# Unemployment Fluctuations

Pascal Michaillat
Brown University
www.pascalmichaillat.org

**Objective :**

- countercyclical unemployment
- procyclical tightness
- "large" unemployment fluctuating

→ elasticity of unemployment rate wrt shock is the same as in data

**Two wage functions :**

- Rigid wage
- Bargained wage

**Matching model with rigid wage**

$W = \omega \cdot a^{\gamma}$ $\qquad \gamma \in [0,1]$

$$L^s(\theta) = \frac{f(\theta)}{s + f(\theta)} \cdot H$$

$$L^d(\theta) = \left[ \frac{a \cdot \alpha}{w \cdot a^{\gamma} \cdot [1+\tau(\theta)]^{\alpha}} \right]^{1/1-\alpha}$$

$$L^d(\theta) = \left[ \frac{a^{1-\gamma} \cdot \alpha}{w \cdot [1+\tau(\theta)]^{\alpha}} \right]^{1/1-\alpha}$$

$1-\gamma > 0$

equilibrium condition : $L^s(\theta) = L^d(\theta)$

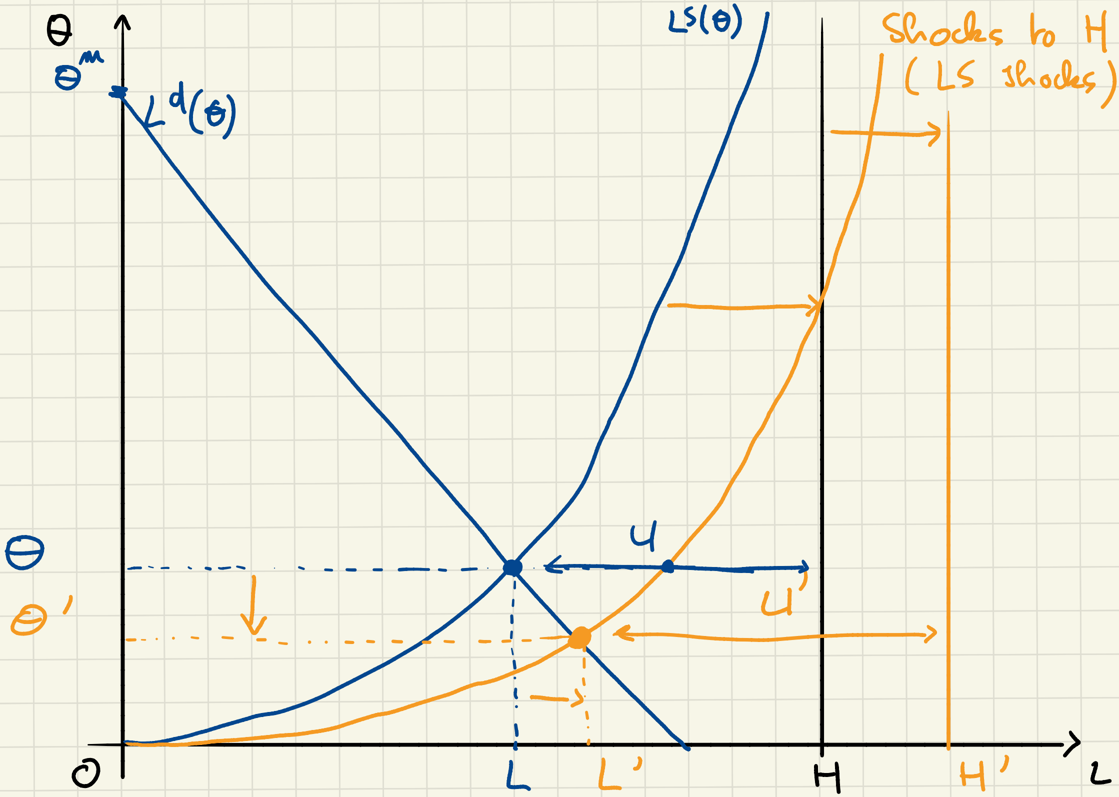
→ gives equilibrium $\theta$.

Which shocks can explain fluctuations.

compare labor-demand shock $a$ with labor-supply shock $H$.

