Mathematical Modeling of the Spread of Opinions Between Populations

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Abstract

This paper explores the mathematical modeling of the exchange of two opinions between two population groups, separated by age. The primary models used in our research are the Leslie Matrix, SIR model, ISAv model, and several type-change graphs. Earlier in our research, the purpose of these models were to depict hypothetical scenarios for different populations with arbitrary ideas, however we are now looking at modeling the spread of opinions on getting vaccinated using data from the ANES government database.

1 Introduction

We are trying to model different age groups and how they interact to exchange ideas. Different countries have different cultures, which influence which age groups interact regularly. For example, in European countries grandparents are much less likely to live with their children and grandchildren compared to in Asian countries. In our paper, we analyze the idea of vaccines by creating two different groups of populations. One group has a positive opinion towards vaccines and the other has an opposing negative opinion towards vaccines. We analyze how the popularity of an idea and the age of an individual determines the concentration of an idea.

2 Methods

In order to study patterns in a population, we must first recognize that all individuals are not the same. Age-structured population models allow us to study interactions between different demographics. Discrete models of age-structured populations were first created by Leonhard Euler in the 18th century. Age-structured models do not consider population density as a factor of survival; they only look to age. The age-structured model can also be written in the form of a matrix, commonly known as the Leslie Matrix. The Leslie matrix demonstrates that the ratio of population sizes in each age class will become constant over time, and after a few fluctuations, the total population size will grow exponentially[3].

2.1 Leslie Matrix

To introduce an discrete age-structured population system with type-changing, we first have to define how will the population grow. The Leslie Matrix mentioned in the introduction part can provide a steady exponential growth. This model can be represented by two equations:

$$\begin{aligned}
x_0(n+1) &= a_0 x_0(n) + a_1 x_1(n) + a_2 x_2(n) + ... \\
&= \sum_{i=1}^{N} a_i x_i(n) \\
x_i(n+1) &= \delta_{i-1} x_{i-1}(n)
\end{aligned}$$

$$\begin{aligned}
dS/dt &= -aS(t)I(t) \\
dI/dt &= aS(t)I(t) - bI(t) \\
dR/dt &= bI(t)
\end{aligned}$$

The variable $x_n(t)$ represents the number of population x at time t. δ_{n-1} is the probability of survival of a population age group to the next year and a_n is the fertility rate of demographic n. If we plot the Leslie Matrix of the total population, it appears as Figure 1.

2.2 SIR Model

Another model which can be used to model the spread of opinions is the SIR model. It is traditionally used to track the spread of infections. The model represents individuals susceptible to contracting the disease, those already infected, and the final category of those who have recovered after being infected. Translating this model to our purpose of studying the transfer of opinions amongst various population demographics, we are using the SI parts of the SIR model, since no one can "recover" from an opinion[1].

Although the Leslie Matrix is a useful base to simulate a steadily growing population, it must be modified to represent the spread of ideas. Thus, we incorporate the SIR model which is typically used to model the spread of infectious diseases. A model of the flow of SIR is shown in Figure 2. The susceptible, infected, and recovered populations are represented in the form of a derivative of S, I, and T. Thus, these equations demonstrate the rate of change for each population.

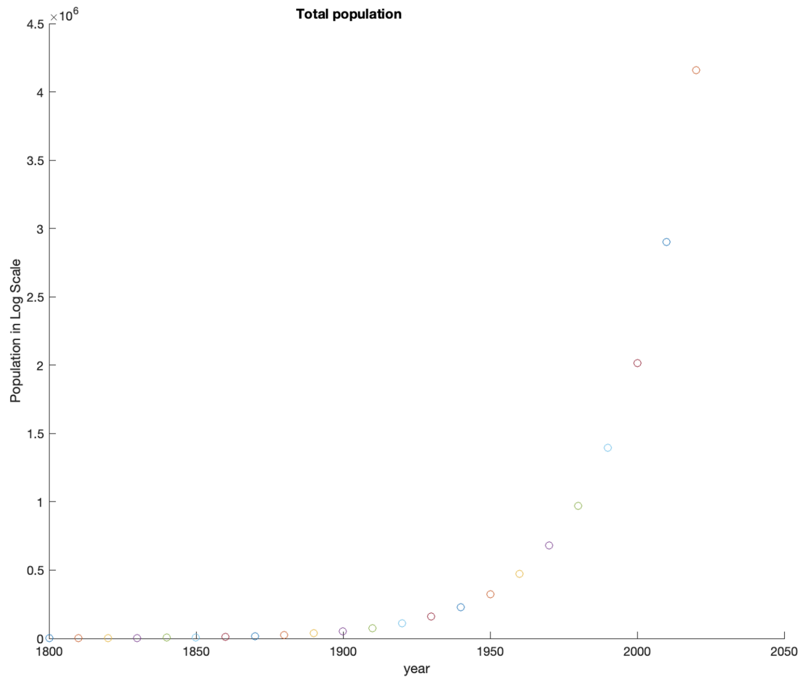


Figure 1: Total Population Graph

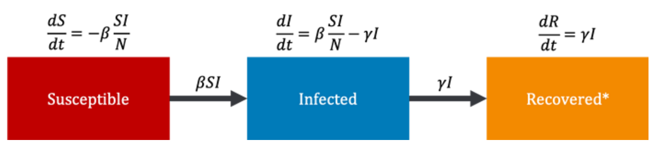


Figure 2: SIR model flowchart

2.3 ISAv Model

Besides SIR, the Infectious-Salmon Anemia-Virus (ISAv) is also a common model to analyze a disease-free system. The difference is that ISAv focuses on severer diseases. Also, in an ISAv model, the period of demographic population cycles won't affect infectious populations, but it could predict the SIR model's infectious populations[4].

