Matching Model of the Labor Market

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Good Model.

① Descriptive

② Economical

③ Guide to the unkwonwn

See Thomas Kuhn.

Matching model:

① - unemployment exists
- vacant jobs exist

- Beveridge curve
- unemployment : countercyclical
- vacancy rate : procyclical
- θ (tightness) = V/U procyclical

② Simple diagrams

Labor demand + Labor supply] Equilibrium

③ - Multipliers

- Job queues in good/bad times

Labor supply: $L^S(\theta, \cancel{W})$

- H: size of labor force $\quad H > 0$
- s: job-separation rate $\quad s > 0$

  US: $s \approx 3.5\%$ per month.

- $m(U, V)$: matching function
- U: # of unemployed workers
- V: # of vacant jobs
- u: unemployment <u>rate</u>
- v: vacancy <u>rate</u>

definition:

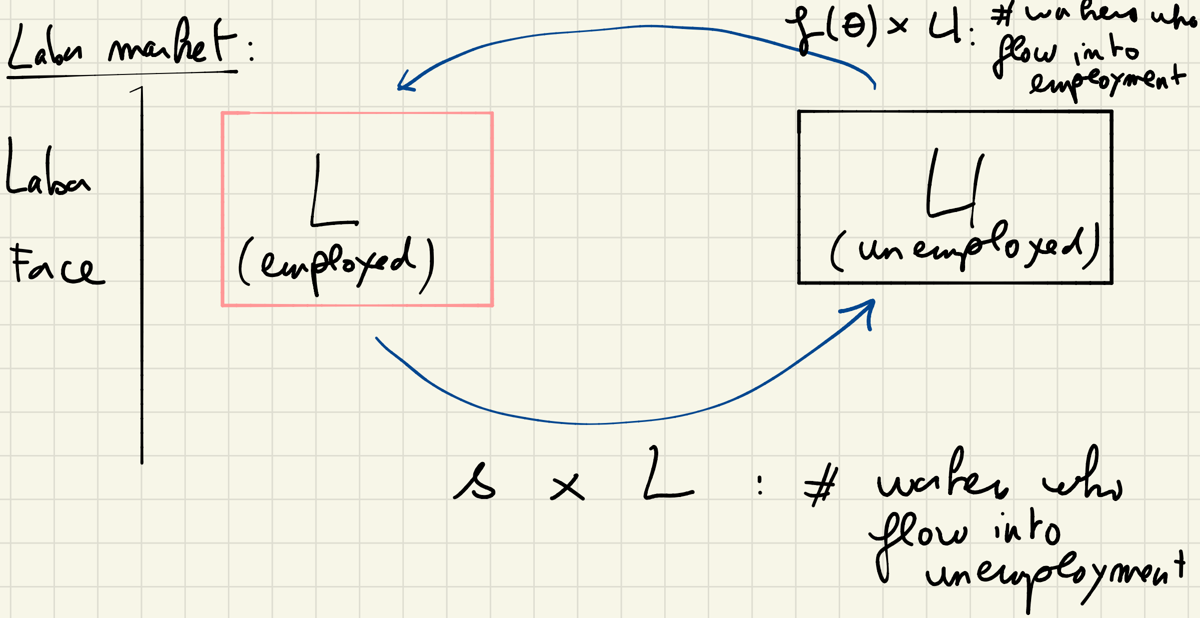
$$u = U / H$$

$$v = V / H$$

Flows on labor market:

$$\underbrace{H}_{\text{labor force}} = \underbrace{L}_{\#\text{ employed}} + \underbrace{U}_{\#\text{ of unemployed}}$$

Labor market:



$s \times L$: inflows into unemployment

$f(\theta) \times U$: outflows from unemployment

assumption: labor market flows are balanced

unemployment rate under balanced flows:

