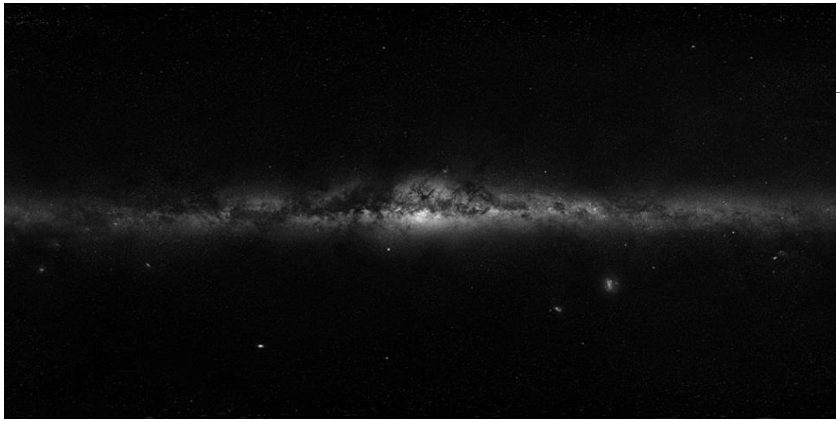
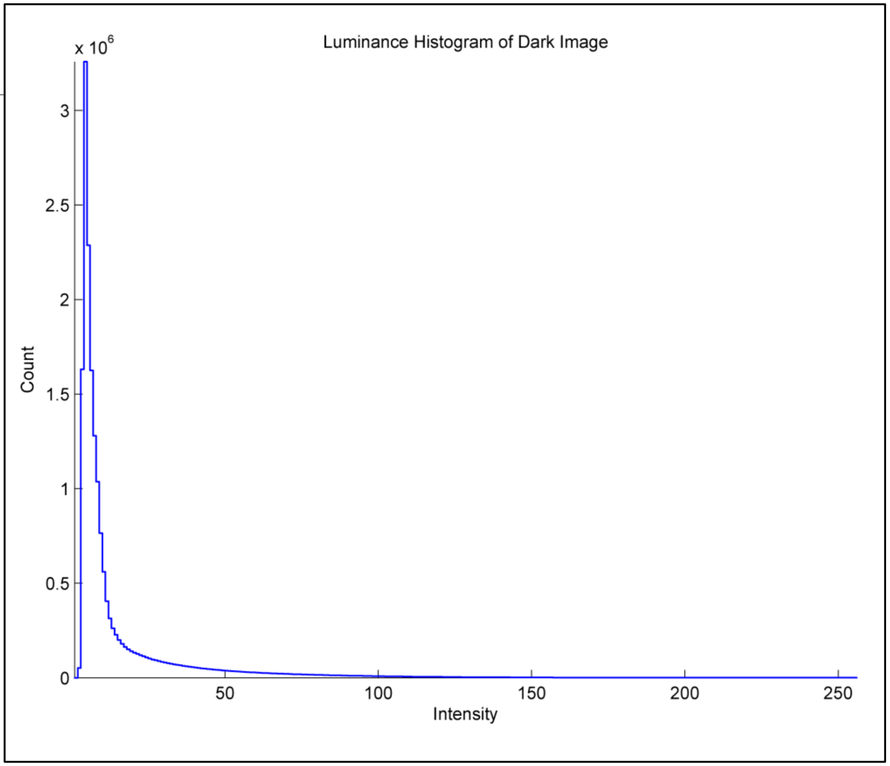
IMAGE HISTOGRAM

Image Histogram

- The histogram of an image is *a tally of the number of pixels at each intensity level or colour*.
- The shape of the histogram is related to *the ranges and groupings* of intensity values in the image.
- In the following monochrome examples notice how the peaks of in the histogram correspond to concentrations of intensities in the image globally.
- In the colour examples the primary that has the largest value at any intensity <u>dominates</u> the image.

Monochrome Intensity Distribution

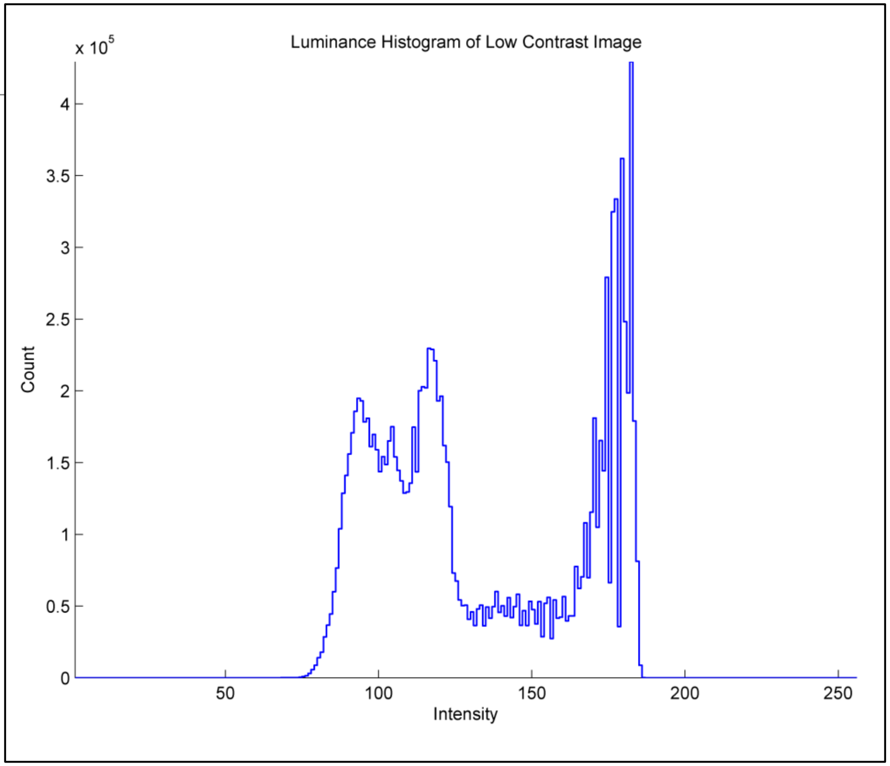




This image is a small, monochrome version of a huge colour mosaic made by the ESO[1]. It contains both celestial hemispheres; it is what you would see in 360° from empty space in the plane of the galaxy above or below the earth.

[1]The Euopean Southern Observatory in the Atacama desert of Chile, http://www.eso.org/public/usa/images/eso0932a/

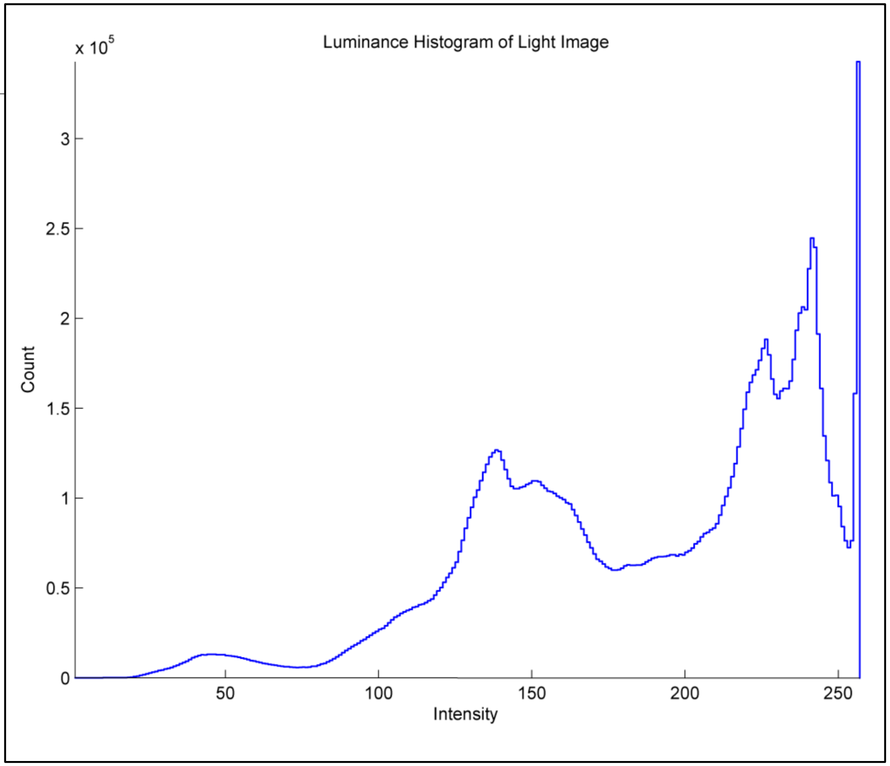
Monochrome Intensity Distribution



This picture, taken in the morning fog, displays low contrast – a narrow range of intensities – with energy at the extremes.

Photographer unknown, downloaded from http://hqwallbase.com/21961-trees-fog-wallpaper-[2]/

Monochrome Intensity Distribution



Castner glacier in the Delta mountains, Alaska. Monochrome extracted from original colour image. Note how the peaks in the histogram correspond to regions in the image.

Photographer unknown, downloaded from https://contest.thesca.org/snow2012/zig-zags-snow

