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Editor: Jared Hoberock
NVIDIA Corporation
jhoberock@nvidia.com

Working Draft, Technical Specification for C++ Extensions for Parallelism

Note: this is an early draft. It's known to be incomplet and incorrekt, and it has lots of bad formatting.

Contents

1 General [parallel.general]

1.1 Scope [parallel.general.scope]

1 This Technical Specification describes requirements for implementations of an interface that computer programs written in the C++ programming language may use to invoke algorithms with parallel execution. The algorithms described by this Technical Specification are realizable across a broad class of computer architectures.

2 This Technical Specification is non-normative. Some of the functionality described by this Technical Specification may be considered for standardization in a future version of C++, but it is not currently part of any C++ standard. Some of the functionality in this Technical Specification may never be standardized, and other functionality may be standardized in a substantially changed form.

3 The goal of this Technical Specification is to build widespread existing practice for parallelism in the C++ standard algorithms library. It gives advice on extensions to those vendors who wish to provide them.

1.2 Normative references [parallel.general.references]

1 The following referenced document is indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IS~~O~~/IEC 14882:—[1], *Programming Languages — C++*

2 ISO/IEC 14882:— is herein called the *C++ Standard*. The library described in ISO/IEC 14882:— clauses 17-30 is herein called the *C++ Standard Library*. The C++ Standard Library components described in ISO/IEC 14882:— clauses 25 , 26.7 and 20.7.2 are herein called the *C++ Standard Algorithms Library*.

3 Unless otherwise specified, the whole of the C++ Standard's Library introduction (C++14 §17) is included into this Technical Specification by reference.

1.3 Namespaces and headers [parallel.general.namespaces]

1 Since the the extensions described in this Technical Specification are experimental and not part of the C++ Standard Library, they should not be declared directly within namespace `std`. Unless otherwise specified, all components described in this Technical Specification are declared in namespace `std::experimental::parallel::v1`.

[*Note:* Once standardized, the components described by this Technical Specification are expected to be promoted to namespace `std`. — *end note*]

2 Unless otherwise specified, references to such entities described in this Technical Specification are assumed to be qualified with `std::experimental::parallel::v1`, and references to entities described in the C++ Standard Library are assumed to be qualified with `std::`.

3 Extensions that are expected to eventually be added to an existing header `<meow>` are provided inside the `<experimental/meow>` header, which shall include the standard contents of `<meow>` as if by

```
#include <meow>
```

1. To be published. Section references are relative to N3797.

1.3.1 Terms and definitions **[parallel.general.defns]**

1 For the purposes of this document, the terms and definitions given in the C++ Standard and the following apply.

2 A *parallel algorithm* is a function template described by this Technical Specification declared in namespace `std::experimental::parallel::v1` with a formal template parameter named `ExecutionPolicy`.

3 Parallel algorithms access objects indirectly accessible via their arguments by invoking the following functions:

- All operations of the categories of the iterators that the algorithm is instantiated with.
- Functions on those sequence elements that are required by its specification.
- User-provided function objects to be applied during the execution of the algorithm, if required by the specification.

These functions are herein called *element access functions*.

[*Example:* The `sort` function may invoke the following element access functions:

- Methods of the random-access iterator of the actual template argument, as per 24.2.7, as implied by the name of the template parameters `RandomAccessIterator`.
- The `swap` function on the elements of the sequence (as per 25.4.1.1 [sort]/2).
- The user-provided `Compare` function object.

— *end example*]

4

2 Execution policies [parallel.execpol]

2.1 In general [parallel.execpol.general]

1 ~~This clause describes classes that represent *execution policies*. An *execution policy* is an object that expresses the requirements on the ordering of functions invoked as a consequence of the invocation of a standard algorithm. Execution policies afford standard algorithms the discretion to execute in parallel.~~ This clause describes classes that are *execution policy* types. An object of an execution policy type indicates to an algorithm whether it is allowed to execute in parallel and expresses the requirements on the element access functions.