Compare labor-demand & labor-supply shocks from labor-market diagram.

Labor-supply shocks :




increase in H:

- $L \uparrow$ : boom / expansion
- $\theta \downarrow$
- $u = s / s + f(\theta)$ so $u \uparrow$
- $Y = a \cdot N^{\alpha}$

$N = L / [1 + \tau(\theta)]$

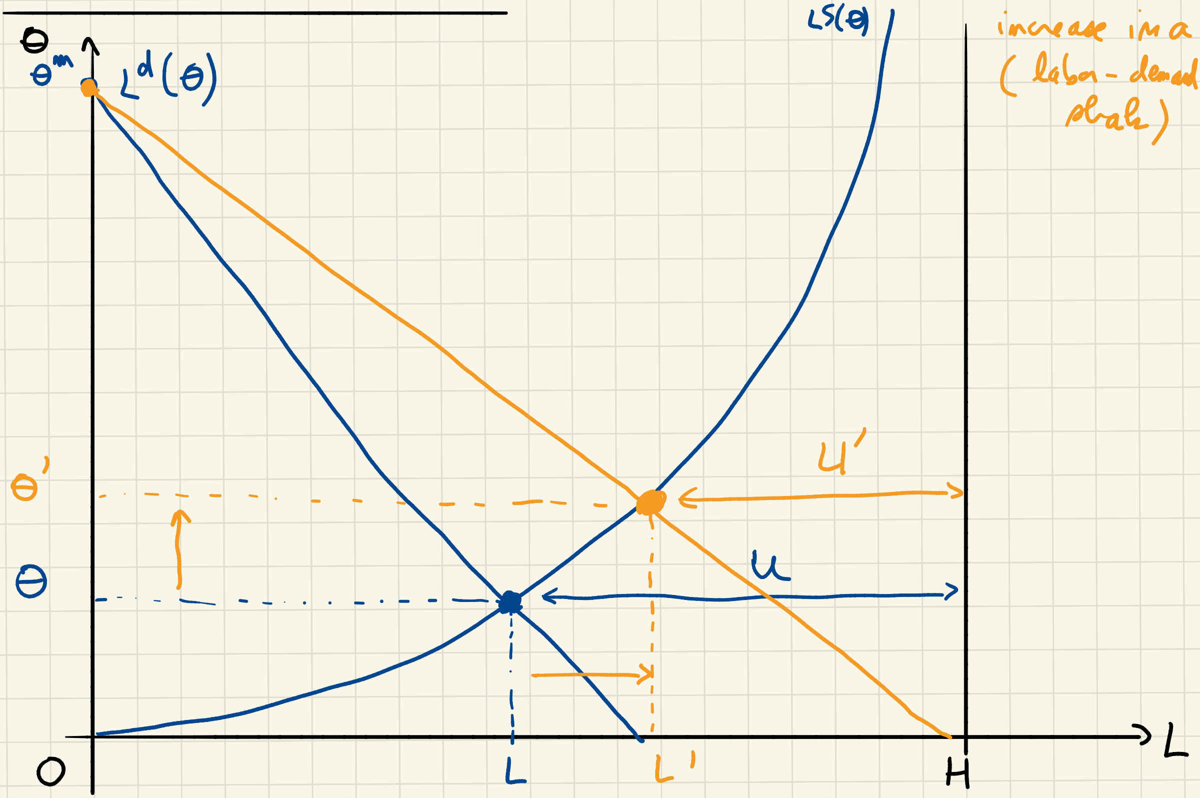
$\theta \downarrow \Rightarrow \begin{cases} \tau(\theta) \downarrow \\ L \uparrow \end{cases} \Rightarrow N \uparrow \Rightarrow Y \uparrow$

- $u$ is <u>procyclical</u>
- $\theta$ is <u>countercyclical</u> ] if H causes business cycles.