$$s \times L = f(\theta) \times U$$

$$s \times (H - U) = f(\theta) \times U \qquad \text{(def. of employment)}$$

$$s \times (1 - u) = f(\theta) \times u \qquad \text{(divided by } H\text{)}$$

$$s = f(\theta) \times u + s \times u = u \times (f(\theta) + s)$$

$$\boxed{u = \frac{s}{s + f(\theta)}}$$

Labor supply: balanced flows: inflows = outflows

$$s \times L = f(\theta) \times U$$

$$s \times L = f(\theta) \times (H - L) \quad \text{(def. of } U\text{)}$$

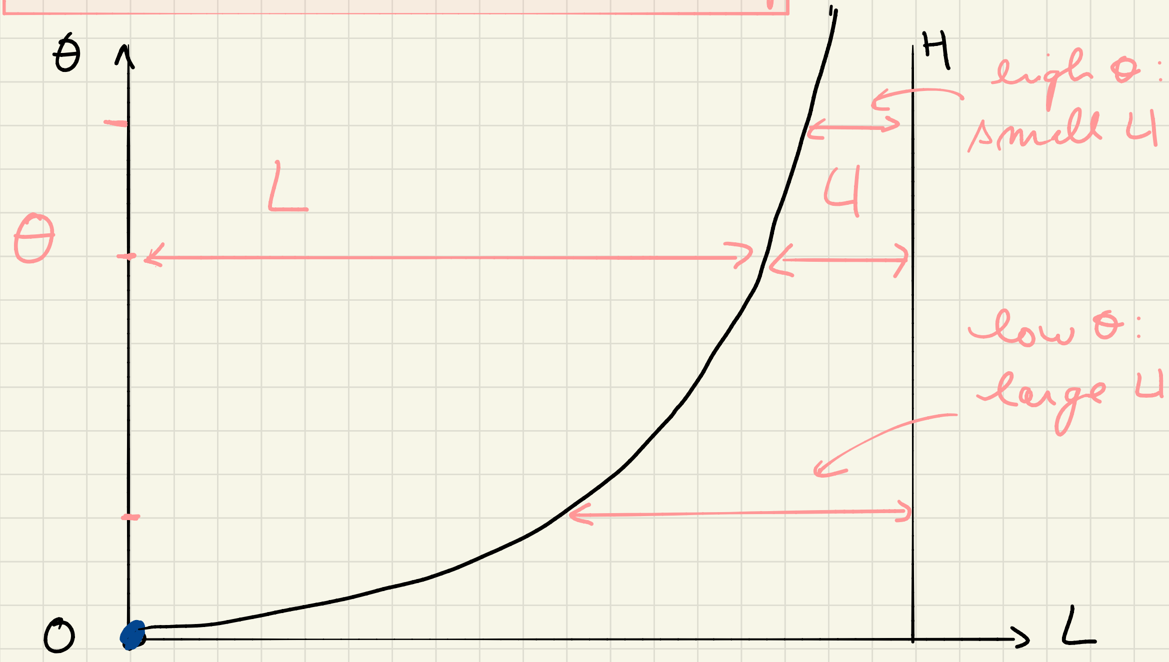
$$s \times L + f(\theta) \times L = f(\theta) \times H$$

$$L \times (s + f(\theta)) = f(\theta) \times H$$

people who want to

$$\boxed{L^s(\theta) = \frac{f(\theta)}{s + f(\theta)} \times H}$$

flows on labor market



$$\theta = 0 : \quad f(0) = 0 \Rightarrow L^s(0) = 0$$

$$L^s(\theta) = \frac{f(\theta)}{s + f(\theta)} \cdot H = \frac{1}{1 + s/f(\theta)} \times H$$

$f(\theta)$: job-finding rate

$$f(\theta) = m(1, \theta) \Rightarrow f'(\theta) > 0$$

- $$\frac{f(\theta)}{s + f(\theta)} < 1 \Rightarrow L^s(\theta) < H$$

- $$\lim_{U \to +\infty} m(U, V) = \lim_{V \to +\infty} m(U, V) = +\infty$$

$$\lim_{\theta \to +\infty} m(1, \theta) = +\infty$$

$$\Rightarrow \lim_{\theta \to \infty} f(\theta) = +\infty$$

$$\Rightarrow \lim_{\theta \to +\infty} \frac{f(\theta)}{s + f(\theta)} = 1$$

$$\Rightarrow \lim_{\theta \to +\infty} L^s(\theta) = H$$

Labor supply: summary:

① $L^S(\theta)$ is increasing in θ

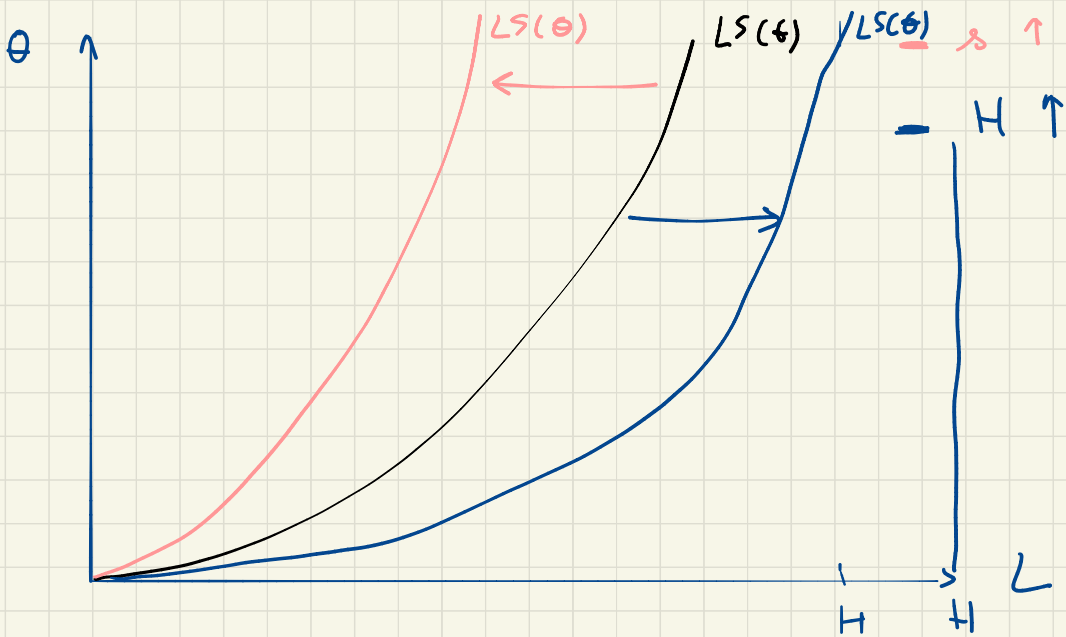
② $L^S(0) = 0$

③ $L^S(\theta) < H$ & $\lim_{\theta \to \infty} L^S(\theta) = H$

Comparative statics:

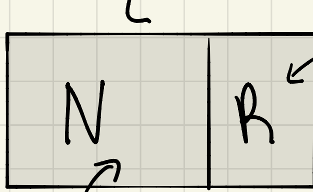
$$L^S(\theta) = \frac{f(\theta)}{\delta + f(\theta)} \cdot H$$

- what happens if $\delta \uparrow$? $L^S(\theta) \downarrow$
- what happens if $H \uparrow$? $L^S(\theta) \uparrow$



Labor demand:

representative firm:



recruiters: HR workers who spend time & effort to fill vacancies

producers: produce goods & services sold by firm

$L = N + R.$

V: # vacancies posted by firms

$r > 0$: recruiting cost
recruiters required to keep a vacancy open per unit time.

$s > 0$: job-separation rate
workers who leave the firm per unit time.

$\tau = R/N$: recruiter-producer ratio

What is τ?

workers lost: $s \times L$

Assumption: labor market flows are balanced
↳ firm: # workers that leave
= # workers that are recruited

⇒ # workers recruited must be $s \times L$

⇒ firm must post enough vacancies V to secure $s \times L$ recruits.

each vacancy is filled w/ proba. $q(\theta)$

$\Rightarrow$ firm must post $V = \dfrac{s \times L}{q(\theta)}$

$\left(q(\theta) \times V = \# \text{ recruits} = s \times L \right)$

$\Rightarrow$ # workers devoted to recruiting:

$$R = r \times V = \frac{r \times s \times L}{q(\theta)} = \frac{r \times s}{q(\theta)} \times (R + N)$$

$$\frac{R}{N} = \frac{r \times s}{q(\theta)} \left(\frac{R}{N} + 1 \right)$$ (divided by N)

$$\tau = \frac{r \times s}{q(\theta)} (1 + \tau)$$

$$\tau \times \left[1 - \frac{r \times s}{q(\theta)} \right] = \frac{r \times s}{q(\theta)}$$

$$\tau \left[q(\theta) - r \times s \right] = r \times s$$

recruiter-producer ratio:

$$\boxed{\tau(\theta) = \frac{r \times s}{q(\theta) - r \times s}}$$

<u>Properties of $\tau(\theta)$:</u>

<u>Recall</u>: $q(\theta) = m\left(\frac{1}{\theta}, 1\right)$ $m(\cdot,\cdot)$: matching function