[*Example:*

```
std::vector<int> v = ...

// standard sequential sort
std::sort(vec.begin(), vec.end());

using namespace std::experimental::parallel;

// explicitly sequential sort
sort(seq, v.begin(), v.end());

// permitting parallel execution
sort(par, v.begin(), v.end());

// permitting vectorization as well
sort(par_vec, v.begin(), v.end());

// sort with dynamically-selected execution
size_t threshold = ...
execution_policy exec = seq;
if (v.size() > threshold)
{
  exec = par;
}

sort(exec, v.begin(), v.end());
```

— *end example*]

[*Note:* Because different parallel architectures may require idiosyncratic parameters for efficient execution, implementations of the Standard Library may provide additional execution policies to those described in this Technical Specification as extensions. — *end note*]

2.2 Header `<experimental/execution_policy>` synopsis [parallel.execpol.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  // 2.3, Execution policy type trait
  template<class T> struct is_execution_policy;
  template<class T> constexpr bool is_execution_policy_v = is_execution_policy<T>::value;

  // 2.4, Sequential execution policy
  class sequential_execution_policy;

  // 2.5, Parallel execution policy
```

```
  class parallel_execution_policy;

  // 2.6, Parallel+Vector execution policy
  class parallel_vector_execution_policy

  // 2.7, Dynamic execution policy
  class execution_policy;
}
}
}
}
```

2.3 Execution policy type trait [parallel.execpol.type]

```
template<class T> struct is_execution_policy { see below };
  : integral_constant<bool, see below> { };
```

1 `is_execution_policy` can be used to detect parallel execution policies for the purpose of excluding function signatures from otherwise ambiguous overload resolution participation.

2 `is_execution_policy<T>` shall be a UnaryTypeTrait with a BaseCharacteristic of `true_type` if `T` is the type of a standard or implementation-defined execution policy, otherwise `false_type`.

3 The behavior of a program that adds specializations for `is_execution_policy` is undefined.

2.4 Sequential execution policy [parallel.execpol.seq]

```
class sequential_execution_policy{ unspecified };
```

1 The class `sequential_execution_policy` is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and require that a parallel algorithm's execution may not be parallelized.

2.5 Parallel execution policy [parallel.execpol.par]

```
class parallel_execution_policy{ unspecified };
```

1 The class `parallel_execution_policy` is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be parallelized.

2.6 Parallel+Vector execution policy [parallel.execpol.vec]

```
class parallel_vector_execution_policy{ unspecified };
```

1 The class `class parallel_vector_execution_policy` is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be vectorized and parallelized.

2.7 Dynamic execution policy [parallel.execpol.dynamic]

```
class execution_policy
{
  public:
    // 2.7.1, execution_policy construct/assign
    template<class T> execution_policy(const T& exec);
    template<class T> execution_policy& operator=(const T& exec);
```

```
    // 2.7.2, execution_policy object access
    template<class T> T* get() noexcept;
    template<class T> const T* get() const noexcept;
};
```

1 The class `execution_policy` is a container for execution policy objects. `execution_policy` allows dynamic control over standard algorithm execution.

[*Example:*

```
std::vector<float> sort_me = ...

using namespace std::experimental::parallel;
execution_policy exec = seq;

if(sort_me.size() > threshold)
{
  exec = std::par;
}

std::sort(exec, std::begin(sort_me), std::end(sort_me));
```

— *end example*]

2 Objects of type `execution_policy` shall be constructible and assignable from objects of type `T` for which `is_execution_policy<T>::value` is `true`.

2.7.1 `execution_policy` construct/assign [parallel.execpol.con]

1 `template<class T> execution_policy(const T& exec);`

2 *Effects:* Constructs an `execution_policy` object with a copy of `exec`'s state.

3 *Remarks:* This constructor shall not participate in overload resolution unless `is_execution_policy<T>::value` is `true`.

4 `template<class T> execution_policy& operator=(const T& exec);`

5 *Effects:* Assigns a copy of `exec`'s state to `*this`.

6 *Returns:* `*this`.

2.7.2 `execution_policy` object access [parallel.execpol.access]

1 `const type_info& type() const noexcept;`

2 *Returns:* `typeid(T)`, such that `T` is the type of the execution policy object contained by `*this`.

3 `template<class T> T* get() noexcept;`
`template<class T> const T* get() const noexcept;`

4 *Returns:* If `target_type() == typeid(T)`, a pointer to the stored execution policy object; otherwise a null pointer.

5 *Requires:* `is_execution_policy<T>::value` is `true`.

2.8 Execution policy objects [parallel.execpol.objects]