⇒ predictions are counterfactual

⇒ H / labor-supply shocks cannot cause business cycles (in matching model)

<u>Labor-demand shocks</u>




<u>increase in a</u> (positive labor-demand shock)

- $L \uparrow$ : boom / expansion / peak
- $\theta \uparrow$
- $u = s / s + f(\theta)$ so $u \downarrow$
- $U = u \cdot H$ so $U \downarrow$
- balanced flows : inflows = outflows

$$s \times L = m(U, V)$$

$$\uparrow \qquad \downarrow \qquad \uparrow$$

$V \uparrow$ and $v = V/H \uparrow$

under shocks to labor productivity:

- $\theta$ is procyclical
- $v$ is procyclical
- $u$ is countercyclical

Beveridge curve

In matching model: shocks to productivity generate realistic business cycle. [under rigid wages]

labor-demand shocks.

Michaillat (2012, p. 1741) : elasticity of $\theta$ wrt to productivity is $\boxed{\varepsilon_a^\theta \approx 8}$.

→ when productivity goes up by 1%, tightness goes up by 8%.

- elasticity of $u$ wrt $a$ is $\varepsilon_a^u \approx 4$
- elasticity of $v$ wrt $a$ is $\varepsilon_a^v \approx 4$

Can we get $\varepsilon_a^\theta$ of 8 in matching model with rigid wages ?

equilibrium condition: $L^s(\theta) = L^d(\theta)$

$$\Leftrightarrow \frac{f(\theta)}{s + f(\theta)} \cdot H = \left[ \frac{\alpha \cdot a^{1-\gamma}}{w\,[1+\tau(\theta)]^\alpha} \right]^{1/1-\alpha}$$

→ $\theta$ is an implicit function of $a$ : $\theta(a)$

→ (implicit function theorem)

$$\varepsilon_a^\theta \equiv \frac{d\ln\theta}{d\ln a}.$$

Cobb - Douglas

$$m(U, V) = \mu \cdot U^\eta \cdot V^{1-\eta} \qquad \eta \in (0,1)$$

$$f(\theta) = \mu \cdot \theta^{1-\eta} \quad \rightarrow \frac{d\ln f}{d\ln\theta} = 1-\eta$$

$$q(\theta) = \mu \cdot \theta^{-\eta} \quad \rightarrow \frac{d\ln q}{d\ln\theta} = -\eta$$

$$\frac{f(\theta)}{s+f(\theta)} = 1 - u = \ell(\theta) : \text{employment rate.}$$

$$\frac{d\ln \ell}{d\ln \theta} = \frac{d\ln f}{d\ln \theta} - \frac{d\ln(s+f)}{d\ln \theta}$$

$$= 1-\eta - \frac{f}{s+f} \times \frac{d\ln f}{d\ln \theta}$$

$$= (1-\eta)\left(1 - \frac{f}{s+f}\right)$$

$$= (1-\eta) \times \frac{s}{s+f}$$

$$\boxed{\frac{d\ln \ell}{d\ln \theta} = (1-\eta) \times u}$$

---

Results on elasticities

Leibniz's notation : $\frac{dx}{df}$

$f(x)$ : any differentiable function

$$\boxed{df = f'(x) \cdot dx} \longrightarrow \frac{df}{dx} = f'(x)$$

elasticity of $f$ with respect to $x$ :

$$\boxed{\varepsilon_x^f = \frac{d\ln f}{d\ln x}}$$

Interpretation

$x$ increases by $1\%$ : what happens to $f$ ?

$$dx = 1\% \times x$$

$$df = f'(x)\, dx = f'(x) \times x \times 1\%$$

percentage change in $f$ :

$$\rightarrow \frac{df}{f} \times 100 = f'(x) \times \frac{x}{f} \times \cancel{1/100} \times \cancel{100}$$

percentage change in $f$ $= \frac{x}{f} f'(x) = \frac{x}{f} \cdot \frac{df}{dx}$

$d \ln f = \frac{1}{f} \cdot df$ ; $d \ln x = \frac{1}{x} \cdot dx$

← derivative of ln

$$\boxed{\frac{d \ln f}{d \ln x} = \frac{x}{f} \cdot \frac{df}{dx}} \qquad \boxed{\frac{d \ln f}{d \ln x} = \frac{x}{f} \cdot f'(x)}$$

|| percentage change in $f$ given by $\frac{d \ln f}{d \ln x} = \varepsilon^f_x$

## Results about elasticities :

- $f(x) = x^{\alpha}$ $\longrightarrow$ $\boxed{\varepsilon^f_x = \alpha}$

proof : $\ln(f) = \alpha \cdot \ln(x) \rightarrow \frac{d \ln f}{d \ln x} = \alpha$