Image vs. Histogram

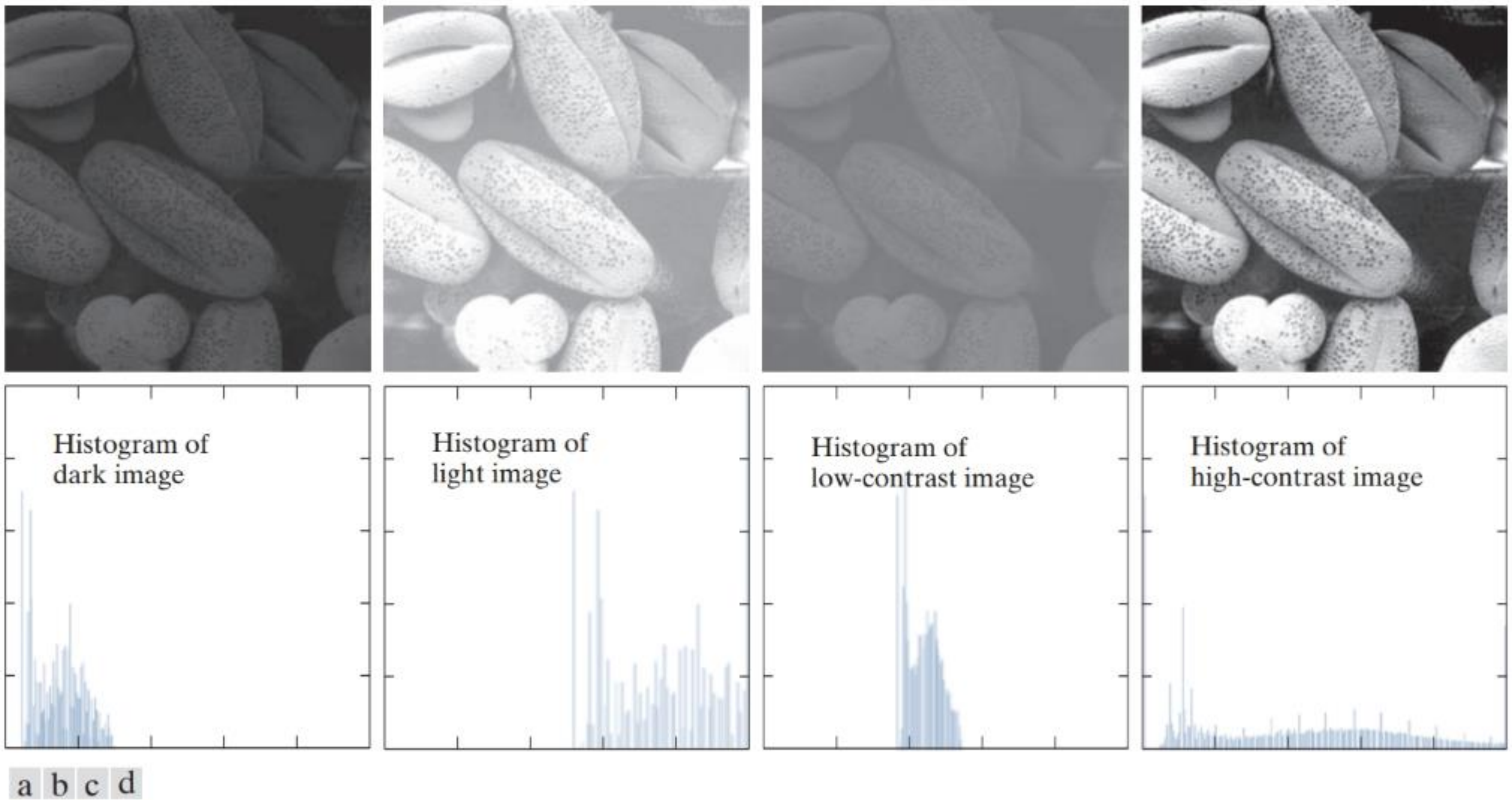


FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

Colour Intensity Distribution

Castle Rock, Sedona, Arizona. There is one histogram for each of red, green, and blue. The red rock's colour is in the midrange of intensities while the greenery is darker. Blue peaks correspond to the haze on the mountainside (dark) and the sky (bright).

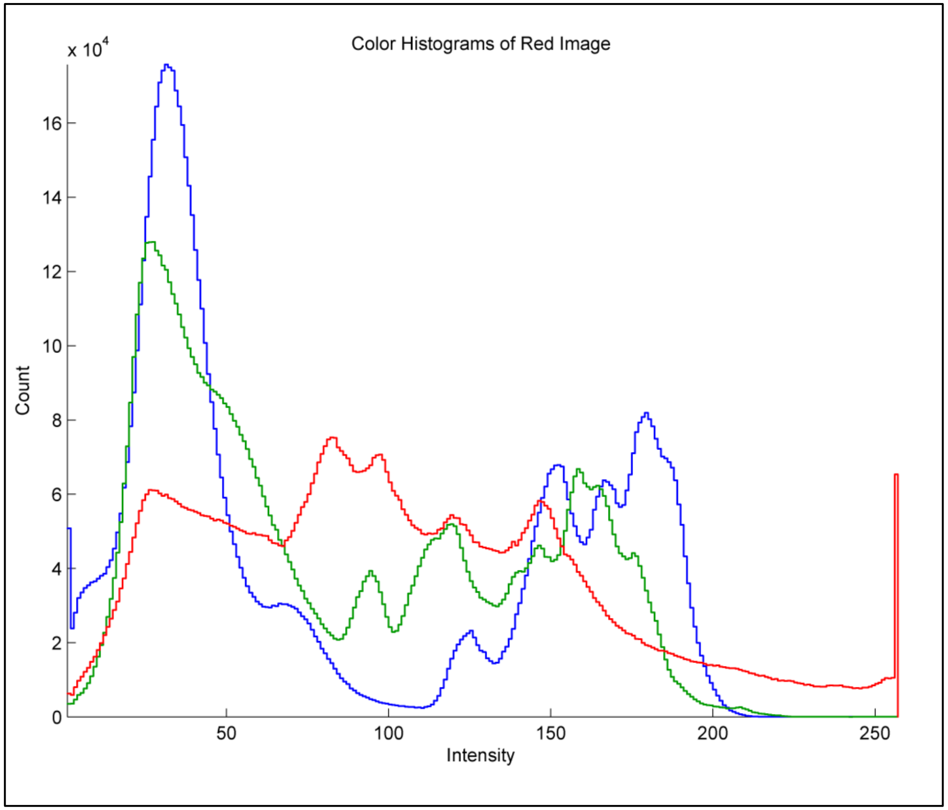
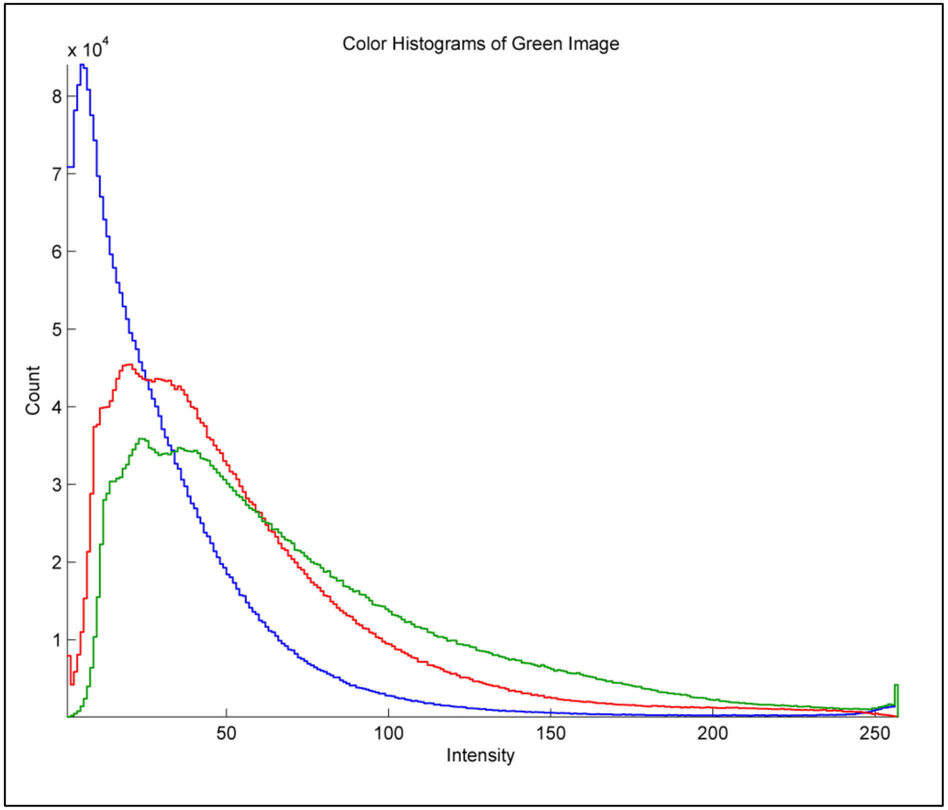


Photo by Edward Chavez, http://www.zensoulstyle.com

Colour Intensity Distribution



Unidentified place in a photo from the website below. Notice that the intensity of green dominates the others over much of the range. Red dominant corresponds to yellow-green regions. Blue dominates in the shadows.

Photographer Unknown, downloaded from http://forum.baboo.com.br/index.php?/gallery/image/20033-floresta-80/

Colour Intensity Distribution

Blue Poison Dart Frog (Dendrobates azureus) in the Frankfurt Zoo, Germany. Dominant colours in increasing intensity: brown, blue, tan brown, blue.

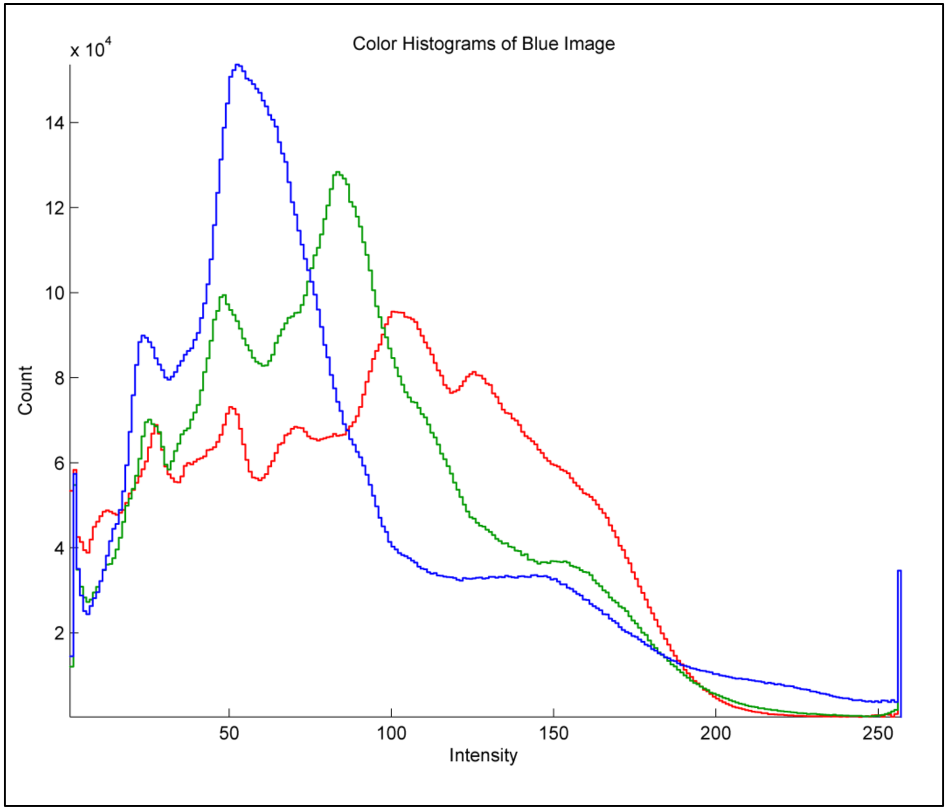
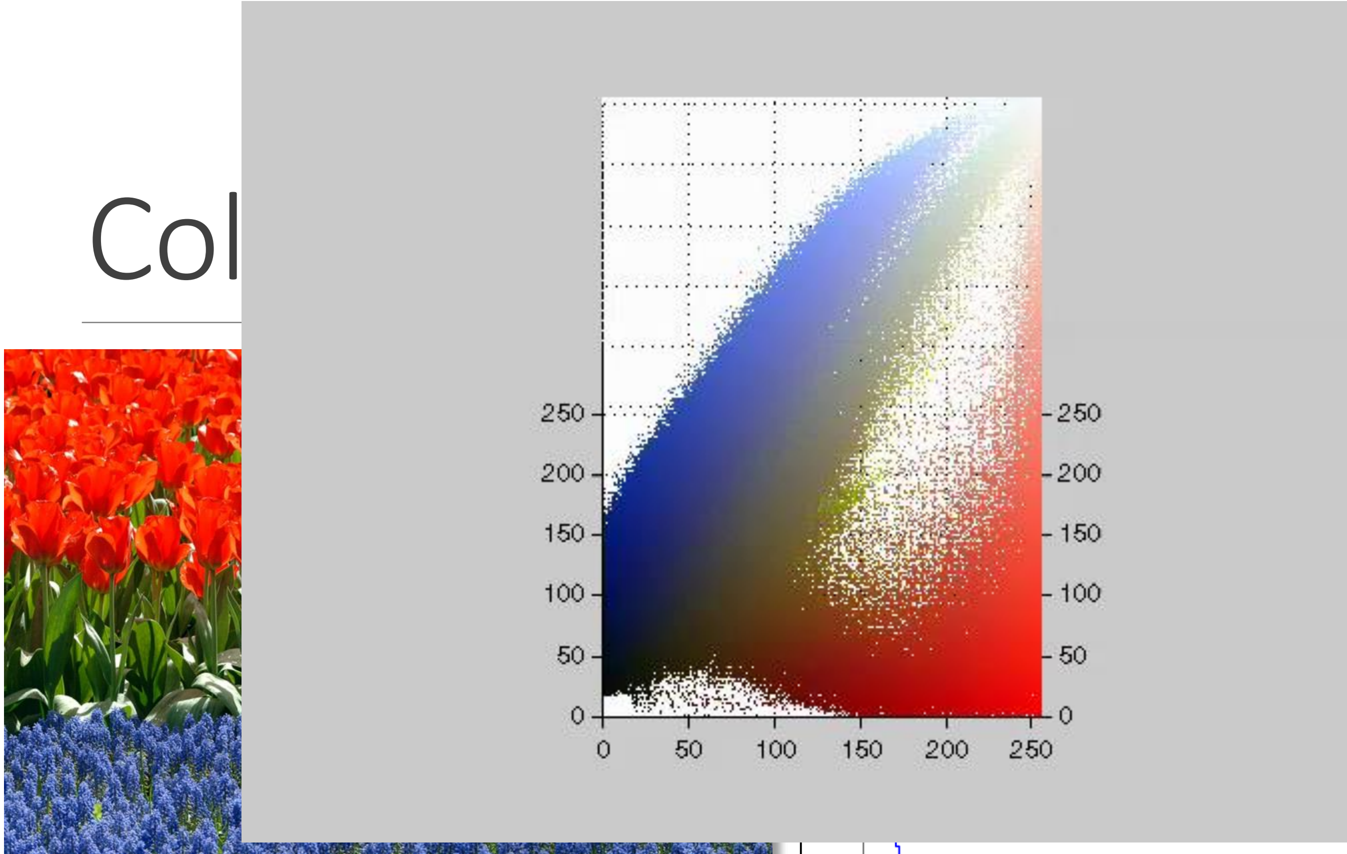


