Frictional and Rationing Unemployment

Pascal Michaillat
Brown University
www.pascalmichaillat.org

Short history of matching models

Model	Production function	Wage function	References
Standard	Linear $Y = a \cdot N$	Bargaining: Surplus sharing $W = (1-\beta) z + \beta a (1 + r\theta)$	Mortensen (1982) Diamond (1982) Pissarides (1985)

Main issue: Tightness, unemployment, vacancies do not fluctuate sufficiently over the business cycle. Formally: elasticity of tightness with respect to productivity (ε^{θ}_{a}) is much too small in calibrated version of model

$z = 0$: $\varepsilon^{\theta}_{a} = 0$

$z = 0.4$: $\varepsilon^{\theta}_{a} = 2/3$

US : $\varepsilon^{\theta}_{a} = 8$

Diagnostic: Wages are too flexible: absorb too much of productivity fluctuations.

→ Shimer (2005)

Solution: Replace wage function to obtain more rigid wages.

Rigid-wage model	Linear $y = a \cdot N$	Rigid wage $W = \omega \cdot a^{\gamma}$, $\gamma < 1$	Hall (2005)

Main issue:

- all unemployment is frictional
- if matching frictions disappear, all unemployment disappears
- if workers search infinitely hard → unemployment disappears
- if recruiting is costless → unemployment disappears

→ does not allow for queues of workers in bad times.

Diagnostic: No lack of jobs: all workers absorbed if no frictions

Solution: Introduce a proper, downward-sloping labor demand → allows for job rationing

Job-rationing model	Concave	Rigid wage	Michaillat (2012)
	$Y = a \cdot N^{\alpha}$ $\alpha < 1$	$W = \omega \cdot a^{\gamma}$ $\gamma < 1$	

2 good properties

① Rigid wage → realistic fluctuations in unemployment

↳ fluctuations are large enough.

② Concave production function → job rationing

unemployment is frictional → not all

→ unemployment does not vanish if matching frictions disappear (recruiting → 0; job-search effort → ∞)

In this model:

$$\boxed{\text{Total unemployment} = \text{Frictional unemployment} + \text{Rationing unemployment}}$$


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All unemployment is frictional in standard + rigid-wage model.

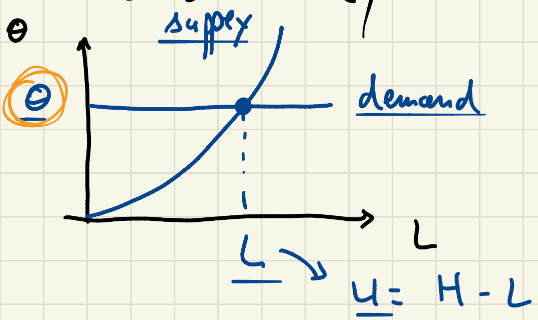
Need to show that if matching frictions disappear, all unemployment disappears.

① Show that if $\boxed{r \to 0}$, then $U \to 0$
(recruiting cost)

② Show that if $\boxed{E \to \infty}$, then $U \to 0$
(job-search effort)

What happens in standard model & rigid-wage model when $r \to 0$?

Describe the equilibrium when $r \to 0$.



Labor demand curves:

$$\boxed{a = [1 + \tau(\theta)] \cdot W}$$

Standard model:

$$a = [1 + \tau(\theta)] \left[(1-\beta) z + \beta a (1 + r\theta) \right]$$

$$\rightarrow \quad a = \left[\frac{q(\theta)}{q(\theta) - \boxed{r}\cdot s}\right]\left[(1-\beta) z + \beta a\left(1 + \boxed{r}\theta\right)\right]$$

$\boxed{r} \downarrow 0$ $\qquad$ $\boxed{r} \downarrow 0$

$\theta^d(r)$. What is $\lim_{r \to 0} \theta^d(r)$?