$q(\theta) > 0$ $q'(\theta) < 0$ $\begin{cases} q(0) \to +\infty \\ q(+\infty) \to 0 \end{cases}$

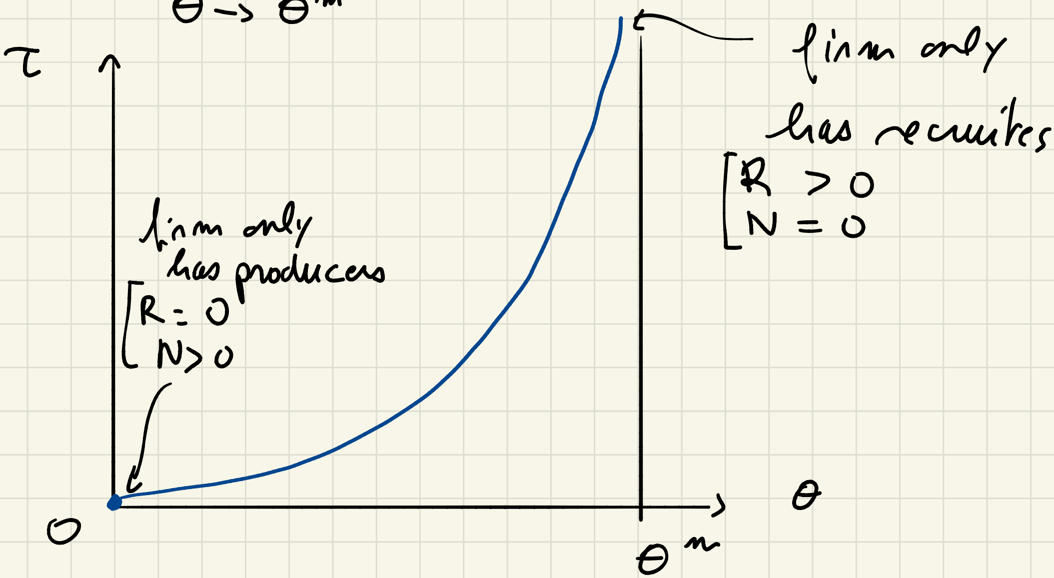
$\tau(\theta) = \frac{r \times s}{\underbrace{q(\theta) - r \times s}}$

- $\tau(0) = \frac{r \times s}{\infty - r \times s} = 0$
- $\tau'(\theta) > 0$
- $\tau(\theta)$ defined $(0, \theta^m)$

θ^m : vertical asymptote for τ.

defined such that $q(\theta^m) = r \times s$

$\lim_{\theta \to \theta^{m-}} \tau(\theta) = +\infty$.



Firm : L workers : R recruiters + N producers

- production function

$$Y = a \times N^{\alpha}$$

Y: output

a: technology level / labor productivity

$\alpha \in [0, 1]$: marginal returns to labor

- $p = 1$: goods / services as numeraire (unit of account)
- $W > 0$: wage paid by firm to all its workers.

(later : bargaining, unions ...)

labor cost : $W \times L = W \times (R + N)$

$$= W \times [1 + \tau(\theta)] \times N$$

because $\boxed{R = \tau(\theta) \times N}$

firm profits $= \pi =$ turnover − labor costs

$$\pi = p \times Y - W \times L$$

$$\pi(N) = a \times \underline{N}^{\alpha} - W \times [1 + \tau(\theta)] \times \underline{N}$$

Objective : $\max_{N>0} \pi(N)$ at any point in time.

$$\pi(0) = 0$$

(for $\alpha < 1$) : $\pi(N)$ is concave.