In the susceptible equation for dS/dt, $-\beta$ means the susceptible population will always decrease, reaching a point where no more people will get infected. This variable is multiplied by SI/N because a greater number of interactions between susceptible and infected people will create a higher chance for the susceptible group to get infected. In the infected equation for dI/dt, β stands for a constant rate that people will become infected, and is the same rate represented in the susceptible equation. It is subtracted by yI, where y is a variable of the rate at which infected populations recover proportional to the number of infected people (I). Lastly, in the recovered equation for dR/dt, the population is equivalent to the yI subtracted in the dI/dt equation.

2.4 Type Change Populations

$$x_1^{(1)}(n+1) = \sum_{i=1}^{N} a_i x_i^{(1)}(n)$$

$$x_i^{(1)}(n+1) = \delta_{i-1} x_{i-1}^{(1)}(n)(1 - B\sum_{j=i}^{N} x_j^{(2)}(n))$$

$x_1^{(1)}$ = first age group of type 1
a_i = fertility rate of age group i
$x_i^{(1)}$ = population size of age group i for type 1
δ_{i-1} = turnover/survival rate of the previous age group
$x_{i-1}^{(1)}$ = population size of previous age group for type 1
B = rate of change from type 1 to type 2
$\sum_{j=i}^{n} x_j^{(2)}$ = population size of people in the same age group or older

$$x_1^{(2)}(n+1) = \sum_{i=1}^{N} a_i x_i^{(2)}(n)$$

$$x_i^{(2)}(n+1) = \delta_{i-1} x_{i-1}^{(2)}(n) + \delta_{i-1} x_{i-1}^{(1)}(n)(B\sum_{j=i}^{N} x_j^{(2)}(n))$$

$x_1^{(2)}$ = age group i of type 1
a_i = fertility rate corresponding to age group i
x_i = turn over rate corresponding to age group i
B = rate of changing from type 1 to type 2

The two equations above model how opinions are shared between populations of type 1 and type 2. Based on the fertility rates of certain age groups, they produce more children who will adopt the same ideas as them, and will therefore yield a larger turnover for people with the same ideas into the next generation.

2.5 Matrix Equations

In order to incorporate the interaction rates between different age groups from real world populations, we create a system of equations that adapts to data matrices, building on the previous equations for type change. Matrices from different countries represent levels of interaction between different age groups, and in order to follow our model of learning from those who are the same age group or older, we developed a model which finds the sum of the interactions between those in the same age group and every age group above. By setting $i \leq j$, we can find each interaction rate following this principle and add them together. Thus, we can replace the previous model which measured interactions based on population sizes with a measure of real world interactions.

$$x_1^{(1)}(n+1) = \sum_{i=1}^{N} a_i x_i^{(1)}(n)$$

$$x_i^{(1)}(n+1) = \delta_{i-1} x_{i-1}^{(1)}(n)(1 - B \sum_{y \leq z}^{N} M_{y,z})$$

$$x_1^{(2)}(n+1) = \sum_{i=1}^{N} a_i x_i^{(2)}(n)$$

$$x_i^{(2)}(n+1) = \delta_{i-1} x_{i-1}^{(2)}(n) + \delta_{i-1} x_{i-1}^{(1)}(n)(B \sum_{y \leq z}^{N} M_{y,z})$$

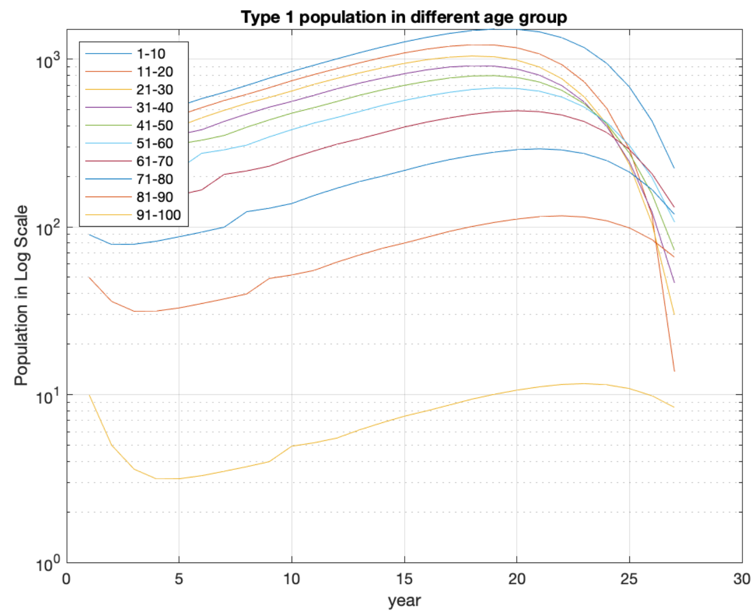
$\sum_{i \leq j}^{n}$ = sum of values in the matrix from i to n, where i is less than or equal to j
$M_{i,j}$ = value in the matrix at column i row j