- $\frac{q(\theta)}{q(\theta) - rs} \rightarrow \frac{q(\theta)}{q(\theta)} = 1 \quad \forall\, \theta$. when $r \to 0$

Labor demand when $r \to 0$:

$$a = (1-\beta) z + \beta a (1 + r\theta) \qquad r \to 0$$

$$a = (1-\beta) z + \beta a + \beta a r \theta$$

$$(1-\beta) a = (1-\beta) z + \beta a r \theta$$

$$\underbrace{(1-\beta)}_{>0}\underbrace{(a - z)}_{>0} = \beta a r \theta \qquad r \downarrow 0, \quad \theta \to \infty$$

$$\lim_{r \to 0} \theta^d(r) = \infty \rightarrow \boxed{\lim_{r \to 0} \theta = +\infty}$$

$$\boxed{\lim_{r \to 0} L = \lim_{\theta \to \infty} L^S(\theta) = H}$$

$$\boxed{\lim_{r \to 0} U = \lim_{r \to 0} H - L = 0}$$

So unemployment vanishes when recruiting costs vanish → <u>All unemployment is frictional</u>.

Rigid-wage model:

Labor demand curve: $a = (1+\tau)\, w \cdot a^{\gamma}$

$$a > w \cdot a^{\gamma}$$

$$\Rightarrow \boxed{\frac{a^{1-\gamma}}{w} > 1}$$

Labor demand curve: $\dfrac{a^{1-\gamma}}{w} = 1 + \dfrac{rs}{q(\theta) - rs}$

$$\frac{a^{1-\gamma}}{w} = \frac{q(\theta)}{q(\theta) - rs}$$

$$\frac{a^{1-\gamma}}{w} = \frac{1}{1 - rs/q(\theta)}$$

$q(\theta) = \mu \cdot \theta^{-\eta}$

under Cobb-Douglas matching function:

$m = \mu \cdot U^{\eta} \cdot V^{1-\eta}$

$$\frac{a^{1-\gamma}}{w} = \frac{1}{1 - \frac{rs}{\mu} \cdot \theta^{\eta}}$$

$$1 - \frac{rs}{\mu}\theta^{\eta} = \frac{w}{a^{1-\gamma}}$$

$$\frac{rs}{\mu} \cdot \theta^{\eta} = \underbrace{1 - \underbrace{w/a^{1-\gamma}}_{\in (0,1)}}_{\in (0,1)}$$

$$\hookrightarrow \frac{\overset{\to 0}{r s}}{\nu} \theta^{\eta} = K > 0 \qquad K \in (0,1)$$
$$K = 1 - \omega / A^{1-\gamma}$$

$\theta \to +\infty$

$$\to \underbrace{\theta^{\eta}}_{\to +\infty} = \frac{K \cdot \nu}{\underbrace{r \cdot s}_{\to 0}} . \text{ So } \boxed{\lim_{r \to 0} \theta^{d}(r) = +\infty}$$

(as in the standard model)

$$\boxed{\lim_{r \to 0} \theta = +\infty}$$

$$\lim_{r \to 0} L = \lim_{r \to 0} L^{S}(\theta) = H$$

$$\Rightarrow \boxed{\lim_{r \to 0} U = \lim_{r \to 0} H - L = 0}$$

No unemployment when matching frictions vanish

$\to$ all unemployment is frictional.

What happens in standard model & rigid-wage model when $E \to \infty$?

E job-search effort by unemployed workers $E > 0$

(previously $E = 1$)

total amount of job-search effort $E \times U$

matching function becomes $m(\underset{\substack{\uparrow \\ \text{total search effort}}}{E \times U}, V)$

labor market tightness becomes $\boxed{\theta = V / E \times U}$

• vacancy-filling rate $= \frac{m}{V} = \frac{m(EU, V)}{V} = m\left(\frac{EU}{V}, 1\right)$

so $= m\left(\frac{1}{\theta}, 1\right)$

$= q(\theta)$ → Labor demand is unchanged.