- $f(x) = A(x) \times B(x)$ $\longrightarrow$ $\boxed{\varepsilon^f_x = \varepsilon^A_x + \varepsilon^B_x}$

proof : $\ln f = \ln A + \ln B$

$$\frac{d \ln f}{d \ln x} = \frac{d \ln A}{d \ln x} + \frac{d \ln B}{d \ln x}$$

- $f(x) = A(x) / B(x) \longrightarrow$ $\boxed{\varepsilon^f_x = \varepsilon^A_x - \varepsilon^B_x}$

proof: $\ln f = \ln A - \ln B$

$$\frac{d\ln f}{d\ln x} = \frac{d\ln A}{d\ln x} - \frac{d\ln B}{d\ln x}$$

- $f(x) = \alpha \cdot A(x) \longrightarrow$ $\boxed{\varepsilon^f_x = \varepsilon^A_x}$

proof: $\ln f = \ln \alpha + \ln A$

$$\underbrace{\frac{d\ln f}{d\ln x}}_{\varepsilon^f_x} = \underbrace{\frac{d\ln \alpha}{d\ln x}}_{0} + \underbrace{\frac{d\ln A}{d\ln x}}_{\varepsilon^A_x}$$

- $f(x) = A(x) + B(x) \longrightarrow$ $\boxed{\varepsilon^f_x = \frac{A}{A+B} \cdot \varepsilon^A_x + \frac{B}{A+B} \cdot \varepsilon^B_x}$

"elasticity of sum = weighted sum of elasticities"

proof: $\ln f = \ln(A+B)$

$$\frac{d\ln f}{d\ln x} = \frac{1}{A+B} \times \left( \frac{dA}{d\ln x} + \frac{dB}{d\ln x} \right)$$

$$= \frac{A}{A+B} \cdot \frac{1}{A} \frac{dA}{d\ln x} + \frac{B}{A+B} \cdot \frac{1}{B} \cdot \frac{dB}{d\ln x}$$

$d\ln A = \frac{dA}{A}$ ; $d\ln B = \frac{dB}{B}$

$$\frac{d\ln f}{d\ln x} = \frac{A}{A+B}\cdot\frac{d\ln A}{d\ln x} + \frac{B}{A+B}\,\frac{d\ln B}{d\ln x}$$

$\uparrow \varepsilon^{f}$ $\qquad \varepsilon^{A} \nearrow$ $\qquad \varepsilon^{B} \nearrow$

---

- $\underbrace{1+\tau(\theta)}_{g(\theta)}$ : $\quad \varepsilon_{\theta}^{g} = \frac{\tau}{1+\tau}\cdot\varepsilon_{\theta}^{\tau}$

- $\tau(\theta) = \frac{rs}{q(\theta)-rs}$ $\qquad \varepsilon_{\theta}^{\tau} = -\left(\frac{q}{q-rs}\times\varepsilon_{\theta}^{q}\right)$

- $\varepsilon_{\theta}^{q} = -\eta$

- $\varepsilon_{\theta}^{\tau} = \eta\cdot\frac{q}{q-rs} = \eta\,(1+\tau)$

- $\varepsilon_{\theta}^{g} = \frac{\tau}{1+\tau}\cdot\eta\cdot(1+\tau)$

$$\Rightarrow \boxed{\varepsilon_{\theta}^{1+\tau} = \tau\cdot\eta}$$

<u>equilibrium:</u> $\quad \frac{f(\theta)}{1+f(\theta)}\cdot H = \left[\frac{\alpha\cdot a^{1-\gamma}}{\omega\,[1+\tau(\theta)]^{\alpha}}\right]^{\frac{1}{1-\alpha}}$

$\theta(a)$ . Consider small change in productivity $d\ln a$

$\rightarrow$ $d\ln\theta$ : induced, small change in $\theta$

$$\boxed{df = f'(x)\,dx} \rightarrow \frac{df}{f} = \frac{1}{f} f'(x)\, dx$$

$$\rightarrow \frac{df}{f} = \frac{x}{f} \cdot f'(x) \frac{dx}{x}$$

$$\rightarrow \boxed{d\ln f = \varepsilon_x^f \cdot d\ln x}$$

extends to multivariate functions.

