12 Trees

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Outline

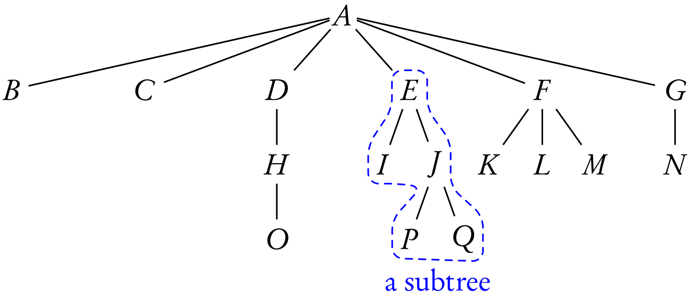
1. **Trees**
   - Concepts and Terms
   - Tree Traversals

2. **Binary Trees**
   - Perfectly Balanced Trees — Insertion

General Trees

A tree of type T is

- either empty, or
- a root node r which contains an element of type T; and zero or more non-empty T trees, called *subtrees*; and there is an edge going from r to the root node of each subtree.



The tree is a recursive data type.

Parents, Children and Siblings

- The root of each subtree is called a *child* of r, and r is the *parent* of each child. The number of children that a node has is called its *degree*.
- Nodes with the same parent are *siblings*.
- A node with no children (0-degree) is called a *leaf*, or an *external node*; otherwise it is called an *internal node*.
- If the order of the siblings are significant, then the tree is called an *ordered tree*.
- An *unordered tree* can be specified by the set of its edges: $\{parent \to child\}$.

$$\{A \to B, A \to C, A \to D, A \to E, A \to F, A \to G, D \to H, E \to I,$$
$$E \to J, F \to K, F \to L, F \to M, G \to N, H \to O, J \to P, J \to Q\}$$

Paths and Depths

- A path from node n_1 to n_k is a sequence of nodes $n_1, n_2, \dots, n_k$ such that

$$n_i \text{ is the parent of } n_{i+1}, \quad \text{for } 1 \leqslant i < k.$$

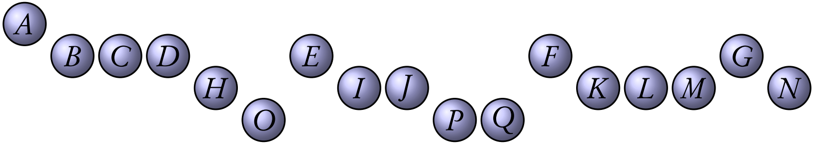
  The number of edges on the path is called its length, that is, $k-1$.
- The *depth* (*level*) of a node is the length of the unique path from the root to the node. The depth of a tree is the depth of the deepest leaf.

  The depth of the root node is 0.
- The *height* of a node is the length of the longest path starting from the node. The height of a tree is the height of its root.

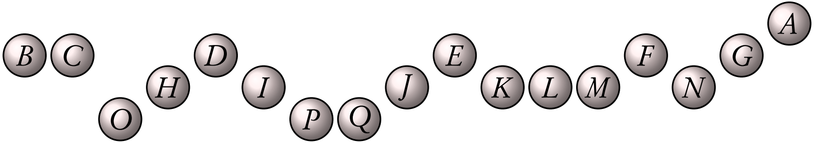
  The depth of a tree equals the height of the tree.
- If there exists a path from node x to node y, then x is the *ancestor* of y and y is a *descendant* of x. If $x \neq y$, then they are called proper ancestor and descendant.
- The number of nodes in a tree is called the *size* of the tree.