3 Results

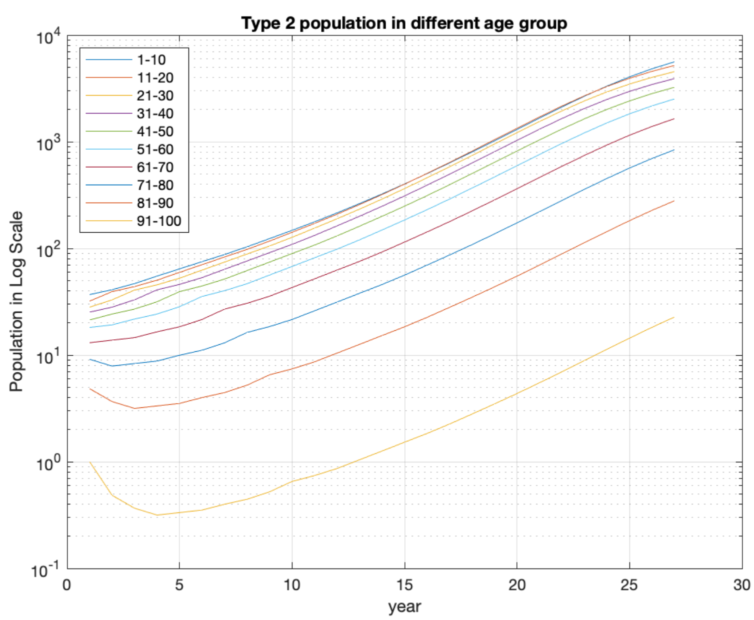
Initially, we hoped to simulate the populations of both types or opinions on the same graph, using percentages to greater show their relative concentrations. Thus, Figure19 shows a simulation of the total population of both type groups and ignores age groups for now. We also played around with the possibility that an idea being studied was relatively new and popular among those in younger age groups.

In order to create a simulation of our model of the type changes, we tested out using 5 age groups for each type and plotted them against each other, labeling type 1 with normal lines and type 2 with dashed lines. As shown in figure 9, type 2 began with a small population in the second age group, and steadily increased as the conversion rate was in a single direction. This caused type 2 populations to exceed type 1 populations following year 40.

Using a matrix of data which includes the interactions between 16 different age groups, we are able to calculate what the interaction will look like for each country[2]. In figure 7, the popularity of an opinion is represented using percentages. All of the type 1 population is shown using normal lines, while type

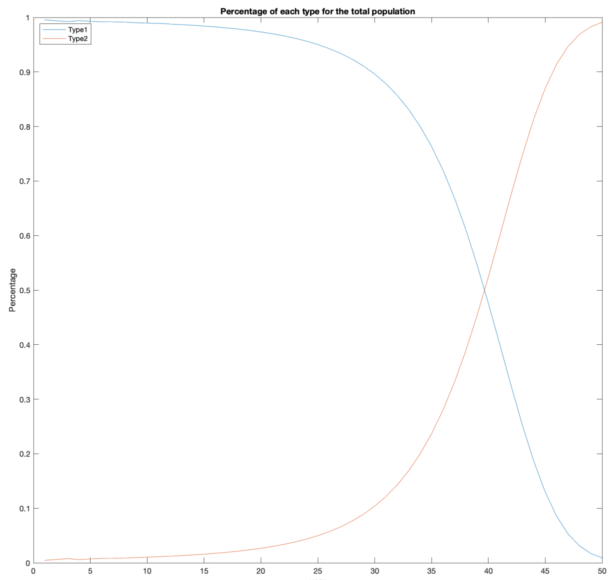


(a) Type 1

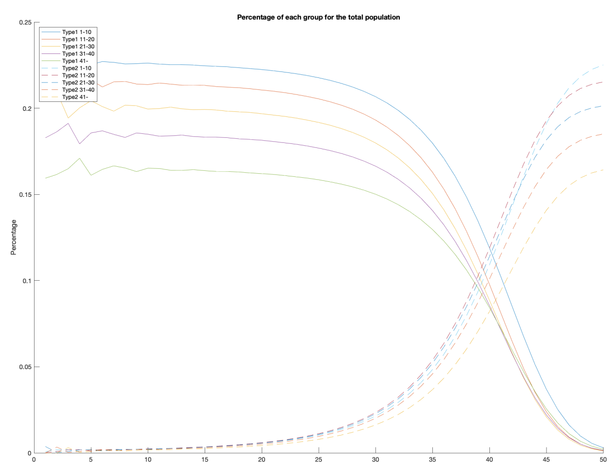


(b) Type 2

Figure 3: Type 1 and Type 2 With 10 Age Groups



(a) Two type change graph percentage



(b) 5 groups type change graph percentage

Figure 4: Type Change Represented In Percentages

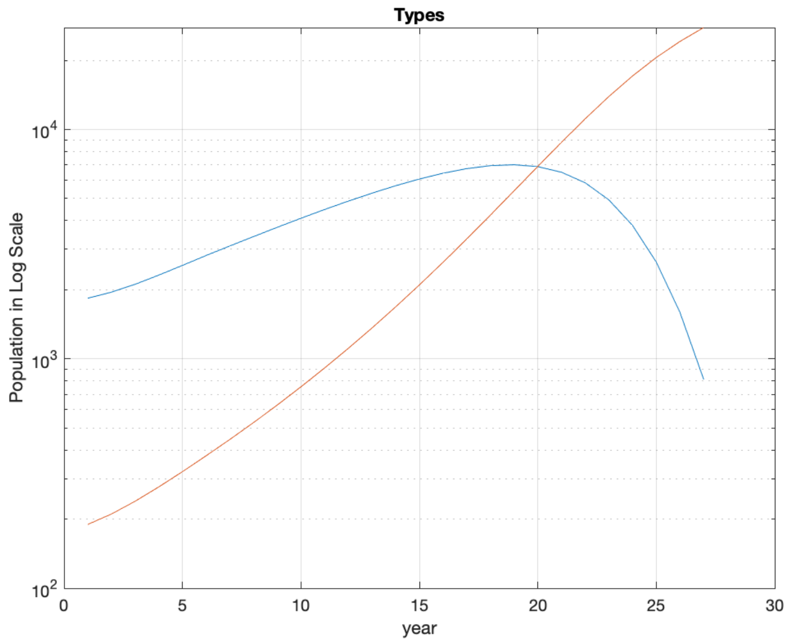


Figure 5: Two type change graph

2 populations are shown using dashed lines. Here, the type 2 population begins with a few people in the younger age groups, but as the idea becomes more popular, the dashed lines take greater percentage than the normal lines.