- $$d\ln L^s = \varepsilon_\theta^{L^s} \cdot d\ln\theta$$

$$L^s = \frac{f(\theta)}{s + f(\theta)} \cdot H \rightarrow \varepsilon_\theta^{L^s} = (1-\eta) \cdot u$$

$$\boxed{d\ln L^s = (1-\eta) \cdot u \cdot d\ln\theta}$$

- $$d\ln L^d(\theta, a) = \varepsilon_\theta^{L^d} \cdot d\ln\theta + \varepsilon_a^{L^d}\, d\ln a$$

$$\left( df(x,y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \right)$$

$$L^d(\theta, a) = \left[ \frac{\alpha \cdot a^{1-\gamma}}{\omega \cdot (1+\tau(\theta))^{\alpha}} \right]^{1/1-\alpha}$$

$$\rightarrow \varepsilon_a^{L^d} = \frac{1}{1-\alpha} \cdot 1-\gamma = \frac{1-\gamma}{1-\alpha}$$

$$\rightarrow \varepsilon_\theta^{L^d} = \frac{-1}{1-\alpha} \cdot \alpha \cdot \varepsilon_\theta^{1+\tau}$$

$$\varepsilon_{\theta}^{L^d} = \frac{-\alpha}{1-\alpha} \cdot \eta \cdot \tau$$

$$\boxed{d\ln L^d = \frac{1-\gamma}{1-\alpha}\, d\ln a - \frac{\alpha}{1-\alpha} \cdot \eta \cdot \tau \cdot d\ln \theta}$$

before shock: $L^s(\theta) = L^d(\theta, a)$

after shock: $L^s(\theta') = L^d(\theta', a')$

$$d\ln L^s = d\ln L^d$$

$$(1-\eta)u \; d\ln\theta = \frac{1-\gamma}{1-\alpha}\, d\ln a - \frac{\alpha}{1-\alpha}\, \eta\, \tau\, d\ln\theta$$

$$(1-\eta)u \; \frac{d\ln\theta}{d\ln a} = \frac{1-\gamma}{1-\alpha} - \alpha/_{1-\alpha} \cdot \eta \cdot \tau \, \frac{d\ln\theta}{d\ln a}$$

$$\left[(1-\eta)u + \frac{\alpha}{1-\alpha}\,\eta\,\tau\right] \frac{d\ln\theta}{d\ln a} = \frac{1-\gamma}{1-\alpha}$$

$$\boxed{\frac{d\ln\theta}{d\ln a} = \frac{1-\gamma}{(1-\alpha)(1-\eta)u + \alpha\,\eta\,\tau}}$$

- $\gamma = 1$ (wage $\propto$ a : flexible wage)

  $\frac{d\ln\theta}{d\ln a} = 0$ : no fluctuations

  $\rightarrow$ need somewhat rigid wage to obtain business-cycle fluctuations $\rightarrow \gamma < 1$.

- $\gamma < 1$ (wages are rigid) : $\frac{d\ln\theta}{d\ln a} > 0$

→ necessity of <u>wage rigidity</u> to obtain business cycle fluctuations in matching model

→ in data : wages are rigid ( $\gamma \simeq 0.5$ )

so model can generate business-cycle fluctuations.

<u>Calibration of model</u> :

$\gamma$ : 0.5

$\eta$ : 0.5

$\alpha$ : 2/3

$u$ : 6 %

$\tau$ : 3 %

$$\varepsilon_a^\theta = \frac{1-\gamma}{(1-\alpha)(1-\eta)\,u + \alpha\,\eta\,\tau}$$

$$= \frac{\cancel{0.5}}{1/3 \cdot \cancel{0.5} \cdot 6\% + 2/3 \cdot \cancel{0.5} \cdot 3\%}$$

$$= \frac{1}{2\% + 2\%}$$

$$\varepsilon_a^\theta = \frac{1}{4\%}$$

$$\boxed{\varepsilon_a^\theta = 25}$$

if we want $\varepsilon_a^\theta = 8$ : what is $\gamma$ ?

