Linear Function

COMP 408 - Linear Algebra
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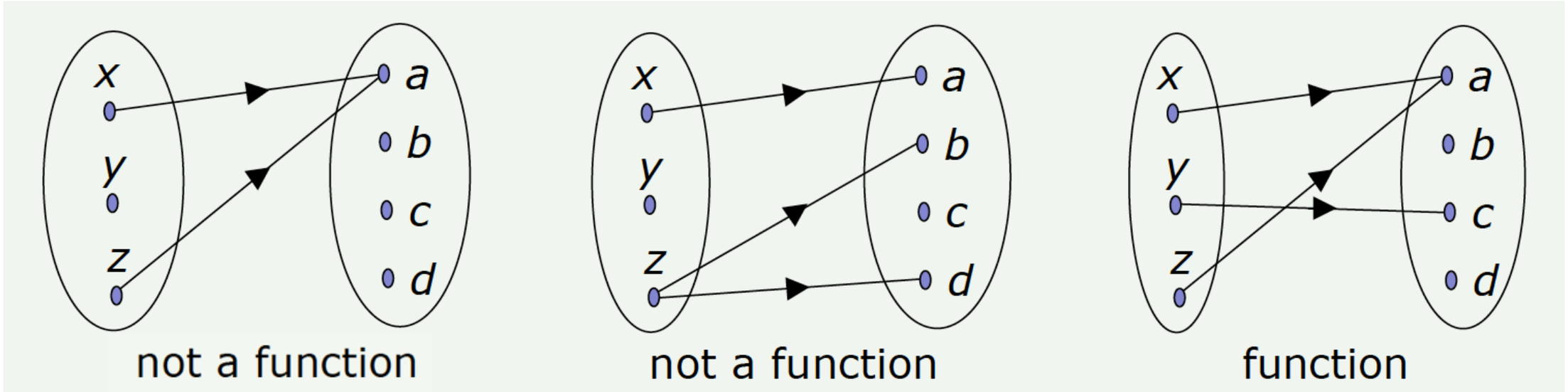
Functions

A ***function*** *f* is a mapping between 2 sets A and B, denoted by *f: A -> B*, such that each $a \in A$ maps to exactly one element in B.

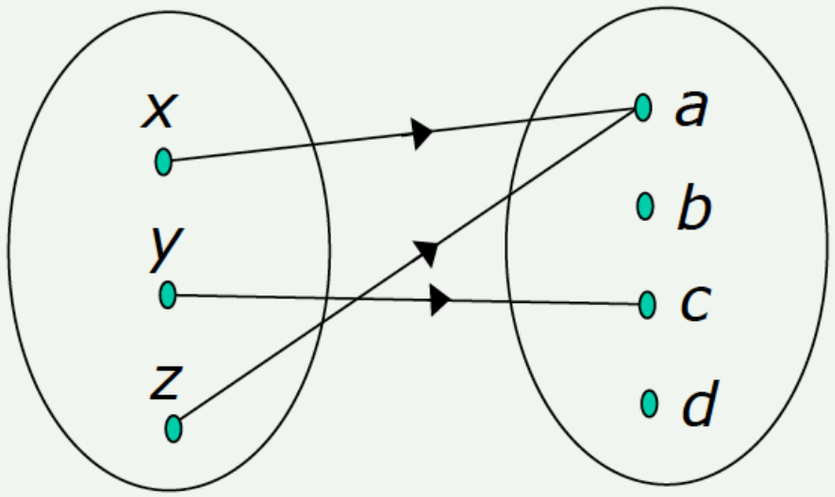


We write *f(a) = b* if the function *f* maps the element $a \in A$ to the element $b \in B$.

Domain and codomain

Let *f* be a function from the sets A to B.

Then we say that A is the ***domain*** of the function *f* and B is the ***codomain*** of the function *f*.



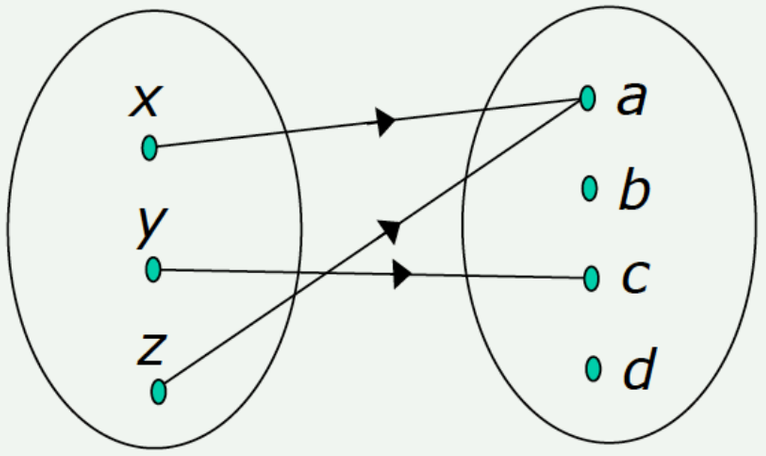
Domain is {x,y,z}
codomain is {a,b,c,d}

We also say *b* is an ***image*** of *a* (or *a* is a ***preimage*** of *b*) when *f(a)* = *b*.

