

## AM2580 HOMEWORK1

Ziao Zhang

### 1. PROBLEM 1

1.(i). Find the smallest time  $t^*$  for which two characteristic lines of Burger's Equation:

$$\begin{cases} u_t + (\frac{u^2}{2})_x = 0 \\ u(x, 0) = \sin(x) \end{cases}$$

will intersect?

1.(ii). Can you generalize the result to the general case:

$$\begin{cases} u_t + f(u)_x = 0 \\ u(x, 0) = u_0(x) \end{cases}$$

### 2. PROBLEM 2

2.(i). How does  $u_x(x(t), t)$  behave along the characteristic line  $x = x(t)$ ?

2.(ii). If  $u_x = 0$  at the foot of the characteristic line  $x = x_0$ , how does  $u_x(x(t), t)$  behave along this characteristic line?

2.(iii). If  $u_x = 0$  at the foot of the characteristic line  $x = x_0$ , how does  $u_{xx}(x(t), t)$  behave along this characteristic line?