TSP Optimization by the Cellular ant algorithm

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ABSTRACT: This paper proposes a new kind of algorithm, the cellular ant algorithm, to solve the Minimum Ratio TSP(MRTSP). It expresses the cellular ant algorithm's mathematics model and detail account steps , get the good effect through experimental method to optimize the function.

Key Word: Cellular ant algorithm, Ant colony algorithm, cellular automata, TSP, Minimum Ratio TSP, function optimization

Ⅰ.INTRODUCTION

Many unsolved famous problems that have great challenges, such as those so called NP-hard problems like the traveling salesman problem（TSP）, degree-constrained minimum spanning tree problem, quadratic assignment problem, graph coloring problem etc. Since these problems have many applications in real situations, it is quite important to find some applicable algorithms. In recent years, there appeared several evolutionary algorithms for solving the NP-hard problems from nature in this field, typically like simulated annealing, genetic algorithm, ant algorithm, etc.

Ant colony algorithm (ACO) was first proposed by M.dorigo in 1991 and published at international magazine in 1996. Compared with other bionic algorithms for TSP, the ant algorithms have some good properties as a searching optimization approach in some test experiments and solving different discrete systems optimization problems successfully.

The concept of cellular automata was first proposed by Von Neumann in simulating system of living with reproducing. Wolfram et al took the method of dynamic system, computational theory and the method of form language for studying the cellular automata. Cellular automata were applied in many fields and have provided effective virtual laboratory in the field of large-scale simulation computing for studying the behavior of systems.

This paper proposes a new algorithm--cellular ant algorithm--for function and discrete systems optimization based on ant algorithm and cellular automata.

Ⅱ.ANT COLONY ALGORITHM(ACO)AND CELLULAR AUTOMATA

ACO is a parallel algorithm in essence. In the research on the behaviors of real ants, we found that the media used to communicate information among ant individuals regarding paths to decide where to go consist of pheromone trails. A moving ant leaves some pheromone on the ground, thus making the path it takes by a trail of this substance. While an isolated ant moves essentially at random, an ant encountering a previously laid trail can detect it and decide with high probability to follow it, thus reinforcing the trail with the pheromone of this following ant. The collective behavior that emerges is a form of autocatalytic behavior where the more ants are following a trail, the more attractive that trail becomes for being followed. The ants move from node to node based on a node transition rule. The node transition rule is probabilistic.

The standard cellular automata is a group include four elements:

$A=(L_d, S, N, f)$

A represent a cellular automata system; L represent cellular space, d is a positive integer, represent the dimension of the cellular automata space;

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S is a set of cellular's limited、discrete state; N represent a combination of all neighbor cellular;

f represent a local conversion function which S^N mapping to S。It expresses:

f: $S_t^N \rightarrow S_{t+1}$

The cellular is the essential part of cellular automata.

III.CELLULAR ANT ALGORITHM

As follows we take the TSP as the instance, discuss the cellular ant algorithm.

A traveling salesman problem(TSP)is a problem in which the salesman must pass each town only one time and the last town is the first one, with the objective to find a permutation of towns that minimizes the cost of the cycle.

The classical TSP mathematics model is as follow:

Definition 1 The set of cities C={c_1 , c_2 , …, c_n }, C is a set of cellular space; L={CellX=(c_1 , …, c_i , …, c_j …, c_n)|ci ∈C,ci≠cj, i,j=1,2,…n}

Definition 2 Cellular neighbor use extended Moore model:

N_{Moore}={CellY|diff(CellY-CellX)≤r CellX,CellY∈L}

Definition 3 Ant neighbor node transfer probability

$$P_{ij} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_k [\tau_{ik}]^{\alpha}[\eta_{ik}]^{\beta}}$$

η_{ij} edge arc(i,j) visibility;

τ_{ij} edge arc(i,j) intensity;

α track relatively essentiality (α≥0);

β visibility relatively essentiality (β≥0)。

Definition 4 Pheromone intension renovate equation:

$$\tau_{ij}^{new} = \rho \ \tau_{ij}^{old} + \sum_k \Delta \ \tau_{ij}^{k}$$

$\Delta\tau_{ij}^{k}$ Ant k edge arc(i,j) Pheromone amount

$$\Delta \ \tau_{ij}^{k} = \begin{cases} \frac{Q}{d_{ij}} \\ 0 \end{cases}$$

Q constant, Ant Pheromone amount

ρ intensity permanence (0≤ρ<1)

Definition 5 Cellular evolvement rules:

According as the definition of cellular neighbor , account the neighbor's aim explain, compare the difference of it, choose the best one.

The cellular ant algorithm main step is as follows:

Step1: nc←0; (nc is the iterative times or search times)

Initiate all the parameter，put m ants to the n peak points;

Step2: Put the ants initiate start to the solution set

Every ant k(k=1,⋯,m), according to probability P move to the neighbor node c, put the peak point to the solution set.

Step3: Account ants aim function Z_k (k=1,…,m);

Register the best solution;

Step4: According as the definition of cellular neighbor, evolvement in the fields of neighbor,record the best solution.