$$8 = \frac{1-\gamma}{1/3 \cdot 1/2 \cdot 6\% + 2/3 \cdot 1/2 \cdot 3\%}$$

$$8 \times [\,1\% + 1\%\,] = 1-\gamma$$

$$16\% = 1-\gamma$$

$$\boxed{\gamma = 0.84}$$

# Matching model with bargained wage

Wage bargaining solution: surplus sharing

$$W = (1-\beta)\, z + \beta \cdot MPL \cdot (1 + r\theta)$$

$$MPL = \alpha \cdot a \cdot N^{\alpha - 1}$$

Linear production function: $\alpha = 1$ $\quad Y = a \cdot N$

$$\rightarrow MPL = a$$

$$\rightarrow \boxed{W = (1-\beta)\, z + \beta a (1 + r\theta)}$$

Labor demand: $\quad (L^{d})^{1-\alpha} = \left[ \dfrac{\alpha \, a}{w \, [1 + \tau(\theta)]^{\alpha}} \right]^{\cancel{\frac{1}{1-\alpha}}}$

As we set $\alpha = 1$: $\quad 1 = \dfrac{a}{w \cdot [1 + \tau(\theta)]}$

Labor demand relation:

$$\boxed{\underbrace{a}_{MPL} = \underbrace{[1 + \tau(\theta)] \cdot W}_{MC}}$$

Combined bargained wage & labor demand:

$$a = [1 + \tau(\theta)] \times [(1-\beta)\, z + \beta a (1 + r\theta)]$$

$$\boxed{1 = [1+\tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]}$$

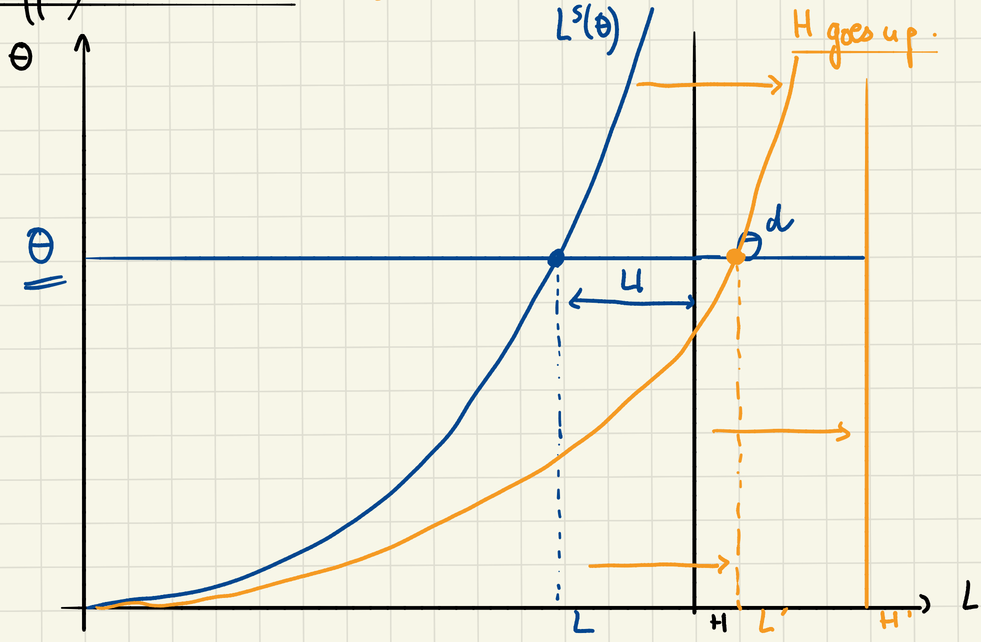
↳ new labor demand: linear prod. function + bargained wage.

↳ • perfectly elastic labor demand

• horizontal labor demand.

