Efficient Unemployment and Unemployment Gap

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CBO: natural rate of unemployment

Taking a trend of unemployment rate + adjustments

Premise: on average, labor market is efficient

Problem: No guarantee that labor market is efficient on average → in matching model, no reason to believe that labor market is efficient.

Phillips-curve approach: accelerationist Phillips curve

(Friedman): target unemployment rate such that inflation remains constant

Problem:
- care about other things than keeping inflation constant
- complete disconnect between inflation & unemployment

Efficient unemployment rate in matching model

Efficient: maximizes social welfare.

Social welfare: sum of welfare of all individuals
utility

Assumption: Cobb-Douglas matching function

Assumptions to simplify social welfare

- Linear utility function over consumption (risk neutral)

→ all individuals value consumption the same

→ can compute aggregate utility from consumption by aggregating consumption = output.

- Disutility from work = disutility from searching for a job → value of time is the same for employed & unemployed workers

→ value of time is not relevant for welfare.

⇒ social welfare is determined solely by aggregate consumption = aggregate output.

Definition efficient unemployment rate is the unemployment rate that maximizes output.

Social planner : benevolent government that can allocate workers between unemployment, producing, & recruiting in order to maximize welfare = output

Social planner is subject to matching function, production function, recruiting process, etc.

Social planner picks <u># vacancies</u> to maximize output $\Leftrightarrow$ picking <u>tightness</u> to maximize output.

<u>Efficient unemployment rate is unemployment rate chosen by social planner</u>

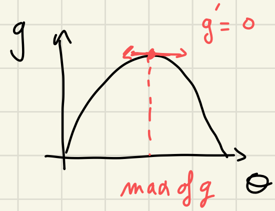
Solution to social planner's problem

$$\max_{V} \; Y \quad \Big| \quad \cdot \; Y = a\,\underline{N}^{\alpha} \qquad \cdot \text{ picking } V \Leftrightarrow \text{picking } \theta$$

$$\Leftrightarrow \max_{\theta} \; N(\theta) \;\longrightarrow\; \begin{cases} \text{workers} = \text{producers} + \text{recruiters} \\ L = [1 + \underbrace{\tau(\theta)}_{\text{recruiter/producer}}]\, N \end{cases}$$

$$\Leftrightarrow \max_{\theta} \; \left.\frac{\overbrace{L^{s}(\theta)}}{1 + \tau(\theta)}\right\} g(\theta)$$

<u>Necessary condition for maximum</u> $\dfrac{dg}{d\theta} = 0$



$$\Leftrightarrow \frac{\theta}{g} \cdot \frac{dg}{d\theta} = 0 \Leftrightarrow \frac{d\ln g}{d\ln\theta} = 0 \Leftrightarrow \boxed{\varepsilon^{g}_{\theta} = 0}$$

$$\varepsilon_\theta^{g} = \varepsilon_\theta^{L^S} - \varepsilon_\theta^{1+\tau}$$

$$(1-\eta)\, u(\theta) \qquad\qquad \eta \cdot \tau(\theta)$$

$$L^S = \frac{f(\theta)}{s+f(\theta)} H$$

$$\varepsilon_\theta^{L^S} = \varepsilon_\theta^{f/s+f}$$

$$\left[\ \varepsilon_\theta^{g} = (1-\eta)\, u(\theta) - \eta\, \tau(\theta)\right.$$

<u>Solution to social planner's problem</u> θ that maximizes social welfare = output = # of producers is given by

$$(1-\eta)\, u(\theta) = \eta\, \tau(\theta)$$

$$\boxed{\frac{u(\theta)}{\tau(\theta)} = \frac{\eta}{1-\eta}}$$

$\rightarrow$ <u>efficient</u> labor market tightness θ^*

$\rightarrow$ efficient unemployment rate $u^* = u(\theta^*) = \dfrac{s}{s+f(\theta^*)}$

$\rightarrow$ efficient level of output $= a\, N(\theta^*)^\alpha$

$$= a \left[\frac{L^S(\theta^*)}{1+\tau(\theta^*)}\right]^\alpha$$

<u>Application to the US labor market</u>

Research on matching function (Petrongolo & Pissarides, 2001).

$$\eta = 0.5 \Rightarrow \eta/_{1-\eta} = 1$$

In practice labor market is efficient when $\boxed{\frac{u}{\tau} = 1}$

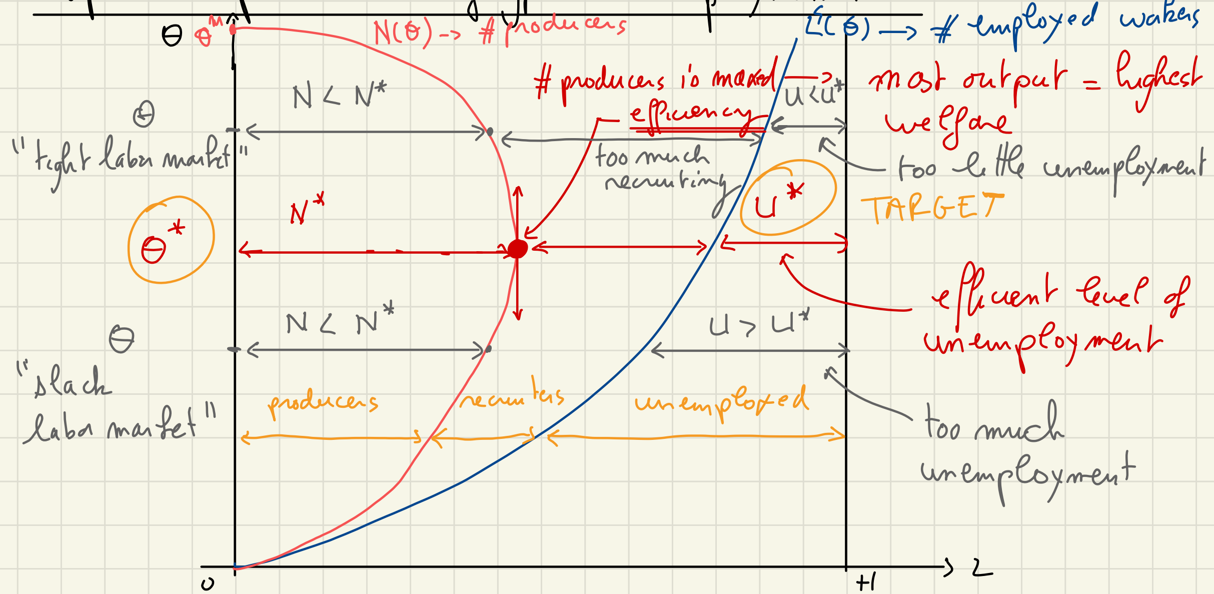
Rule of thumb.

\# unemployed workers = # recruiters

idle

non-productive

Graphical representation of efficient unemployment rate



In real world wage function may not guarantee that $\theta = \theta^* \to$ firms may not have the incentive to post a # of vacancies such that $\theta = \theta^*$ and $u = u^*$ and so on $\to$ government intervention may be needed to bring labor market closer to efficiency.

Efficient unemployment rate in Beveridgean models

Assumption Model admits a Beveridge curve

$$\boxed{v = v(u)}$$

where the function $v(u)$ is decreasing, convex

Examples:
- Matching model
- Mismatch model
- Stock-flow matching model

Advantage Many countries have a Beveridge curve → method is applicable

Two key parameters.
- recruiting cost r workers/vacancy
- social value of unemployment time / employment time z

$z > 0$ or $z < 0$.

Social welfare. $SW = \underbrace{(H - L) - V \times r}_{\text{\# employed workers} \,-\, \text{\# recruiters} \,=\, \text{producers} \,=\, \text{output}} + \underbrace{z\, U}_{\text{"output from unemployment"}}$

Social welfare / capita = $\frac{SW}{H} = (1-u) - v \times r + z \cdot u$

↳ $\boxed{sw(u) = (1-u) + z\,u - v(u) \times r}$

Efficient unemployment rate u^* maximizes

$$sw(u) = \underbrace{\underbrace{1-(1-z)\,u}_{\text{linear}} - \underbrace{v(u) \times r}_{\text{convex}}}_{\text{concave function}}$$

Necessary condition for a maximum of the social welfare function is : $\frac{dsw}{du} = 0$ (first-order condition)

