 2018. He is currently pursuing the M.E. degree in Computer Science and Technology with the School of Computer Science and Technology, Tiangong University, Tianjin, China. His current research interests include stability, passivity, synchronization and neural networks.

Jin-Liang Wang received the Ph.D. degree in control theory and control engineering from the School of Automation Science and Electrical Engineering, Beihang University, Beijing, China, in January 2014. In January 2014, he joined the School of Computer Science and Technology, Tiangong University, Tianjin, China, as an Associate Professor, where he has been promoted to a Professor since March 2018. In 2014, he was a Program Aid with Texas A & M University at Qatar, Doha, Qatar, for two months. From June 2015 to July 2015 and from July 2016 to August 2016, he was a Postdoctoral Research Associate with Texas A & M University at Qatar. From June 2017 to September 2017, he was an Associate Research Scientist in Texas A & M University at Qatar. He has authored two books entitled *Analysis and control of coupled neural networks with reaction–diffusion terms* (Springer, 2017) and *Analysis and control of output synchronization for complex dynamical networks* (Springer, 2018). His current research interests include passivity, synchronization, cooperative control, complex networks, coupled neural networks, coupled reaction–diffusion neural networks, and multiagent systems. Dr. Wang currently serves as an Associate Editor for the *Neurocomputing* and *IEEE Access*, was a Managing Guest Editor for the Special Issue of *Dynamical behaviors of coupled neural networks with reaction–diffusion terms: analysis, control and applications* in Neurocomputing.