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Editor: Jared Hoberock

NVIDIA Corporation jhoberock@nvidia.com

Working Draft, Technical Specification for C++ Extensions for Parallelism

Note: this is an early draft. It's known to be incomplet and incorrekt, and it has lots of bad formatting.

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1 General

[parallel.general]

1.1 Scope

[parallel.general.scope]

This Technical Specification describes requirements for implementations of an interface that computer programs written in the C++ programming language may use to invoke algorithms with parallel execution. The algorithms described by this Technical Specification are realizable across a broad class of computer architectures.

- ² This Technical Specification is non-normative. Some of the functionality described by this Technical Specification may be considered for standardization in a future version of C++, but it is not currently part of any C++ standard. Some of the functionality in this Technical Specification may never be standardized, and other functionality may be standardized in a substantially changed form.
- ³ The goal of this Technical Specification is to build widespread existing practice for parallelism in the C++ standard algorithms library. It gives advice on extensions to those vendors who wish to provide them.

1.2 Normative references

[parallel.general.references]

- ¹ The following referenced document is indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.
 - ISO/IEC 14882: $-^1$, Programming Languages C++
- ² ISO/IEC 14882:— is herein called the *C++ Standard*. The library described in ISO/IEC 14882:— clauses 17-30 is herein called the *C++ Standard Library*. The C++ Standard Library components described in ISO/IEC 14882:— clauses 25 and, 26.7 and 20.7.2 are herein called the *C++ Standard Algorithms Library*.
- ³ Unless otherwise specified, the whole of the C++ Standard's Library introduction (C++14 §17) is included into this Technical Specification by reference.

1.3 Namespaces and headers

[parallel.general.namespaces]

- Since the extensions described in this Technical Specification are experimental and not part of the C++ Standard Library, they should not be declared directly within namespace std. Unless otherwise specified, all components described in this Technical Specification are declared in namespace std::experimental::parallel::y1.
 - [Note: Once standardized, the components described by this Technical Specification are expected to be promoted to namespace std. $end\ note$]
- ² Unless otherwise specified, references to such entities described in this Technical Specification are assumed to be qualified with std::experimental::parallel::v1, and references to entities described in the C++ Standard Library are assumed to be qualified with std::.
- 3 Extensions that are expected to eventually be added to an existing header <meow> are provided inside the <experimental/meow> header, which shall include the standard contents of <meow> as if by

#include <meow>

1. To be published. Section references are relative to N3937.

§ 1.3

1.3.1 Terms and definitions

[parallel.general.defns]

¹ For the purposes of this document, the terms and definitions given in the C++ Standard and the following apply.

- ² A *parallel algorithm* is a function template described by this Technical Specification declared in namespace std::experimental::parallel::v1 with a formal template parameter named ExecutionPolicy.
- ³ Parallel algorithms access objects indirectly accessible via their arguments by invoking the following functions:
 - All operations of the categories of the iterators that the algorithm is instantiated with.
 - Functions on those sequence elements that are required by its specification.
 - <u>User-provided function objects to be applied during the execution of the algorithm, if</u> required by the specification.

These functions are herein called *element access functions*.

[*Example:* The sort function may invoke the following element access functions:

- Methods of the random-access iterator of the actual template argument, as per 24.2.7, as implied by the name of the template parameters RandomAccessIterator.
- The swap function on the elements of the sequence (as per 25.4.1.1 [sort]/2).
- The user-provided Compare function object.
- end example]

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§ 1.3.1

2 Execution policies

[parallel.execpol]

2.1 In general

[parallel.execpol.general]

¹ This clause describes classes that represent *execution policies*. An *execution policy* is an object that expresses the requirements on the ordering of functions invoked as a consequence of the invocation of a standard algorithm. Execution policies afford standard algorithms the discretion to execute in parallel. This clause describes classes that are *execution policy* types. An object of an execution policy type indicates to an algorithm whether it is allowed to execute in parallel and expresses the requirements on the element access functions.

```
[ Example:
```

```
std::vector<int> v = ...
 // standard sequential sort
 std::sort(vec.begin(), vec.end());
 using namespace std::experimental::parallel;
 // explicitly sequential sort
 sort(seq, v.begin(), v.end());
 // permitting parallel execution
 sort(par, v.begin(), v.end());
 // permitting vectorization as well
 sort(par vec, v.begin(), v.end());
 // sort with dynamically-selected execution
 size t threshold = \dots
 execution policy exec = seq;
 if (v.size() > threshold)
 {
   exec = par;
 sort(exec, v.begin(), v.end());
— end example ]
```