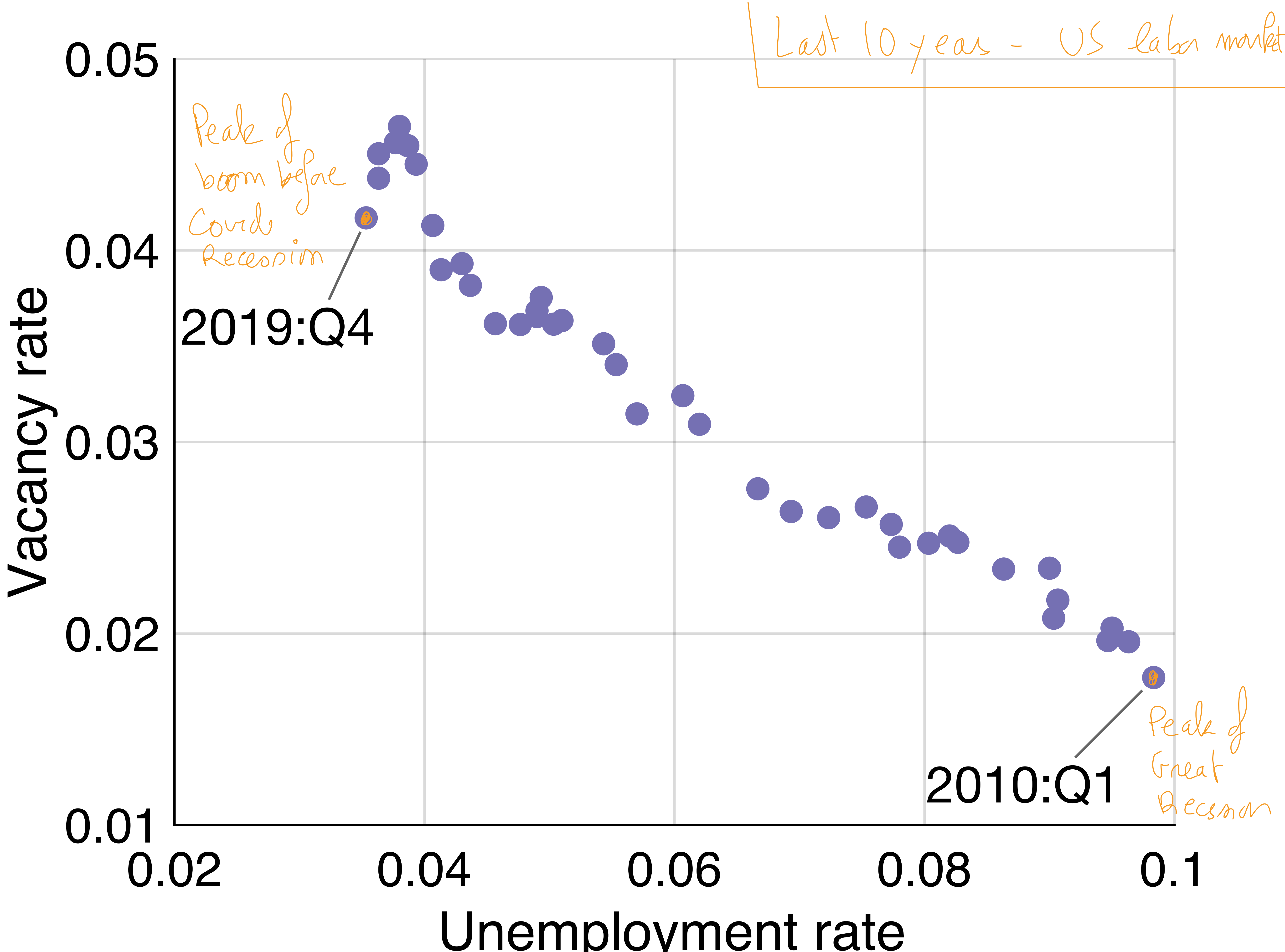
Since $sw(u)$ is concave necessary condition is also sufficient → any u such that $dsw/du = 0$ is a maximum (maximum will be unique)

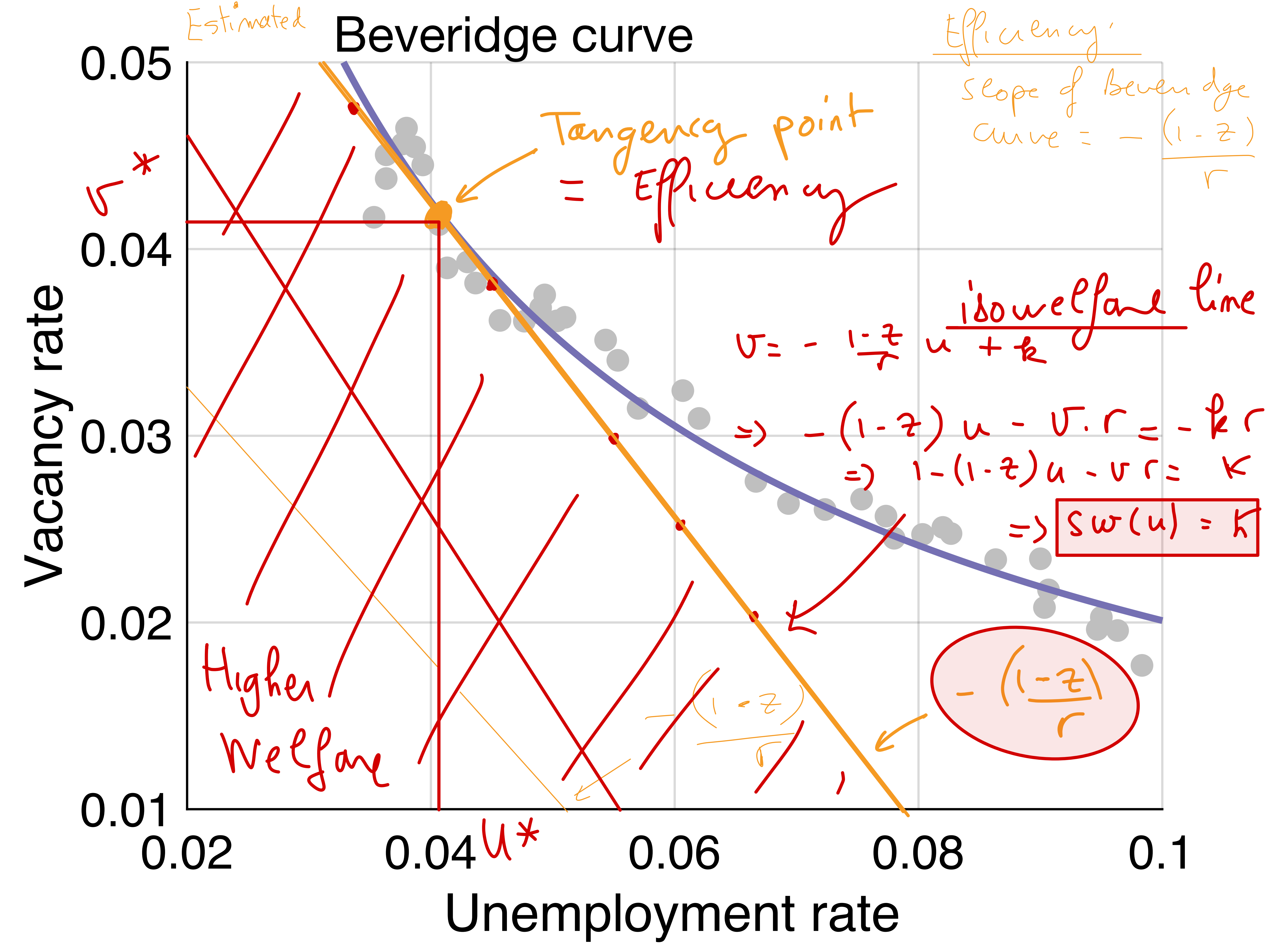
Efficient unemployment rate satisfies

$$\frac{dsw}{du} = -(1-z) - v'(u) \times r = 0$$

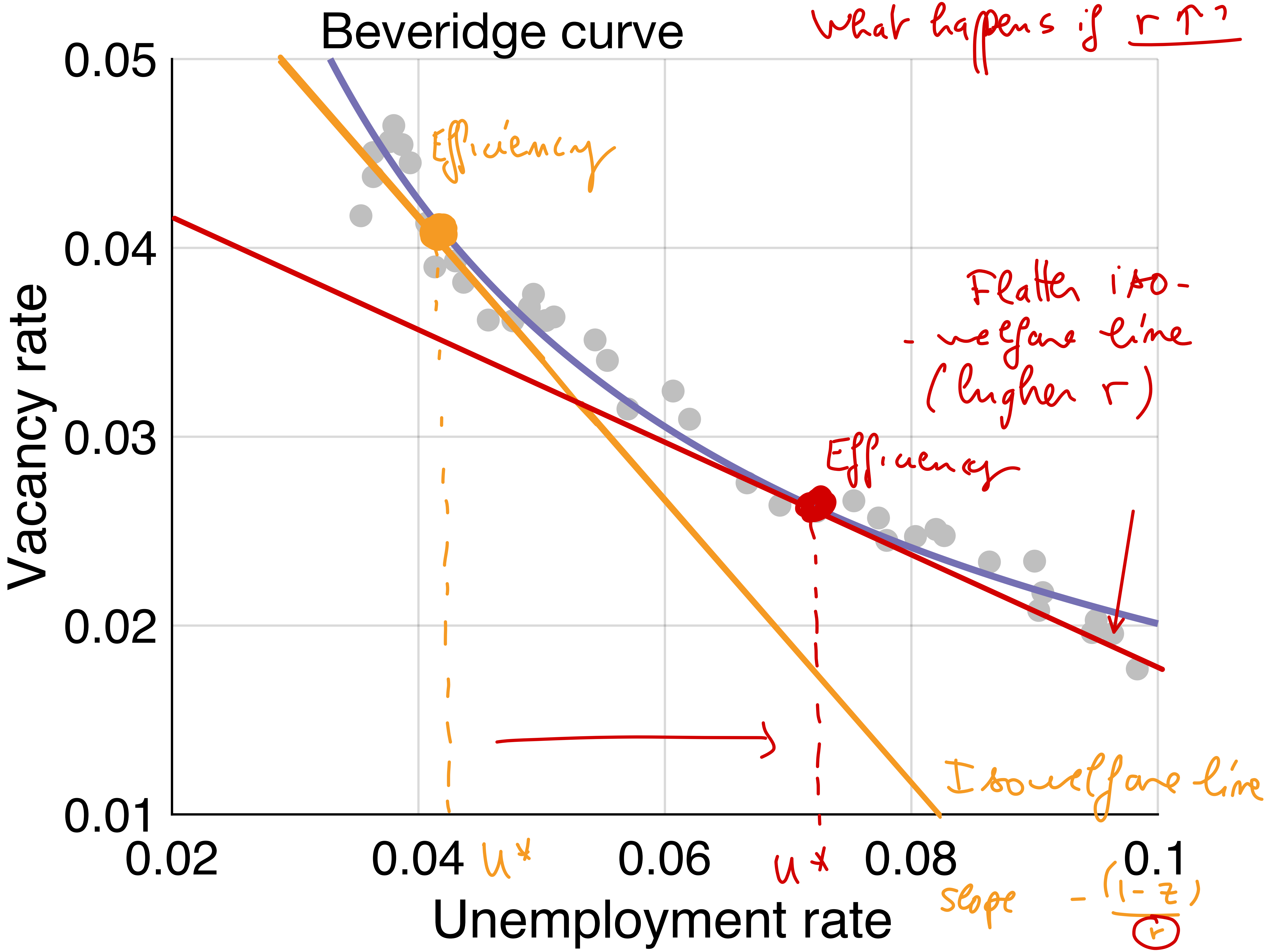
$$\Rightarrow \boxed{v'(u^*) = -\frac{1-z}{r}}$$