3.1 Comparing Countries

Based on social contact matrices for 16 age groups, we selected 12 countries to compare. The matrix data allows us to analyze the rate of social exchange in these different countries, and their differences can be analyzed based on locations, demographic and household structure, and other metrics like work participation and school enrollment.

4 Discussion

Using our models and code to simulate the interactions between different opinions of populations and study how age groups factor into that interaction can be used in a variety of fields. For example, the popularity of vaccines among the public can be studied based on mathematical models that predict dangerous levels of negative sentiment. These models could be used to prepare programs needed to educate the public on the importance of vaccines and cre-

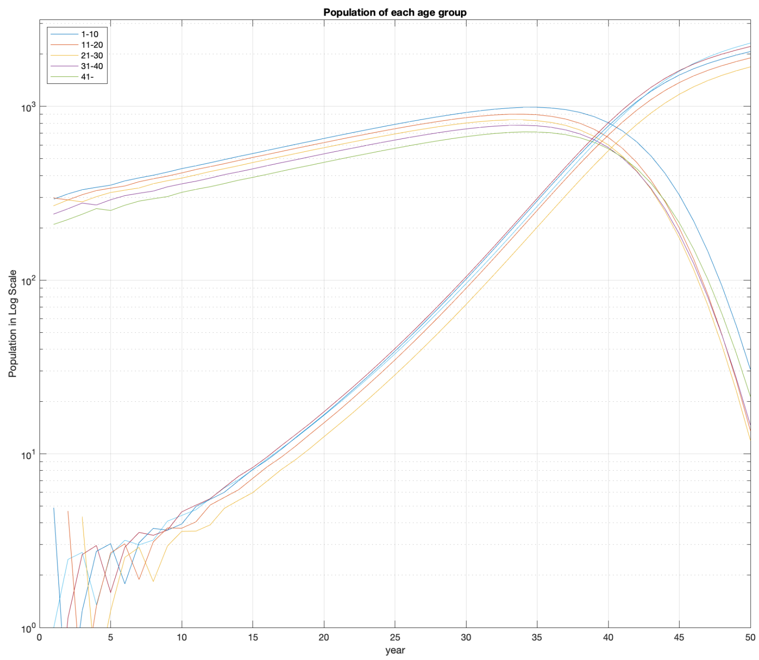


Figure 6: Population Of Each Age Group

ate target numbers for which demographics are the most vulnerable. Although our research has focused on using interaction rates from real world matrices for individual countries, further research can be done comparing our simulated models of interaction and their proposed outcomes to the real results of census data which show how the opinions of people in different countries change over time. These comparisons could be used to further legitimize or improve upon the models to account for more complex variables besides levels of interaction.

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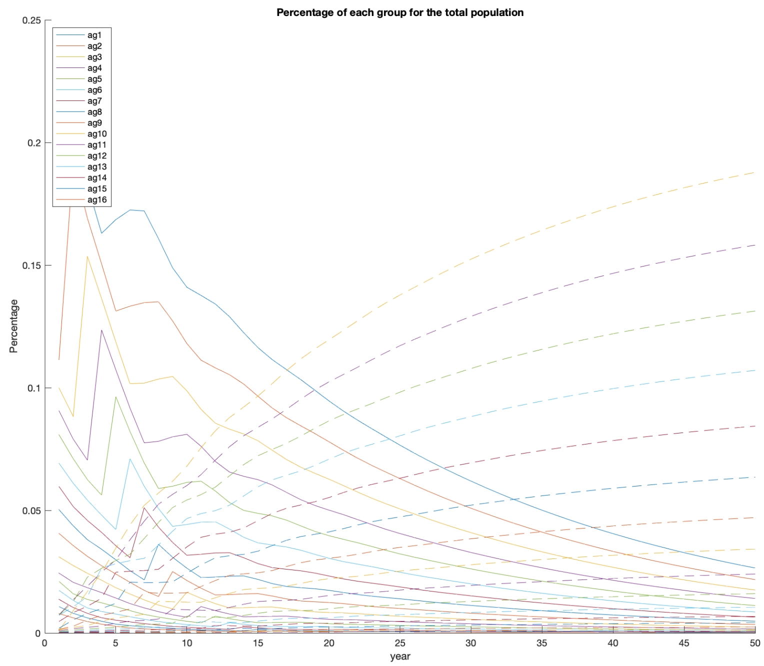

Figure 7: Matrix Albania

References

[1] Zhao Cao. The discrete age-structured seit model with application to tuberculosis transmission in china. *ScienceDirect*, 2012.

[2] K Prem, AR Cook, and M Jit. Projecting social contact matrices in 152 countries using contact surveys and demographic data. *PLOS Computational Biology*, 2017.

