ECON 2080, part 1
Spring 2022
Pascal Michaillat
Brown University

Midterm Exam

Handed out: Wednesday 9 March, 9:00 am.
Due: Wednesday 9 March, 12:00 pm.
Total: 20 points.
Reminder: No calculators, tablets, computers, notes, or books.

For the multiple-choice questions, just state your answer. If there are multiple correct anwers, please list all the correct answers.

For the long problems, please provide your derivations. Unless specifically asked to, you do not need to interpret your answers.

As a matter of general test-taking strategy, please do not devote too much time to any one question until you have devoted quality time to the other questions.

On all questions, please clearly indicate what your final answer is, preferably by boxing it. Multiple or ambiguous answers, containing wrong or irrelevant statements mixed in with correct statements, will be graded with skepticism.

Exams will be collected promptly. Please stop writing exactly when you are asked to. We will be on the look-out for those continuing to write past this time, and deduct points.

If you finish very early, you are welcome to leave quietly. If you finish within 10 minutes of the end, please do not leave or start shuffling out of consideration to other students.

Do not do work you intend to hand in on the exam itself; do so solely in your bluebook. Make sure to write your name on your bluebook(s).

Best of luck!

Question A (1 point)

We define labor-market tightness as $\theta = V/U$. What is the rate at which unemployed workers find a job when the matching function is $m(U, V) = (U^{-\gamma} + V^{-\gamma})^{-\frac{1}{\gamma}}$ with $\gamma > 0$?

1. $f(\theta) = (1 + \theta^{\gamma})^{-\frac{1}{\gamma}}$
2. $f(\theta) = (1 + \theta^{-\gamma})^{\gamma}$
3. $f(\theta) = (1 + \theta^{-\gamma})^{-\frac{1}{\gamma}}$
4. $f(\theta) = (1 + \theta^{\gamma})^{\gamma}$
5. $f(\theta) = (1 - \theta^{\gamma})^{-\frac{1}{\gamma}}$
6. None of the above

Question B (1 point)

We define labor-market tightness as $\theta = V/U$. What is the are at which vacancies are filled when the the matching function is $m(U, V) = (U^{-\gamma} + V^{-\gamma})^{-\frac{1}{\gamma}}$ with $\gamma > 0$?

1. $q(\theta) = (1 + \theta^{-\gamma})^{-\frac{1}{\gamma}}$
2. $q(\theta) = (1 + \theta^{\gamma})^{\gamma}$
3. $q(\theta) = (1 + \theta^{\gamma})^{-\frac{1}{\gamma}}$
4. $q(\theta) = (1 + \theta^{-\gamma})^{\gamma}$
5. $q(\theta) = (1 - \theta^{-\gamma})^{-\frac{1}{\gamma}}$
6. None of the above

Question C (1 point)

Consider the matching function $m(U, V) = (U^{-\gamma} + V^{-\gamma})^{-\frac{1}{\gamma}}$ with $\gamma > 0$. How do the the job-finding rate $f(\theta)$ and vacancy-filling rate $q(\theta)$ behave at the limit?

1. $f(0) = 0, q(0) = 0, \lim_{\theta \to \infty} f(\theta) = 1, \lim_{\theta \to \infty} q(\theta) = 1.$
2. $f(0) = 1, q(0) = 1, \lim_{\theta \to \infty} f(\theta) = 0, \lim_{\theta \to \infty} q(\theta) = 0.$
3. $f(0) = 0, q(0) = 1, \lim_{\theta \to \infty} f(\theta) = 1, \lim_{\theta \to \infty} q(\theta) = 0.$
4. $f(0) = 1, q(0) = 0, \lim_{\theta \to \infty} f(\theta) = 0, \lim_{\theta \to \infty} q(\theta) = 1.$
5. $f(0) = 0, q(0) = \infty, \lim_{\theta \to \infty} f(\theta) = \infty, \lim_{\theta \to \infty} q(\theta) = 0.$
6. $f(0) = \infty, q(0) = 0, \lim_{\theta \to \infty} f(\theta) = 0, \lim_{\theta \to \infty} q(\theta) = \infty.$

Question D (1 point)

Consider a matching function $m(U, V)$ that has constant returns to scale and is increasing in U and V. What is the most you can say about the job-finding rate and vacancy-filling rate?