Tree Traversals

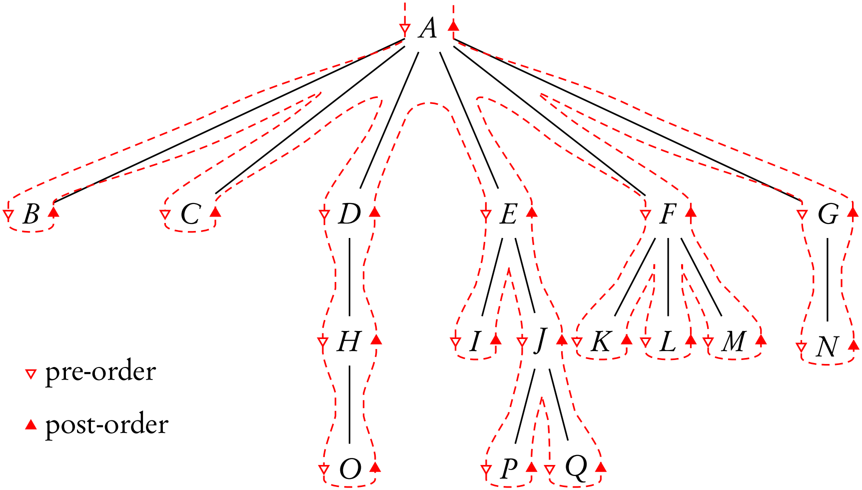
Pre-order traversal: 1) visit the root node;
2) recursively traverse each subtree of the root.



Post-order traversal: 1) recursively traverse each subtree of the root;
2) visit the root node.

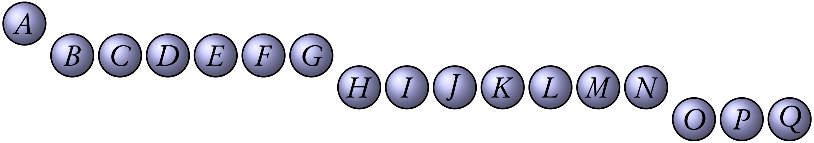


Euler Tour Traversal



Tree Traversals — Depth First and Breadth First

- Pre-order and post-order traversals are cases of *depth first search*, which can be performed recursively.
- Alternatively, we usually use FIFOs (queues) to perform *breadth first search*. A *BFS* visits the tree nodes in increasing depths.
  - Push-back the root node into an empty queue;
  - While the queue is not empty, do
    Pop a node from the queue, and visit it;
    Push-back all of its children (if any) into the queue.

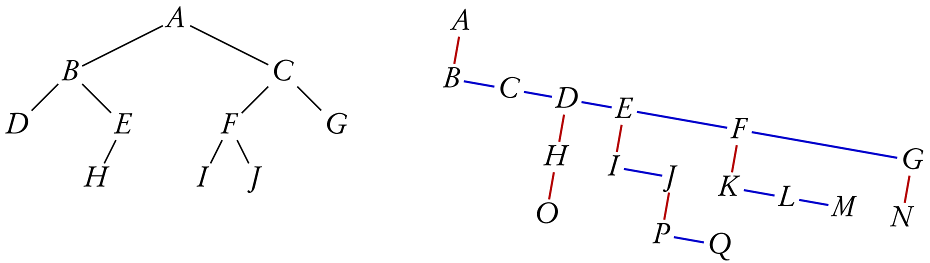


Binary Trees

A binary tree is

- either *empty*, or
- a node with exactly two sub (binary) trees (may all be empty). The two subtrees are called *left* subtree and *right* subtree. It is an ordered tree.

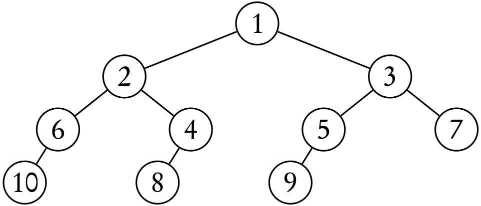
Binary trees are special cases of trees, however, we can encode general trees as binary trees.



Perfectly Balanced (Binary) Trees

If for every node in a tree, the size difference of its left and right subtrees is at most 1, then the tree is a perfectly balanced tree.

An n-node perfectly balanced tree has depth $\lfloor \log n \rfloor$.