↳ labor demand $\theta^d(a)$

Labor supply shock: shock to H




H goes up:

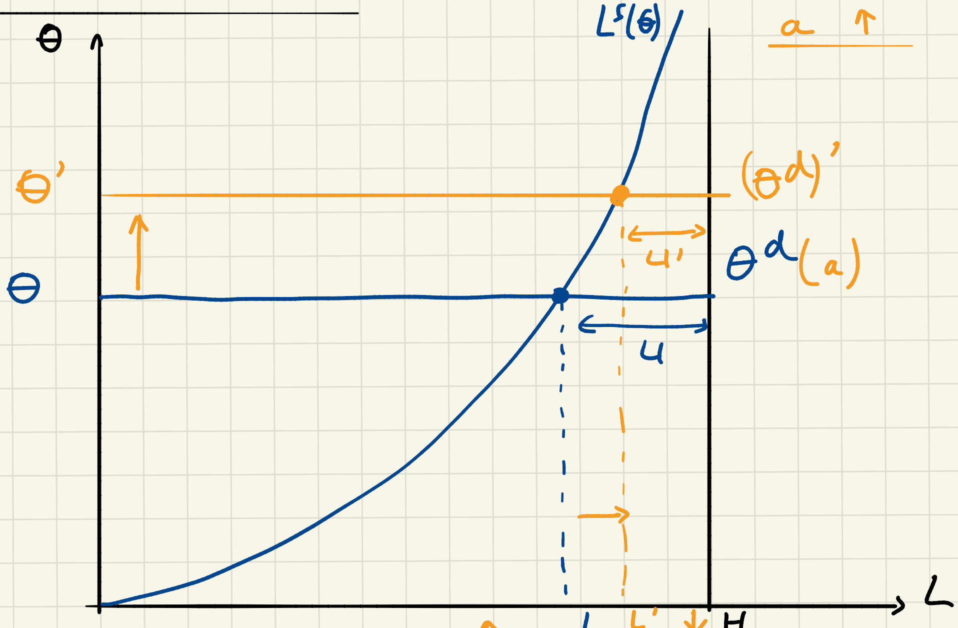
- $L \uparrow$ : boom / expansion ($Y \uparrow$)
- $\theta \rightarrow$
- $u = s / [s + f(\theta)]$ rate $u$ remains same
- $U = u \times H$ $U \uparrow$
- $v = \theta \times u$ $v$ remains same
- $V = v \times H$ $V \uparrow$

→ $\theta$, $u$, $v$ are acyclical

→ not a realistic business cycle

Labor demand shock : shock to $a$

$a \uparrow$




Labor demand $\theta^d$ : $1 = [1 + \tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1 + r\theta)\right]$

if $a \uparrow$ : $\theta^d(a) \uparrow$

After an increase in a:

- $L \uparrow$ : boom / expansion
- $\theta \uparrow$ : $\theta$ procyclical
- $\begin{cases} u \downarrow \\ U \downarrow \end{cases}$ : $u$ countercyclical

- $\underset{\text{(inflows)}}{s \times \overset{\uparrow}{L}} = m(\underset{\downarrow}{U}, \underset{\text{(outflows)}}{\overset{\uparrow}{V}})$

$\rightarrow \begin{cases} V \uparrow \\ v \uparrow \end{cases}$ : $v$ procyclical

Under labor-demand shocks, fluctuations have realistic features (qualitatively)
Not the case with labor-supply shocks.

---

Objective: compute $\varepsilon_a^\theta$.

Labor demand relation:

$$1 = \underbrace{[1+\tau(\theta)]\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]}_{RHS(\theta,a)}$$

Study impact of an infinitesimal change in productivity: $d\ln a$

→ Generates infinitesimal change in tightness: $d\ln\theta$

$$d\ln RHS = \varepsilon_a^{RHS} \cdot d\ln a + \varepsilon_\theta^{RHS} \cdot d\ln\theta = 0$$

so $$\boxed{\frac{d\ln\theta}{d\ln a}} = -\frac{\varepsilon_a^{RHS}}{\varepsilon_\theta^{RHS}}$$

• First, compute $\varepsilon_a^{RHS} = \dfrac{\partial \ln RHS}{\partial \ln a}$.

$$RHS = [1+\tau(\theta)]\cdot\left[(1-\beta)\frac{z}{a} + \beta(1+r\theta)\right]$$

$$\frac{\partial \ln RHS}{\partial \ln a} = \frac{(1-\beta)(z/a)}{(1-\beta)(z/a)+\beta(1+r\theta)}\cdot(-1)$$

$$\frac{\partial \ln RHS}{\partial \ln a} = \frac{-(1-\beta)\,z}{(1-\beta)\,z+\beta a\,(1+r\theta)} = \varepsilon_a^{RHS}.$$