necessary & sufficient condition to find max $\pi(N)$:

$$\pi'(N) = 0$$

$$a \times \alpha \times N^{\alpha-1} - w\,(1+\tau(\theta)) = 0$$

$$N^{\alpha-1} = \frac{w \times [1+\tau(\theta)]}{a \cdot \alpha}$$

$$N^{1-\alpha} = \frac{a \cdot \alpha}{w \times [1+\tau(\theta)]}$$

$$[1+\tau(\theta)] \times N = [1+\tau(\theta)] \times \left[\frac{a \cdot \alpha}{w \times [1+\tau(\theta)]}\right]^{\frac{1}{1-\alpha}}$$

$$L = \left[\frac{a \times \alpha \times (1+\tau(\theta))^{1-\alpha}}{w \times \cancel{(1+\tau(\theta))}}\right]^{1/(1-\alpha)}$$

$$\boxed{L^d(\theta, w) = \left[\frac{\underline{a} \cdot \alpha}{\underline{w} \times [1+\tau(\theta)]^{\alpha}}\right]^{1/(1-\alpha)}}$$

Properties of labor demand:

- $\theta = 0$: $\tau(\theta) = 0 \Rightarrow L^d(0,w) = \boxed{\left(\frac{a\alpha}{w}\right)^{1/1-\alpha}}$

- $\frac{\partial L^d}{\partial \theta} < 0$ $\quad \theta \uparrow \Rightarrow \tau(\theta) \uparrow$

$\Rightarrow (1+\tau(\theta))^\alpha \uparrow$

$\Rightarrow \frac{a\alpha}{w\,(1+\tau(\theta))^\alpha} \downarrow$

$\Rightarrow$ since $1/(1-\alpha) > 0$

$\left(\frac{a\alpha}{w(1+\tau(\theta)^\alpha)}\right)^{1/1-\alpha} \downarrow$

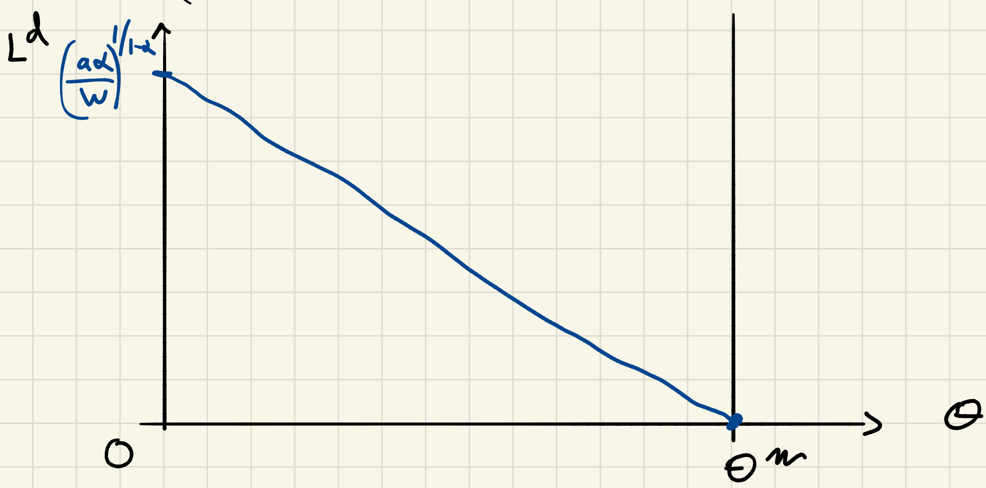
$\parallel$

$L^d \downarrow$

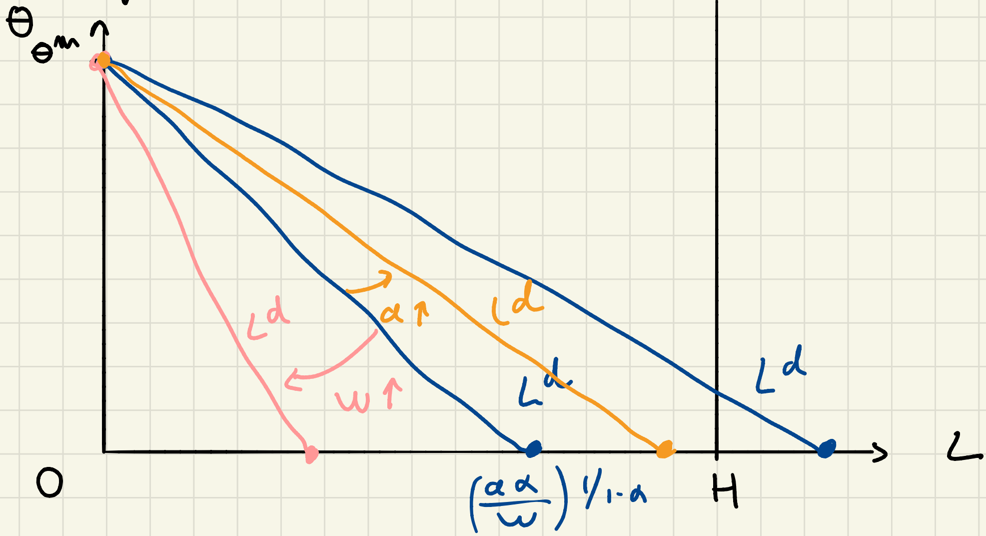
- at $\theta = \theta^m$: $\tau(\theta) \to \infty$