```
constexpr sequential_execution_policy      seq{};
constexpr parallel_execution_policy        par{};
constexpr parallel_vector_execution_policy par_vec{};
```

1 The header `<experimental/execution_policy>` declares a global object associated with each type of execution policy defined by this Technical Specification.

3 Parallel exceptions [parallel.exceptions]

3.1 Exception reporting behavior [parallel.exceptions.behavior]

1 During the execution of a standard parallel algorithm, if temporary memory resources are required and none are available, the algorithm throws a `std::bad_alloc` exception.

2 During the execution of a standard parallel algorithm, if the ~~application of a function object~~invocation of an element access function terminates with an uncaught exception, the behavior of the program is determined by the type of execution policy used to invoke the algorithm:

If ~~t~~the execution policy object is of type `class parallel_vector_execution_policy`, `std::terminate` shall be called.
If ~~t~~the execution policy object is of type `sequential_execution_policy` or `parallel_execution_policy`, the execution of the algorithm terminates with an `exception_list` exception. All uncaught exceptions thrown during the ~~application of user-provided function objects~~invocations of element access functions shall be contained in the `exception_list`.

[*Note:* For example, the number of invocations of the user-provided function object in `for_each` is unspecified. When `for_each` is executed sequentially, only one exception will be contained in the `exception_list` object. — *end note*]

[*Note:* These guarantees imply that, unless the algorithm has failed to allocate memory and terminated with `std::bad_alloc`, all exceptions thrown during the execution of the algorithm are communicated to the caller. It is unspecified whether an algorithm implementation will "forge ahead" after encountering and capturing a user exception. — *end note*]

[*Note:* The algorithm may terminate with the `std::bad_alloc` exception even if one or more user-provided function objects have terminated with an exception. For example, this can happen when an algorithm fails to allocate memory while creating or adding elements to the `exception_list` object. — *end note*]
If ~~t~~the execution policy object is of any other type, the behavior is implementation-defined.

3

3.2 Header `<experimental/exception_list>` synopsis [parallel.exceptions.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {

  class exception_list : public exception
  {
    public:
      typedef unspecified                   iterator;

      size_t size() const noexcept;
      iterator begin() const noexcept;
      iterator end() const noexcept;

      const char* what() const override noexcept;
  };
```

```
}
}
}
}
```

1 The class `exception_list` owns a sequence of `exception_ptr` objects. The parallel algorithms may use the `exception_list` to communicate uncaught exceptions encountered during parallel execution to the caller of the algorithm.

2 The type `exception_list::iterator` shall fulfill the requirements of `ForwardIterator`.

3
```
size_t size() const noexcept;
```

4 *Returns:* The number of `exception_ptr` objects contained within the `exception_list`.

5 *Complexity:* Constant time.

6
```
iterator begin() const noexcept;
```

7 *Returns:* An iterator referring to the first `exception_ptr` object contained within the `exception_list`.

8
```
iterator end() const noexcept;
```

9 *Returns:* An iterator that is past the end of the owned sequence.

10
```
const char* what() const noexcept override;
```

11 *Returns:* An implementation-defined NTBS.

4 Parallel algorithms [parallel.alg]

4.1 In general [parallel.alg.general]

This clause describes components that C++ programs may use to perform operations on containers and other sequences in parallel.