Range

Let *f* be a function from the sets A to B.

The ***range*** of *f* is the subset of B defined as follows: $b \in B$ belongs to the range if and only if it has a preimage under *f.*



Range is {a,c}

Example

Consider the function f: $\mathbf{R}^+ \to \mathbf{R}$

$$x \to 2 - \sqrt{x}$$

Domain is $\mathbf{R}^+$ and codomain is $\mathbf{R}$.

Range is $]-\infty, 2[$.

Question: If the domain of *f* is changed to $\mathbf{R}$, is *f* still a function? Why?

Linear function

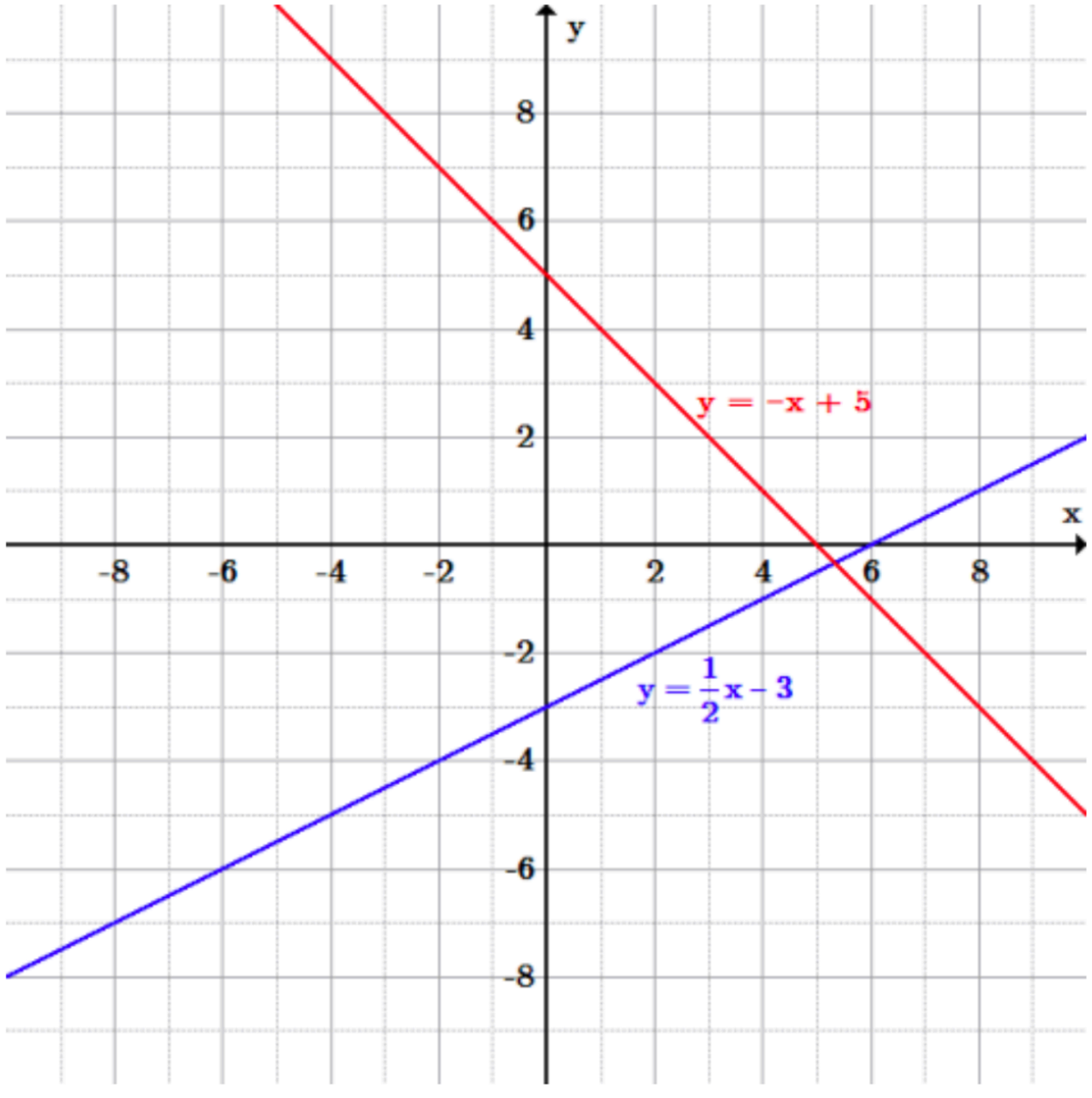
A ***linear function*** is an algebraic equation in which each ***term*** is either a ***constant*** or ***the product of a constant and (the first power of) a single variable***.