1. The rates are functions of labor-market tightness.
2. Both rates are increasing in tightness.
3. Both rates are decreasing in tightness.
4. The job-finding rate is decreasing in tightness and the vacancy-filling rate is increasing in tightness.
5. The job-finding rate is increasing in tightness and the vacancy-filling rate is decreasing in tightness.
6. The matching function is too general to say anything.

Question E (1 point)

Imagine that the government implements training programs to increase the skills and productivity of workers. In the matching model with fixed wage, this policy would

1. Shift the labor-demand curve upward
2. Shift the labor-demand curve downward
3. Shift the labor-supply curve leftward
4. Shift the labor-supply curve rightward
5. Rotate the labor-demand curve upward
6. Rotate the labor-demand curve downward
7. Have no effect on labor demand and labor supply

Question F (1 point)

Imagine that the government implements training programs to increase the skills and productivity of workers. In the matching model with Nash bargaining, this policy would

1. Raise the real wage
2. Lower the real wage
3. Shift the labor-supply curve leftward
4. Shift the labor-supply curve rightward
5. Have no effect on real wage and labor supply

Question G (1 point)

In the matching model, which of the following parameters and variables are negatively influencing labor demand?

1. Labor market tightness and productivity
2. Wage and productivity
3. Wage and labor market tightness
4. Labor force and wage
5. Labor force and recruiting cost
6. None of the above

Question H (1 point)

Consider a negative labor-demand shock in the matching model with rigid wage. Then:

1. The unemployment rate is low.
2. The probability of losing a job in a given month is high.
3. The probability of losing a job in a given month is low.
4. The probability of finding a job in a given month is low.
5. The probability of finding a job in a given month is high.
6. The probability of filling a vacancy in a given month is high.
7. The probability of filling a vacancy in a given month is low.
8. None of the above.

Question I (1 point)

Consider a matching model with linear production function, Cobb-Douglas matching function, and Nash bargaining. Let η be the elasticity of the matching function with respect to unemployment and $1-\eta$ the elasticity of the matching function with respect to vacancies. Let β be workers' bargaining power and $1-\beta$ be firms' bargaining power. Let r be the recruiting cost, measured in recruiters per vacancy. Let z be the social value of nonwork relative to work: $z=0$ means that unemployed workers do not contribute to social welfare; $z=1$ means that unemployed workers contribute as much to social welfare as employed workers. Which of the following statements are correct?

1. The efficient labor-market tightness is increasing in η.
2. The efficient labor-market tightness is decreasing in η.

3. The efficient labor-market tightness is increasing in z.
4. The efficient labor-market tightness is decreasing in z.
5. The efficient labor-market tightness is increasing in r.
6. The efficient labor-market tightness is decreasing in r.
7. The efficient labor-market tightness is increasing in β.
8. The efficient labor-market tightness is decreasing in β.

Question J (1 point)

Consider a matching model with job rationing and rigid wages. What happens over the business cycle?

1. In bad times, rationing unemployment is high but frictional unemployment is low.
2. In bad times, rationing unemployment is low but frictional unemployment is high.
3. In bad times, both rationing unemployment and frictional unemployment are high.
4. An increase in public employment has a larger effect on total employment in bad times than in good times.
5. An increase in public employment has a larger effect on total employment in good times than in bad times.
6. Optimal unemployment insurance is more generous in bad times than in good times.
7. Optimal unemployment insurance is more generous in good times than in bad times.
8. The labor market always operate efficiently so policies do not need to be adjusted over the business cycle.