• job-finding rate $= \frac{m}{U} = \frac{m(EU, V)}{U} = E\,\frac{m(EU, V)}{EU}$

$= E\, m\left(1, \frac{V}{EU}\right)$

$= E\, m(1, \theta)$

$$\boxed{\text{job-finding rate} = E\, f(\theta)}$$

→ Labor supply is changed

$f(\theta)$ job-finding rate per unit of effort & per unit time.

Compute new labor supply curve

with balanced flows, $s \times L = U \times E \times f(\theta)$

$$s \times L = E \times f(\theta) \times (H - L)$$

$$[s + E f(\theta)] \times L = E \times f(\theta) \times H$$

$$\boxed{L^s(\theta, E) = \frac{E\, f(\theta)}{s + E\, f(\theta)} \cdot H}$$

– Same properties of L^s with respect to θ

$L^s(0, E) = 0$

$\frac{\partial L^s}{\partial \theta} > 0$

$$\lim_{\theta \to \infty} L^s(\theta, E) = H$$

- Properties of L^s with respect to E

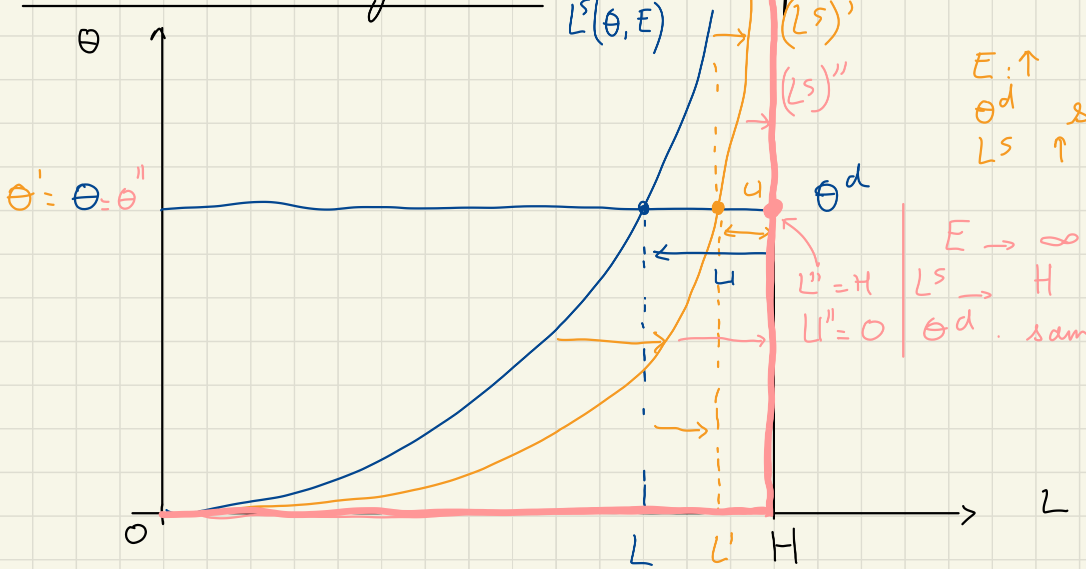
$$L^s(\theta, 0) = 0$$

$$\frac{\partial L^s}{\partial E} > 0$$

$$\lim_{E \to \infty} L^s(\theta, E) = H$$

Standard matching model

What happens when $E \to \infty$?



