Spatial Filtering

- BACKGROUND
- SMOOTHING FILTERS
- SHARPENING FILTERS

Image Processing Methods in Spatial Domain

Spatial domain refers to the image plane itself.

Image processing methods in spatial domain may be divided into 2 main categories

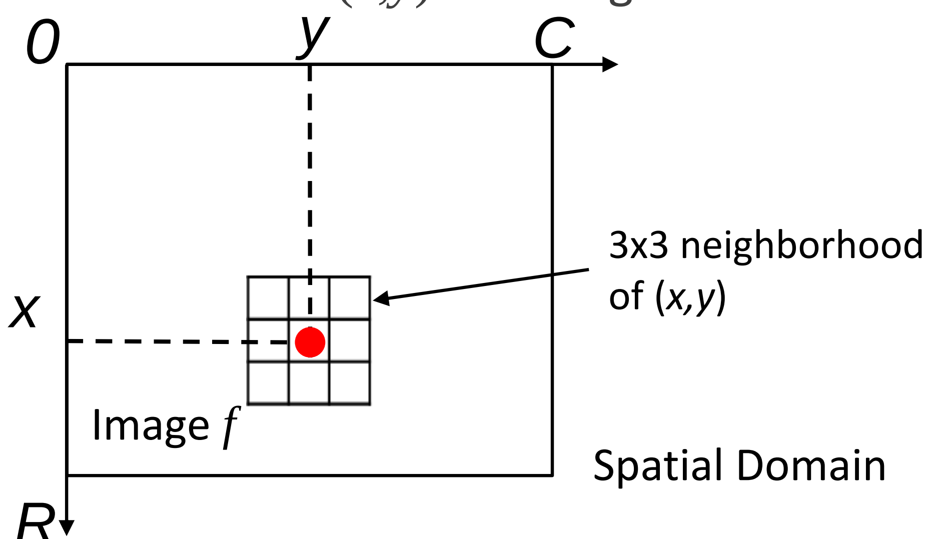
1. **Point operations/Intensity transformation**
   - operate on *single pixels* of an image
   - principally for the purpose of contrast manipulation and image thresholding
2. **Spatial filtering**
   - process the pixel in a small *neighborhood of pixels* around the given pixel
   - deals with performing operations, such as image sharpening

Image Processing Methods in Spatial Domain

The spatial domain processes can be denoted by the expression,

$$g(x,y) = T\left[f(x,y) \right]$$

Where $f(x,y)$ is the input image, $g(x,y)$ is the output image and T is the operator on f defined over the (x,y) or a neighbourhood of point (x,y).