4.1.1 Requirements on user-provided function objects [parallel.alg.general.user]

1 Function objects passed into parallel algorithms as objects of type `BinaryPredicate`, `Compare`, and `BinaryOperation` shall not directly or indirectly modify objects via their arguments.

4.1.2 Effect of execution policies on algorithm execution [parallel.alg.general.exec]

1 Parallel algorithms have template parameters named `ExecutionPolicy` which describe the manner in which the execution of these algorithms may be parallelized and the manner in which they apply ~~user-provided function objects~~the element access functions.

2 The ~~applications of function objects~~invocations of element access functions in parallel algorithms invoked with an execution policy object of type `sequential_execution_policy` execute in sequential order in the calling thread.

3 The ~~applications of function objects~~invocations of element access functions in parallel algorithms invoked with an execution policy object of type `parallel_execution_policy` are permitted to execute in an unordered fashion in unspecified threads, and indeterminately sequenced within each thread. [*Note:* It is the caller's responsibility to ensure correctness, for example that the invocation does not introduce data races or deadlocks. — *end note*]

[*Example:*

```
using namespace std::experimental::parallel;
int a[] = {0,1};
std::vector<int> v;
for_each(par, std::begin(a), std::end(a), [&](int i) {
  v.push_back(i*2+1);
});
```

The program above has a data race because of the unsynchronized access to the container `v`. — *end example*]

[*Example:*

```
using namespace std::experimental::parallel;
std::atomic<int> x = 0;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int n) {
  x.fetch_add(1, std::memory_order_relaxed);
  // spin wait for another iteration to change the value of x
  while (x.load(std::memory_order_relaxed) == 1) { }
});
```

The above example depends on the order of execution of the iterations, and is therefore undefined (may deadlock). — *end example*]

[*Example:*

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
```

```
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int) {
  m.lock();
  ++x;
  m.unlock();
});
```

The above example synchronizes access to object `x` ensuring that it is incremented correctly. — *end example*]

4 The ~~applications of function objects~~invocations of element access functions in parallel algorithms invoked with an execution policy of type `parallel_vector_execution_policy` are permitted to execute in an unordered fashion in unspecified threads, and unsequenced with respect to one another within each thread. [*Note:* This means that multiple function object invocations may be interleaved on a single thread. — *end note*]

~~[*Note:* As a consequence, function objects governed by the `parallel_vector_execution_policy` policy must not synchronize with each other. Specifically, they must not acquire locks. — *end note*]~~ [*Note:* This overrides the usual guarantee from the C++ standard, Section 1.9 [intro.execution] that function executions do not interleave with one another. — *end note*]

Since `parallel_vector_execution_policy` allows the execution of element access functions to be interleaved on a single thread, synchronization, including the use of mutexes, risks deadlock. Thus the synchronization with `parallel_vector_execution_policy` is restricted as follows:

A standard library function is *vectorization-unsafe* if it is specified to synchronize with another function invocation, or another function invocation is specified to synchronize with it, and if it is not a memory allocation or deallocation function. Vectorization-unsafe standard library functions may not be invoked by element access functions invoked by `parallel_vector_execution_policy` algorithms.

[*Note:* Implementations must ensure that internal synchronization inside standard library routines does not induce deadlock. — *end note*]

5 [*Example:*

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
int a[] = {1,2};
for_each(par_vec, std::begin(a), std::end(a), [&](int) {
  m.lock();
  ++x;
  m.unlock();
});
```

The above program is invalid because the applications of the function object are not guaranteed to run on different threads. — *end example*]

[*Note:* The application of the function object may result in two consecutive calls to `m.lock` on the same thread, which may deadlock. — *end note*]

[*Note:* The semantics of the `parallel_execution_policy` or the `parallel_vector_execution_policy` invocation allow the implementation to fall back to sequential execution if the system cannot parallelize an algorithm invocation due to lack of resources. — *end note*]

6 Algorithms invoked with an execution policy object of type `execution_policy` execute internally as if invoked with the contained execution policy object.