[*Note:* Because different parallel architectures may require idiosyncratic parameters for efficient execution, implementations of the Standard Library <u>shouldmay</u> provide additional execution policies to those described in this Technical Specification as extensions. — *end note*]

2.2 Header <experimental/execution_policy> synopsis [parallel.execpol.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
   // 2.3, Execution policy type trait
```

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```
template<class T> struct is_execution_policy;
  template<class T> constexpr bool is_execution_policy_v = is_execution_policy<T>::value;

// 2.4, Sequential execution policy
  class sequential_execution_policy;

// 2.5, Parallel execution policy
  class parallel_execution_policy;

// 2.6, Parallel+Vector execution policy
  class vector_execution_policyparallel_vector_execution_policy;

// 2.7, Dynamic execution policy
  class execution_policy;
}
}
}
```

2.3 Execution policy type trait

[parallel.execpol.type]

is_execution_policy can be used to detect parallel execution policies for the purpose of excluding function signatures from otherwise ambiguous overload resolution participation.

If T is the type of a standard or implementation-defined execution policy, is_execution_policy<T> shall be publicly derived from integral_constant<bool,true>, otherwise from integral_constant<bool,false>.

 $\underline{\text{is_execution_policy}}{\text{T> shall be a UnaryTypeTrait with a BaseCharacteristic of true_type if T is the type of a standard or implementation-defined execution policy, otherwise false_type.}$

The behavior of a program that adds specializations for is execution policy is undefined.

2.4 Sequential execution policy

[parallel.execpol.seq]

```
namespace std {
namespace experimental {
namespace parallel {
   class sequential_execution_policy{ unspecified };
}
```

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```
}
}
```

The class sequential_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and require that a parallel algorithm's execution may not be parallelized.

2.5 Parallel execution policy

[parallel.execpol.par]

```
namespace std {
namespace experimental {
namespace parallel {
   class parallel_execution_policy{ unspecified };
}
}
}
```

¹ The class parallel_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be parallelized.

2.6 <u>Parallel+</u>Vector execution policy

[parallel.execpol.vec]

```
namespace std {
namespace experimental {
namespace parallel {
   class vector_execution_policyparallel_vector_execution_policy{ unspecified };
}
}
}
```

The class class vector_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be vectorized and parallelized.

2.7 Dynamic execution policy

[parallel.execpol.dynamic]

§ 2.7

```
template<class T> execution_policy(const T& exec);
template<class T> execution_policy& operator=(const T& exec);

// 2.7.2, execution_policy object access
template<class T> T* get() noexcept;
template<class T> const T* get() const noexcept;
};

}
```

¹ The class execution_policy is a container for execution policy objects. execution_policy allows dynamic control over standard algorithm execution.

[Example:

```
std::vector<float> sort_me = ...

using_namespace_std::experimental::parallel;
std::execution_policy exec = std::seq;

if(sort_me.size() > threshold)
{
    exec = std::par;
}

std::sort(exec, sort_me.begin(), sort_me.end());
std::sort(exec, std::begin(sort_me), std::end(sort_me));
- end example]
```

Objects of type execution_policy shall be constructible and assignable from objects of type T for which is execution policy<T>::value is true.

2.7.1 execution policy construct/assign

[parallel.execpol.con]

- 1 template<class T> execution_policy(const T& exec);
 - ² Effects: Constructs an execution_policy object with a copy of exec's state.

```
Requires: is execution policy<T>::value is true.
```

Remarks: This constructor shall not participate in overload resolution unless is execution policy<T>::value is true.

- 3 template<class T> execution_policy& operator=(const T& exec);
 - ⁴ Effects: Assigns a copy of exec's state to *this.
 - ⁵ Returns: *this.

Requires: is_execution_policy<T>::value iS true.

§ 2.7.1

2.7.2 execution policy object access

[parallel.execpol.access]

```
const type_info& type() const noexcept;
```

² *Returns:* typeid(T), such that T is the type of the execution policy object contained by *this.