[3] Mark Rees and Stephen P. Ellner. Age-structured and stage-structured population dynamics. *Princeton: Princeton University Press*, pages 155–165, 2009.

[4] P. van den Driessche and Abdul-Aziz Yakubu. Age structured discrete-time disease models with demographic population cycles. 14(1):308–331.

Country	Size of Type 2 After 50 Years
Albania	7.0543e+06
Argentina	7.0594e+6
Australia	7.0447e+6
Canada	7.0283e+6
China	7.0565e+06
France	7.0213e+06
India	7.0542e+06
Iraq	7.0592e+06
Japan	7.0167e+6
Jamaica	7.0587e+06
Mexico	7.0600e+06
Morocco	7.0570e+06

Table 1: Rate of Exchange of Ideas Based on Type 2 Population

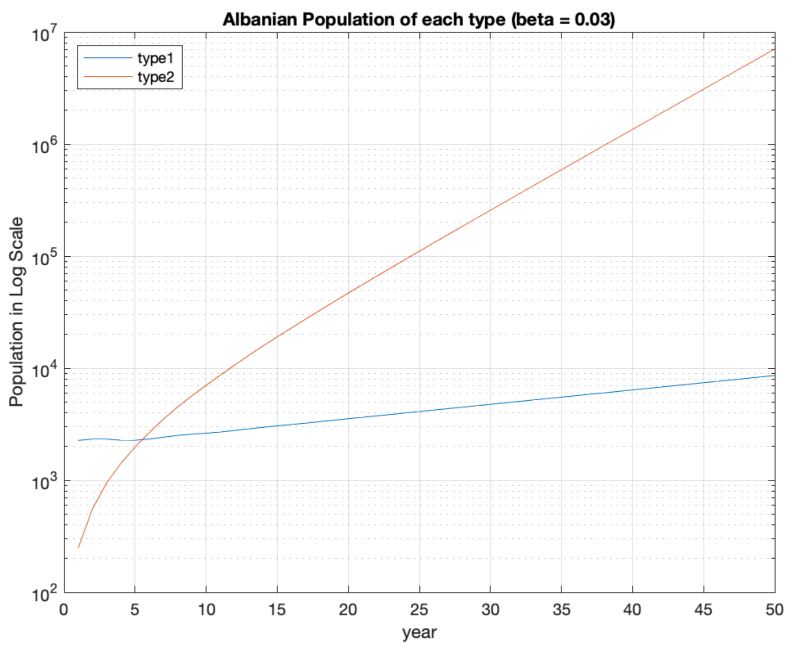

Figure 8: Albania