Photo by Wikipedia user, Quartl: http://en.wikipedia.org/wiki/File:Dendrobates_azureus_qtl1.jpg/ .

Col on

Notice the Necker cube illusion[1] as the virtual cube rotates.



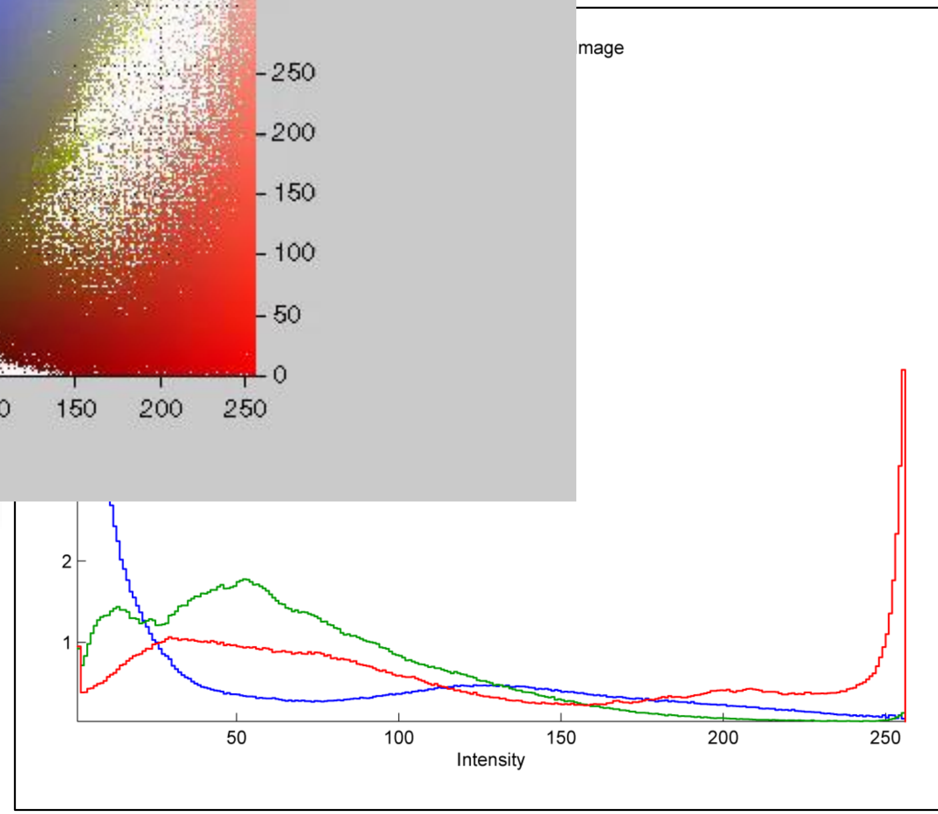


Photo taken in the gardens at Keukenhof, Holland, The Netherlands. RGB primaries dominant at different intensities: blue shadows, green tulip stems, blue hyacinths, red tulip flowers.

Photo by Jim Pyre: http://thedude.com/archives/2005/04/amsterdam.html

Histogram of monochrome image

- The histogram of an image is a tally of the number of pixels at each intensity level or colour. For a monochrome image **I**,

$$H_{\mathbf{I}}(g) = \#\{\mathbf{p} | \mathbf{I} = g\}.$$

- The value of the histogram at *g* is the number of pixels for which image **I** has intensity level *g*. For an 8-bit image, *H* has 256 values

$$H_{\mathbf{I}} : \{0, \ldots, 255\} \rightarrow \{0, \ldots, R \times C\}.$$

- If **I** is an $R \times C$ image and all its pixels have the same intensity, g_0, then $H(g_0) = R \times C$ and $H(g) = 0$ for all intensities $g \neq g_0$.

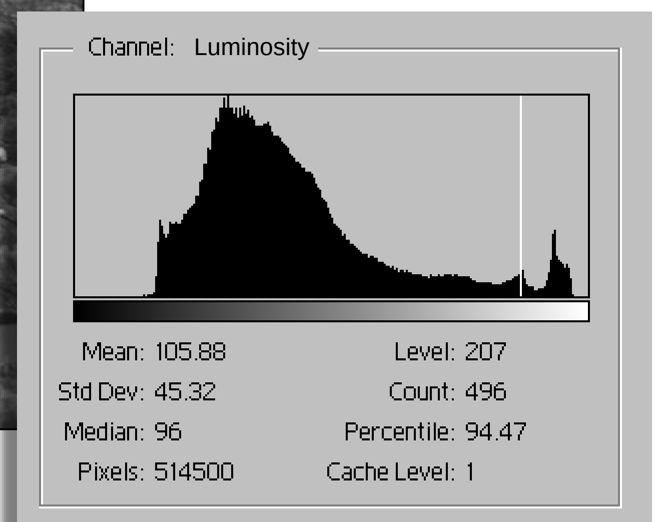
Histogram of monochrome image

- If **I** is a 1-band (monochrome) image, then
- the pixel **I**(x,y) is an 8-bit integer between 0 and 255.
- The histogram, $h_{\mathbf{I}}$, of **I** is:
  - a 256-element array, $h_{\mathbf{I}}$, where
  - $h_{\mathbf{I}}(g)$ is an integer for g = 1, 2,..., 256, such that
  - $h_{\mathbf{I}}(g)$ = number of pixels in **I** that have value g-1
  - Or $h_{\mathbf{I}}(g+1)$ = number of pixels in **I** that have value g (Matlab/Scilab).

In Matlab an array of length n has indices from 1 to n. In many computer languages, *e.g.* "C" or "C++" an n-element array is indexed from 0 to n-1.

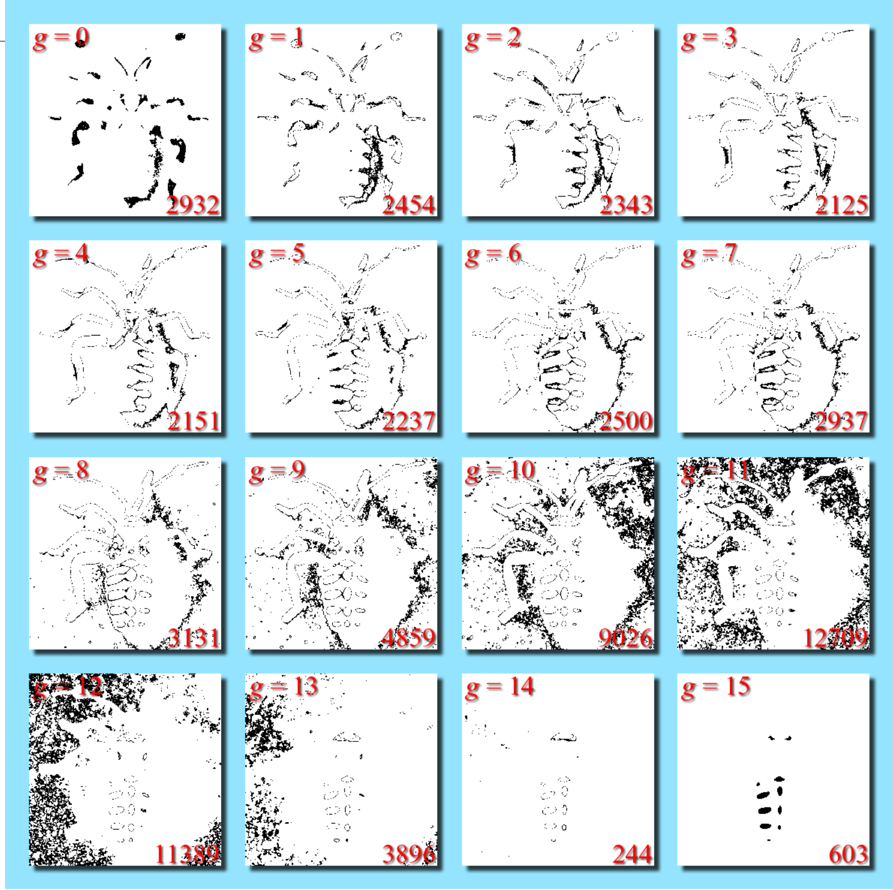
Histogram of monochrome image

$h_I(g)$ = the number of pixels in I with intensity level, $g-1$.