$L^d \to 0$

$$\lim_{\theta \to (\theta^m)^-} L^d(\theta, w) = 0$$



market diagram (L, θ):



Comparative statics:

- $w \uparrow \Rightarrow L^d(\theta) \downarrow$ (higher wage)
- $a \uparrow \Rightarrow L^d(\theta) \uparrow$ (higher productivity)

Matching model

- firms maximize profits given θ: want to employ

$$L^d(\theta) - L^d(\theta) = \left[\frac{a\cdot\alpha}{W\cdot(1+\tau(\theta))^\alpha}\right]^{1/1-\alpha}$$

- workers expect an employment level given θ:

$$L^S(\theta) - L^S(\theta) = \frac{f(\theta)}{s+f(\theta)}\cdot H\ .$$

- assumptions: matching function m, production function $y = a\cdot N^\alpha$; labor market w/ balanced flows

$$\begin{tikzcd} \boxed{E} \arrow[rr, bend right, "E-U"'] & & \boxed{U} \arrow[ll, bend right, "U-E"'] \end{tikzcd} \qquad E\text{-}U = U\text{-}E$$

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unemployment follows a differential equation:

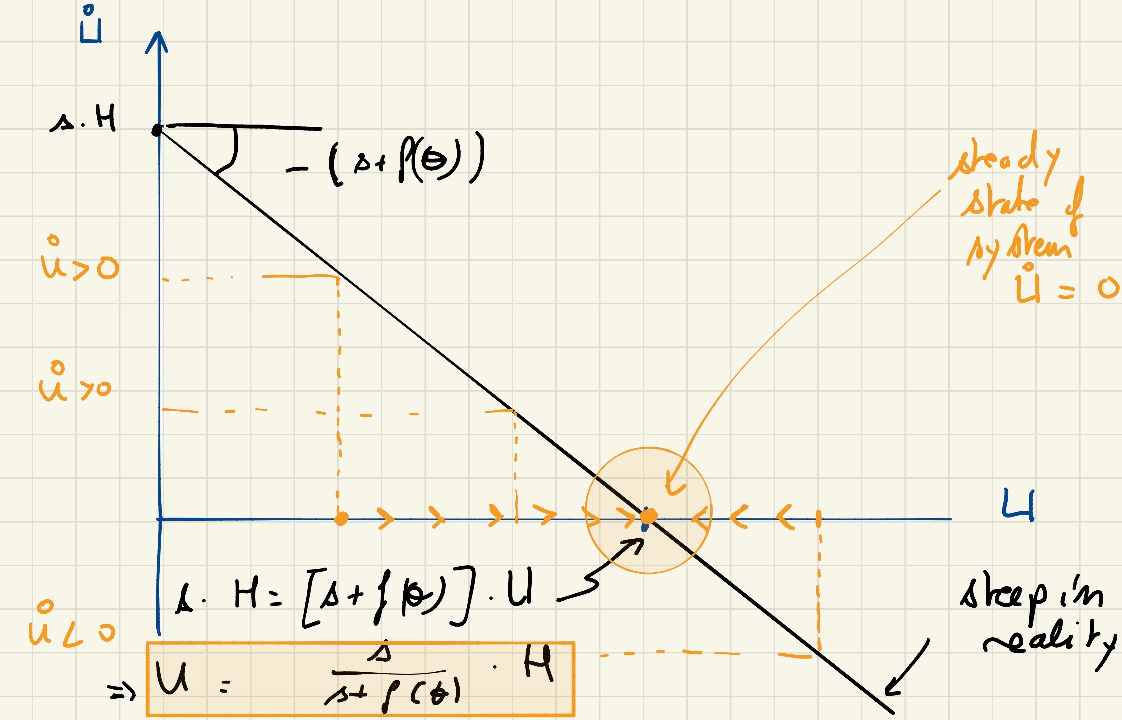
$$\dot{U} = s\cdot L - f(\theta)\,U$$

$$\dot{U} = s\cdot(H-U) - f(\theta)\cdot U$$

$$\dot{U} = \underline{s\cdot H} - (\underline{s+f(\theta)})\cdot U$$

if large: $\dot{U} = 0$ almost all the time.