- Slope of Beveridge curve $= -(1-z)/r$
- $v(u)$ is convex → $v'(u)$ is strictly increasing → efficient unemployment rate is unique
- $r \uparrow \Rightarrow u^* \uparrow$ | • $z \uparrow \to u^* \uparrow$

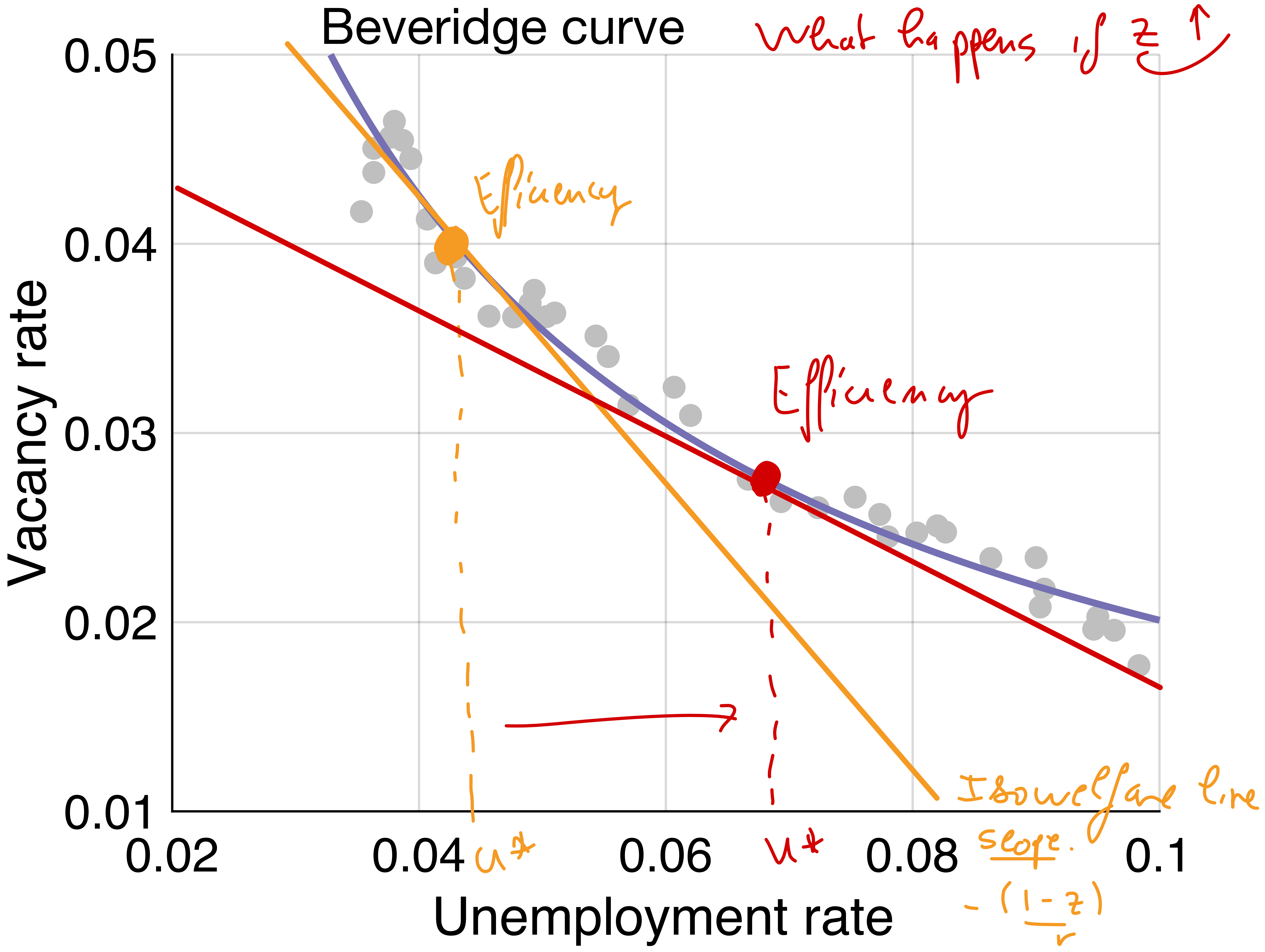

Last 10 years - US labor market
Peak of boom before Covid Recession
2019:Q4
Peak of Great Recession
2010:Q1
Vacancy rate
0.05
0.04
0.03
0.02
0.01
Unemployment rate
0.02
0.04
0.06
0.08
0.1



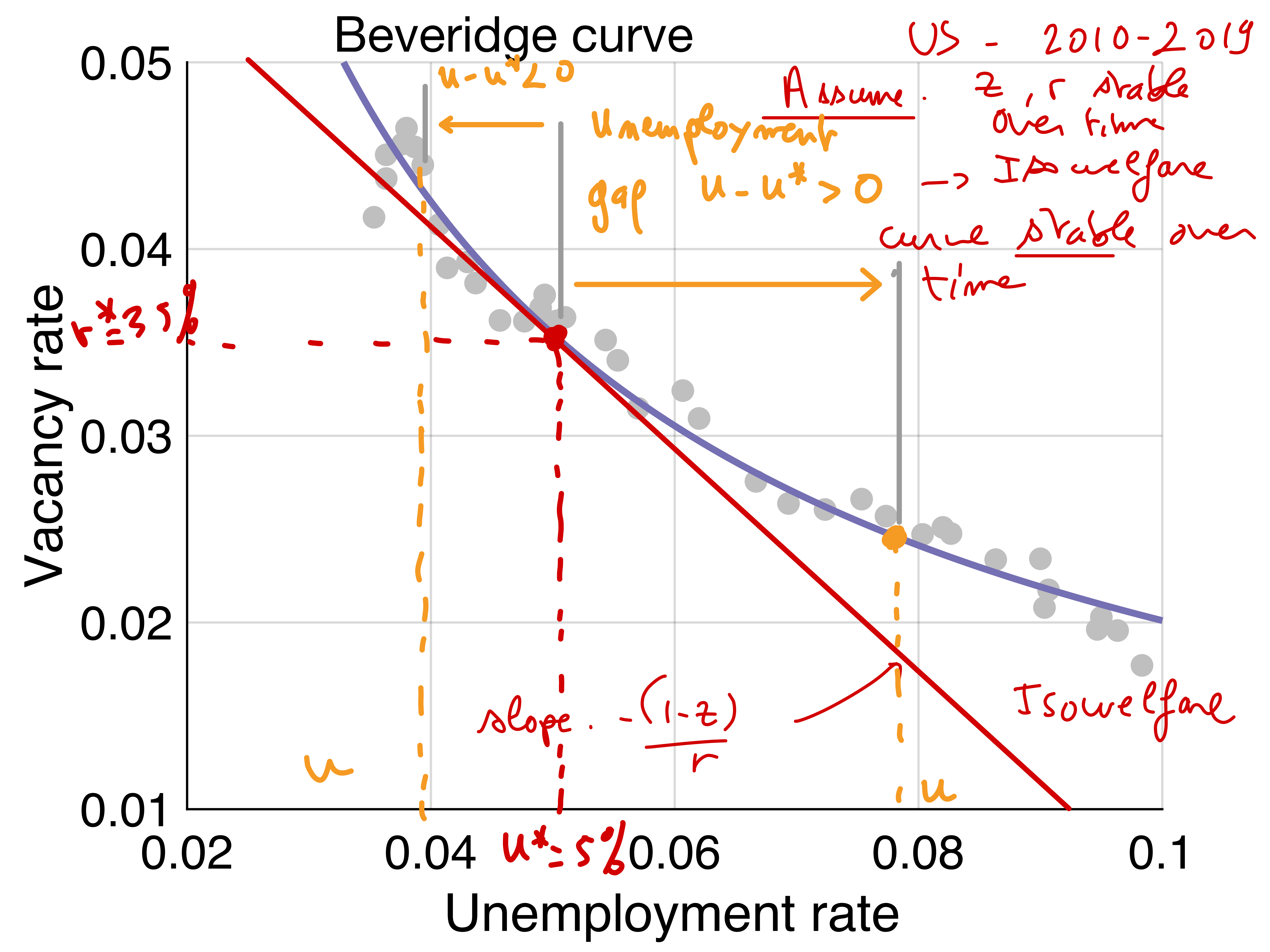
Estimated
Beveridge curve
Efficiency:
slope of Beveridge curve = $-\frac{(1-z)}{r}$
Tangency point = Efficiency
v^*
isowelfare line
$v = -\frac{1-z}{r}u + k$
$\Rightarrow -(1-z)u - v.r = -kr$
$\Rightarrow 1-(1-z)u - vr = k$
$\Rightarrow \boxed{sw(u) = k}$
$-\frac{(1-z)}{r}$
$-\frac{(1-z)}{r}$
Higher Welfare
u^*
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1