Histogram of monochrome image

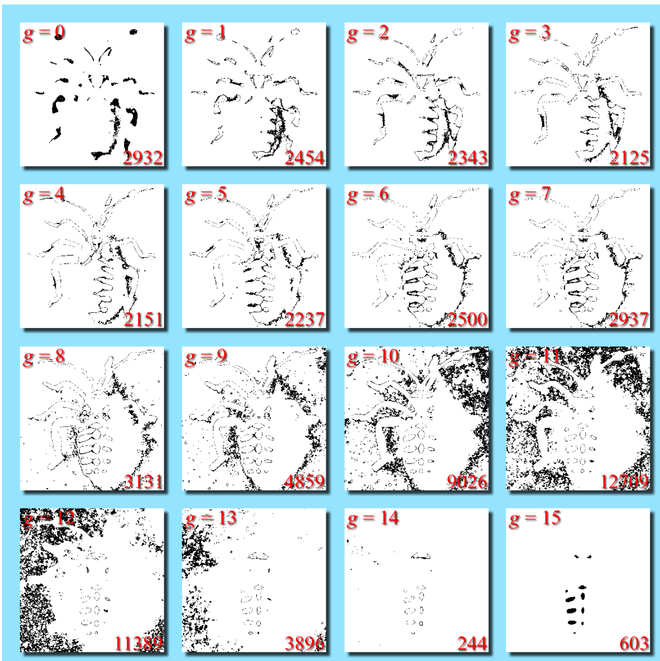
16-level (4-bit) image



lower RHC: number of pixels with intensity *g*

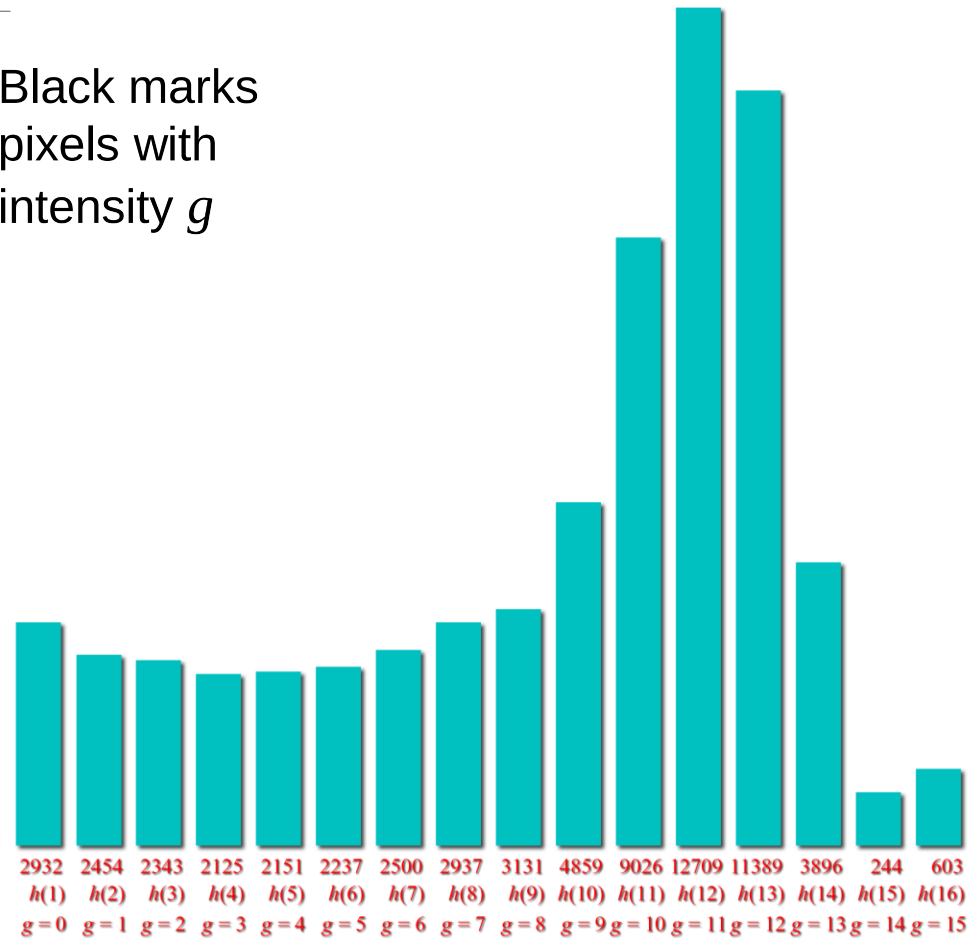
black marks pixels with intensity *g*

Histogram of monochrome image

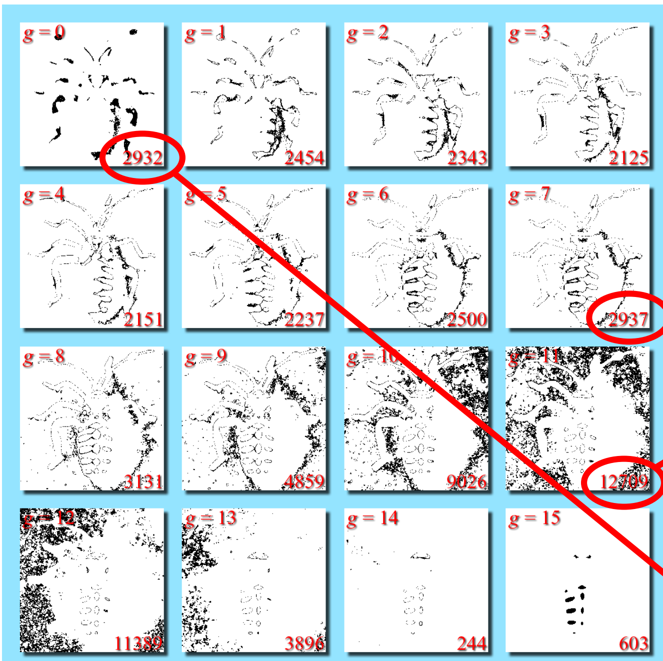


Black marks pixels with intensity g

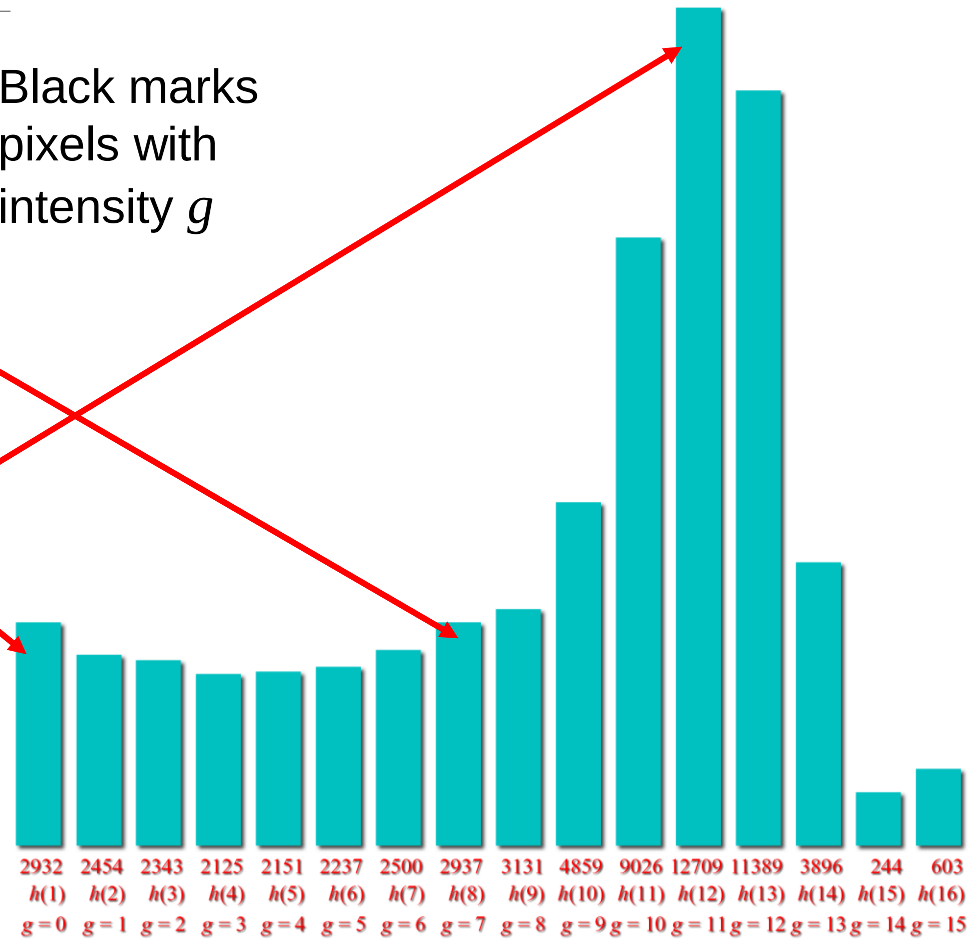
Plot of histogram:
number of pixels with intensity g



Histogram of monochrome image



Black marks pixels with intensity g

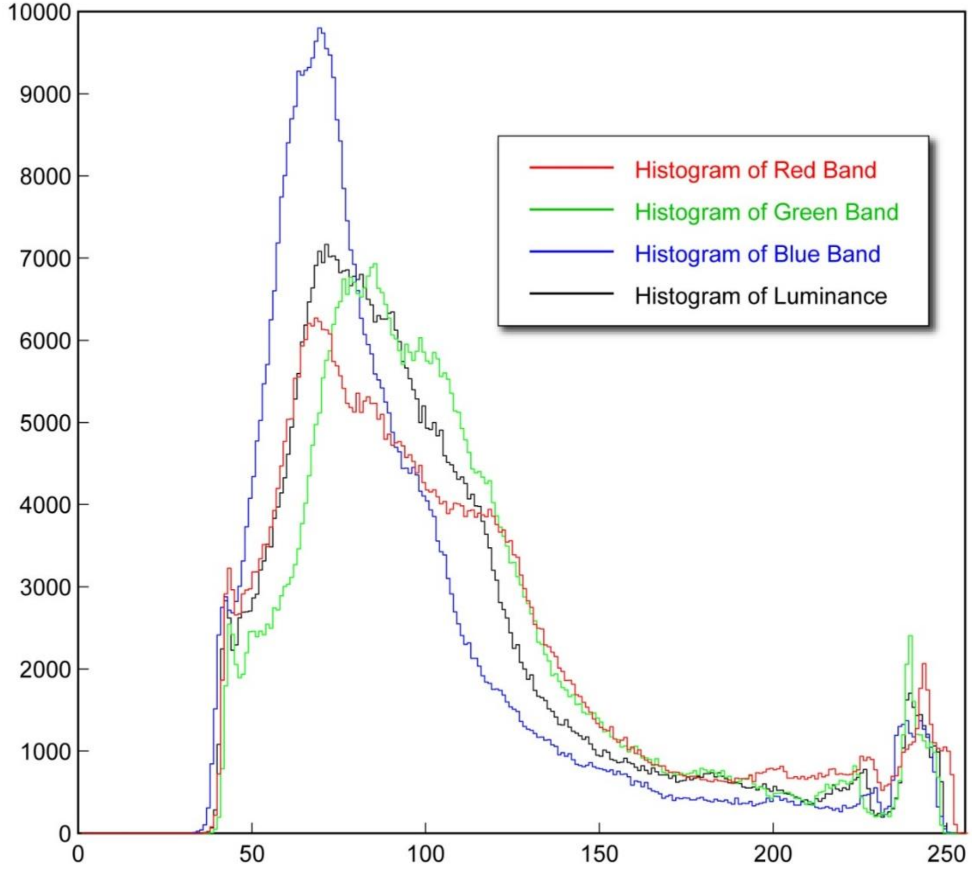


Plot of histogram:
number of pixels with intensity g

Histogram of colour image

- If **I** is a 3-band image (truecolour, 24-bit)
- then **I**(r,c,b) is an integer between 0 and 255.
- Either **I** has 3 histograms:
  - $h_{\mathbf{R}}(g+1)$ = # of pixels in **I**(:,:,1) with intensity value g
  - $h_{\mathbf{G}}(g+1)$ = # of pixels in **I**(:,:,2) with intensity value g
  - $h_{\mathbf{B}}(g+1)$ = # of pixels in **I**(:,:,3) with intensity value g
- or 1 vector-valued histogram, $h(1,g,b)$ where
  - $h(1,g+1,1)$ = # of pixels in **I** with red intensity value g
  - $h(1,g+1,2)$ = # of pixels in **I** with green intensity value g
  - $h(1,g+1,3)$ = # of pixels in **I** with blue intensity value g

Histogram of colour image



Value Image

How to extract a monochrome intensity image from a colour image.