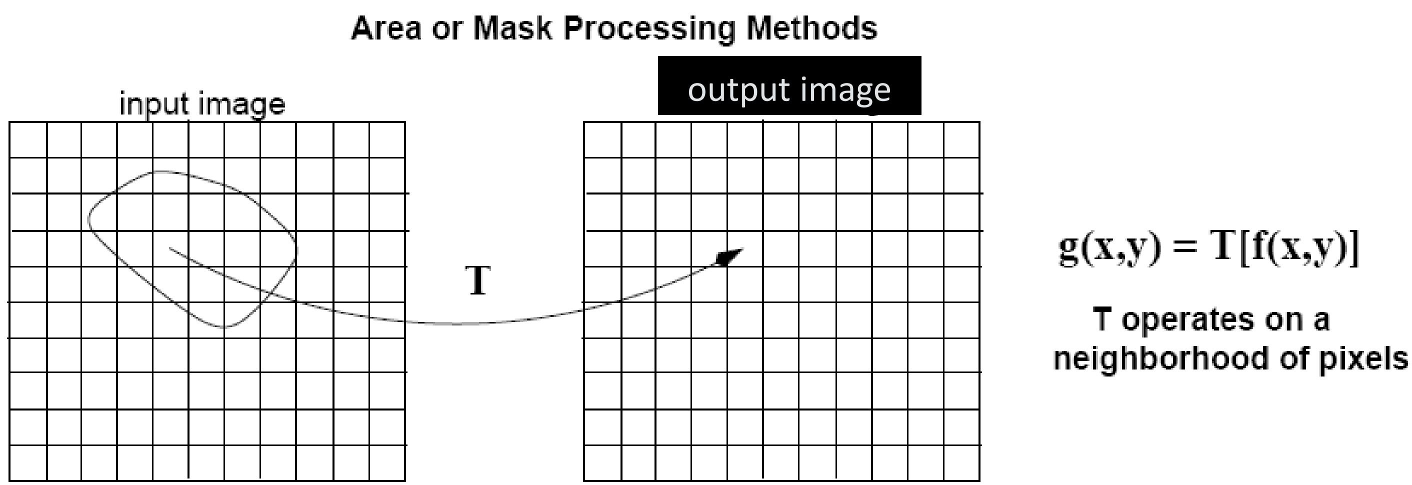
Spatial Filtering

Spatial filtering consists of:

(1) A local neighborhood in the input image (eg. a small square)

(2) A predefined operation (*T*) that is performed on the pixels inside the neighborhood.

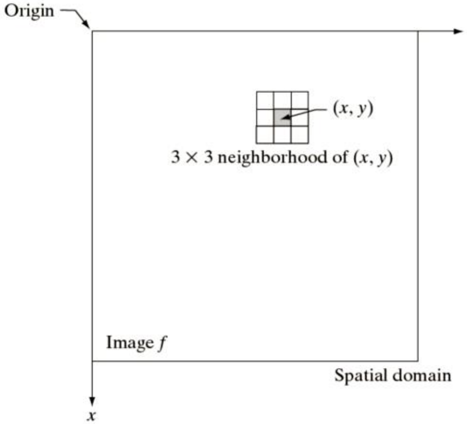
The predefined operations (*T*) is embodied in a *sub image* with the same dimensions as the neighborhood.



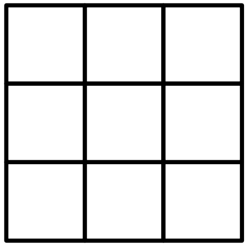
Spatial Filtering

- Typically, the neighborhood and sub-image are square and their size are much smaller than the image.
  - e.g., 3x3 or 5x5 pixels
  - The sub image is called a *mask, filter or kernel*.
  - The values in the mask are referred to as *coefficients or weights*, rather than pixels.
- The filtering is performed by shifting the mask over the whole image so that the center of the mask visits each pixel in the input image.