What happens if r ↑?
Beveridge curve
0.05
0.04
0.03
0.02
0.01
Vacancy rate
0.02
0.04
0.06
0.08
0.1
Unemployment rate
Efficiency
Efficiency
Flatter iso-
- welfare line
(higher r)
Isowelfare line
u*
u*
slope $-\frac{(1-z)}{r}$



Beveridge curve
What happens if z ↑
Efficiency
Efficiency
Isowelfare line
slope.
$-\frac{(1-z)}{r}$
u^*
u^*
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1



Beveridge curve
US - 2010-2019
Assume: z, r stable over time
→ Isowelfare curve stable over time
$u - u^* < 0$
Unemployment gap $u - u^* > 0$
$r^* = 3.5\%$
slope: $-\frac{(1-z)}{r}$
Isowelfare
u
u
$u^* = 5\%$
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1

Formula for the efficient labor market tightness

Condition for labor market efficiency: $\boxed{v'(u^*) = -\frac{1-z}{r}}$

Tightness $\theta = v/u$

Beveridge elasticity (normalized to be positive) $\boxed{\varepsilon = -\frac{d\ln v}{d\ln u}}$

$$\varepsilon = -\frac{u}{v}\cdot\frac{dv}{du} = -\frac{v'(u)}{\theta}$$

Efficiency condition $\underbrace{-\theta\frac{v'(u)}{\theta}}_{\varepsilon} = \frac{1-z}{r}$

Efficiency condition $\boxed{\theta^* = \frac{1-z}{\varepsilon\, r}}$

3 key factors

- z value of unemployment

$z\uparrow \Rightarrow \theta^*\downarrow,\ u^*\uparrow$

- r recruiting cost

$r\uparrow \Rightarrow \theta^*\downarrow,\ u^*\uparrow$

- ε elasticity of Beveridge curve

$\varepsilon\uparrow \Rightarrow \theta^*\downarrow,\ u^*\uparrow$

Application to US labor market

- 2.5% of labor costs devoted to recruiting (US 1997)

  ↳ 2.5% of workers are recruiters

  $$\boxed{\tau \simeq 0.7}$$

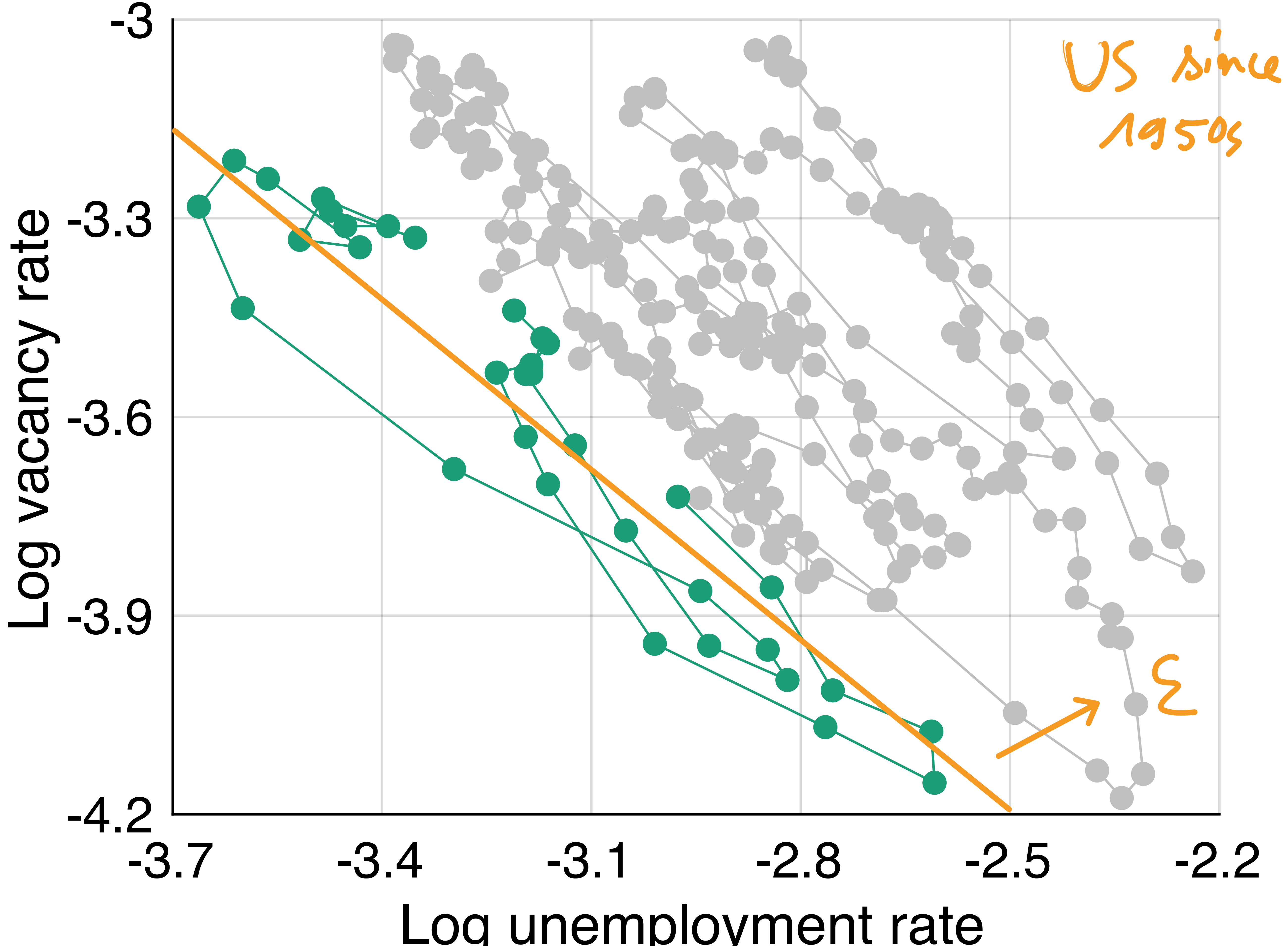
- 13% - 35% of lost earnings ($\simeq$ labor productivity) replaced by leisure & home production

  $13\% \leq z \leq 35\% \rightarrow \boxed{z \simeq 1/4}$

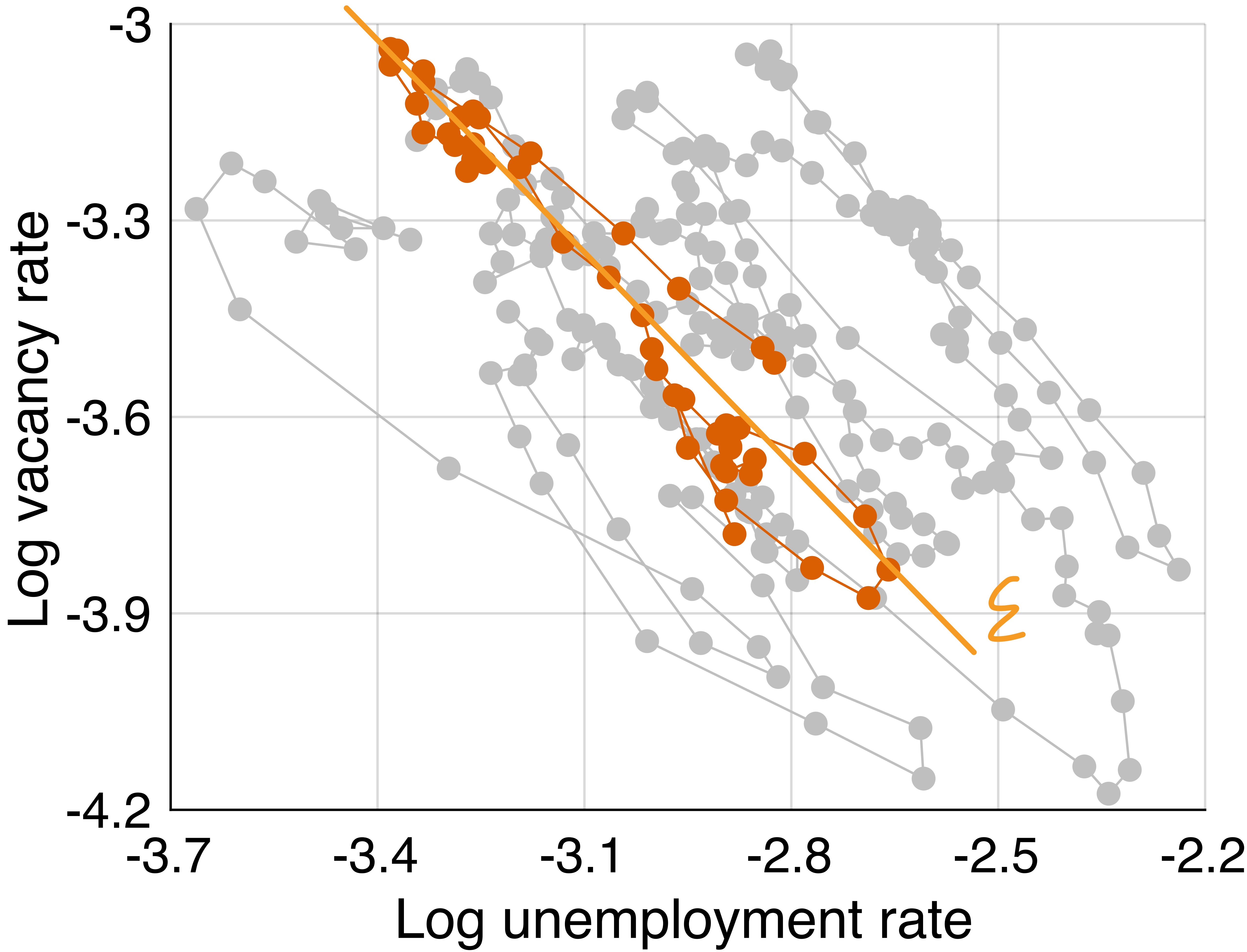
- $\varepsilon = \dfrac{d\ln v}{d\ln u}$