If I is a rgb image, then I's value image, V, has one band that is the pixel-wise average of I's R, G, & B bands:

$$\mathbf{V}(r,c)=\frac{1}{3}\left[\mathbf{R}(r,c)+\mathbf{G}(r,c)+\mathbf{B}(r,c)\right].$$

This is easily computed in Matlab/Scilab by

```
V=sum(I,3)/3;
```

The 3 in the 2^{nd} argument of sum tells it to act along dimension 3 of the image – across the colour bands.

How to extract a monochrome intensity image from a colour image.

Luminance Image

I's luminance image, **L**, is a 1-band image that is a specific, weighted, pixel-wise average of **I**'s **R**, **G**, and **B** bands:

$$\mathbf{L}(r,c) = 0.299 \times \mathbf{R}(r,c) + 0.587 \times \mathbf{G}(r,c) + 0.114 \times \mathbf{B}(r,c)$$

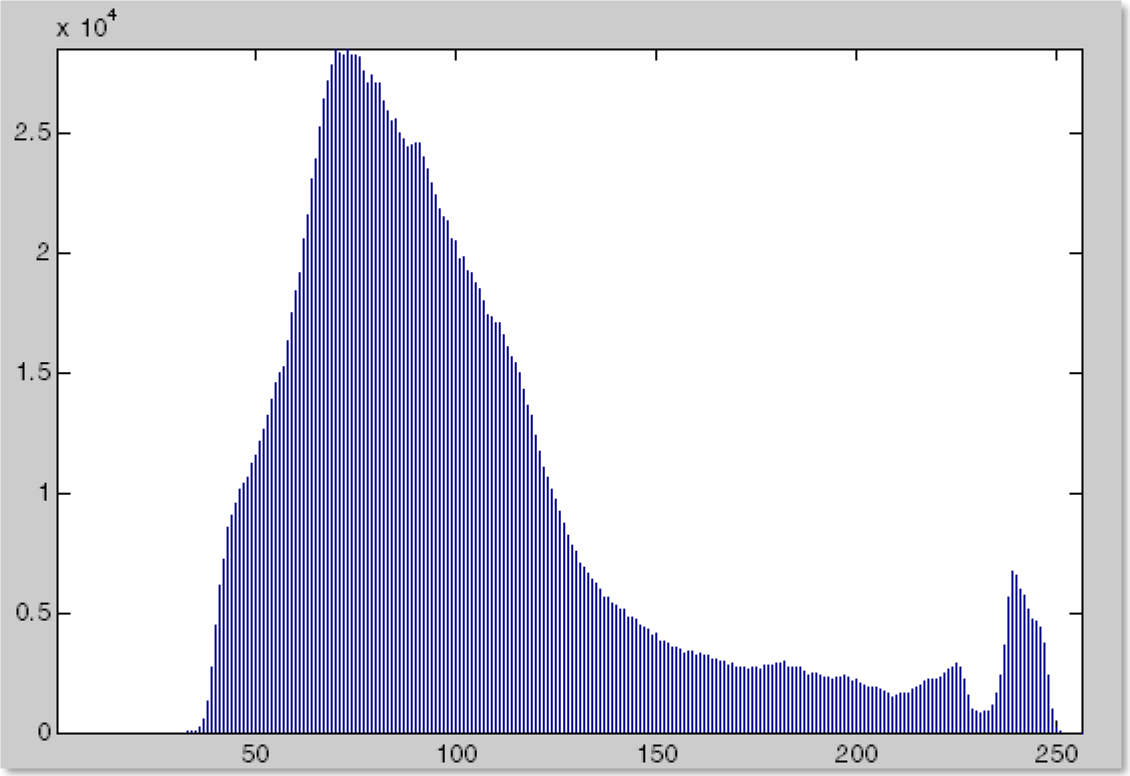
The numbers were derived by the NTSC[1] to weight each colour band according to the relative intensity resolution that colour by the human eye. The following Matlab code will compute it

```
I=double(I); %%unit8 to double
L=unit8(0.299*I(:,:,1)+ 0.587*I(:,:,2)+
0.114*I(:,:,3) %%weighted sum of c
```

[1] National Television System Committee, 1953, http://en.wikipedia.org/wiki/NTSC

Histogram of value image

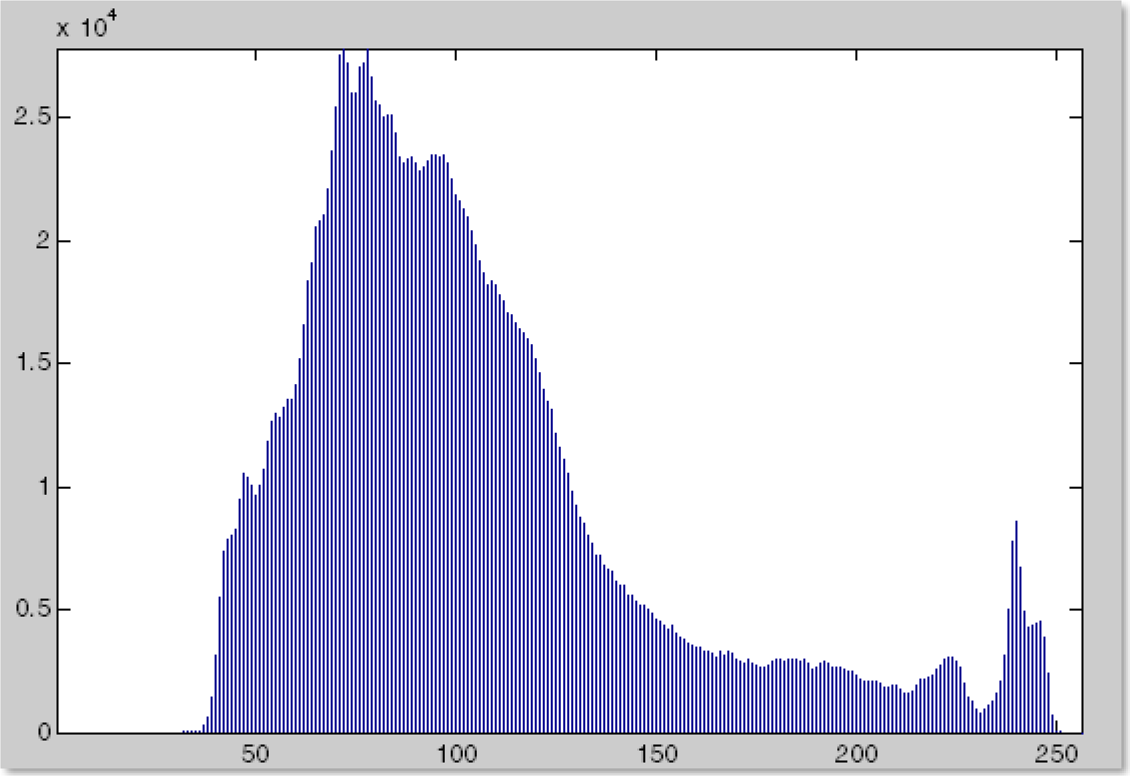
Value image, V.



Histogram of the value image.

Histogram of luminance image

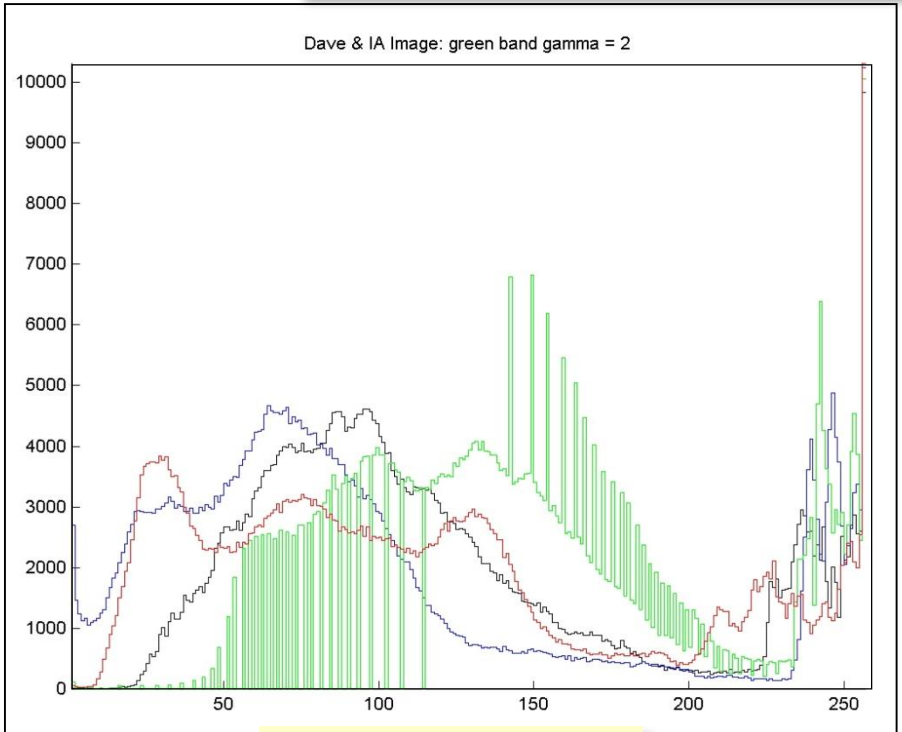
Luminance image, L.



