Vector Multiplication

COMP 408 - Linear Algebra
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Magnitude and Direction

The ***magnitude*** of a vector is the distance from the endpoint of the vector to the origin, that is, it's ***length***.

The magnitude of a vector $\vec{a}$, denoted by $|\vec{a}|$, can be computed by the Pythagorean theorem.

Example: $\vec{a} = [4, 3]$ and so $|\vec{a}| = \sqrt{(4^2 + 3^2)} = 5$.

A ***unit vector***, denoted by ^ on top, is a vector of magnitude 1. Unit vectors can be used to express the direction of a vector independent of its magnitude.

Example: The unit vector that corresponds to the direction of $\vec{a} = [4, 3]$ is $\hat{a} = [4, 3] / |\vec{a}| = [4/5, 3/5]$.

Linear independence

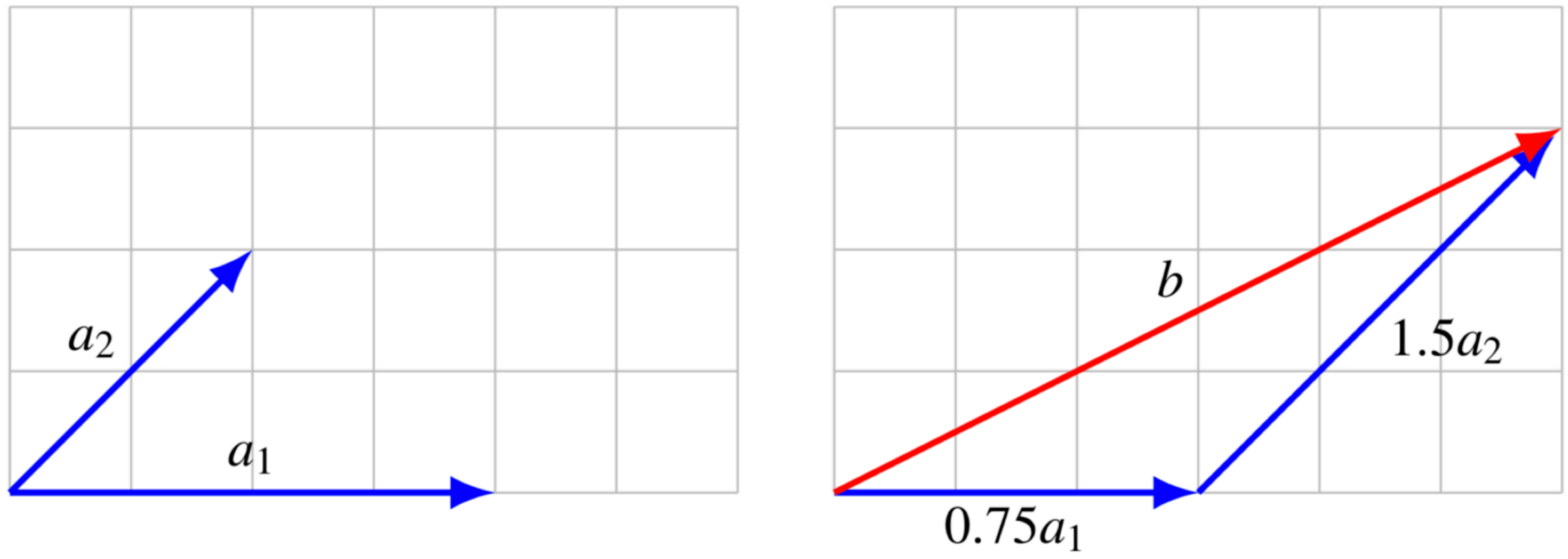
A family of vectors is ***linearly independent*** if no one of the vectors can be created by any linear combination of the other vectors in the family.

In other words, if two vectors point in different directions, they are said to be linearly independent.

If two vectors point in the same direction, then we can multiply one of the vector with a scalar to get the other vector, and the two vectors are said to be ***linearly dependent***.

Linear independence

Example: The below vector b is said to be linearly dependent to the vector a_1 and a_2.



Dot Product

A ***dot product*** (or ***scalar product***) is the numerical product of the lengths of two vectors, multiplied by the cosine of the angle between them, that is $\vec{a} \cdot \vec{b} = |\vec{a}|\ |\vec{b}|\ cos\ \theta$, where θ represents the angle between the two vectors.

A simply way to calculate a dot product is by multiplying the components of each vector separately and then adding these products together.

Example: $\vec{a} = [4, 3]$, $\vec{b} = [1, 2]$

$\vec{a} \cdot \vec{b} = (4 \text{ x } 3) + (3 \text{ x } 2) = 11$

Orthogonality

Two vectors are ***orthogonal*** to one another if the dot product of those two vectors is equal to zero.

Since $cos(90°) = 0$, as the angle between the two vectors opens up to approach 90°, the dot product of the two vectors will approach 0, regardless of the vector magnitudes.

Orthogonality can be considered as a mathematically precise way of saying ***perpendicular***.