7 The semantics of parallel algorithms invoked with an execution policy object of implementation-defined type are implementation-defined.

4.1.3 `ExecutionPolicy` algorithm overloads [parallel.alg.overloads]

1 The Parallel Algorithms Library provides overloads for each of the algorithms named in Table 1, corresponding to the algorithms with the same name in the C++ Standard Algorithms Library. For each algorithm in Table 1, if there are overloads for corresponding algorithms with the same name in the C++ Standard Algorithms Library, the overloads shall have an additional template type parameter named `ExecutionPolicy`, which shall be the first template parameter. In addition, each such overload shall have the new function parameter as the first function parameter of type `ExecutionPolicy&&`.

2 Unless otherwise specified, the semantics of `ExecutionPolicy` algorithm overloads are identical to their overloads without.

3 Parallel algorithms shall not participate in overload resolution unless `is_execution_policy<decay_t<ExecutionPolicy>>::value` is `true`.

Table 1 — Table of parallel algorithms

`adjacent_difference`	`adjacent_find`	`all_of`	`any_of`
`copy`	`copy_if`	`copy_n`	`count`
`count_if`	`equal`	`exclusive_scan`	`fill`
`fill_n`	`find`	`find_end`	`find_first_of`
`find_if`	`find_if_not`	`for_each`	`for_each_n`
`generate`	`generate_n`	`includes`	`inclusive_scan`
`inner_product`	`inplace_merge`	`is_heap`	`is_heap_until`
`is_partitioned`	`is_sorted`	`is_sorted_until`	`lexicographical_compare`
`max_element`	`merge`	`min_element`	`minmax_element`
`mismatch`	`move`	`none_of`	`nth_element`
`partial_sort`	`partial_sort_copy`	`partition`	`partition_copy`
`reduce`	`remove`	`remove_copy`	`remove_copy_if`
`remove_if`	`replace`	`replace_copy`	`replace_copy_if`
`replace_if`	`reverse`	`reverse_copy`	`rotate`
`rotate_copy`	`search`	`search_n`	`set_difference`
`set_intersection`	`set_symmetric_difference`	`set_union`	`sort`
`stable_partition`	`stable_sort`	`swap_ranges`	`transform`
`uninitialized_copy`	`uninitialized_copy_n`	`uninitialized_fill`	`uninitialized_fill_n`
`unique`	`unique_copy`		

[*Note:* Not all algorithms in the Standard Library have counterparts in Table 1. — *end note*]

4.2 Definitions [parallel.alg.defns]

1 Define *`GENERALIZED_SUM`*`(op, a1, ..., aN)` as follows:

`a1` when `N` is `1`

`op(`*`GENERALIZED_SUM`*`(op, b1, ..., bK),` *`GENERALIZED_SUM`*`(op, bM, ..., bN))` where `b1, ..., bN` may be any permutation of `a1, ..., aN` and `1 < K+1 = M ≤ N`.

2

3 Define *`GENERALIZED_NONCOMMUTATIVE_SUM`*`(op, a1, ..., aN)` as follows:

`a1` when `N` is `1`

`op(`*`GENERALIZED_NONCOMMUTATIVE_SUM`*`(op, a1, ..., aK),` *`GENERALIZED_NONCOMMUTATIVE_SUM`*`(op, aM, ..., aN)` where `1 < K+1 = M ≤ N`.