E : ↑
θ^d same
L^s ↑

$E \to \infty$
$L^s \to H$
θ^d . same

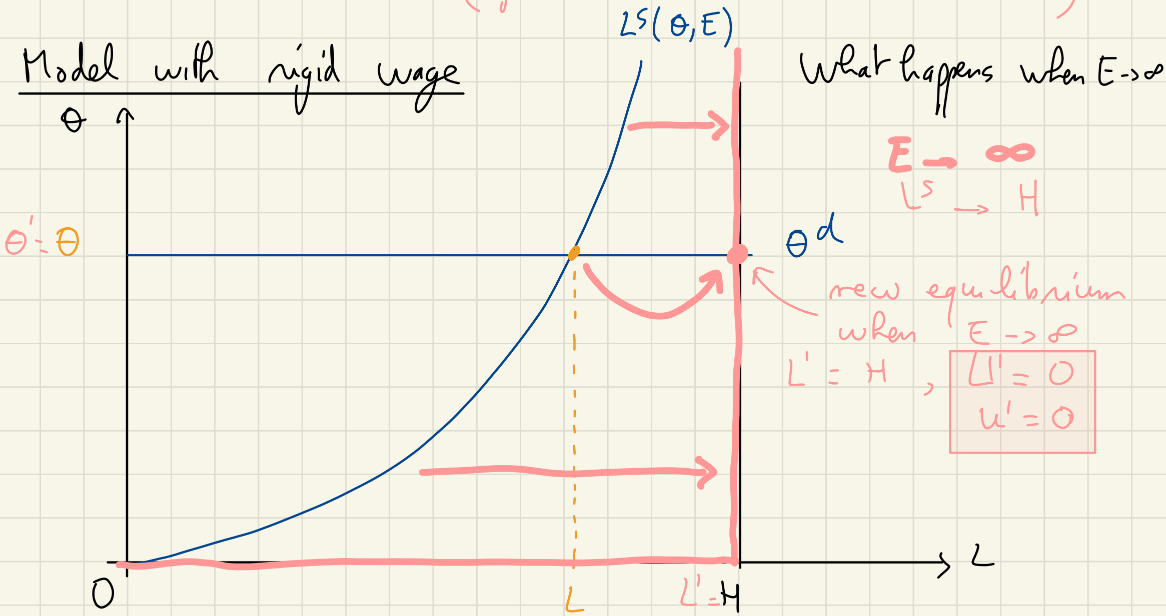
When E ↑ L ↑, U ↓, u ↓, $\theta \to$

When $E \to \infty$ $L = H$, $\boxed{U = u = 0}$, $\theta \to$

. If people really want jobs, unemployment would vanish

- All unemployment is frictional in standard model
- There is { no lack of job in standard model
  { no job rationing
- Model is not consistent with queues of workers in recessions → standard model does not describe recessions well (fails Kuhn's 1st criterion)

Model with rigid wage



Even with rigid wages unemployment vanishes when people search hard enough for jobs → no lack of jobs; model cannot describe queues of workers in bad times, all unemployment is frictional. Exactly as in the standard model

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Introducing job rationing into the matching model

Assumption 1. concave production function $Y = a \cdot N^{\alpha}$
$0 < \alpha < 1$

Assumption 2: rigid wage $W = \omega \; a^{\gamma}$
$0 < \gamma < 1$

What happens when recruiting cost $r \to 0$?

- Labor supply stays the same $L^s(\theta) = \dfrac{f(\theta)}{s + f(\theta)} H$
- Labor demand changes

$$L^d(\theta, r) = \left[\frac{\alpha \; a^{1-\gamma}}{\omega \; \underbrace{[1 + \tau(\theta)]^{\alpha}}} \right]^{1/1-\alpha}$$

$$\tau(\theta) = \frac{r \, s}{q(\theta) - r \, s} \longrightarrow 1 + \tau(\theta) = \frac{q(\theta)}{q(\theta) - r \, s}$$

$\forall \, \theta \in (0, +\infty)$. $\boxed{\lim_{r \to 0} 1 + \tau(\theta) = 1}$