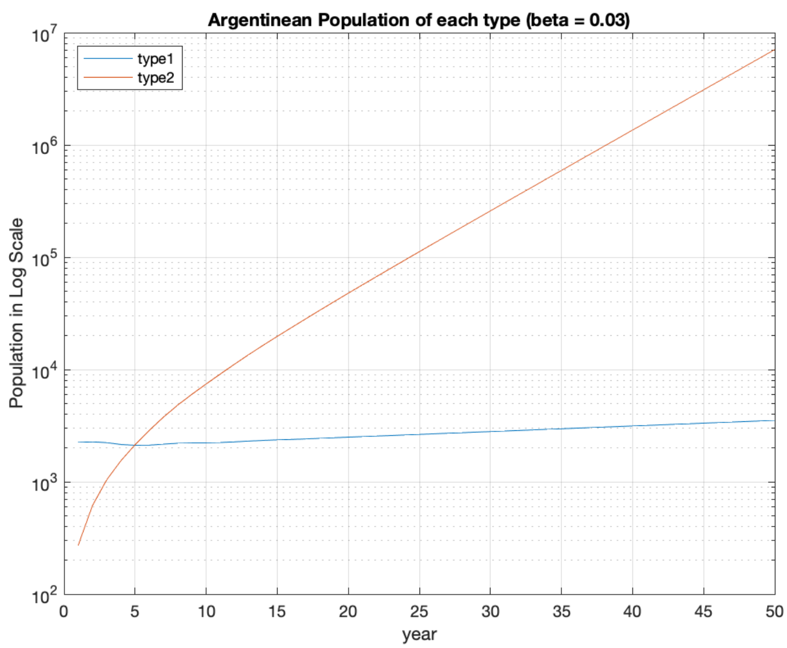

Figure 9: Argentina

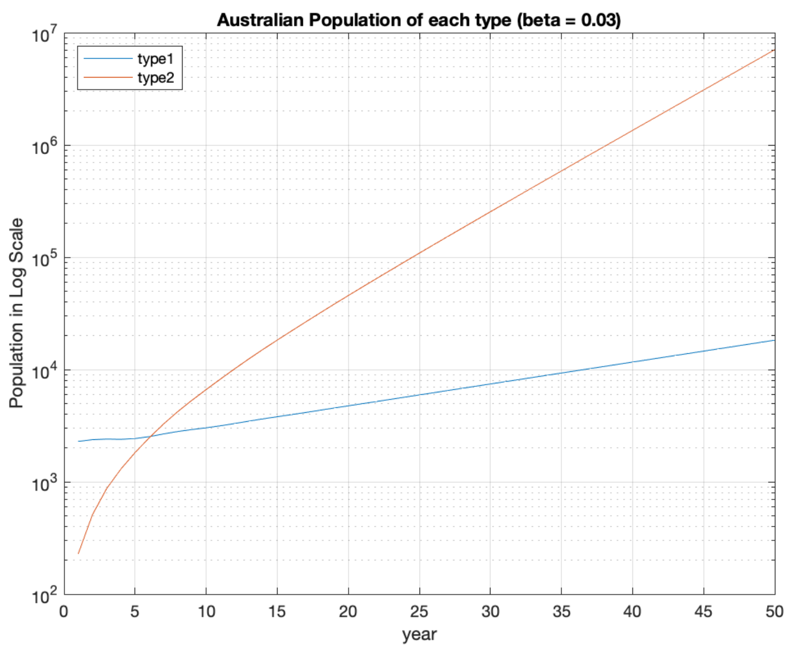

Figure 10: Australia

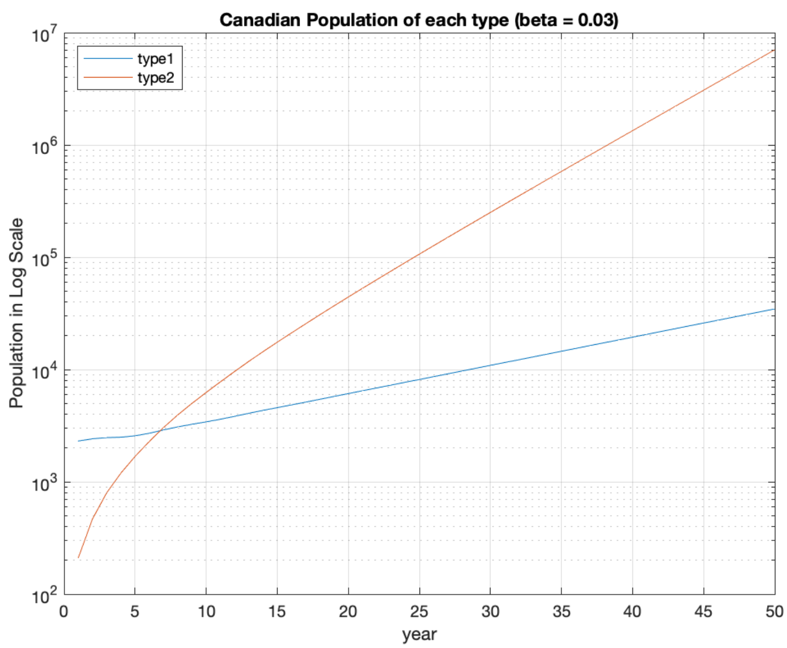

Figure 11: Canada

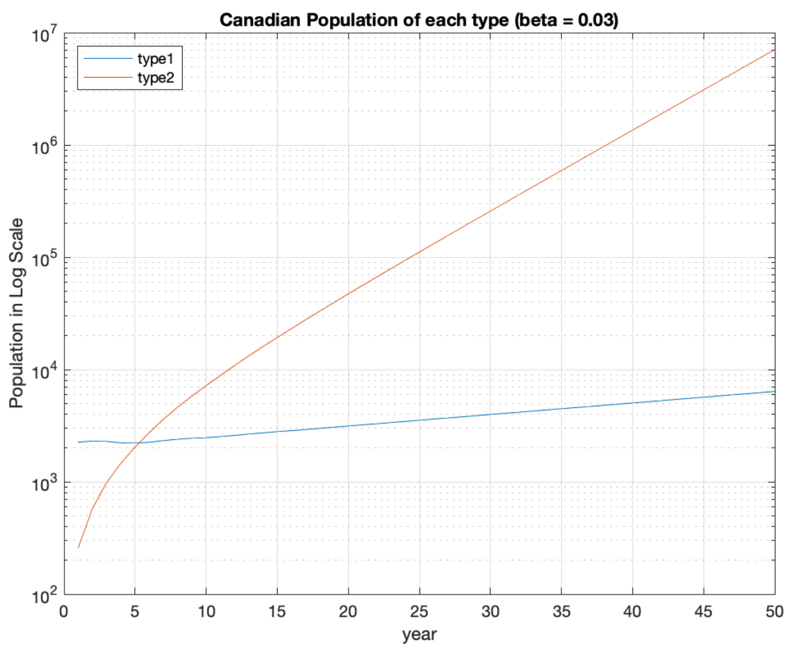

Figure 12: China

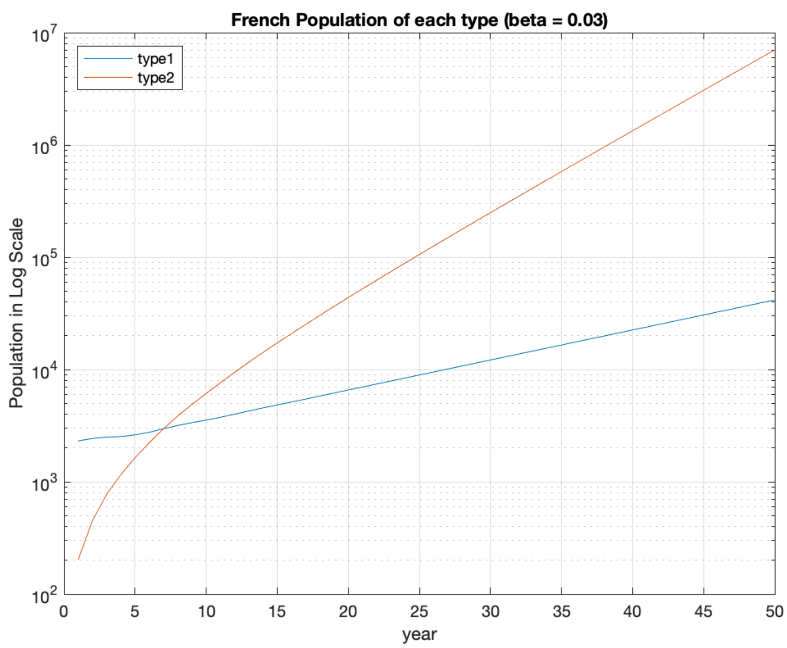

Figure 13: France

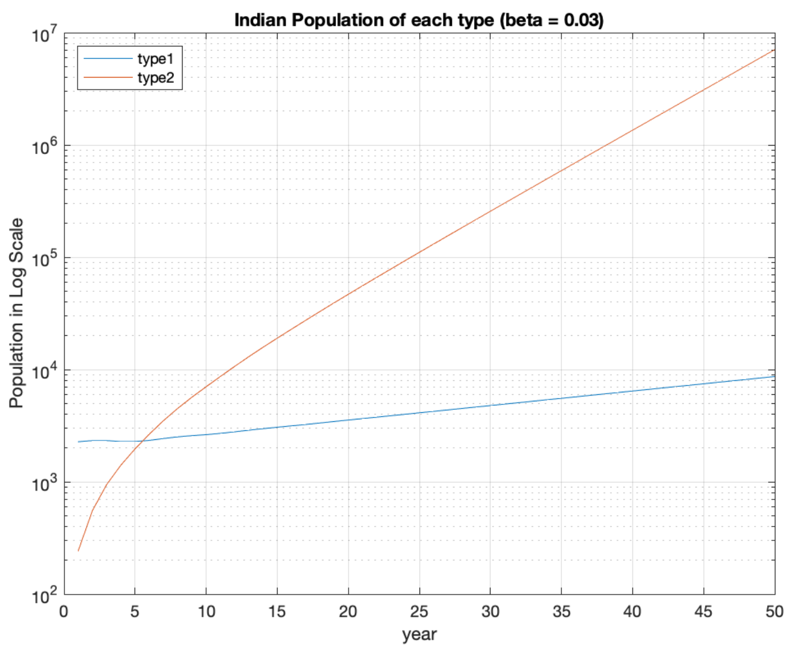