4

4.3 Non-Numeric Parallel Algorithms [parallel.alg.ops]

4.3.1 Header `<experimental/algorithm>` synopsis [parallel.alg.ops.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  template<class ExecutionPolicy,
           class InputIterator, class Function>
    void for_each(ExecutionPolicy&& exec,
                  InputIterator first, InputIterator last,
                  Function f);
  template<class InputIterator, class Size, class Function>
    InputIterator for_each_n(InputIterator first, Size n,
                             Function f);
}
}
}
}
```

4.3.2 For each [parallel.alg.foreach]

1
```
template<class ExecutionPolicy,
            class InputIterator, class Function>
     void for_each(ExecutionPolicy&& exec,
                   InputIterator first, InputIterator last,
                   Function f);
```

2 *Effects:* Applies `f` to the result of dereferencing every iterator in the range `[first,last)`. [*Note:* If the type of `first` satisfies the requirements of a mutable iterator, `f` may apply nonconstant functions through the dereferenced iterator. — *end note*]

3 *Complexity:* Applies `f` exactly `last - first` times.

4 *Remarks:* If `f` returns a result, the result is ignored.

5 *Notes:* Unlike its sequential form, the parallel overload of `for_each` does not return a copy of its `Function` parameter, since parallelization may not permit efficient state accumulation.

6 *Requires:* Unlike its sequential form, the parallel overload of `for_each` requires `Function` to meet the requirements of `CopyConstructible`.

7
```
template<class InputIterator, class Size, class Function>
     InputIterator for_each_n(InputIterator first, Size n,
                              Function f);
```

8 *Requires:* `Function` shall meet the requirements of `MoveConstructible` [*Note:* `Function` need not meet the requirements of `CopyConstructible`. — *end note*]

9 *Effects:* Applies `f` to the result of dereferencing every iterator in the range `[first,first + n)`, starting from `first` and proceeding to `first + n - 1`. [*Note:* If the type of `first` satisfies the requirements of a mutable iterator, `f` may apply nonconstant functions through the dereferenced iterator. — *end note*]

10 *Returns:* `first + n` for non-negative values of `n` and `first` for negative values.

11 *Remarks:* If `f` returns a result, the result is ignored.

12
```
template<class ExecutionPolicy,
            class InputIterator, class Size, class Function>
            InputIterator for_each_n(ExecutionPolicy && exec,
                                     InputIterator first, Size n,
                                     Function f);
```

13 *Effects:* Applies `f` to the result of dereferencing every iterator in the range `[first,first + n)`, starting from `first` and proceeding to `first + n - 1`. [*Note:* If the type of `first` satisfies the requirements of a mutable iterator, `f` may apply nonconstant functions through the dereferenced iterator. — *end note*]

14 *Returns:* `first + n` for non-negative values of `n` and `first` for negative values.

15 *Remarks:* If `f` returns a result, the result is ignored.

16 *Notes:* Unlike its sequential form, the parallel overload of `for_each_n` requires `Function` to meet the requirements of `CopyConstructible`.

4.4 Numeric Parallel Algorithms [parallel.alg.numeric]

4.4.1 Header `<experimental/numeric>` synopsis [parallel.alg.numeric.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  template<class InputIterator>
    typename iterator_traits<InputIterator>::value_type
      reduce(InputIterator first, InputIterator last);
  template<class InputIterator, class T>
    T reduce(InputIterator first, InputIterator last, T init);
  template<class InputIterator, class T, class BinaryOperation>
    T reduce(InputIterator first, InputIterator last, T init,
             BinaryOperation binary_op);

  template<class InputIterator, class OutputIterator,
           class T>
    OutputIterator
      exclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init);
```

```
  template<class InputIterator, class OutputIterator,
           class T, class BinaryOperation>
    OutputIterator
      exclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init, BinaryOperation binary_op);

  template<class InputIterator, class OutputIterator>
    OutputIterator
      inclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result);
  template<class InputIterator, class OutputIterator,
           class BinaryOperation>
    OutputIterator
      inclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     BinaryOperation binary_op);
  template<class InputIterator, class OutputIterator,
           class BinaryOperation, class T>
    OutputIterator
      inclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     BinaryOperation binary_op, T init);
}
}
}
}
```

4.4.2 Reduce [parallel.alg.reduce]

1
```
template<class InputIterator>
     typename iterator_traits<InputIterator>::value_type
       reduce(InputIterator first, InputIterator last);
```

2 *Effects:* Same as `reduce(first, last, typename iterator_traits<InputIterator>::value_type{})`.