Input image & neighborhood



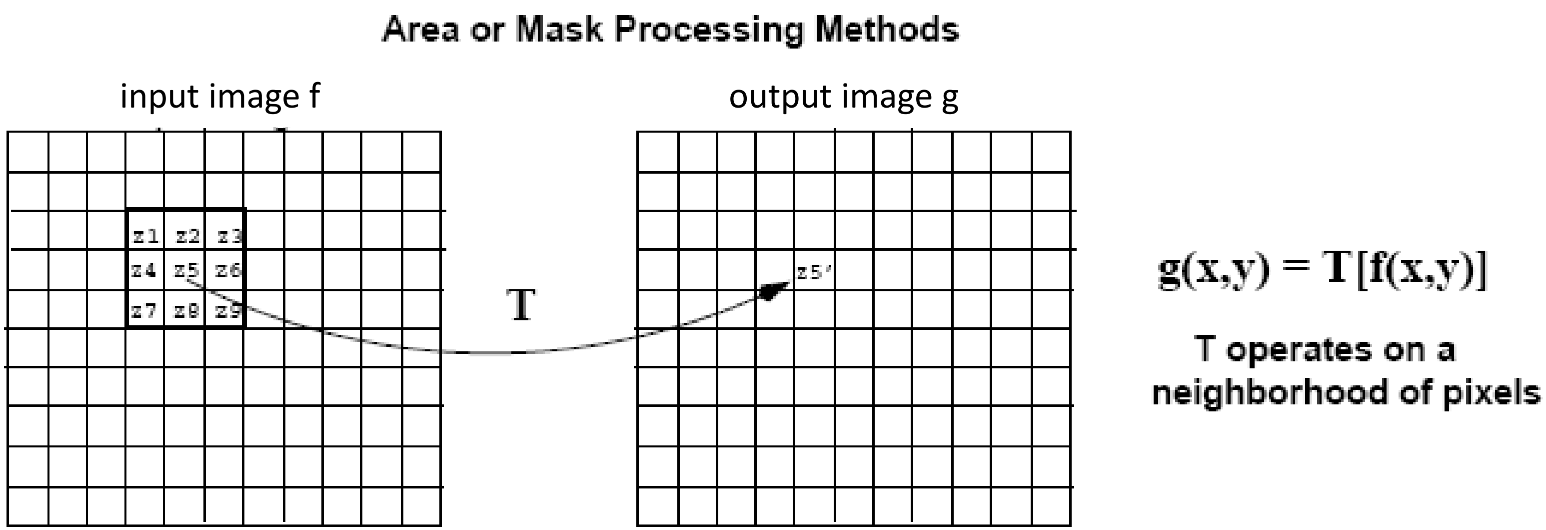
Mask



Spatial Filtering

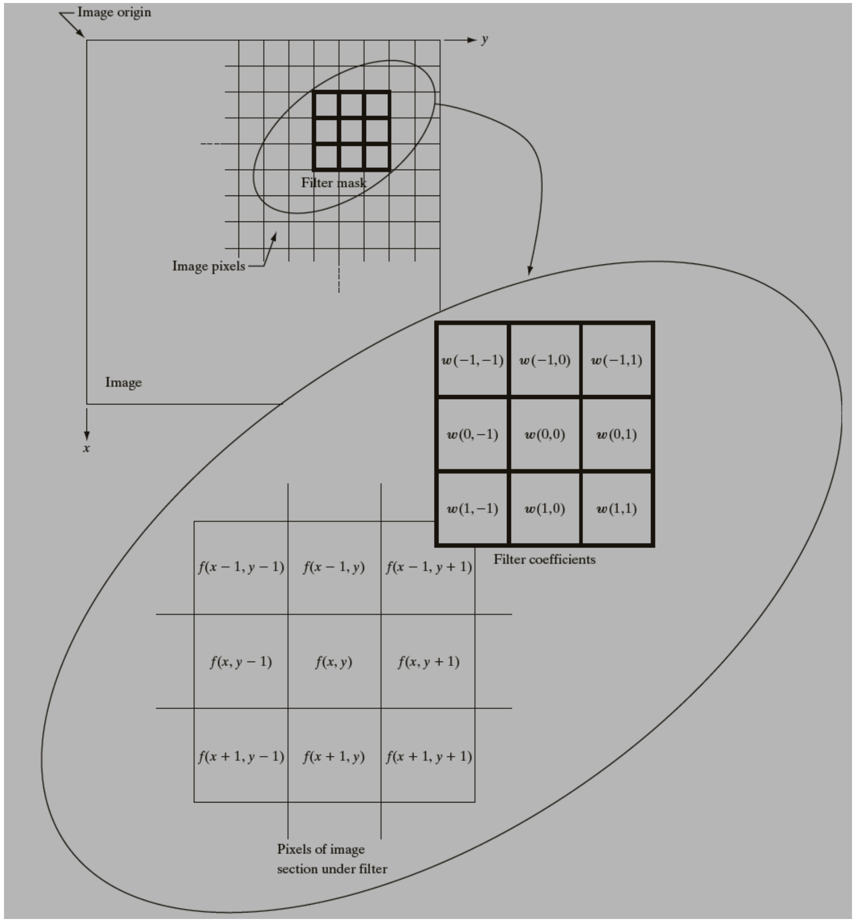
Example: apply a mask of weights

w1	w2	w3
w4	w5	w6
w7	w8	w9



z5′=R=w1*z1+w2*z2+…+w9*z9

Spatial Filtering



Assume the origin of the mask is the center of the mask.
A filtered image is generated as the centre of the mask moves to every pixel in the input image.