  Slope of curve: $\ln v$ versus $\ln u$.

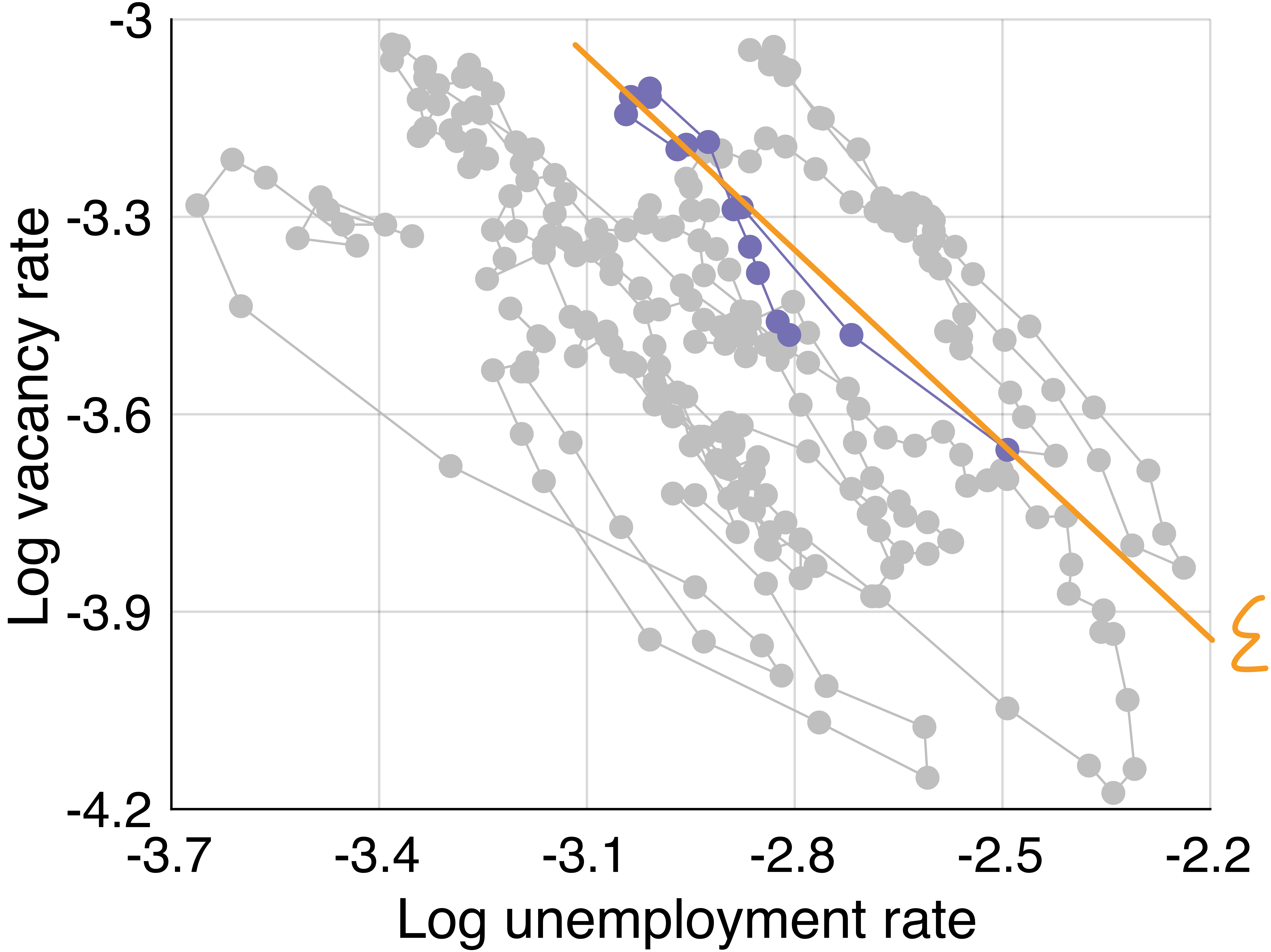
  Coefficient in regression.



US since 1950s
ε
-3
-3.3
-3.6
-3.9
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate



-3
-3.3
-3.6
-3.9
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate
ε


-3
-3.3
-3.6
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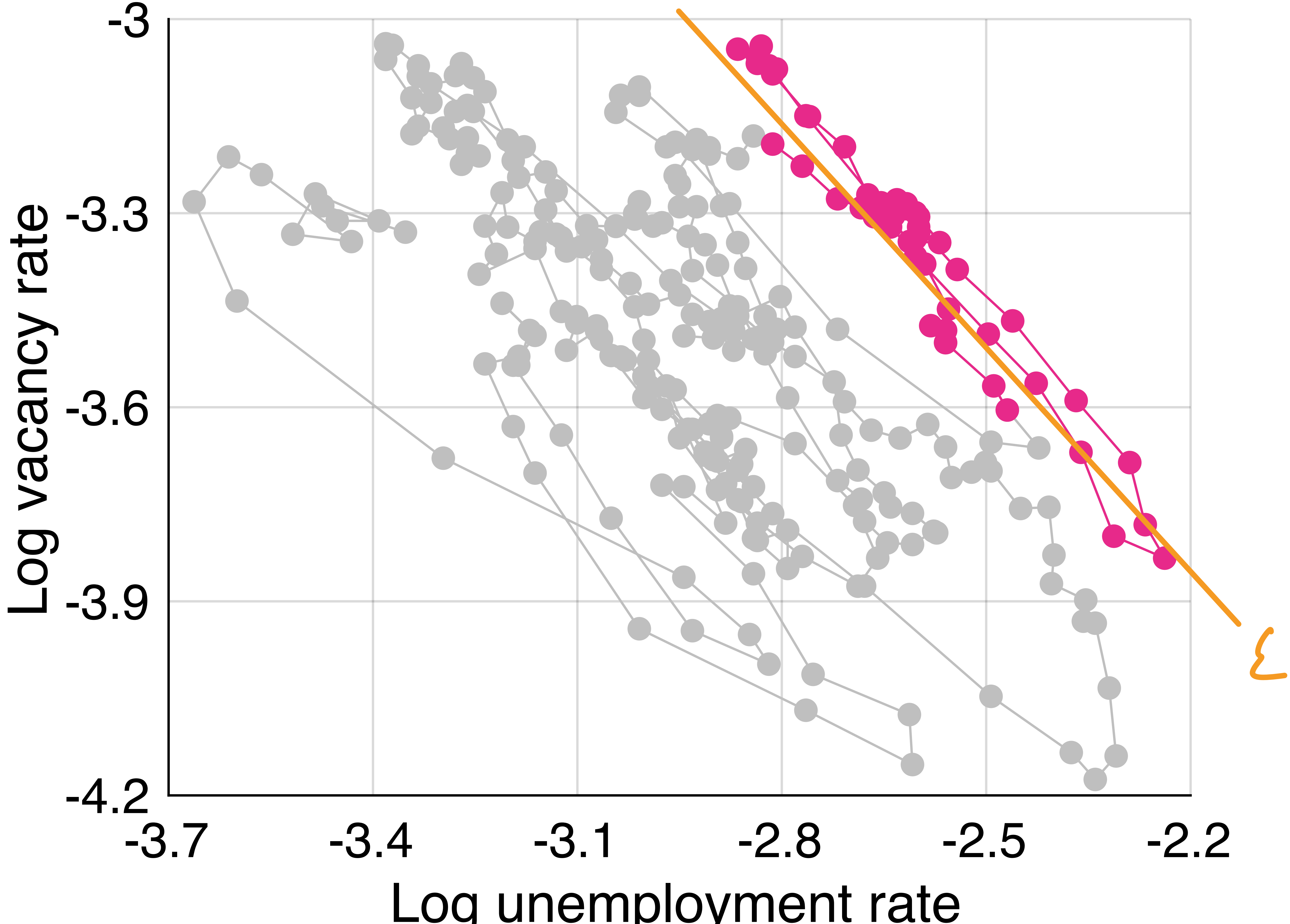
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate
ε