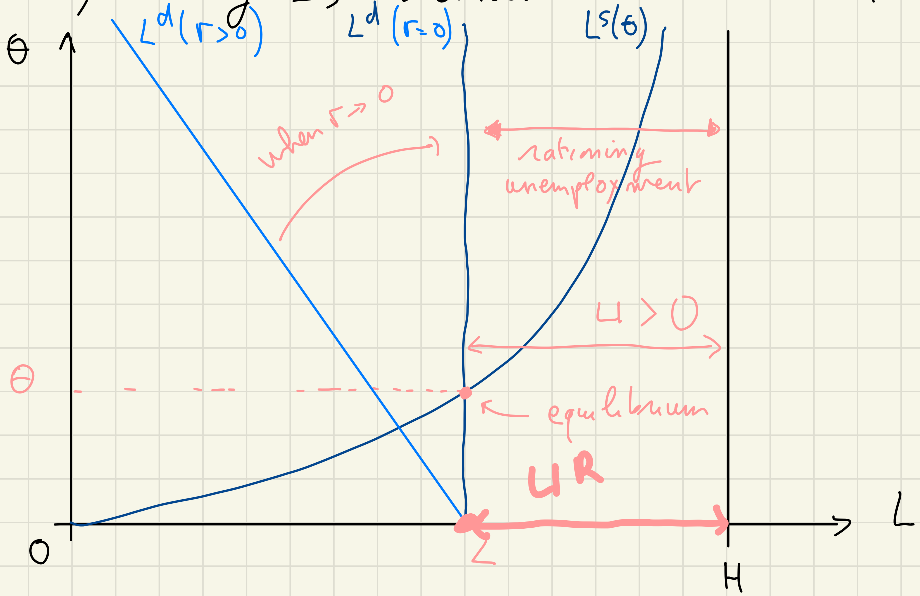
- $\tau(\theta) \to 0$ when $r \to 0$
- # of recruiters $\to 0$ when $r \to 0$

When $r \to 0$, $L^d(\theta) = \left[\dfrac{\alpha \; a^{1-\gamma}}{\omega} \right]^{1/1-\alpha}$

↳ L^d does not depend on tightness θ

↳ labor demand curve is <u>vertical</u> in diagram

↳ labor demand determine employment / output by itself → demand-determined economy



In standard model + in rigid-wage model $u \to 0$ when $r \to 0$ all unemployment is <u>frictional</u>

<u>Here</u> It is possible that $u > 0$ even when $r \to 0$.

↳ even when matching frictions vanish, (recruiting is free) firms do not want to hire all workers in the labor force

↳ there is a lack of job in economy

= <u>job rationing</u>

Rationing unemployment. $U^R = H - L^d(\quad r=0)$

$$L^d(r=0) = \left[\frac{\alpha\, a^{1-\gamma}}{w}\right]^{1/1-\alpha}$$

$$L^d(\theta, r>0) = \left[\frac{\alpha\, a^{1-\gamma}}{w\,[1+\tau(\theta)]^{\alpha}}\right]^{1/1-\alpha}$$

- $\theta = 0 \rightarrow \tau(\theta) = 0 \rightarrow [1+\tau(\theta)]^{\alpha} = 1$

$$\boxed{L^d(\underline{\theta=0}, r>0) = L^d(r=0)}$$

→ labor demand when $r=0$ & when $r>0$
have the same intercept w/ x-axis

→ U^R is also $\boxed{U^R = H - L^d(\theta=0)}$

When is rationing unemployment $U^R > 0$?

$U^R > 0$ when $L^d(\theta=0) < H$

when $\left[\frac{\alpha\, a^{1-\gamma}}{w}\right]^{1/1-\alpha} < H$

when $\frac{w}{\alpha\, a^{1-\gamma}} > H^{\alpha-1}$

when $w . a^{\gamma} > \alpha\, a\, H^{\alpha-1}$ —

when $\boxed{W > MPL(H)}$

MPL of least productive worker, that is the H^{th} worker

Then we see some <u>job rationing</u>, because some workers are less productive than they are paid

~ "classical unemployment", unemployment caused by <u>wage being too high</u>

<u>Here</u> when productivity is low enough job rationing will appear and $U^R > 0$.