we assume that $\dot{U} = 0$ all the time



$$\Rightarrow \boxed{U = \frac{s}{s+f(\theta)} \cdot H}$$

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given θ : - firms employ $L^d(\theta)$
- $L^s(\theta)$ have jobs

<u>but what is θ ?</u>

<u>Neoclassical labor market:</u>

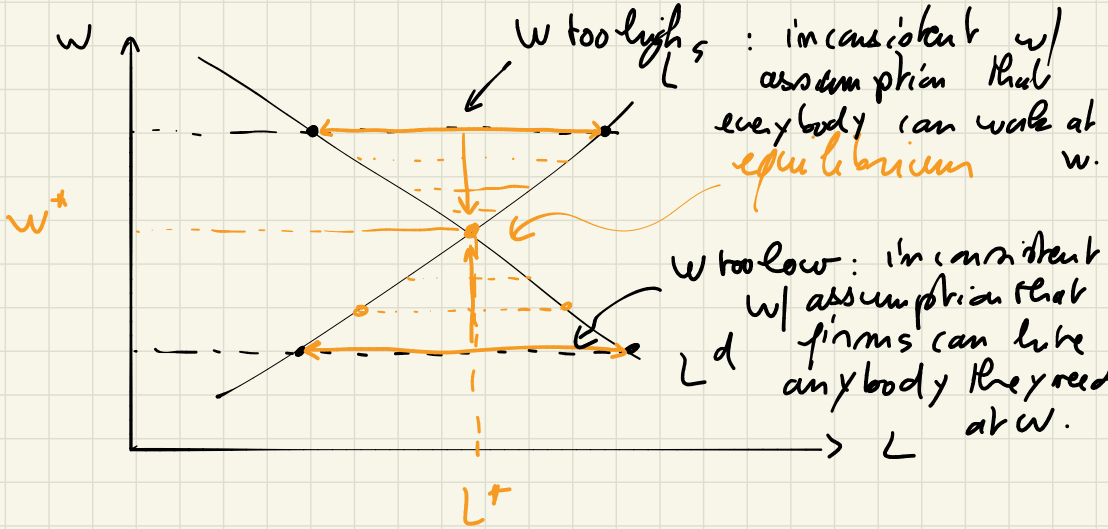
given wage w : - firms employ $L^d(w)$
- $L^s(w)$ workers want a job

but what is w ?

W is such that "labor market clears"

"supply = demand"

- auctioneer - "invisible hand of the market"

- 

- internally consistent (Kuhn)

↳ requires $L^s(w) = L^d(w)$

<u>equilibrium condition</u> : condition for internal consistency

Matching model :

equilibrium condition: to ensure internal consistency

→ to ensure that tightness θ taken as given by firms & workers is realized

$V(\theta)$: # vacancies posted by firms that take θ as given

$U(\theta)$: # unemployed workers that take θ as given

$$\boxed{\text{equilibrium condition: } \quad \underbrace{\frac{V(\theta)}{U(\theta)}}_{\text{realized tightness}} = \overbrace{\theta}^{\text{tightness taken as given}}}$$

$$V(\theta) = \frac{s \times \overbrace{L^d(\theta)}^{\text{\# hire at } \theta}}{q(\theta)}$$

$$U(\theta) = H - L^s(\theta)$$

equilibrium imposes : $\dfrac{V(\theta)}{U(\theta)} = \theta$

$$\Leftrightarrow \frac{s \times L^d(\theta)}{q(\theta)} \times \frac{1}{H - L^s(\theta)} = \theta$$