Step5: According as the renovate equation modify track intensity;

Step6: All edge arc(i,j), $\Delta\tau_{ij}$←0, nc←nc+1;

Step7: If nc <scheduled iterative numbers and do not have degenerate action, turn to step 2;

Step8: Output the current best solution.

IV.MINIMUM RATIO TSP(MRTSP) OPTIMIZATION

Minimum Ratio TSP(MRTSP)，expresses from classic TSP, it is a distortion question, assume that you can get some benefits moving from one city to the others, expresses as Pij，The MRTSP is to confirm the best path, which make the least ratio of overall journey and all benefits.

G=（V,E），V={1,2,⋯,10} is the vertexes， E is the edge set，distance matrix D=[dij]n*n，profits matrix P=[pij]n*n, Minimum Ratio TSP mathematics model is as follows:

$$\text{Min } Z = \frac{\sum_{i \neq j} d_{ij} \cdot x_{ij}}{\sum_{i \neq j} p_{ij} \cdot x_{ij}}$$

$$\text{s.t} \begin{cases} \sum_{j=1}^{n} x_{ij} = 1, & i \in V \quad (1) \\ \sum_{i=1}^{n} x_{ij} = 1, & j \in V \quad (2) \\ \sum_{i \in S} \sum_{j \in S} x_{ij} \le |S| - 1, & \forall S \subset V \quad (3) \\ x_{ij} \in \{0,1\} \end{cases}$$

If the distance matrix D=[dij]n*n:

$$D = \begin{pmatrix} \infty & 1 & 862 & 273 & 319 & 373 & 83 & 71 & 60 & 918 \\ 1 & \infty & 775 & 698 & 718 & 163 & 467 & 826 & 482 & 875 \\ 862 & 775 & \infty & 773 & 493 & 828 & 142 & 501 & 593 & 775 \\ 273 & 698 & 773 & \infty & 771 & 558 & 682 & 956 & 999 & 677 \\ 319 & 718 & 493 & 771 & \infty & 86 & 496 & 510 & 954 & 339 \\ 373 & 163 & 828 & 558 & 86 & \infty & 706 & 775 & 198 & 914 \\ 83 & 467 & 142 & 682 & 496 & 706 & \infty & 590 & 690 & 94 \\ 71 & 826 & 501 & 956 & 510 & 775 & 590 & \infty & 281 & 162 \\ 60 & 482 & 593 & 999 & 594 & 198 & 690 & 281 & \infty & 544 \\ 918 & 875 & 775 & 677 & 339 & 914 & 94 & 162 & 544 & \infty \end{pmatrix}$$

If the profits matrix P=[pij]n*n:

$$P = \begin{pmatrix} \infty & 32 & 203 & 672 & 162 & 426 & 475 & 841 & 294 & 368 \\ 32 & \infty & 328 & 845 & 307 & 330 & 247 & 280 & 150 & 288 \\ 203 & 328 & \infty & 917 & 888 & 21 & 144 & 22 & 10 & 651 \\ 672 & 845 & 917 & \infty & 709 & 207 & 593 & 645 & 244 & 296 \\ 162 & 307 & 888 & 709 & \infty & 773 & 882 & 574 & 685 & 8 \\ 426 & 330 & 21 & 207 & 773 & \infty & 989 & 750 & 144 & 785 \\ 475 & 247 & 144 & 593 & 882 & 989 & \infty & 865 & 288 & 629 \\ 841 & 280 & 22 & 645 & 574 & 750 & 865 & \infty & 878 & 274 \\ 294 & 150 & 10 & 244 & 685 & 144 & 288 & 878 & \infty & 961 \\ 368 & 288 & 651 & 296 & 8 & 785 & 629 & 274 & 961 & \infty \end{pmatrix}$$

Then , use the standard ant algorithm and cellular ant algorithm separately by iterative 1000 or 10000 times to solve the question.

From the table , two kind of functions can get the least ratio 0.40871, TSP trace loop is {1,4,2,6,5,3,7,10,9,8}, Total cost is 2845,total benefit is 6961, but the cellular ant algorithm's probability which to get the best result is higher than the standard ant algorithm, and the cellular ant algorithm's average result of iterativing 1000 times is better than standard ant algorithm's average result of iterativing 10000 times.

Table 1 The result of two algorithm

Function	iterative times	Test data			Average result	Best result
Standard Ant algorithm	1000	0.55382	0.40871	0.48907	0.49677	0.40871
		0.46637	0.47959	0.59979		
	10000	0.45094	0.40871	0.49325	0.48696	0.40871
		0.56467	0.56280	0.40871		
Cellular Ant algorithm	1000	0.51355	0.47759	0.46637	0.47980	0.40871
		0.52057	0.50354	0.40871		
	10000	0.47959	0.52148	0.46637	0.46066	0.40871
		0.40871	0.47800	0.47959		

In short, the paper provides a new kind of algorithm for NP-hard problems such as Minimum Ratio TSP(MRTSP)and gives the definition and steps of the algorithm. Practically, This paper solves the classical TSP by cellular ant algorithm through typical instances. The computational results show the effectiveness of the algorithm in experimental method.

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