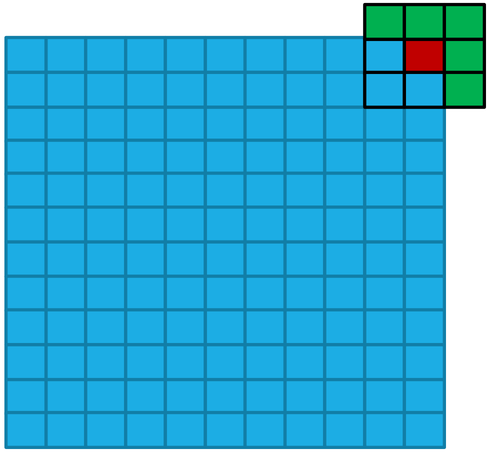
Border problem

The values of pixels outside the image but involved in the filtering process need to be estimated.

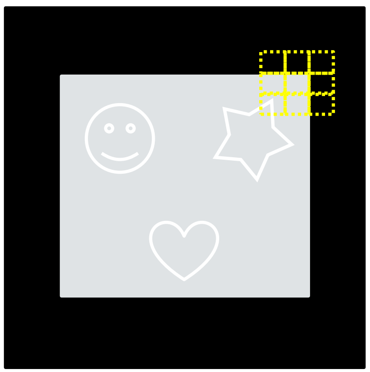
Solutions

1. change mask size along the border

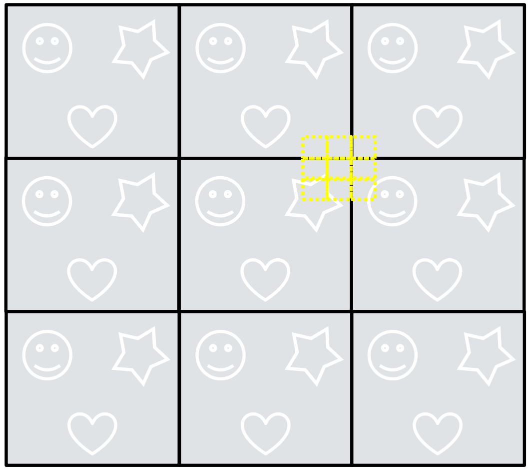
2. enlarge the image
   - Fill with zeros
   - Periodic extension of the image
   - Mirroring the borders.



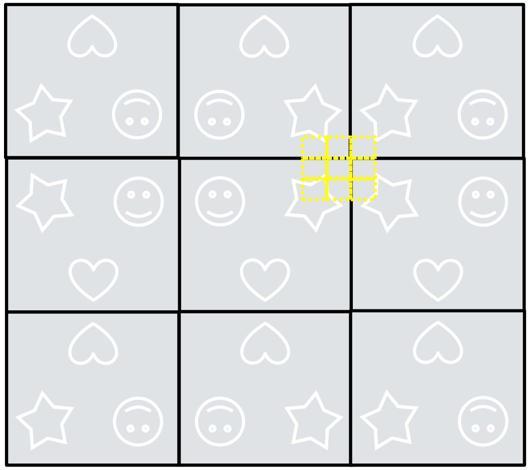
Border problem



Fill with zeros



Periodic extension



Mirroring

Linear vs Non-Linear Spatial Filtering

A filtering method is linear when the output is a weighted sum of the input pixels.

w1	w2	w3
w4	w5	w6
w7	w8	w9

z5'=R=w1*z1+w2*z2+…+w9*z9

Methods that do not satisfy the above property are called non-linear.