Log vacancy rate
-3
-3.3
-3.6
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-4.2
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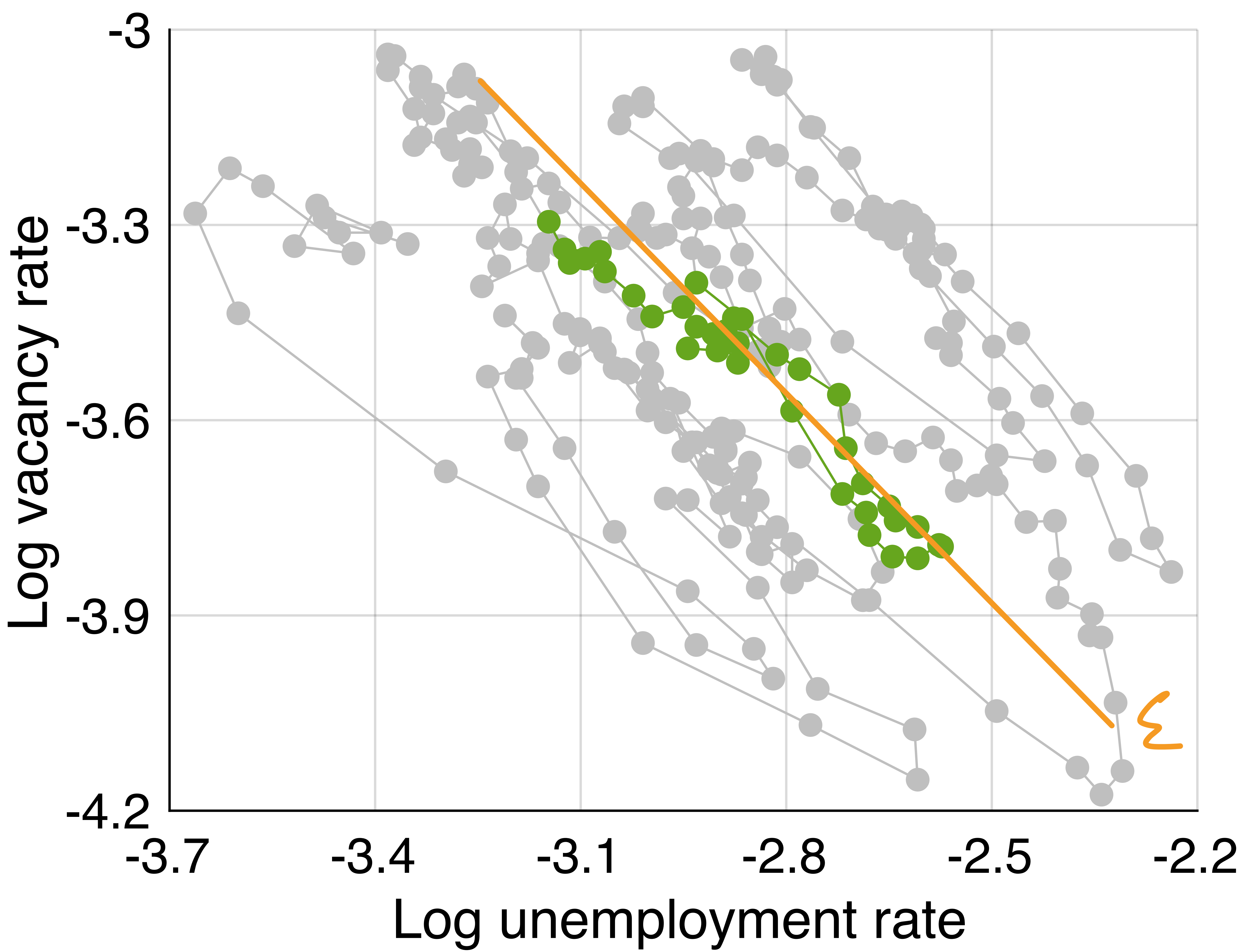
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate


-3
-3.3
-3.6
-3.9
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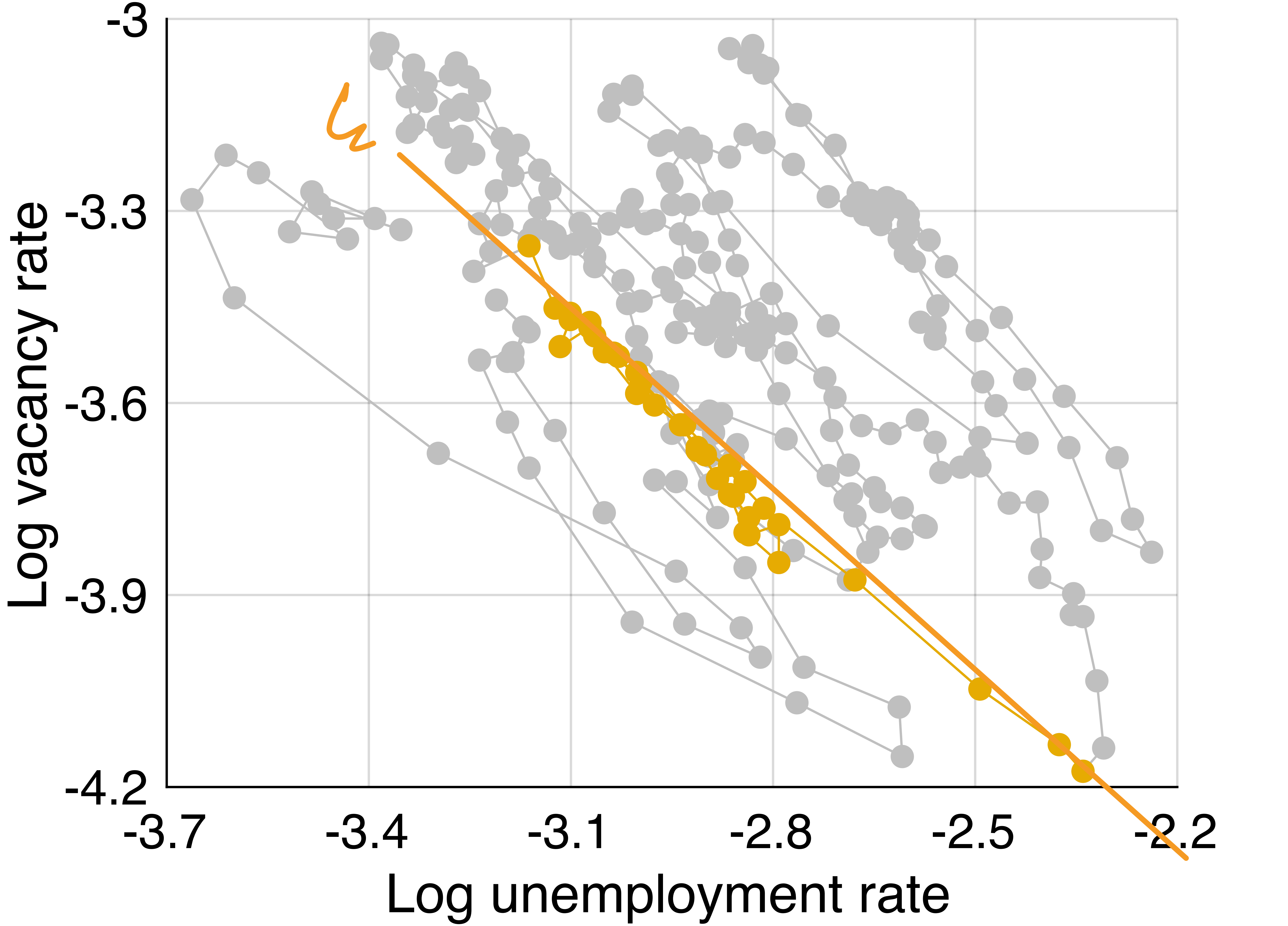
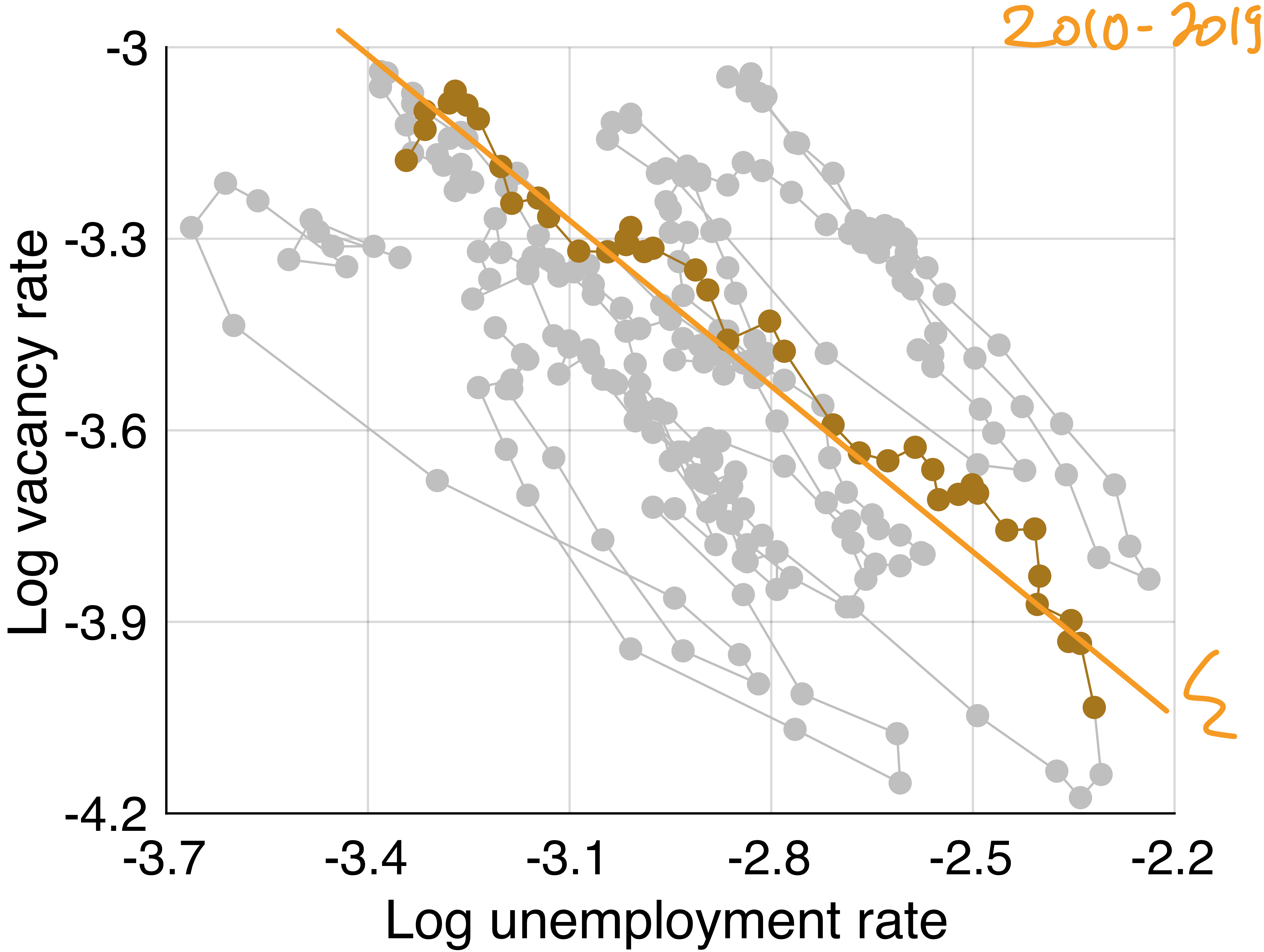
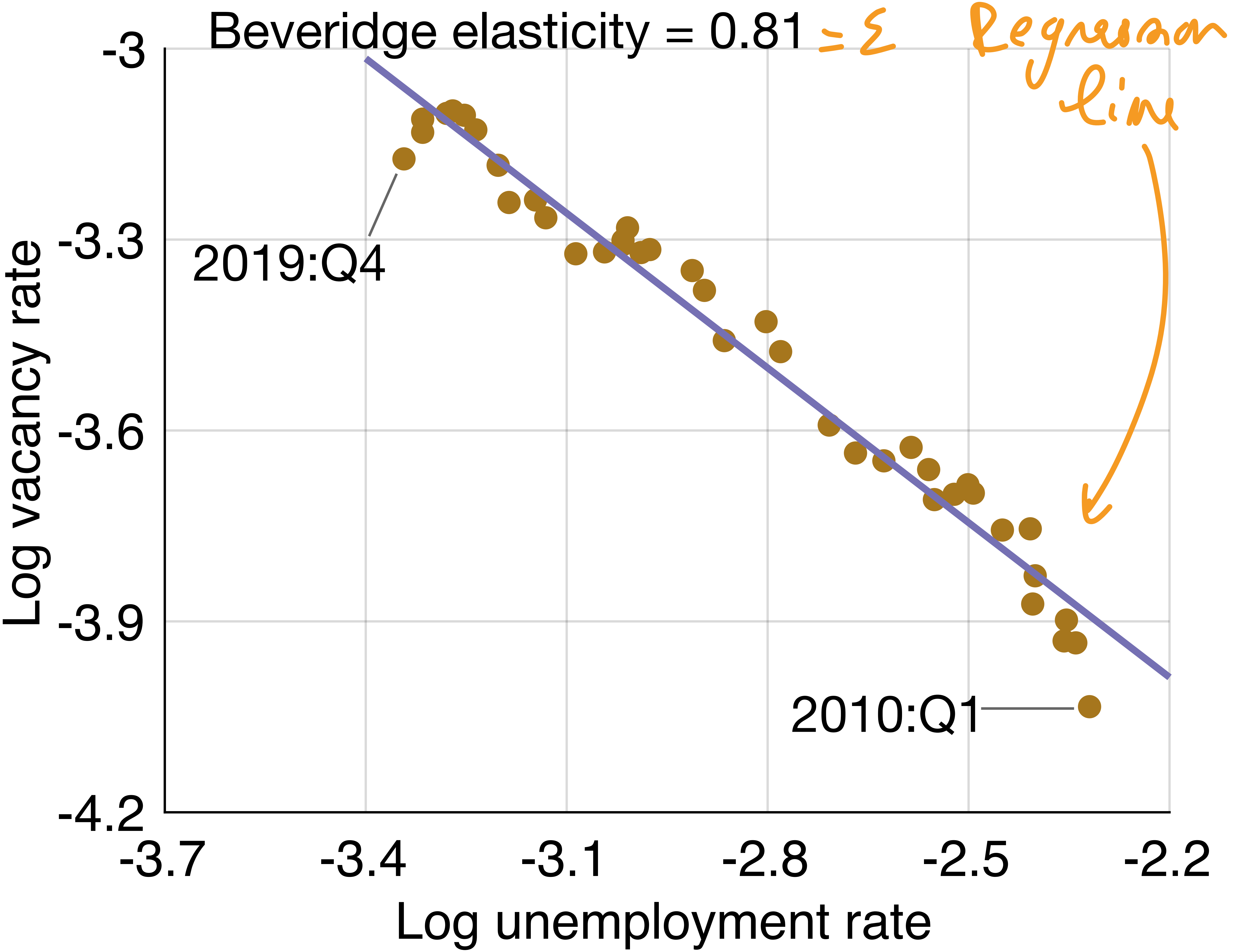
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate
ε

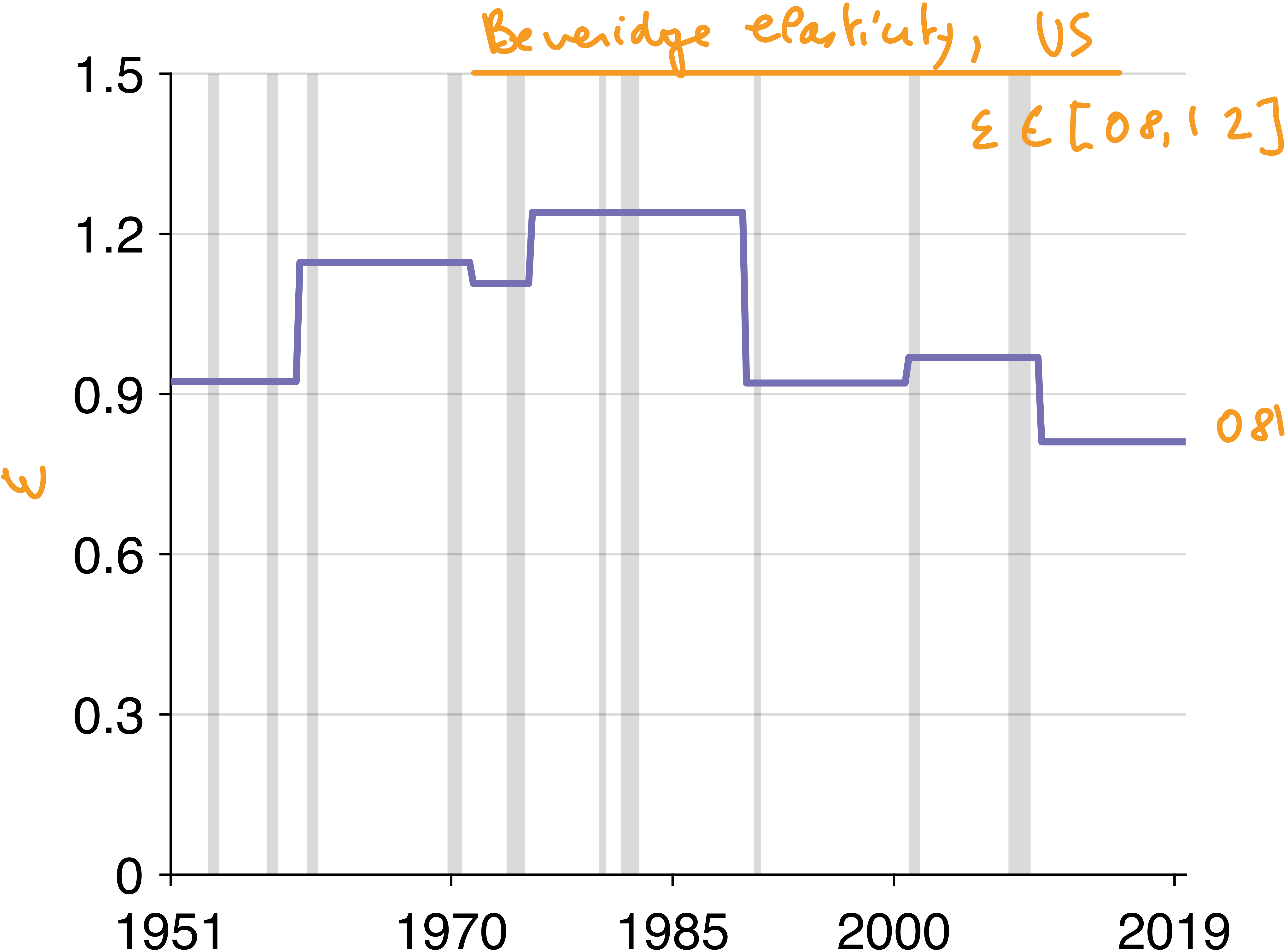


-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log vacancy rate
Log unemployment rate