```
3 template<class T> T* get() noexcept;
    template<class T> const T* get() const noexcept;
```

4 Returns: If target_type() == typeid(T), a pointer to the stored execution policy object; otherwise a null pointer.

Requires: is_execution_policy<T>::value is true.

2.8 Execution policy objects

[parallel.execpol.objects]

The header <experimental/execution_policy> declares a global object associated with each type of execution policy defined by this Technical Specification.

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3 Parallel exceptions

[parallel.exceptions]

3.1 Exception reporting behavior

[parallel.exceptions.behavior]

During the execution of a standard parallel algorithm, I temporary memory resources are required by the algorithm and none are available, the algorithm throws a std::bad_alloc exception.

- During the execution of a standard parallel algorithm, if the application of a function objectinvocation of an element access function terminates with an uncaught exception, the behavior of the program is determined by the type of execution policy used to invoke the algorithm:
 - If the execution policy object is of type class vector_execution_policyparallel_vector_execution_policy, std::terminate shall be called.
 - If the execution policy object is of type sequential_execution_policy or parallel_execution_policy, the execution of the algorithm terminates with an exception_list exception. All uncaught exceptions thrown during the application of userprovided function objects invocations of element access functions shall be contained in the exception_list.

[*Note:* For example, the number of invocations of the user-provided function object in for_each is unspecified. When for_each is executed sequentially, only one exception will be contained in the exception_list object. — *end note*]

[Note: These guarantees imply that, unless the algorithm has failed to allocate memory and terminated with std::bad_alloc, all exceptions thrown during the execution of the algorithm are communicated to the caller. It is unspecified whether an algorithm implementation will "forge ahead" after encountering and capturing a user exception.

— end note]

[Note: The algorithm may terminate with the std::bad_alloc exception even if one or more user-provided function objects have terminated with an exception. For example, this can happen when an algorithm fails to allocate memory while creating or adding elements to the exception list object. — end note]

 If the execution policy object is of any other type, the behavior is implementationdefined.

3

3.2 Header <experimental/exception_list> synopsis

[parallel.exceptions.synopsis]

§ 3.2

```
typedef const value type&
                                                                                        const reference;
           typedef implementation-defined
                                                                                        const iterator;
          typedef const iterator
                                                                                        iterator:
        typedef typename iterator traits::difference type difference type;
          typedef size t
                                                                                        size type;
          typedef unspecified
                                                       iterator;
           size t size() const noexcept;
           iterator begin() const noexcept;
           iterator end() const noexcept;
           const char* what() const noexcept override;
        private:
           std::list<exception ptr> exceptions ; // exposition only
      };
    }
}
}
<sup>1</sup> The class exception list is a container owns a sequence of exception ptr objects. The parallel
  algorithms may use the exception list to communicate uncaught exceptions encountered during
  parallel execution to the caller of the algorithm.
<sup>2</sup> The type exception_list::iterator shall fulfill the requirements of ForwardIterator.
           size t size() const noexcept;
   <sup>4</sup> Returns: The number of exception_ptr objects contained within the exception_list.
   <sup>5</sup> Complexity: Constant time.
           exception list::iterator begin() const noexcept;
   <sup>7</sup> Returns: An iterator referring to the first exception ptr object contained within the
      exception list.
           exception list::iterator end() const noexcept;
   9 Returns: An iterator which is the past-the-end value for the exception list that is past the
      end of the owned sequence.
           const char* what() const noexcept override;
   11 Returns: An implementation-defined NTBS.
```

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§ 3.2 11

4 Parallel algorithms

[parallel.alg]

4.1 In general

[parallel.alg.general]

This clause describes components that C++ programs may use to perform operations on containers and other sequences in parallel.

4.1.1 Requirements on user-provided function objects

[parallel.alg.general.user]

Function objects passed into parallel algorithms as objects of type BinaryPredicate, Compare, and BinaryOperation shall not directly or indirectly modify objects via their arguments.

4.1.2 Effect of execution policies on algorithm execution [parallel.alg.general.exec]

- Parallel algorithms have template parameters named ExecutionPolicy which describe the manner in which the execution of these algorithms may be parallelized and the manner in which they apply user-provided function objects the element access functions.
- ² The applications of function objects invocations of element access functions in parallel algorithms invoked with an execution policy object of type sequential_execution_policy execute in sequential order in the calling thread.
- The applications of function objects invocations of element access functions in parallel algorithms invoked with an execution policy object of type parallel_execution_policy are permitted to execute in an unordered fashion in unspecified threads, and indeterminately sequenced within each thread. [*Note:* It is the caller's responsibility to ensure correctness, for example that the invocation does not introduce data races or deadlocks. end note]

[Example:

```
using namespace std::experimental::parallel;
int a[] = {0,1};
std::vector<int> v;
for_each(par, std::begin(a), std::end(a), [&](int i) {
   v.push_back(i*2+1);
});
foo_bar
```

The program above has a data race because of the unsynchronized access to the container v. — $end\ example$]

[Example:

```
using namespace std::experimental::parallel;
std::atomic≤int> x = 0;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int n) {
    x.fetch_add(1, std::memory_order_relaxed);
    // spin wait for another iteration to change the value of x
    while (x.load(std::memory_order_relaxed) == 1) { }
});
```

The above example depends on the order of execution of the iterations, and is therefore undefined (may deadlock). - end example]

§ 4.1.2

```
[ Example:
```

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above example synchronizes access to object x ensuring that it is incremented correctly. — $end\ example$]

⁴ The applications of function objects invocations of element access functions in parallel algorithms invoked with an execution policy of type

vector_execution_policyparallel_vector_execution_policy are permitted to execute in an unordered fashion in unspecified threads, and unsequenced with respect to one another within each thread. [Note: This means that multiple function object invocations may be interleaved on a single thread. — end note]

[Note: As a consequence, function objects governed by the vector_execution_policy policy must not synchronize with each other. Specifically, they must not acquire locks. — end note] [Note: This overrides the usual guarantee from the C++ standard, Section 1.9 [intro.execution] that function executions do not interleave with one another. — end note]

Since parallel_vector_execution_policy allows the execution of element access functions to be interleaved on a single thread, synchronization, including the use of mutexes, risks deadlock. Thus the synchronization with parallel vector execution policy is restricted as follows:

A standard library function is *vectorization-unsafe* if it is specified to synchronize with another function invocation, or another function invocation is specified to synchronize with it, and if it is not a memory allocation or deallocation function. Vectorization-unsafe standard library functions may not be invoked by user code called from parallel_vector_execution_policy algorithms.

[*Note:* Implementations must ensure that internal synchronization inside standard library routines does not induce deadlock. — *end note*]

[Example:

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
int a[] = {1,2};
for_each(par_vec, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above program is invalid because the applications of the function object are not guaranteed to run on different threads. — *end example*]

[Note: The application of the function object may result in two consecutive calls to m.lock on the same thread, which may deadlock. — end note]

§ 4.1.2

[Note: The semantics of the parallel execution policy or the

<u>vector_execution_policyparallel_vector_execution_policy</u> invocation allow the implementation to fall back to sequential execution if the system cannot parallelize an algorithm invocation due to lack of resources. — *end note*]

A parallel algorithm invoked with an execution policy object of type parallel_execution_policy or vector_execution_policy may apply iterator member functions of a stronger category than its specification requires. In this case, the application of these member functions are subject to provisions 3. and 4. above, respectively.

[Note: For example, an algorithm whose specification requires InputIterator but receives a concrete iterator of the category RandomAccessIterator may use operator[]. In this case, it is the algorithm caller's responsibility to ensure operator[] is race-free. — end note]

- 6 Algorithms invoked with an execution policy object of type execution_policy execute internally as if invoked with instances of type sequential_execution_policy, parallel_execution_policy, or an implementation-defined execution policy type depending on the dynamic value of the execution_policy object. the contained execution policy object.
- ⁷ The semantics of parallel algorithms invoked with an execution policy object of implementation-defined type are <u>unspecifiedimplementation-defined</u>.

4.1.3 ExecutionPolicy algorithm overloads

[parallel.alg.overloads]

- Parallel algorithms coexist alongside their sequential counterparts as overloads distinguished by a formal template parameter named ExecutionPolicy. This is the first template parameter and corresponds to the parallel algorithm's first function parameter, whose type is ExecutionPolicy&&. The Parallel Algorithms Library provides overloads for each of the algorithms named in Table 1, corresponding to the algorithms with the same name in the C++ Standard Algorithms Library. For each algorithm in Table 1, if there are overloads for corresponding algorithms with the same name in the C++ Standard Algorithms Library, the overloads shall have an additional template type parameter named ExecutionPolicy, which shall be the first template parameter. In addition, each such overload shall have the new function parameter as the first function parameter of type ExecutionPolicy&&.
- ² Unless otherwise specified, the semantics of ExecutionPolicy algorithm overloads are identical to their overloads without.
- ³ Parallel algorithms have the requirement is_execution_policy<ExecutionPolicy>::value is true shall not participate in overload resolution unless is_execution_policy<decay_t<ExecutionPolicy>>::value is true.

The algorithms listed in Table 1 shall have ExecutionPolicy overloads.

Table 1 — Table of parallel algorithms

	Tubic 1 Tubic of	Jaraner argoritmins	
<u>adjacent_difference</u>	adjacent_find	all_of	any_of
сору	copy_if	copy_n	count
count_if	equal	exclusive_scan	fill
fill_n	find	find_end	find_first_of
find_if	find_if_not	for_each	for_each_n
generate	generate_n	includes	inclusive_scan
<u>inner_product</u>	inplace_merge	is_heap	is_heap_until
is_partitioned	is_sorted	is_sorted_until	lexicographical_compare
max_element	merge	min_element	minmax_element
mismatch	move	none_of	nth_element
partial_sort	partial_sort_copy	partition	partition_copy
reduce	remove	remove_copy	remove_copy_if

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```
remove if
                    replace
                                             replace copy
                                                                replace_copy_if
replace if
                    reverse
                                             reverse copy
                                                                rotate
rotate_copy
                    search
                                             search n
                                                                set difference
                    set symmetric difference set union
set intersection
                                                                sort
stable partition
                    stable sort
                                             swap ranges
                                                                transform
uninitialized copy uninitialized copy n
                                             uninitialized fill uninitialized fill n
unique
                    unique copy
```