Problem A (6 points)

Consider a matching model with a labor force of size H. The matching function is Cobb-Douglas: $m(U, V) = \omega \cdot U^{\eta} \cdot V^{1-\eta}$, where U is the number of unemployed workers, V is the number of vacant jobs, and $\eta \in (0, 1)$ is the matching elasticity. All workers are paid at a minimum wage $w > 0$. Firms have a production function $y(N) = a \cdot N^{\alpha}$, where a governs labor productivity, N denotes the number of producers in the firm, and $\alpha \in (0, 1)$ indicates diminishing marginal returns to labor. Firms incur a recruiting cost of $r > 0$ recruiters per vacancy and face a job-destruction rate $s > 0$. The labor market tightness is $\theta = V/U$.

1. Compute the job-finding rate $f(\theta)$ and vacancy-filling rate $q(\theta)$. Assuming that

labor-market flows are balanced, compute the recruiter-producer ratio $\tau(\theta)$. Compute the elasticities of f, q, and $1+\tau$ with respect to θ.

2. Assuming that labor-market flows are balanced, compute labor supply $L^s(\theta, H)$. Compute the elasticities of L^s with respect to θ and H.
3. Firms choose employment to maximize flow profits: $y(N) - [1+\tau(\theta)] \cdot w \cdot N$. Compute the labor demand $L^d(\theta)$ by solving this maximization problem. Compute the elasticity of L^d with respect to θ.
4. Characterize tightness $\theta(H)$ and unemployment rate $u(H)$ in the model. Illustrate with a diagram how $\theta(H)$ and $u(H)$ are determined.
5. Denote the elasticities of tightness $\theta(H)$ and unemployment rate $u(H)$ with respect to H as ϵ_H^θ and ϵ_H^u. Compute ϵ_H^θ and ϵ_H^u. Interpret the signs of these elasticities.
6. What is the sign of the derivative $d|\epsilon_H^\theta|/da$? (You do not need to compute the derivative: just find its sign.) There are times when people are strongly opposed to immigration, and other times when immigration is a nonissue. Based on the sign of the derivatives, when is it likely that opposition to immigration is particularly strong? As far as you know, is this prediction supported by historical evidence?

Problem B (4 points)

Consider a one-period matching model with a labor force of size 1, a mass 1 of firms, and a government. All workers are initially unemployed. Private firms and the government post vacancies and match with workers. Then production occurs. The matching function is $m(V) = \sqrt{V}$, where V is the total number of vacancies posted by firms and the government. Given that all workers are initially unemployed, the labor market tightness equals the aggregate number of vacancies: $\theta = V$.

Firms incur a recruiting cost of $r > 0$ recruiters per vacancy. Firms have a production function $y(N) = 2 \times a \times \sqrt{N}$, where a governs labor productivity and N denotes the number of producers in the firm. We denote by F the total number of workers in the firm. The firm pays a rigid wage $w = \sqrt{a}$ to all their F workers. Each firm choose employment F to maximize profits.

The government employs $G > 0$ workers. Aggregate employment is the sum of public and private employment: $L = G + F$. The share of public employment in the labor market is denoted by $\sigma = G/L$.

1. The labor supply $L^s(\theta)$ gives the number of worker who find a job (either in the public or private sector) through the matching process when tightness is θ. Given

the expression of $L^s(\theta)$. What is the elasticity of $L^s(\theta)$ with respect to θ.

2. The aggregate labor demand $L^d(\theta, G)$ is the sum of the private labor demand $F^d(\theta)$ and the public labor demand G. Compute $L^d(\theta, G)$. What are the elasticities of $L^d(\theta, G)$ with respect to θ and with respect to G?
3. Compute an expression for the government multiplier $\lambda = dL/dG$. Is the multiplier λ positive or negative? Is $|\lambda|$ more or less than 1? Interpret these findings.
4. What is the sign of the derivative $d\lambda/da$? What does this result imply for the effectiveness of fiscal policy over the business cycle? As far as you know, does the result seem realistic?