To build a perfectly balanced tree from scratch, we insert new nodes to the tree as follows:

- If it is an empty tree, we make the new node the root;
- Otherwise, we recursively insert the node to the right, and then swap the left and right subtrees on the way back.

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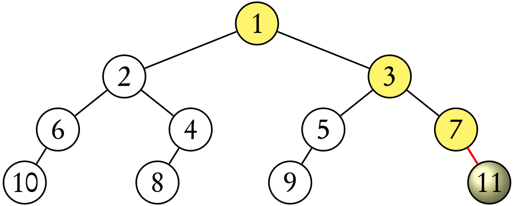

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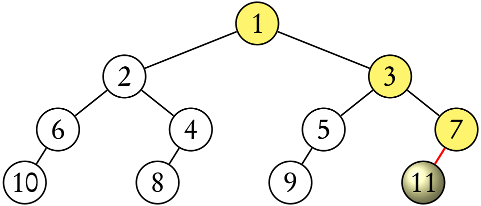

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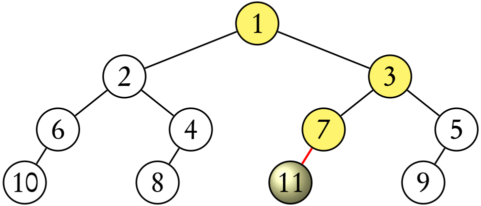

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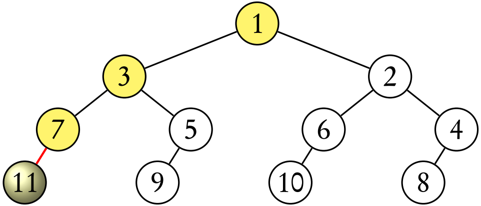

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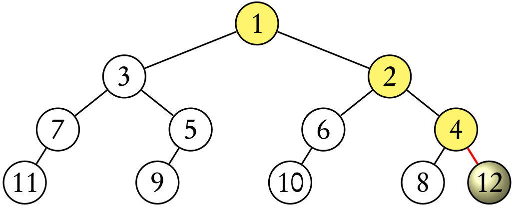

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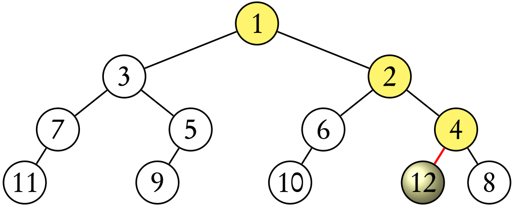

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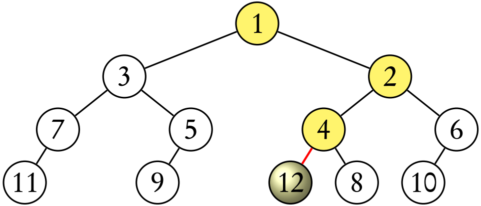

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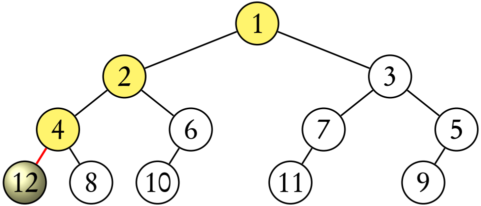

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Representing Binary Trees

- A binary tree can be represented as a reference to a tree node, and we can use `None` to represent an empty tree.

```
class Node:
    def __init__(self, elm):
        self.elm = elm
        self.left = self.right = None
```

- The preorder generator function of such a binary tree can be recursively defined as follows.

```
def preorder(root):
    if root is not None:
        yield root.elm
        yield from preorder(root.left)
        yield from preorder(root.right)
```

Node Insertion of Perfectly Balanced Trees

The function *insert_bal* returns the new root node of the tree after the insertion of node *p*.

```
def insert_bal(root, p):
    if root is None:
        p.left = p.right = None
        return p
    else:
        root.left, root.right = insert_bal(root.right, p), root.left
        return root
```