[Note: Not all algorithms in the Standard Library have counterparts in Table 1. — end note]

4.2 Definitions

[parallel.alg.defns]

```
Define GENERALIZED_SUM(op, a1, ..., aN) as follows:

— a1 when N is 1
— op(GENERALIZED_SUM(op, b1, ..., bMK), GENERALIZED_SUM(op, bM, ..., bN)) where
— b1, ..., bN may be any permutation of a1, ..., aN and
— 0 < M < N.
— 1 < K+1 = M ≤ N.

Define GENERALIZED_NONCOMMUTATIVE_SUM(op, a1, ..., aN) as follows:
— a1 when N is 1
— op(GENERALIZED_NONCOMMUTATIVE_SUM(op, a1, ..., aMK), GENERALIZED_NONCOMMUTATIVE_SUM(op, aM, ..., aN) where 0 < M < N 1 < K+1 = M ≤ N.</pre>
```

4.3 Non-Numeric Parallel Algorithms

[parallel.alg.ops]

4.3.1 Header <experimental/algorithm> synopsis

[parallel.alg.ops.synopsis]

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4.3.2 For each

7

[parallel.alg.foreach]

- ² Effects: Applies f to the result of dereferencing every iterator in the range [first,last). [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- ³ Complexity: Applies f exactly last first times.
- ⁴ *Remarks:* If f returns a result, the result is ignored.
- ⁵ *Notes:* Unlike its sequential form, the parallel overload of for_each does not return a copy of its Function parameter, since parallelization may not permit efficient state accumulation.
- ⁶ Requires: Unlike its sequential form, the parallel overload of for_each requires Function to meet the requirements of CopyConstructible.

- ⁸ Requires: Function shall meet the requirements of MoveConstructible [Note: Function need not meet the requirements of CopyConstructible. end note]
- ⁹ Effects: Applies f to the result of dereferencing every iterator in the range [first,first + n), starting from first and proceeding to first + n 1. [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- 10 Returns: first + n for non-negative values of n and first for negative values.
- 11 *Remarks:* If f returns a result, the result is ignored.

Effects: Applies f to the result of dereferencing every iterator in the range [first,first + n), starting from first and proceeding to first + n - 1. [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. — end note]

Returns: first + n for non-negative values of n and first for negative values.

Remarks: If f returns a result, the result is ignored.

Notes: Unlike its sequential form, the parallel overload of for_each_n requires Function to meet the requirements of CopyConstructible.

§ 4.3.2

4.4 Numeric Parallel Algorithms

[parallel.alg.numeric]

4.4.1 Header <experimental/numeric> synopsis

[parallel.alg.numeric.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  template<class InputIterator>
    typename iterator traits<InputIterator>::value type
      reduce(InputIterator first, InputIterator last);
  template<class InputIterator, class T>
    T reduce(InputIterator first, InputIterator last, T init);
  template<class InputIterator, class T, class BinaryOperation>
    T reduce(InputIterator first, InputIterator last, T init,
             BinaryOperation binary op);
template<class InputIterator, class OutputIterator>
---OutputIterator
exclusive scan(InputIterator first, InputIterator last,
           OutputIterator result);
  template<class InputIterator, class OutputIterator,
           class T>
    OutputIterator
      exclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init);
  template<class InputIterator, class OutputIterator,
           class T, class BinaryOperation>
    OutputIterator
      exclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init, BinaryOperation binary op);
  template<class InputIterator, class OutputIterator>
    OutputIterator
      inclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result);
  template<class InputIterator, class OutputIterator,
           class BinaryOperation>
    OutputIterator
      inclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     BinaryOperation binary op);
  template<class InputIterator, class OutputIterator,
           class T, class BinaryOperation, class T>
    OutputIterator
      inclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init, BinaryOperation binary op, T init);
```

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```
}
}
}
```