- e.g. z5'=max(zk,k=1,2,…9)

Linear Spatial Filtering Methods

Two main linear spatial filtering methods:

- **Correlation**
  - The process of moving a filter over the image and computing the ***sum of products*** at each location.
  - Correlation is a function of displacement of the filter.
- **Convolution**
  - The same process except that the filter is firstly rotated by 180 degree.

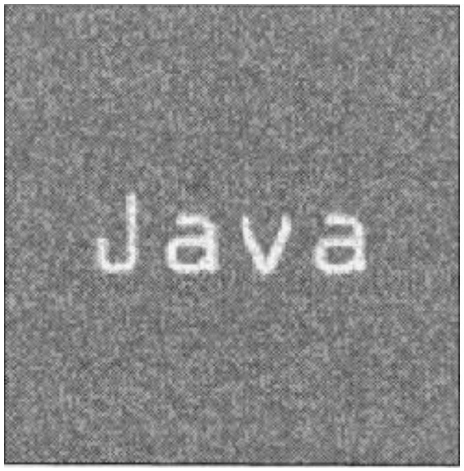
Correlation

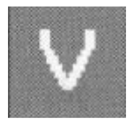
The correlation of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$, denoted by $w(x, y) \bullet f(x, y)$

$$w(x, y) \bullet f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) \times f(x + s, y + t)$$

Where $a = (m - 1)/2, b = (n - 1)/2$， and m and n are odd integers.

Correlation





Often used in applications where we need to measure the similarity between images or parts of images
(e.g., pattern matching).