For example, a common equation, ***y = mx + b***, is a linear function because it meets both criteria with *x* and *y* as ***variables*** and *m* and *b* as ***constants***.

It is linear: the ***exponent*** of the *x* term is a ***one*** (first power), and it follows the definition of a ***function***: for each input (*x*) there is exactly one output (*y*).

Graphs of linear functions

A linear function is a function whose graph is a straight line.

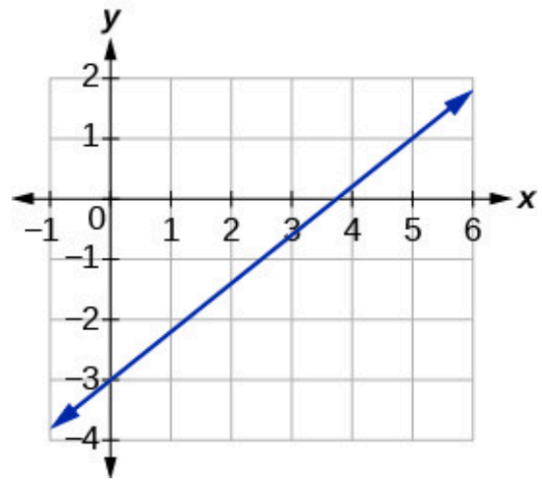


Slope

The ***slope*** of a line is a number that describes both the ***direction*** and the ***steepness*** of the line.

The slope of a line can be calculated with the formula $m = (y_2 - y_1) / (x_2 - x_1)$, where (x_1 , y_1) and (x_2, y_2) are points on the line.

Example: What is the slope of the following line?



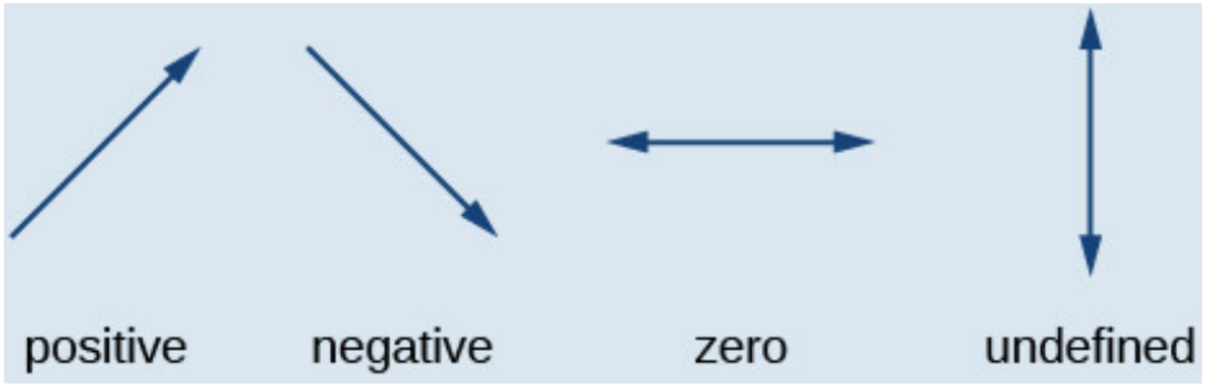
Two points on the line are (5, 1) and (0, -3), thus the slope is (1 -(-3)) / (5 - 0) = 4/5.

Slope

The direction of a line is either ***increasing***, ***decreasing***, ***horizontal*** or ***vertical***.

A line is ***increasing*** if it goes up from left to right which implies that the slope is ***positive***. A line is ***decreasing*** if it goes down from left to right and the slope is negative.

If a line is ***horizontal*** the slope is zero and the function y is equal to a constant function ($y = c$). If a line is ***vertical*** the slope is undefined (or infinity).



Slope

The ***steepness***, or ***incline***, of a line is measured by the ***absolute value*** of the slope.

A slope with a greater absolute value indicates a ***steeper*** line.

For example, a line with a slope of –9 is steeper than a line with a slope of 7 .

y-intercept

The ***y-intercept*** of a line is a point at which a line crosses the y-axis of a Cartesian grid.

A linear function can have none, one, or infinitely many y-intercepts.

If the function is a vertical line, it will have no y-intercept unless its equation is $\boldsymbol{x = 0}$, in which case it will have infinitely many y-intercept. If the line is non-vertical, it will have one y-intercept.

Exercise: Find the y-intercept of $y = 1/2\ x + 2$?

Slope-intercept form and standard form

The ***slope-intercept form*** of a line is given by $y = mx + b$ where m is the slope of the line and b is the y-intercept.

The ***standard form*** of a linear equation is written as: $Ax + By = C$, where A, B and C are constants.

Exercise: Find the slope-intercept form and standard form of the following line?