Job rationing appears when $w\, a^{\gamma} > \alpha\, a\, H^{\alpha-1}$

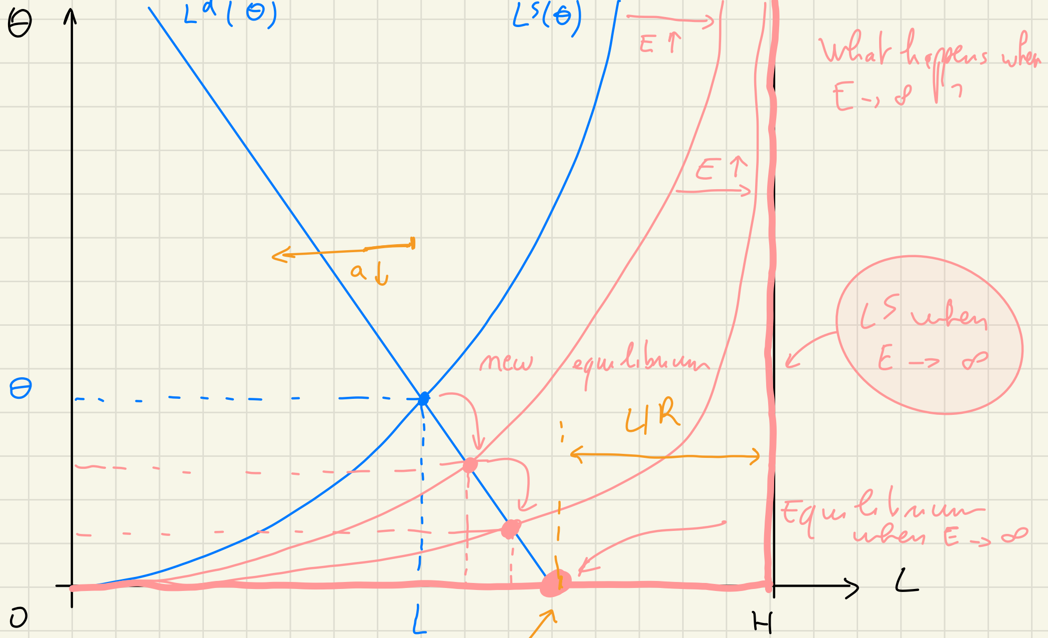
$$\rightarrow \frac{w}{\alpha} H^{1-\alpha} > a^{1-\gamma}$$

$$\rightarrow \boxed{\left[\frac{w}{\alpha} H^{1-\alpha}\right]^{1/1-\gamma} > a}$$

$a^R > 0$ threshold for labor productivity below which there is job rationing.