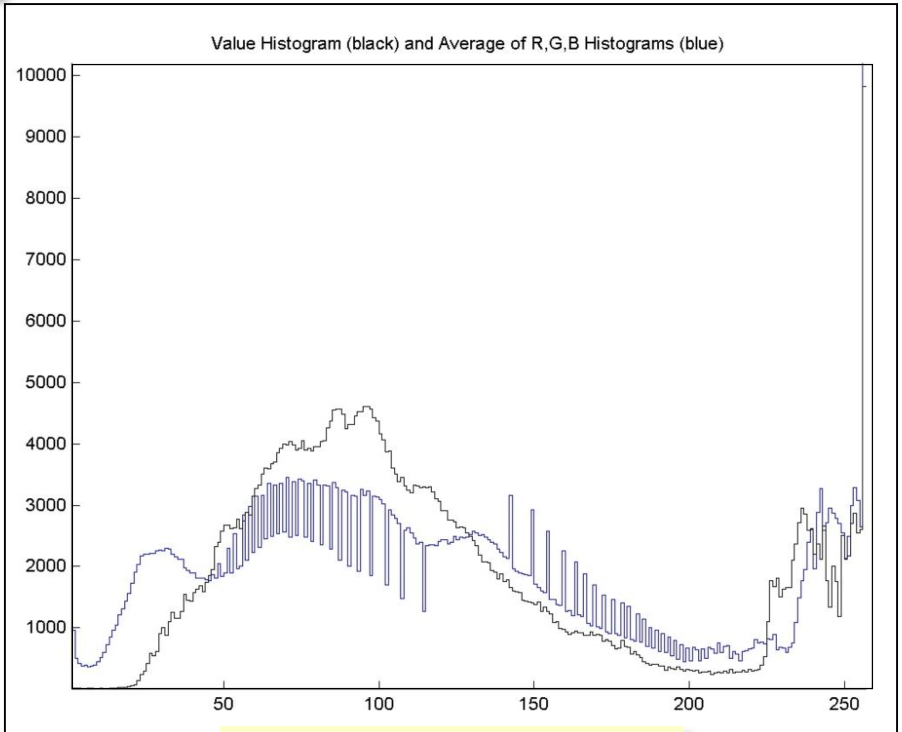
Histogram of the luminance image.

Value Histogram vs. Average of R,G,&B Histograms

The value histogram is **NOT** the average of the three 1-D colour intensity histograms.



R,G,B,&V histograms



V & avg. of R,G,&B histograms

Multi-Band Histogram Calculator in Matlab/Scilab

```
% Multi-band histogram calculator
function h=histogram(I)

[r,c,b]=size(I);

% initialize the histogram
h=zeros(1,256,b);


% range through the intensity values
for g=0:255
   h(1,g+1,:) = sum(sum(I==g,1),2);% accumulate
end


end
```

Multi-Band Histogram Calculator in Matlab/Scilab

```
% Multi-band histogram calculator
function h=histogram(I)

[r,c,b]=size(I);

% initialize the histogram
h=zeros(1,256,b);


% range through the intensi
for g=0:255
    h(1,g+1,:) = sum(sum(I==g,1),2);% accumulate
end


end
```

Loop through all intensity levels (0-255)
Tag the elements that have value g.
The result of `I==g` is an rxcxb *logical* array that has a 1 wherever $\mathbf{I}(r,c,b)$ = g and 0 everywhere else.
Compute the number of 1s in each band of the image for intensity g.
Store that value in the 1x256xb histogram at `h(1, g+1, b)`.

`sum(sum(I==g,1),2)` computes one number for each band in the image.

If `b==3`, then `h(1,g+1,:)` contains 3 numbers: the number of pixels in bands 1, 2, & 3 that have intensity g.

The probability density function of an Image

Here we consider a 1-band (monochrome) image **I**

Let $A = \sum_{g=0}^{255} h_{\mathbf{I}}(g+1)$.

Note that since $h_{\mathbf{I}}(g+1)$ is the number of pixels in **I** with value g,

A is the total number of pixels in **I**. That is if **I** is R rows by C columns then $A = R' \; C$.

Then,

$$p_{\mathbf{I}}(g+1) = \frac{1}{A} h_{\mathbf{I}}(g+1)$$

is the gray level probability density function of **I**.

pdf
[*lower case*]

This is the probability that an arbitrary pixel from **I** has value g.

The probability density function of an Image

- $p_{\mathbf{I}}(g+1)$ is the fraction of pixels in an image that have intensity value g.
- $p_{\mathbf{I}}(g+1)$ is the probability that a pixel has intensity value g.
- Whereas the sum of the histogram $h_{\mathbf{I}}(g+1)$ over all g from 0 to 255 is equal to the number of pixels in the image, the sum of $p_{\mathbf{I}}(g+1)$ over all g is 1.
- $p_{\mathbf{I}}$ is the normalized histogram of the band.

The Cumulative Distribution Function of an Image

Let α be the value of a randomly selected pixel from **I**. Let g be a specific grey level. The probability that $\alpha \le g$ is given by

$$P_{\mathbf{I}}(g+1) = \sum_{\alpha=0}^{g} p_{\mathbf{I}}(\alpha+1) = \frac{1}{A}\sum_{\alpha=0}^{g} h_{\mathbf{I}}(\alpha+1)$$

where $h_{\mathbf{I}}(\alpha+1)$ is the number of pixel whose grey level is α in **I**.
$p_{\mathbf{I}}(\alpha+1)$ is the probability that any given pixel from I has the value equal to α

CDF
[upper case]

This is the probability that any given pixel from **I** has value less than or equal to g.

The Cumulative Distribution Function of an Image

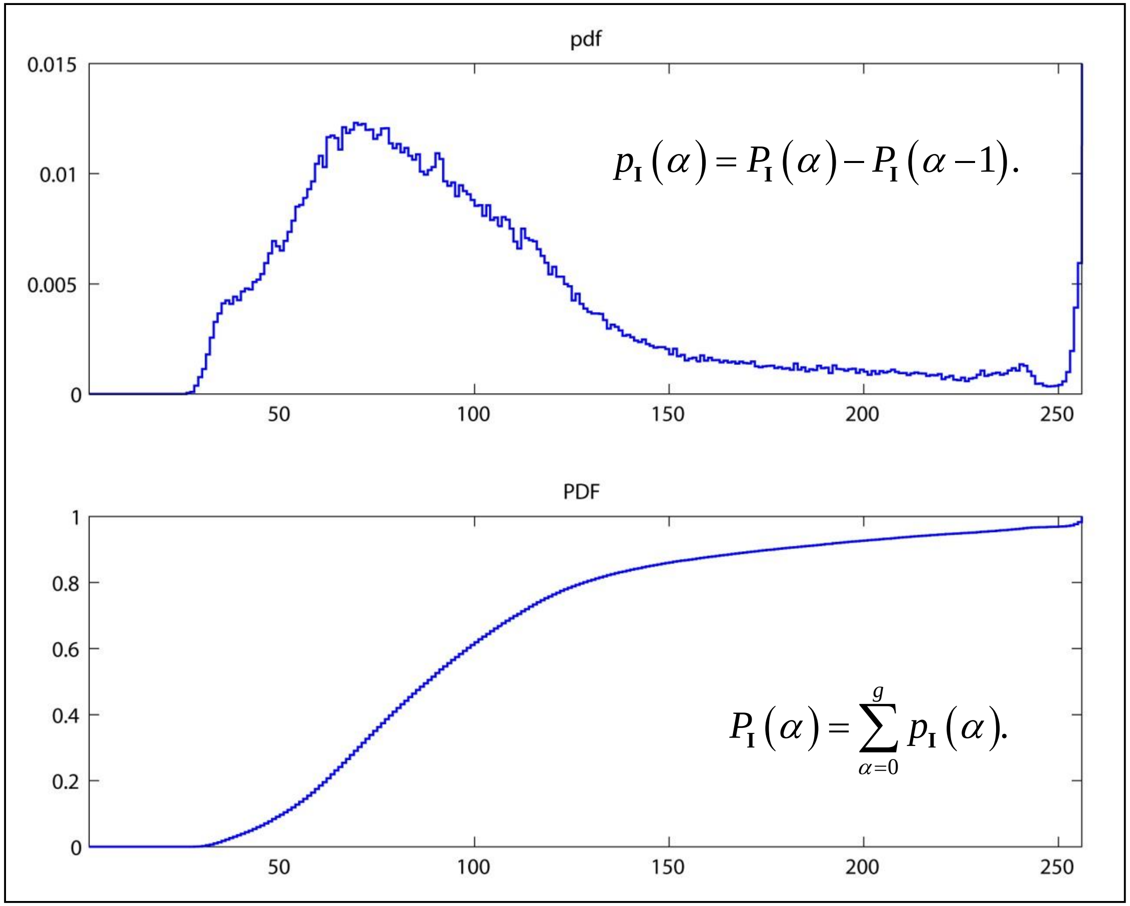
- $P_{\mathbf{I}}(g+1)$ is the fraction of pixels in (a specific band of) an image that have intensity values less than or equal to g.
- $P_{\mathbf{I}}(g+1)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to g.
- $P_{\mathbf{I}}(g+1)$ is the cumulative (or running) sum of $p_{\mathbf{I}}(g+1)$ from 0 through g inclusive.
- $P_{\mathbf{I}}(1) = p_{\mathbf{I}}(1)$ and $P_{\mathbf{I}}(256) = 1$; $P_{\mathbf{I}}(g+1)$ is nondecreasing.

Note: the Probability Distribution Function (PDF, capital letters) and the Cumulative Distribution Function (CDF) are exactly the same things. Both PDF and CDF will refer to it. However, pdf (small letters) is the *density* function.

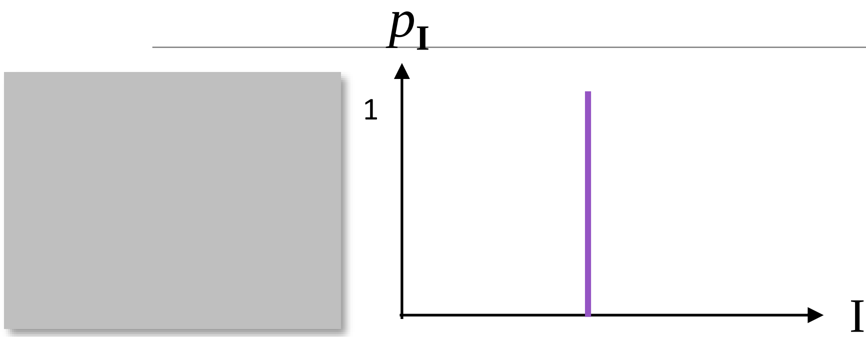
The pdf vs. the CDF

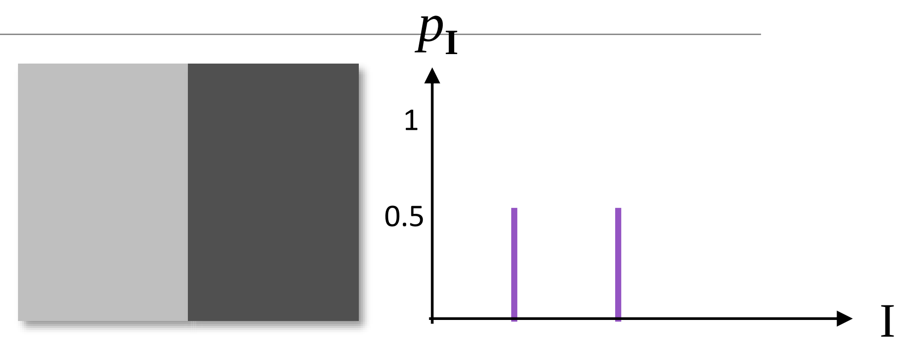
pdf (p_{I}) = backward difference of CDF.

