- Chapter 3 [Each 10 points]

P52: 3.1-1

By definition of $\Theta(g(n))$: $f(n) \in g(n)$ if

$c_1 g(n) \le f(n) \le c_2 g(n)$

let c_1 = max

let c_2 = 2 * max (For when both f(n) and g(n) are equal)

s.t. $\max[f(n), g(n)] \le f(n) \le 2 * \max[f(n), g(n)]$

and $\max[f(n), g(n)] \in \Theta(f(n) + g(n))$

which we know from our text can be used interchangeably with

$\max[f(n), g(n)] = \Theta(f(n) + g(n))$

P53:

3.1-4

$2^{n+1} = O(2^n)$ is true, because there is a constant multiple of *c*, that exists

s.t. $2^{n+1} \le c * 2^n$ when $c \ge 2$

$2^{2n} = O(2^n)$ is false, because when solving for c,

$2^n * 2^n \le c * 2^n$

$2^n \le c$

This inequality cannot be true because there is no constant, *c*, that guarantees it to be true

3.1-8

$\Omega(g(n, m))$ = {f(n, m) : there exist positive constants c, n_0 and m_0 such that

$0 \le cg(n, m) \le f(n, m)$ for all $n \ge n_0$ or $m \ge m_0$}

$\Theta(g(n, m))$ = {f(n, m) : there exist positive constants c_1, c_2, n_0, and m_0 such that

$0 \le c_1 g(n, m) \le f(n, m) \le c_2 g(n, m)$ for all $n \ge n_0$ or $m \ge m_0$}

P61: 3-1

(a)

If $k \ge d$, then $p(n) = O(n^k)$

If $(O(n^k))$ = {p(n) : there exist positive constants c and n_0 such that

$0 \le p(n) \le c * (n^k)$ for all $n \ge n_0$

given $k \ge d$, and Let $c = Xa_i$ s.t. $X > 1$

}

Thus the inequality hold true, and $p(n) = O(n^k)$ when $k \geq d$.

(c)
If $k = d$, then $p(n) = \Theta(n^k)$

If $(\Theta(n^k)) = \{p(n)$: there exist positive constants c_1, c_2, and n_0 such that
$0 \leq c_1 * (n^k) \leq p(n) \leq c_2 * (n^k)$ for all $n \geq n_0$
given $k \geq d$, and Let $c_1 = Xa_i$ s.t. $X < 1$ and $c_2 = Ya_i$ s.t. $Y > 1$
}
Thus the inequality holds true, and $p(n) = \Theta(n^k)$ when $k = d$.

(e)
If $k < d$, then $p(n) = \omega(n^k)$

If $(\omega(n^k)) = \{p(n)$: for any positive constant $c > 0$, there exists a constant $n_0 > 0$ such that
$0 \leq c * (n^k) < p(n)$ for all $n \geq n_0$
given $k < d$, and Let $c = Xa_i$ s.t. $X < 1\}$
Thus the inequality holds true, and $p(n) = \omega(n^k)$ when $k < d$.
P61-62: 3-3
a)
$2^{2^{\wedge}(n+1)} \rightarrow 2^{2^{\wedge}n} \rightarrow (n+1)! \rightarrow n! \rightarrow e^n \rightarrow n * 2^n \rightarrow 2^n \rightarrow (3/2)^n \rightarrow n^{\lg\lg n} = (\lg n)^{\lg n} \rightarrow$
$(\lg n)! \rightarrow n^3 \rightarrow n^2 = 4^{\lg n} \rightarrow n \lg n = \lg(n!) \rightarrow n = 2^{\lg n} \rightarrow (\sqrt{2})^{\lg n} \rightarrow 2^{\sqrt{(2 \lg n)}} \rightarrow$
$\lg^2 n \rightarrow \ln n \rightarrow \sqrt{(\lg n)} \rightarrow \ln \ln n \rightarrow 2^{\lg * n} \rightarrow \lg * (\lg n) = \lg * n \rightarrow \lg(\lg * n) \rightarrow$
$1 = n^{1/\lg n}$

b)
Let $f(n) =$ { n^2 when n is even
{ 0 when n is odd

- Chapter 4 [Each 10 points]
P74-75: 4.1-5

```
void linearSubMax (int A[], int low, int high, int* leftSub, int* rightSub, int *max) {
    *max = 0;
    int currentSum = 0;
    *leftSub = 0;
    *rightSub = 0;
    int left = 0;
    for (int i = low; i < high; i++)
    {
        currentSum = currentSum + A[i];
        if (i == low)
        {
            *max = currentSum;
        }
        if (currentSum > * max)
        {
            *max = currentSum;
            *leftSub = left;
            *rightSub = i;
        }
        if (currentSum < 0)
        {
            currentSum = 0;
            left = i + 1;
        }
    }
}
```

P87: 4.3-1
The value of $O(n^2) = (n(n+1))/2$
Assume “$T(n) = T(n-1) + n$” is equal to “$O(n^2)$”
then if $n = 1$, $T(n)$ should equal a constant value,
which is $T(1) = (1(1+1)) / 2) = 1$
which can also be shown as $T(1) = T(0) + 1 = 0 + 1$.
With $T(1)$ as the base case,
Assuming $T(n) = (n(n+1))/2$, and
$T(n) = T(0) + 1 + 2 + 3 + 4 \dots (n-2) + (n-1) + n$
then $T(n) = O(n^2)$

P92: 4.4-2
$T(n) \quad = \sum_{i=0}^{\lg n}(n/2^i)^2$
$\quad\quad\quad = O(n^2)$
$T(n) \quad = T(n/2) + n^2$

$T(n) = cn^2/4 + n^2$
$= (c/4 + 1) * n^2$
Let $c \geq c/4 + 1$

$T(n) \leq cn^2$

P96-97: 4.5-3
a = 1 and b = 2,
According to the master method, if $f(n) = \Theta(n^{logb(a)})$, then $T(n) = \Theta(n^{logb(a)}lgn)$

$n^{log2(1)} = 1$
and $f(n) = \Theta(1)$, so $f(n) = \Theta(n^{logb(a)})$,
so $T(n) = \Theta(1*lgn)$
$T(n) = \Theta(lgn)$

P107: 4-1
a)
$a = 2, b = 2, f(n) = n^4$
$n^{log2(2)} = n$
$T(n) = \Theta(n^4)$
b)
$a = 1, b = 10/7, f(n) = n$
$n^{log10/7(1)} = 1$
$T(n) = \Theta(n)$
c)
$a = 16, b = 4, f(n) = n^2$
$n^{log4(16)} = n^2$
$T(n) = \Theta(n^2lgn)$

d)
$a = 7, b = 3, f(n) = n^2$
$n^{log3(7)} = n^{1.77}$
$T(n) = \Theta(n^2)$

e)
$a = 7, b = 2, f(n) = n^2$
$n^{log2(7)} = n^{2.80}$

$T(n) = \Theta(n^{2.80})$

f)

$a = 2$, $b = 4$, $f(n) = \sqrt{n}$

$n^{\log 4(2)} = \sqrt{n}$

$T(n) = \Theta(\sqrt{n}\ \lg n)$

g)

$T(n) = T(n-2) + n^2$

$T(n) = T(n-4) + (n-2) + n^2$

$T(n) = T(n-6) + (n-4)^2 + (n-2)^2 + n^2$

$T(n) = {}^{n/2}{}_{i=0}\Sigma(n - 2i)^2$

$T(n) = \Theta(n^3)$