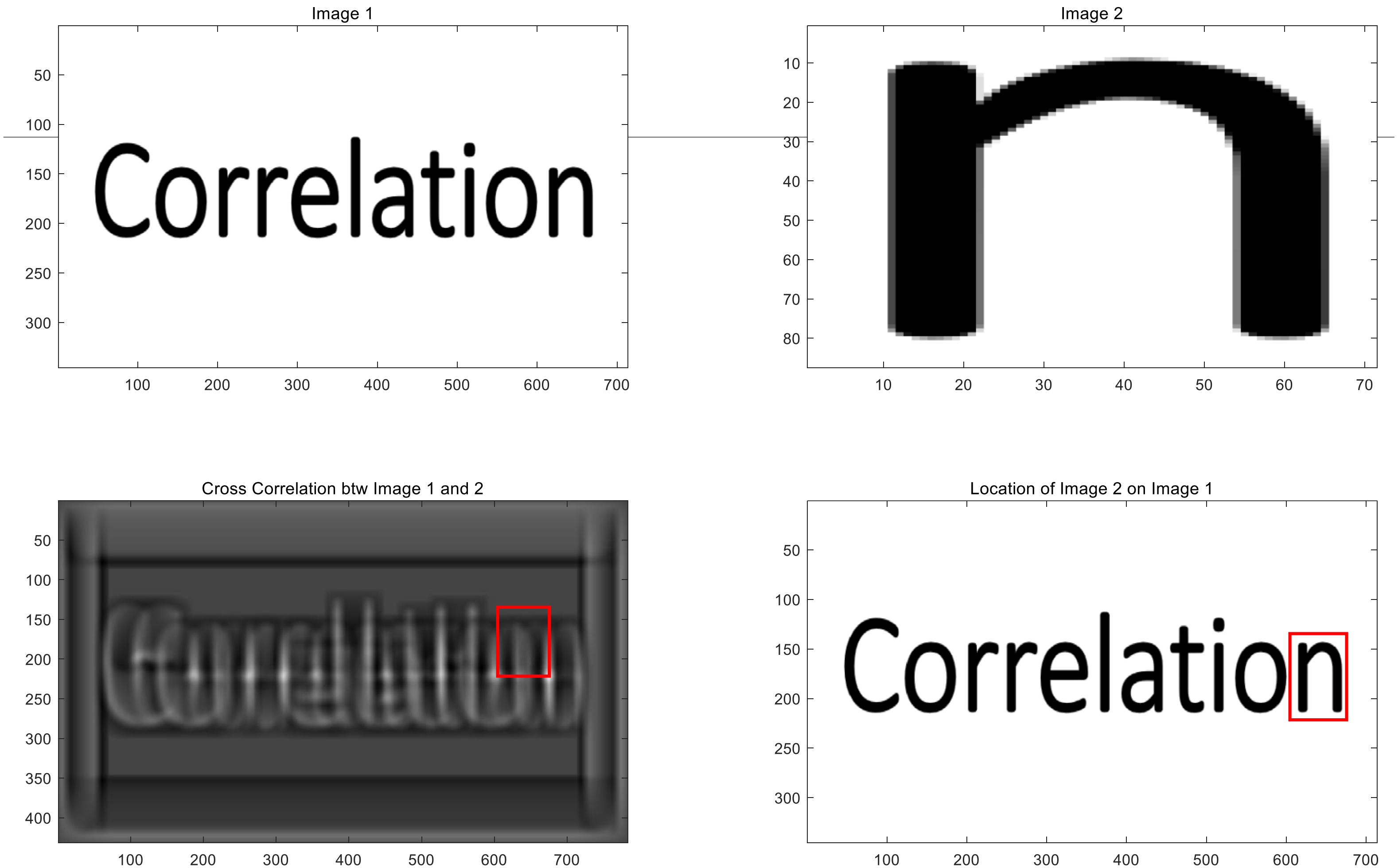


Image 1
50
100
150
200
250
300
100 200 300 400 500 600 700
Correlation
Image 2
10 20 30 40 50 60 70 80
10 20 30 40 50 60 70
Cross Correlation btw Image 1 and 2
50 100 150 200 250 300 350 400
100 200 300 400 500 600 700
Location of Image 2 on Image 1
50 100 150 200 250 300
100 200 300 400 500 600 700
Correlation

Convolution

The convolution of a filter $w(x, y)$ of size $m \times n$ with an image $f(x, y)$, denoted by $w(x, y) \circ f(x, y)$

Rotate f by 180 degree

$$w(x, y) \circ f(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) \times f(x - s, y - t)$$

Note:

1. For notational simplicity, f **is flipped** both horizontally and vertically instead of w.
2. If $w(x, y)$ is symmetric, that is $w(x, y) = w(-x, -y)$, then convolution is equivalent to correlation!

Convolution-2D illustration

f

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

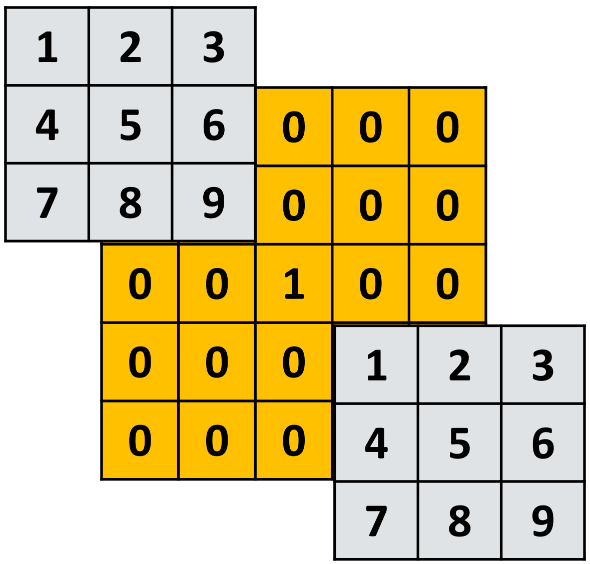
w after flipping

1	2	3
4	5	6
7	8	9

*Mask is flipped vertically and horizontally, and the origin of the mask is the center of the mask by default.