<u>What happens in the model when job-search effort $E \to \infty$?</u>

$\theta = Y / E \times U$



$E \to \infty$ but $L < H$ so $U > 0$

-> queues of workers at factory gates

-> Great Depression

-> model consistent with existence of queues on labor market in bad times

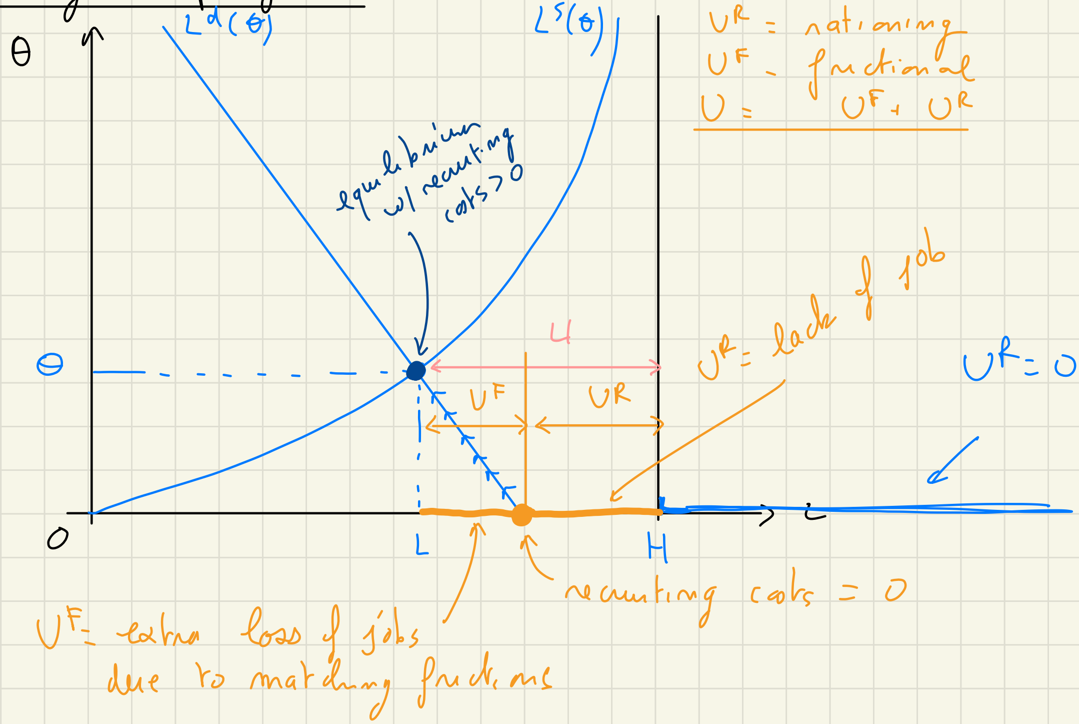
-> when $E \to \infty$ amount of unemployment = rationing unemployment U^R.

-> $U^R > 0$ whenever demand is weak enough $\Leftrightarrow$ productivity is low enough.

-> whether $E \to \infty$ or $r \to 0$: $\boxed{U = U^R}$

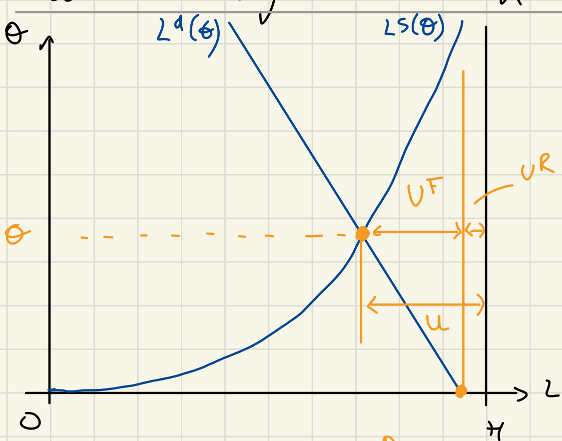
Decomposition of unemployment between frictional & rationing unemployment

U^R = rationing
U^F = frictional
$U = U^F + U^R$



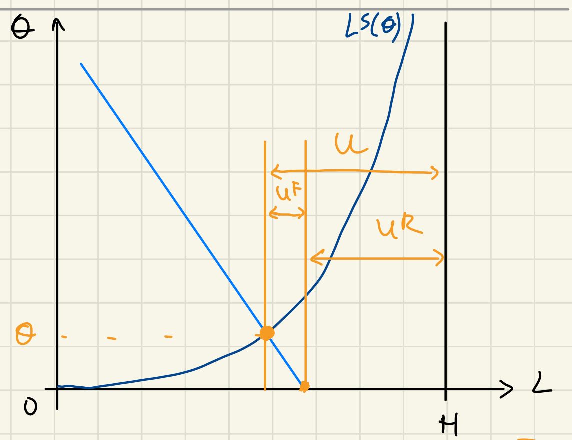
Frictional + rationing unemployment over the business cycle.