3

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4.4.2 Reduce

[parallel.alg.reduce]

² Effects: Same as reduce(first, last, typename iterator_traits<InputIterator>::value_type{}).

```
Returns: reduce(first, last, typename iterator_traits<InputIterator>::value_type{})
```

Requires: typename iterator_traits<InputIterator>::value_type{} shall be a valid expression. The operator+ function associated with iterator_traits<InputIterator>::value_type shall not invalidate iterators or subranges, nor modify elements in the range [first,last).

Complexity: O(last - first) applications of operator+.

Notes: The primary difference between reduce and accumulate is that the behavior of reduce may be non-deterministic for non-associative or non-commutative operator+.

```
template<class InputIterator, class T>
    T reduce(InputIterator first, InputIterator last, T init);
```

4 Effects: Same as reduce(first, last, init, plus<>()).

```
Returns: reduce(first, last, init, plus<>())
```

Requires: The operator+ function associated with T shall not invalidate iterators or subranges, nor modify elements in the range [first,last).

```
Complexity: O(last - first) applications of operator+.
```

Notes: The primary difference between reduce and accumulate is that the behavior of reduce may be non-deterministic for non-associative or non-commutative operator+.

- 6 Returns: GENERALIZED_SUM(binary_op, init, *first, ..., *(first + last first 1)*(first + (last first) 1)).
- Requires: binary_op shall not invalidate iterators or subranges, nor modify elements in the range [first,last).
- 8 Complexity: O(last first) applications of binary op.
- ⁹ *Notes:* The primary difference between reduce and accumulate is that the behavior of reduce may be non-deterministic for non-associative or non-commutative operator+binary op.

§ 4.4.2

4.4.3 Exclusive scan

[parallel.alg.exclusive.scan]

² Effects: Same as exclusive scan(first, last, result, init, plus<>()).

```
Returns: exclusive_scan(first, last, result, init, plus<>())
```

Requires: The operator+ function associated with iterator_traits<InputIterator>::value_type shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result + (last - first)).

Complexity: O(last - first) applications of operator+.

Notes: The primary difference between exclusive_scan and inclusive_scan is that exclusive_scan excludes the ith input element from the ith sum. If the operator+ function is not mathematically associative, the behavior of exclusive scan may be non-deterministic.

- Effects: Assigns through each iterator i in [result, result + (last first)) the value of GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, init, *first, ..., (*first + i - result - 1)*(first + (i - result) - 1)).
- ⁵ Returns: The end of the resulting range beginning at result.
- ⁶ Requires: binary_op shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last first)].
- ⁷ Complexity: O(last first) applications of binary_op.
- ⁸ Notes: The primary difference between exclusive_scan and inclusive_scan is that exclusive_scan excludes the ith input element from the ith sum. If binary_op is not mathematically associative, the behavior of exclusive scan may be non-deterministic.

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4.4.4 Inclusive scan

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[parallel.alg.inclusive.scan]

² Effects: Same as inclusive_scan(first, last, result, plus<>()).

```
Returns: inclusive scan(first, last, result, plus<>())
```

Requires: The operator+ function associated with iterator_traits<InputIterator>::value_type shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last - first)).

Complexity: O(last - first) applications of operator+.

Notes: The primary difference between exclusive_scan and inclusive_scan is that exclusive_scan excludes the ith input element from the ith sum. If the operator+ function is not mathematically associative, the behavior of inclusive_scan may be non-deterministic.

- 4 Effects: Assigns through each iterator i in [result, result + (last first)) the value of GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, *first, ..., (*first + i result)*(first + (i result))) or GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, init, *first, ..., (*first + i result)*(first + (i result))) if init is provided.
- ⁵ *Returns:* The end of the resulting range beginning at result.
- 6 Requires: binary_op shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last first)).
- ⁷ Complexity: O(last first) applications of binary op.
- ⁸ Notes: The primary difference between exclusive_scan and inclusive_scan is that inclusive_scan includes the ith input element in the ith sum. If binary_op is not mathematically associative, the behavior of inclusive_scan may be non-deterministic.

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