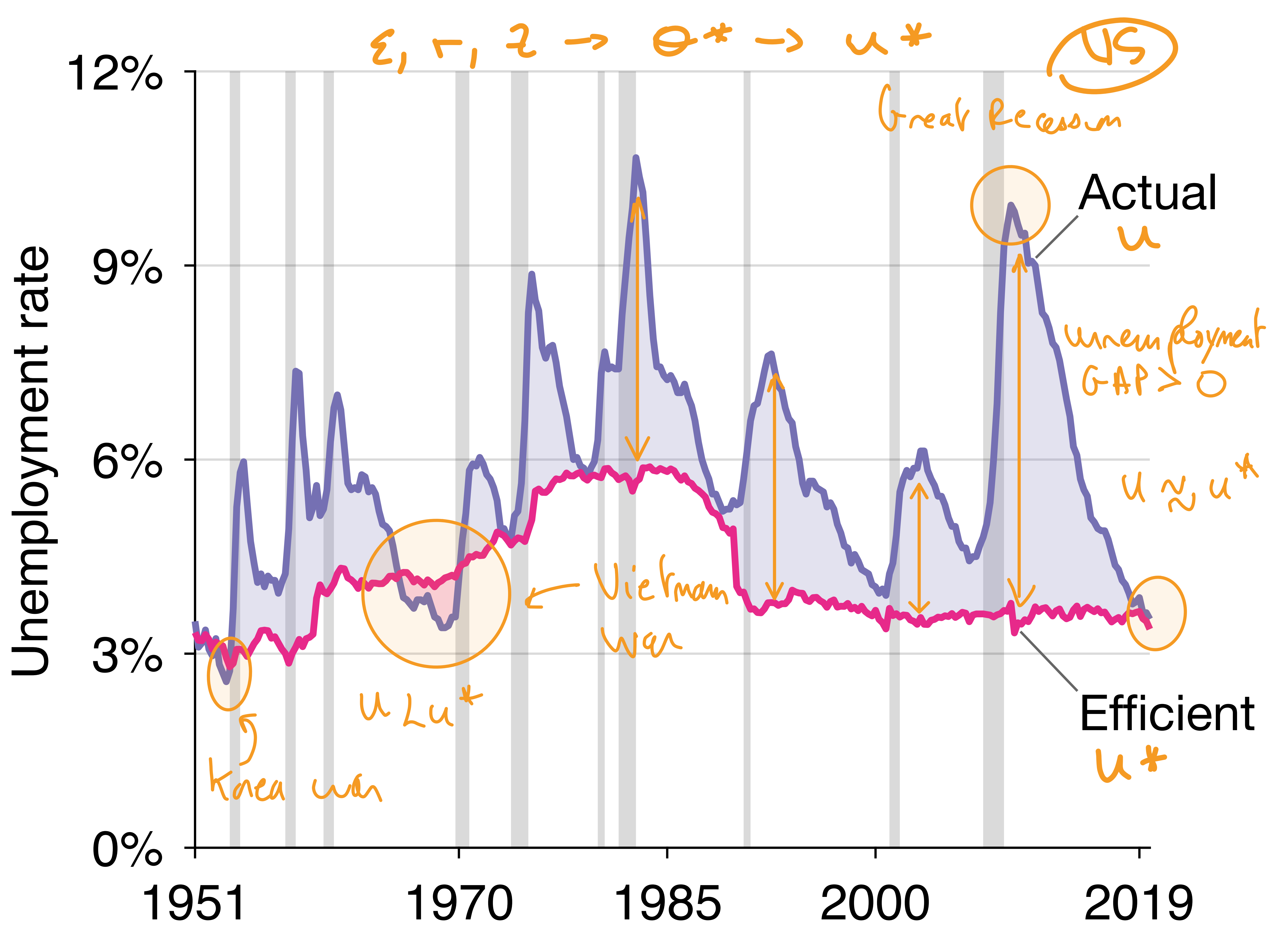

2010-2019
Log vacancy rate
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate

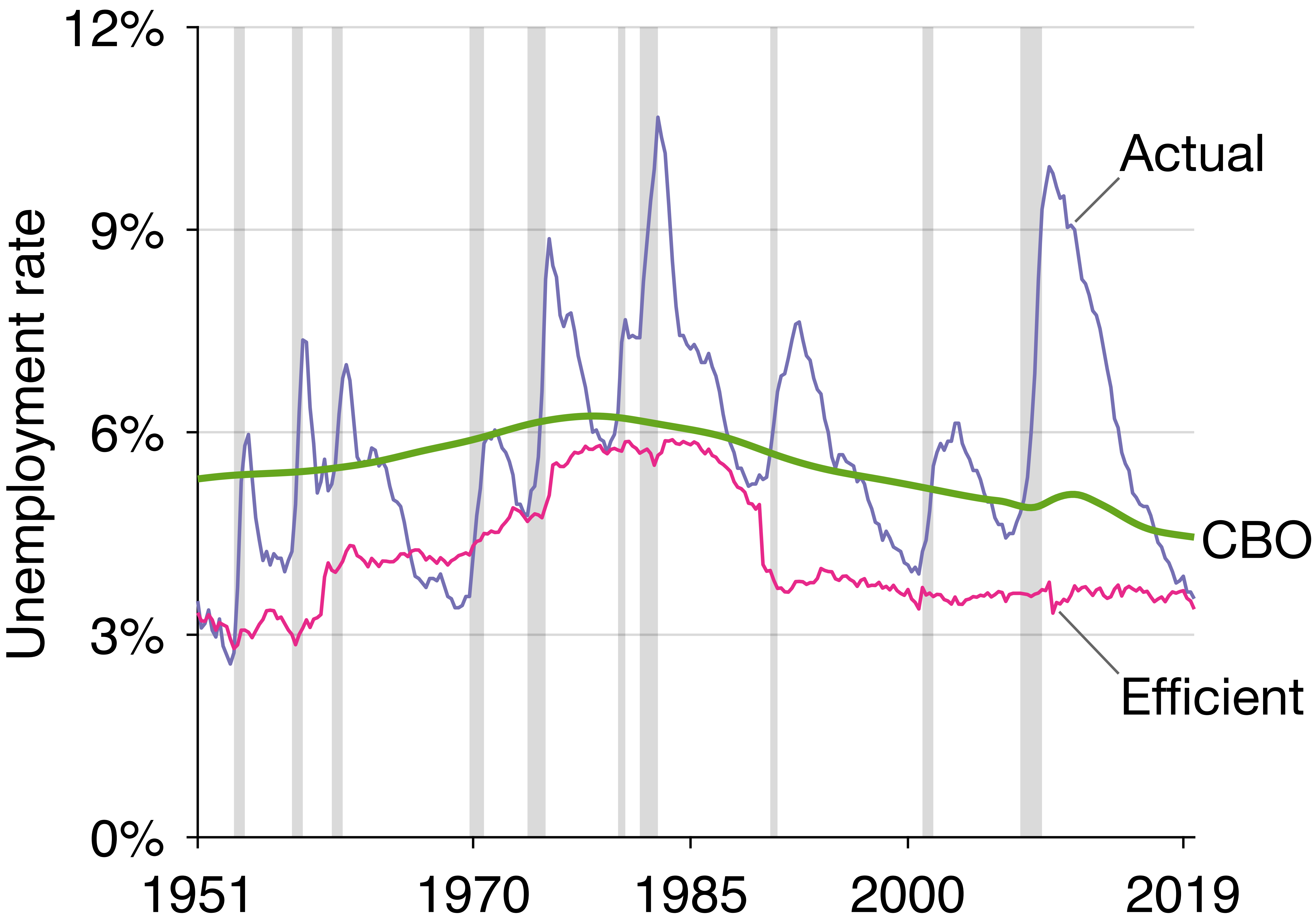


Beveridge elasticity = 0.81
$\approx \varepsilon$
Regression line
2019:Q4
2010:Q1
Log vacancy rate
Log unemployment rate
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2



Beveridge elasticity, US
ε ∈ [0.8, 1.2]
1.5
1.2
0.9
0.6
0.3
0
ε
0.81
1951
1970
1985
2000
2019


ε, r, z → θ* → u*
US
Great Recession
12%
9%
6%
3%
0%
Unemployment rate
Actual
u
unemployment GAP > 0
u ≈ u*
Efficient
u*
Vietnam war
u < u*
Korea war
1951
1970
1985
2000
2019



Unemployment rate
12%
9%
6%
3%
0%
1951
1970
1985
2000
2019
Actual
CBO
Efficient