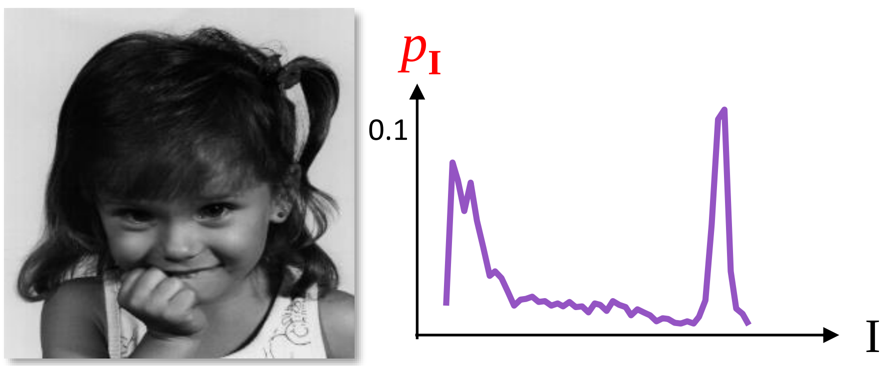
CDF (P_{I}) = running sum of pdf.

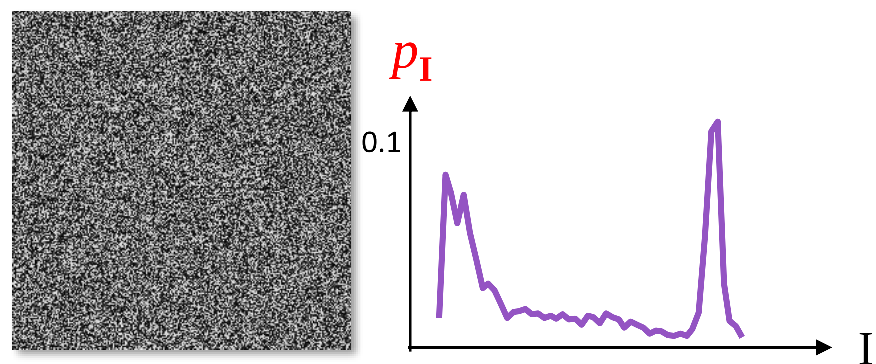


The image histogram doesn't fully represent the image.









Pixel permutation of
the left image

Image Statistics

Image Mean: $E\{\mathbf{I}\}$

$$E\{\mathbf{I}\} = \frac{1}{A}\sum_r \sum_c \mathbf{I}(c,r) = \frac{1}{A}\sum_g gH_{\mathbf{I}}(g) = \sum_g gp_{\mathbf{I}}(g)$$

Image Standard Deviation (s.t.d.):

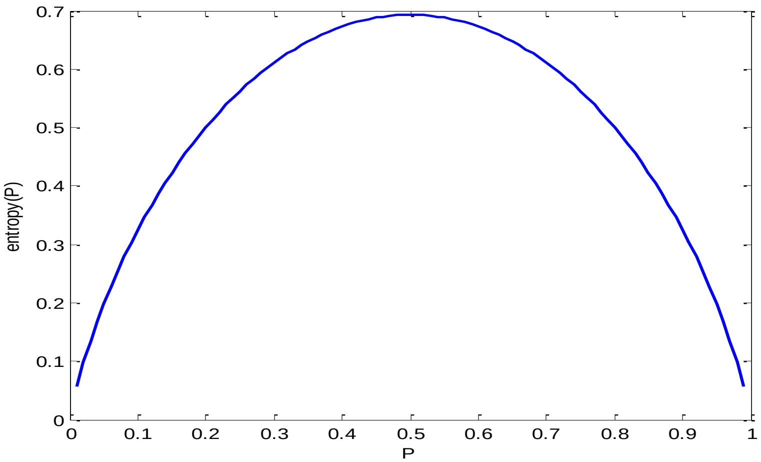
$$\sigma(I) = \sqrt{E(I^2) - E^2(I)}$$

Where $E(I^2) = \sum_g g^2 p_{\mathbf{I}}(g)$ and A is the number of pixels in $\mathbf{I}$.

Image Entropy

- The image entropy specifies the uncertainty in the image values.
- It measures the averaged amount of information required to encode the image values.

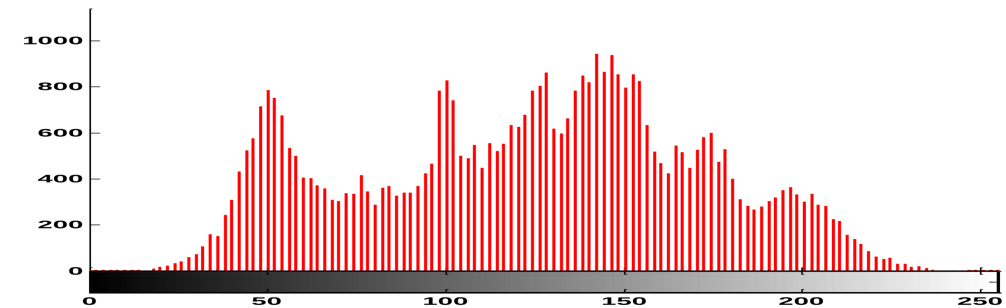
$$Entropy(\mathbf{I}) = -\sum_{g} p_{\mathrm{I}}(g) \log p_{\mathrm{I}}(g)$$



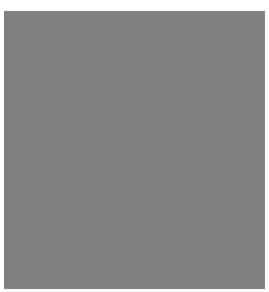
Entropy of a 2-value variable

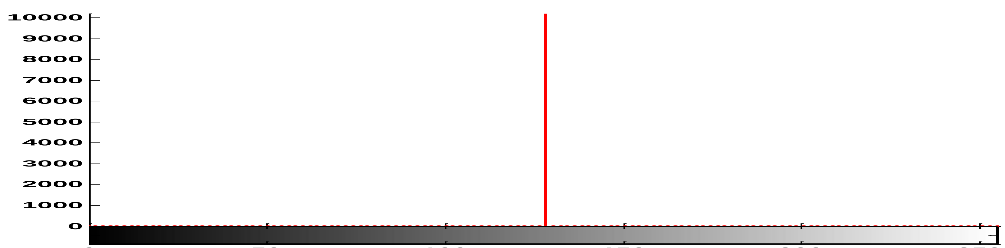
Image Entropy

- An infrequent event provides more information than a frequent event.
- Entropy is a measure of histogram dispersion.



entropy=7.4635





entropy=0

Histogram Usage

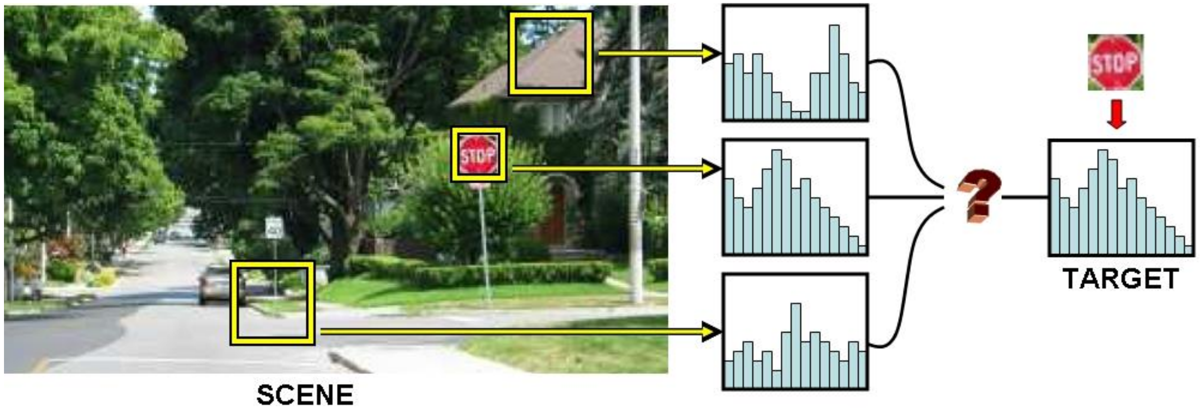
- Digitizing parameters
- Measuring image properties:
  - Average
  - Variance
  - Entropy
  - Contrast
  - Area (for a given grey-level range)
- Threshold selection
- Image distance
- Image Enhancement
  - Histogram equalization
  - Histogram stretching
  - Histogram matching

Adaptive Histogram

In many cases histograms are needed for local areas in an image

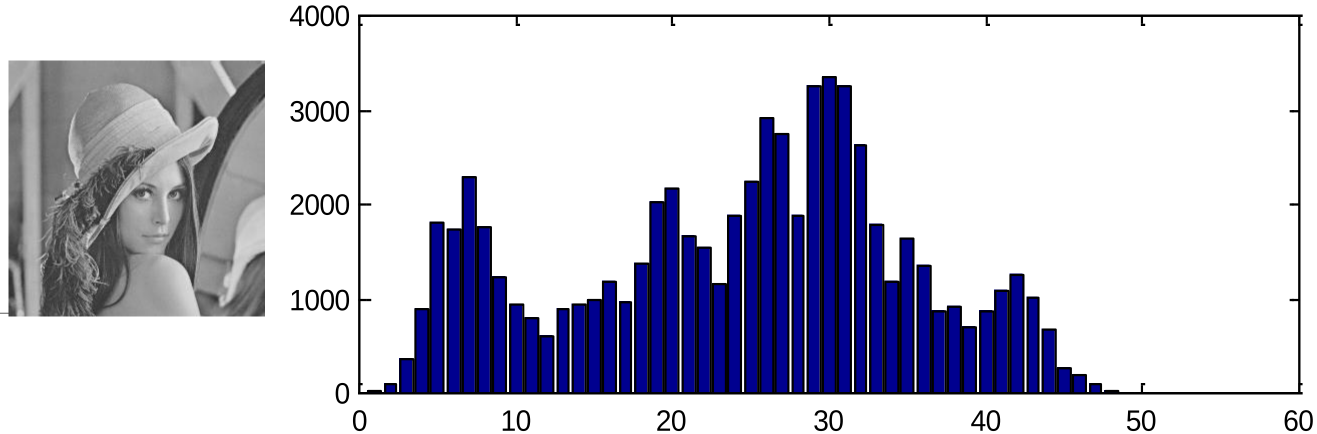
Examples:

- Pattern detection
- adaptive enhancement
- adaptive thresholding
- tracking



Example: Auto-Focus

- In some optical equipment (e.g. slide projectors) inappropriate lens position creates a blurred ("out-of-focus") image
- We would like to automatically adjust the lens
- How can we measure the amount of blurring?