• Second, compute $\varepsilon_\theta^{RHS} = \dfrac{\partial \ln RHS}{\partial \ln \theta}$.

$$\varepsilon_\theta^{RHS} = \underbrace{\eta\cdot\tau(\theta)}_{\varepsilon_\theta^{1+\tau}} + \frac{\beta(1+r\theta)}{(1-\beta)(z/a)+\beta(1+r\theta)}$$

$$* \ \frac{r\theta}{1+r\theta}\cdot 1$$

$$\varepsilon_\theta^{RHS} = \eta \cdot \tau(\theta) + \frac{\beta r \theta a}{(1-\beta) z + \beta a (1+r\theta)}.$$

- Third, combine results to compute $\varepsilon_a^\theta$:

$$\frac{d\ln\theta}{d\ln a} = \frac{(1-\beta) z}{(1-\beta) z + \beta a \,(1+r\theta)} \cdot \frac{(1-\beta) z + \beta a \,(1+r\theta)}{\beta r \theta a + \eta \tau \cdot \frac{a}{1+\tau}}$$

$$a = (1+\tau)\big((1-\beta) z + \beta a (1+r\theta)\big) \qquad \text{(labor demand)}$$

$$\frac{a}{1+\tau} = (1-\beta) z + \beta a \,(1+r\theta)\Big]$$

$$\boxed{\frac{d\ln\theta}{d\ln a} = \frac{(1-\beta)\cdot z}{a \cdot \left[\beta r\theta + \eta \cdot \frac{\tau}{1+\tau}\right]}}$$

- $z$: value from unemployment

$z = 0$ : $\frac{d\ln\theta}{d\ln a} = 0$ : no business-cycle fluctuations

$\theta, L, u, v$ : do not respond to productivity $a$.

Intuition: if $z = 0$ : $W$ is proportional to $a$

$\rightarrow$ $w$ is flexible : $W$ absorbs fluctuations in $a$ so $\theta^d$ is independent of $a$

Algebraically: $z = 0 \Rightarrow w = \beta a (1 + r\theta)$

Labor demand relation: $a = [1 + \tau(\theta)]\, w$

$$\Rightarrow 1\ \cancel{a} = [1 + \tau(\theta)]\, \beta\, \cancel{a}\, (1 + r\theta)$$

$$\Rightarrow 1 = [1 + \tau(\theta)]\, \beta\, (1 + r\theta)$$

$\hookrightarrow$ independent of $a$

Shimer (2005):

$z = 0.4$

$a = 1$ (normalization)

$\eta = 0.5$

$\beta = 0.5$ (tradition)

$\tau = 3\%$

$r\theta = 0.6$

Labor demand relation:

$$a = (1+\tau)\left((1-\beta)z + \beta a (1 + r\theta)\right)$$

$$1 = (1.03)\left(\tfrac{1}{2} \cdot 0.4 + \tfrac{1}{2} \cdot 1 \cdot (1 + r\theta)\right)$$

$$1 = (1.03)\left(0.2 + 0.5\,(1 + r\theta)\right)$$

$$r\theta = \left[\frac{1}{1.03} - 0.2\right] \times 2 - 1 \simeq 0.6$$

Calibrated value of $\frac{d\ln\theta}{d\ln a}$ ?

$$\frac{d\ln\theta}{d\ln a} = \frac{0.4 \times 0.5}{0.5 \times 0.6 + 0.5 \cdot 0.03/1.03} = \frac{0.2}{0.3 + 0.015}$$

$$\frac{d\ln\theta}{d\ln a} \simeq 2/3 > 0 \quad \text{once } \underline{z > 0}$$

BUT $\boxed{\dfrac{d\ln\theta}{d\ln a} \ll 8}$

Fluctuations in $\theta$, $u$, $v$ are much smaller than in US data.

→ Model with surplus sharing as wage function is inappropriate to describe business cycle on labor market.

→ Violates criterion # 1 for a good model (Kuhn)

See T. Bewley (1999).