Convolution-2D illustration

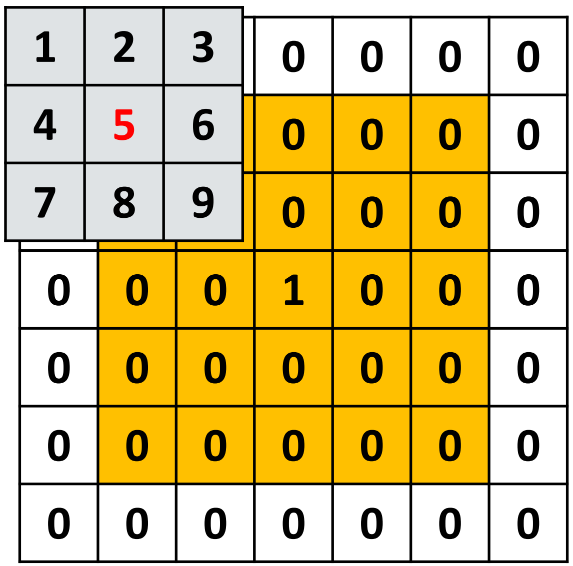
Initial position of *w*



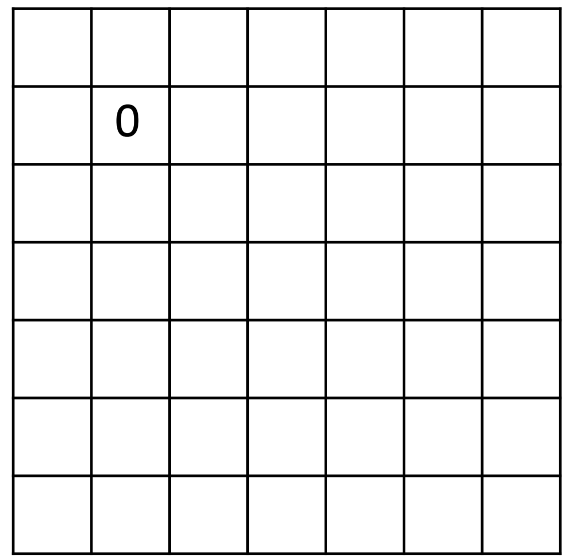
finish position of *w*

Convolution-2D illustration

Filling the zeros