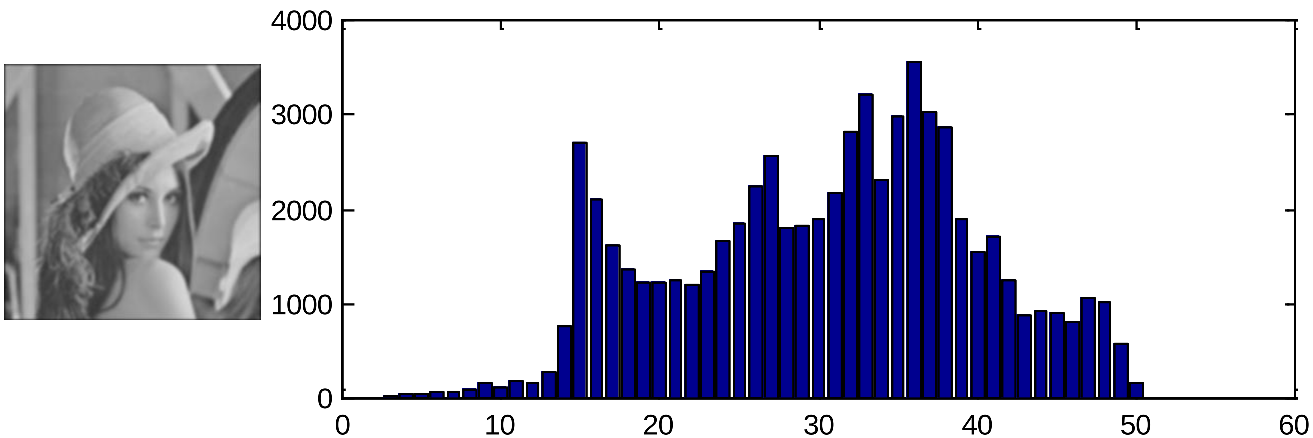
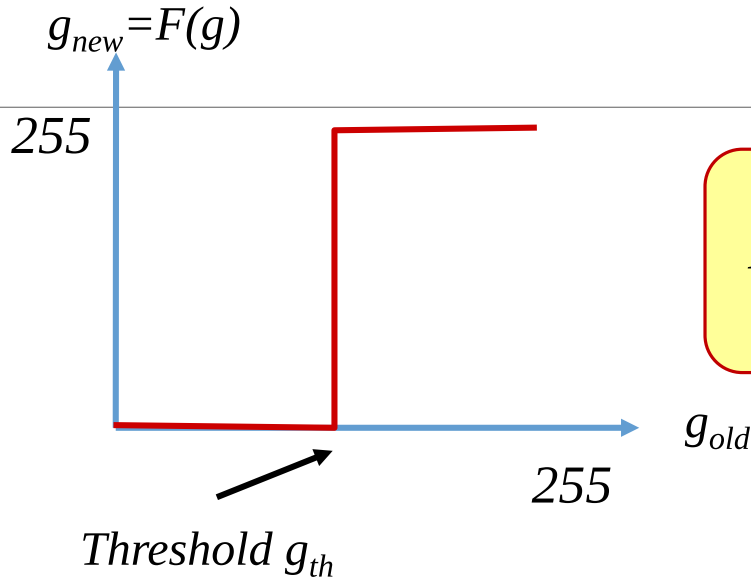


Image mean is not affected by blurring.

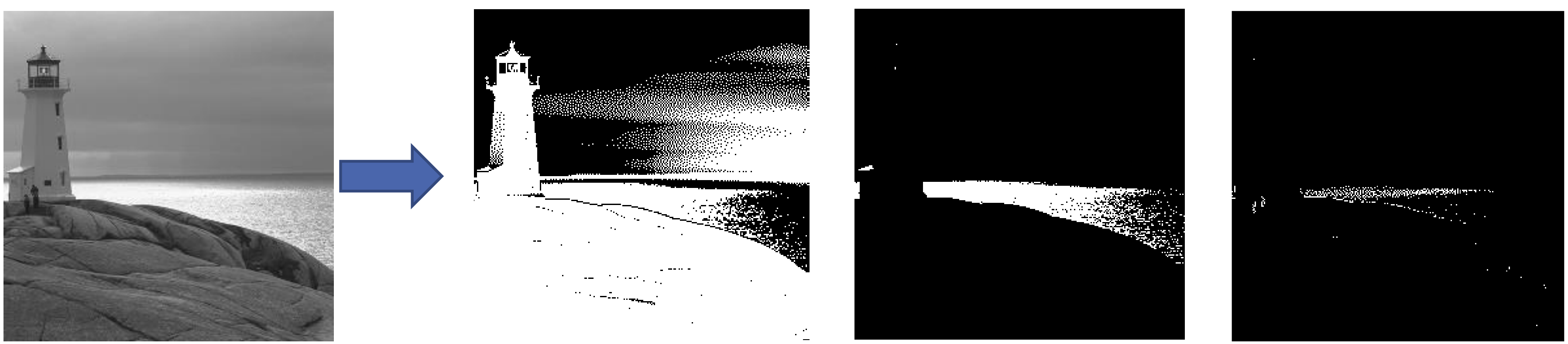
Image s.t.d (entropy) is decreased by blurring.

Algorithm: Adjust lens according to the changes in the histogram s.t.d.

Thresholding



$$F\left(g\right)=\begin{cases}0 & \forall g \le g_{th}\\ 255 & \forall g > g_{th}\end{cases}$$

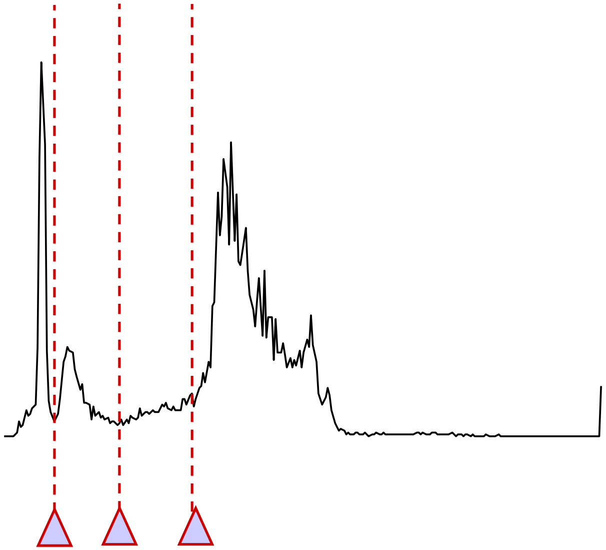


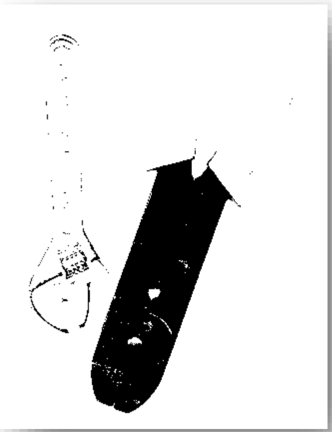
Original *g_{th}low to high*

Segmentation using Thresholding

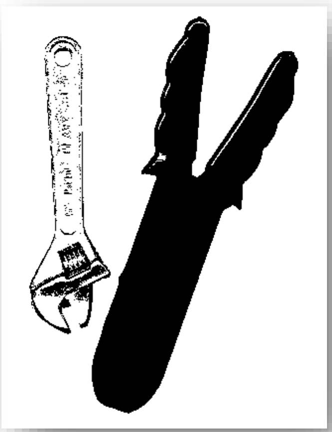
Original

Histogram

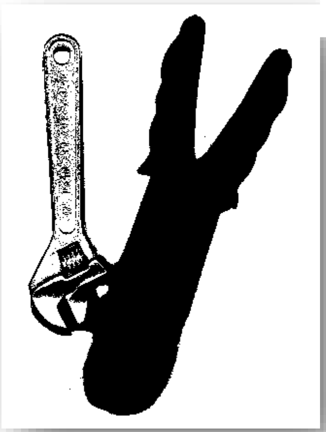




Threshold = 21



Threshold = 50



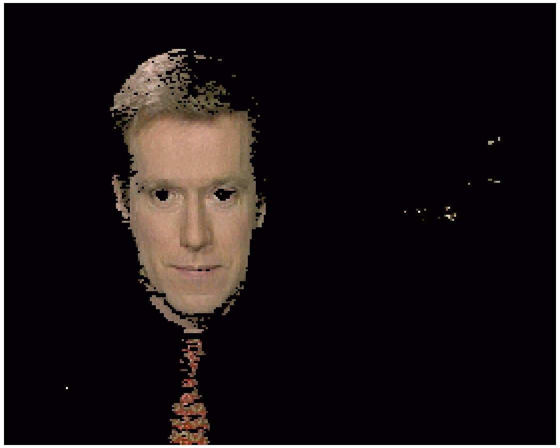
Threshold = 75

Colour Segmentation

Segmentation is based on colour values.

Apply clustering in colour space (e.g. k-means).

Segment each pixel to its closest cluster.



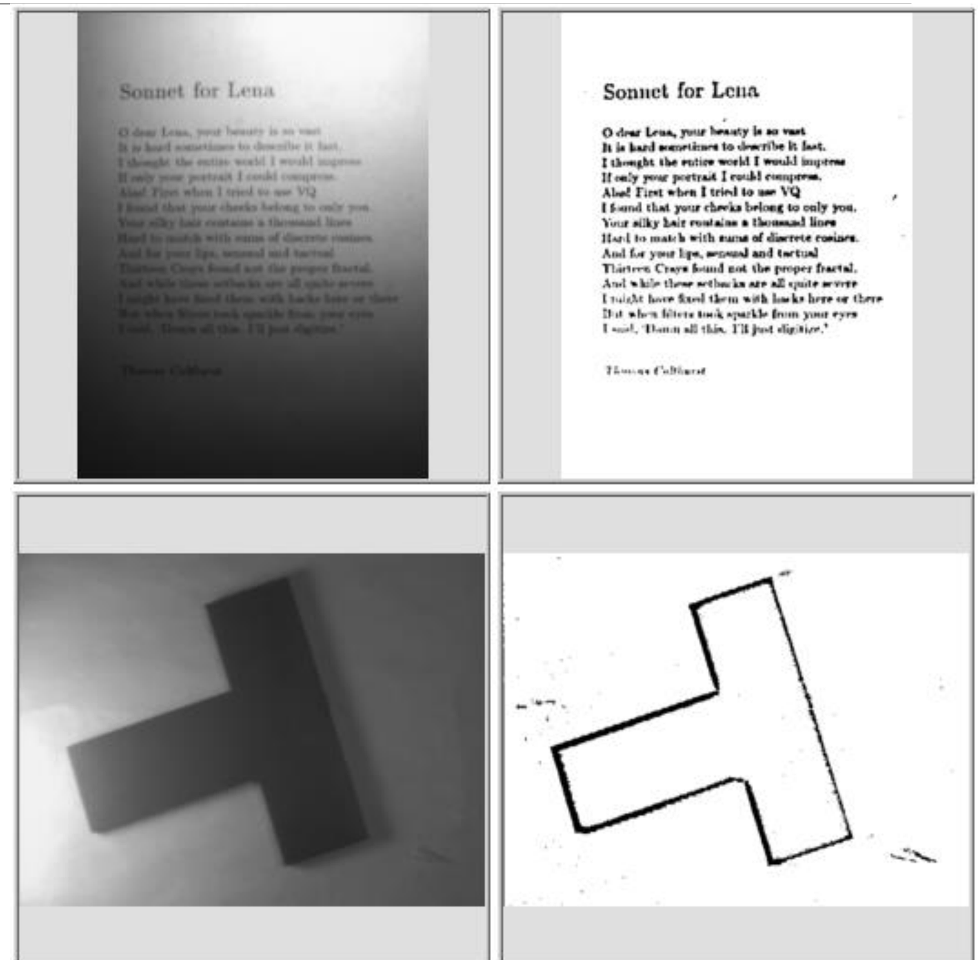
https://www.mathworks.com/help/images/ref/imsegkmeans.html

Think on this...

To threshold images with varying lighting conditions

Will normal thresholding work well?

How can we achieve good segmentation?



Q&A