Figure 14: India

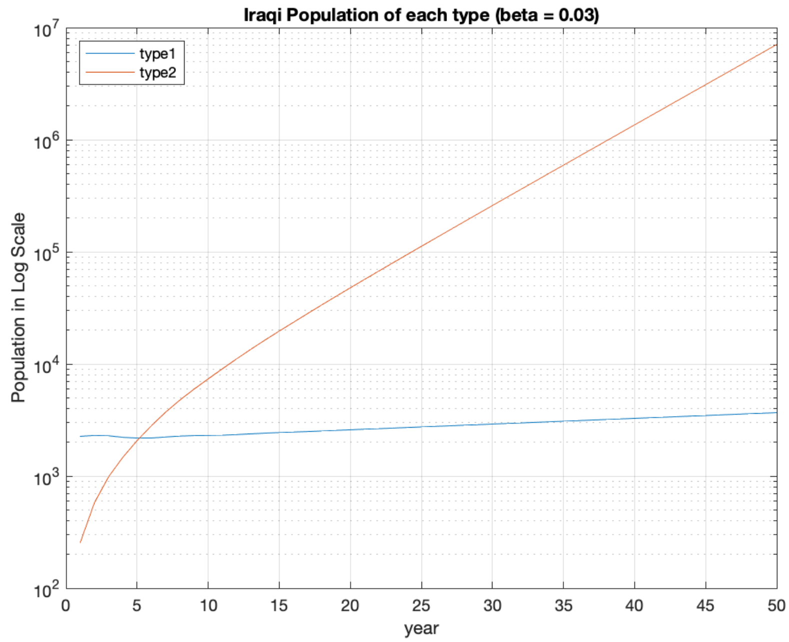

Figure 15: Iraq

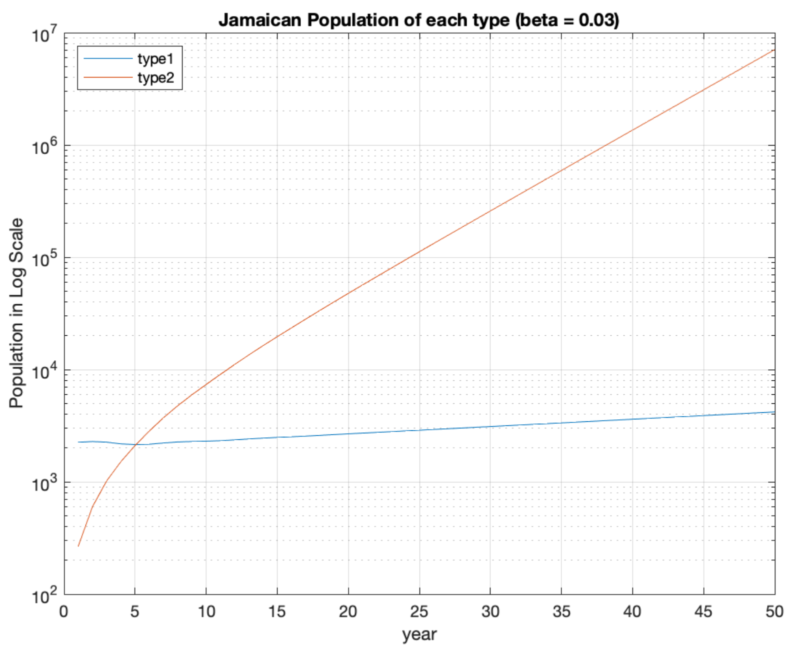

Figure 16: Jamaica

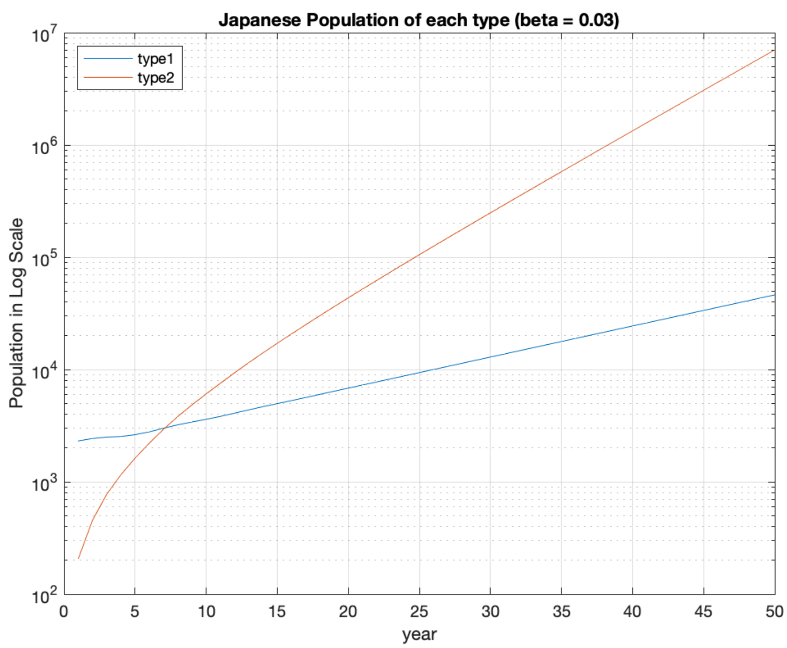

Figure 17: Japan

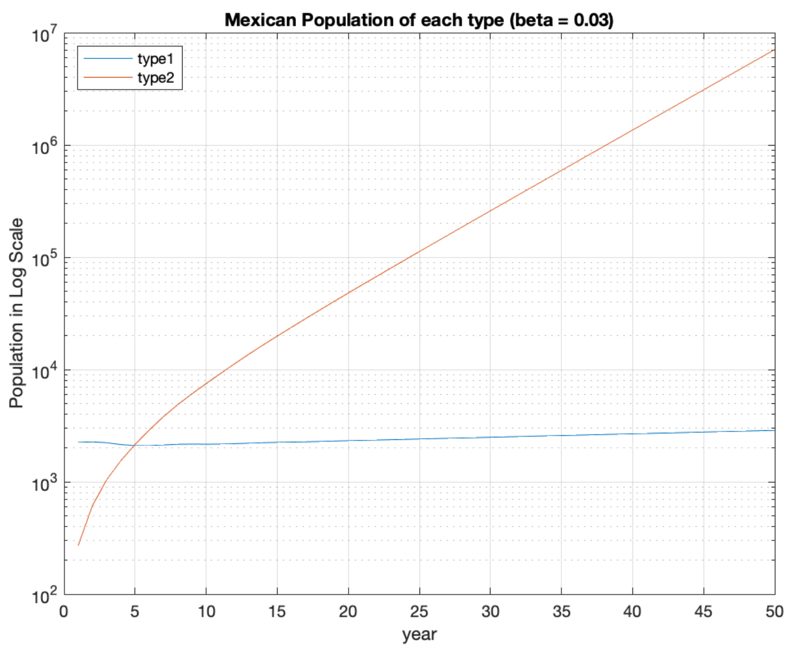

Figure 18: Mexico

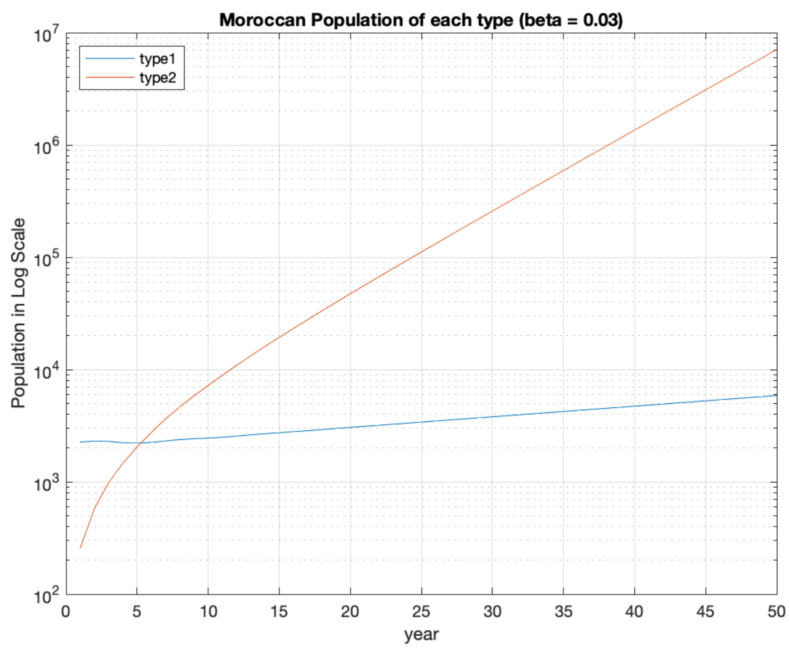

Figure 19: Morocco