- $q(\theta) = f(\theta) / \theta$

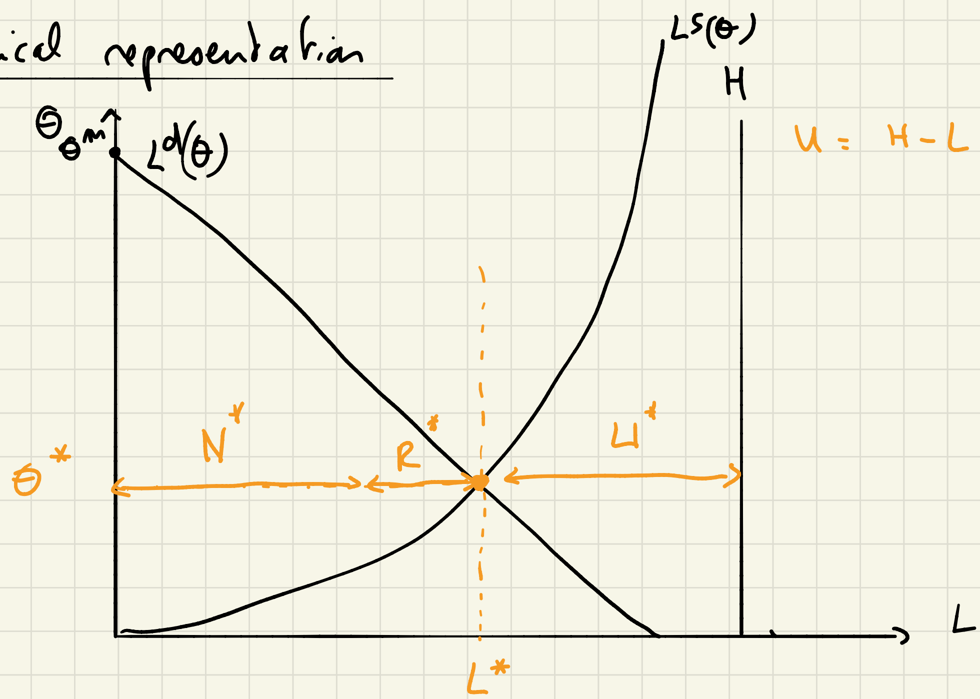
- $H - L^s(\theta) = H\left(1 - \frac{f(\theta)}{s + f(\theta)}\right) = H \cdot \frac{s}{s + f(\theta)}$

$$\Leftrightarrow \cancel{\theta} \cdot L^d(\theta) \cdot \frac{\cancel{s}}{f(\theta)} \cdot \frac{s + f(\theta)}{\cancel{s}} \cdot \frac{1}{H} = \cancel{\theta}$$

$$\Leftrightarrow \frac{L^d(\theta)}{\underbrace{\frac{f(\theta)}{s + f(\theta)} \cdot H}_{L^s(\theta)}} = 1$$

$$\Leftrightarrow \boxed{L^d(\theta) = L^s(\theta)}$$

Graphical representation



from tightness θ^*: infer values of all variables in the model

$$L^* = L^d(\theta^*) = L^s(\theta^*) = \boxed{\frac{f(\theta^*)}{s+f(\theta^*)} \cdot H = L^*}$$

$$U^* = H - L^* = \boxed{\frac{s}{s+f(\theta^*)} \cdot H = U^*}$$

$$u^* = U^*/H = 1 - L^*/H = \boxed{\frac{s}{s+f(\theta^*)} \text{ "} u^*}$$

$$L = N + \overset{\tau(\theta)\cdot N}{R} = [1+\tau(\theta)] \cdot N$$

$$N^* = \frac{L^*}{1+\tau(\theta^*)} = \boxed{\frac{1}{1+\tau(\theta^*)} \cdot \frac{f(\theta^*)}{s+f(\theta^*)} \cdot H = N^*}$$

$$R^* = \tau(\theta^*) \cdot N^* = \boxed{\frac{\tau(\theta^*)}{1+\tau(\theta^*)} \cdot \frac{f(\theta^*)}{s+f(\theta^*)} \cdot H = R^*}$$

$$V^* = \theta^* \cdot U^* = \boxed{\frac{\theta^* \cdot s}{s+f(\theta^*)} \cdot H = V^*}$$

$$v^* = \frac{V^*}{H} = \boxed{\frac{s \cdot \theta^*}{s+f(\theta^*)} = v^*}$$