Good times: high labor demand

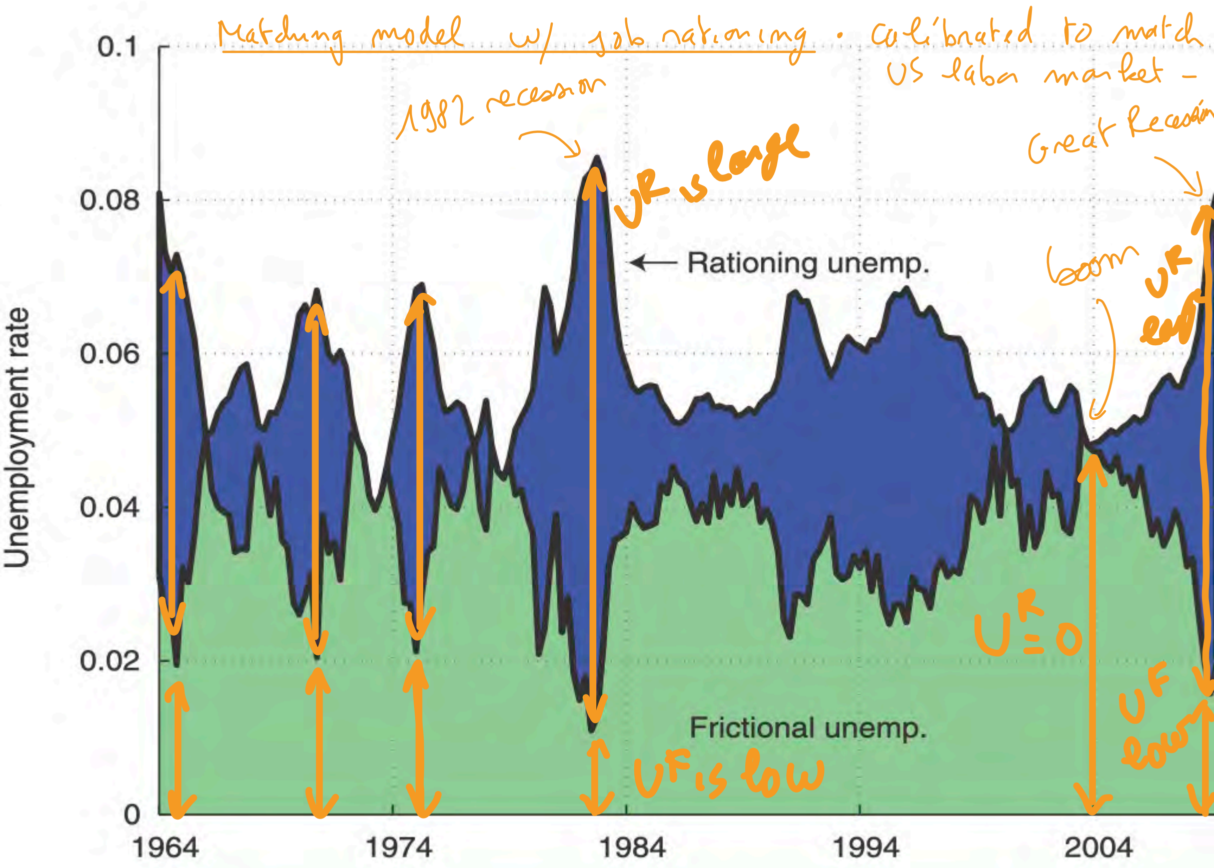


High U^F, low U^R, low U

Bad times: low labor demand



high U^R, high U, low U^F


Matching model w/ job rationing · calibrated to match US labor market –
1982 recession
Great Recession
Rationing unemp.
Frictional unemp.
uR is large
uF is low
boom
uR = 0
Unemployment rate
0.1
0.08
0.06
0.04
0.02
0
1964
1974
1984
1994
2004