3
```
template<class InputIterator, class T>
     T reduce(InputIterator first, InputIterator last, T init);
```

4 *Effects:* Same as `reduce(first, last, init, plus<>())`.

5
```
template<class InputIterator, class T, class BinaryOperation>
     T reduce(InputIterator first, InputIterator last, T init,
              BinaryOperation binary_op);
```

6 *Returns:* *GENERALIZED_SUM*`(binary_op, init, *first, ..., *(first + (last - first) - 1))`.

7 *Requires:* `binary_op` shall not invalidate iterators or subranges, nor modify elements in the range `[first,last)`.

8 *Complexity:* O(`last - first`) applications of `binary_op`.

9 *Notes:* The primary difference between `reduce` and `accumulate` is that the behavior of `reduce` may be non-deterministic for non-associative or non-commutative `binary_op`.

4.4.3 Exclusive scan [parallel.alg.exclusive.scan]

1
```
template<class InputIterator, class OutputIterator,
            class T>
     OutputIterator
       exclusive_scan(InputIterator first, InputIterator last,
                      OutputIterator result,
                      T init);
```

2 *Effects:* Same as `exclusive_scan(first, last, result, init, plus<>())`.

3
```
template<class InputIterator, class OutputIterator,
            class T, class BinaryOperation>
     OutputIterator
       exclusive_scan(InputIterator first, InputIterator last,
                      OutputIterator result,
                      T init, BinaryOperation binary_op);
```

4 *Effects:* Assigns through each iterator `i` in `[result,result + (last - first))` the value of *`GENERALIZED_NONCOMMUTATIVE_SUM`*`(binary_op, init, *first, ..., *(first + (i - result) - 1))`.

5 *Returns:* The end of the resulting range beginning at `result`.

6 *Requires:* `binary_op` shall not invalidate iterators or subranges, nor modify elements in the ranges `[first,last)` or `[result,result + (last - first))`.

7 *Complexity:* O(`last - first`) applications of `binary_op`.

8 *Notes:* The difference between `exclusive_scan` and `inclusive_scan` is that `exclusive_scan` excludes the `i`th input element from the `i`th sum. If `binary_op` is not mathematically associative, the behavior of `exclusive_scan` may be non-deterministic.

4.4.4 Inclusive scan [parallel.alg.inclusive.scan]

1
```
template<class InputIterator, class OutputIterator>
     OutputIterator
       inclusive_scan(InputIterator first, InputIterator last,
                      OutputIterator result);
```

2 *Effects:* Same as `inclusive_scan(first, last, result, plus<>())`.

3
```
template<class InputIterator, class OutputIterator,
            class BinaryOperation>
     OutputIterator
       inclusive_scan(InputIterator first, InputIterator last,
                      OutputIterator result,
                      BinaryOperation binary_op);

   template<class InputIterator, class OutputIterator,
            class BinaryOperation, class T>
     OutputIterator
       inclusive_scan(InputIterator first, InputIterator last,
                      OutputIterator result,
                      BinaryOperation binary_op, T init);
```

4 *Effects:* Assigns through each iterator `i` in `[result,result + (last - first))` the value of *`GENERALIZED_NONCOMMUTATIVE_SUM`*`(binary_op, *first, ..., *(first + (i - result)))` or *`GENERALIZED_NONCOMMUTATIVE_SUM`*`(binary_op, init, *first, ..., *(first + (i - result)))` if `init` is provided.

5 *Returns:* The end of the resulting range beginning at `result`.

6 *Requires:* `binary_op` shall not invalidate iterators or subranges, nor modify elements in the ranges `[first,last)` or `[result,result + (last - first))`.

7 *Complexity:* O(`last - first)` applications of `binary_op`.

8 *Notes:* The difference between `exclusive_scan` and `inclusive_scan` is that `inclusive_scan` includes the `i`th input element in the `i`th sum. If `binary_op` is not mathematically associative, the behavior of `inclusive_scan` may be non-deterministic.