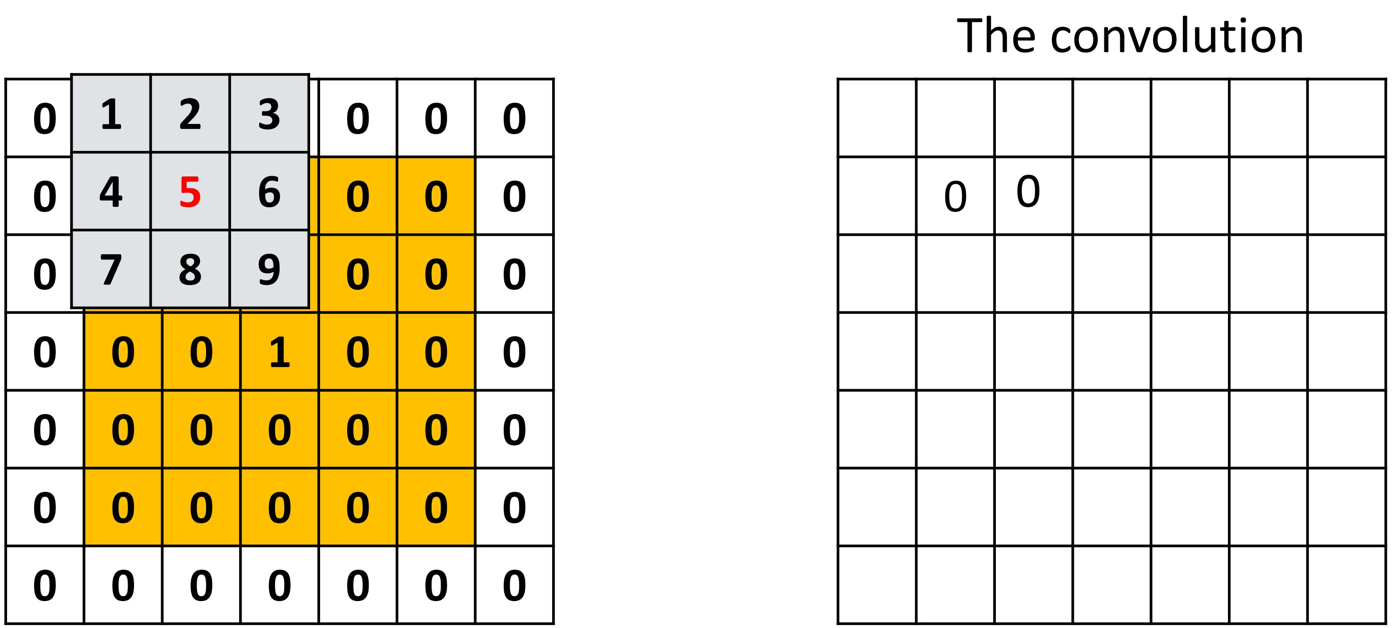


The convolution



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

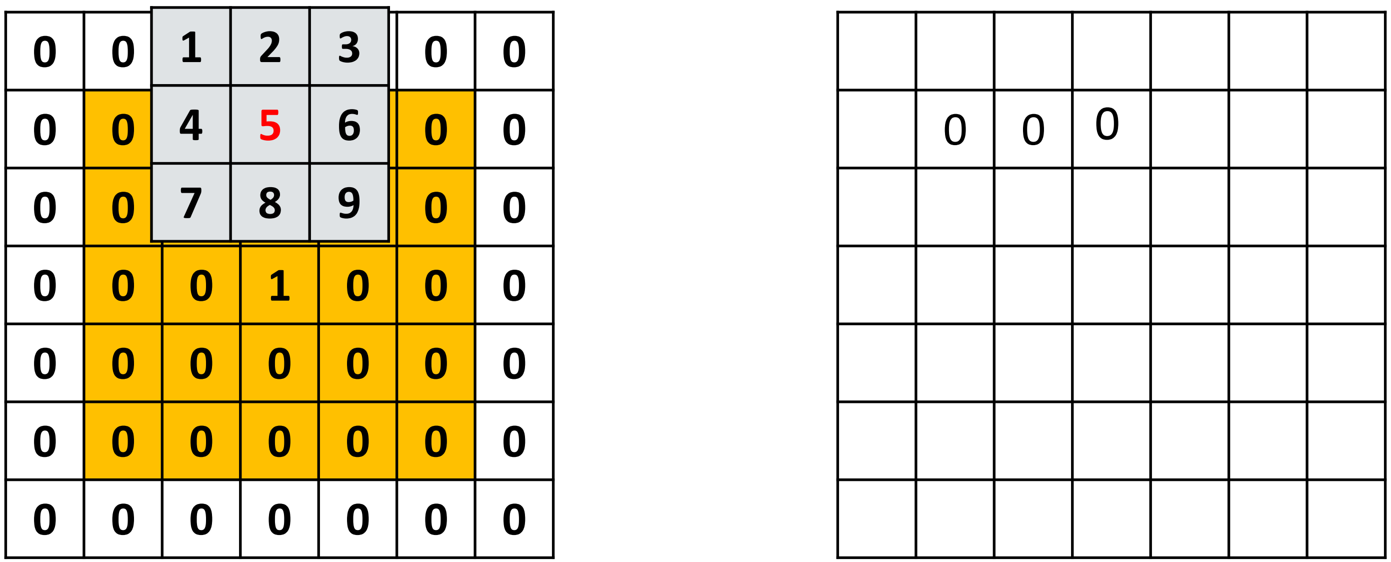
Convolution-2D illustration



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

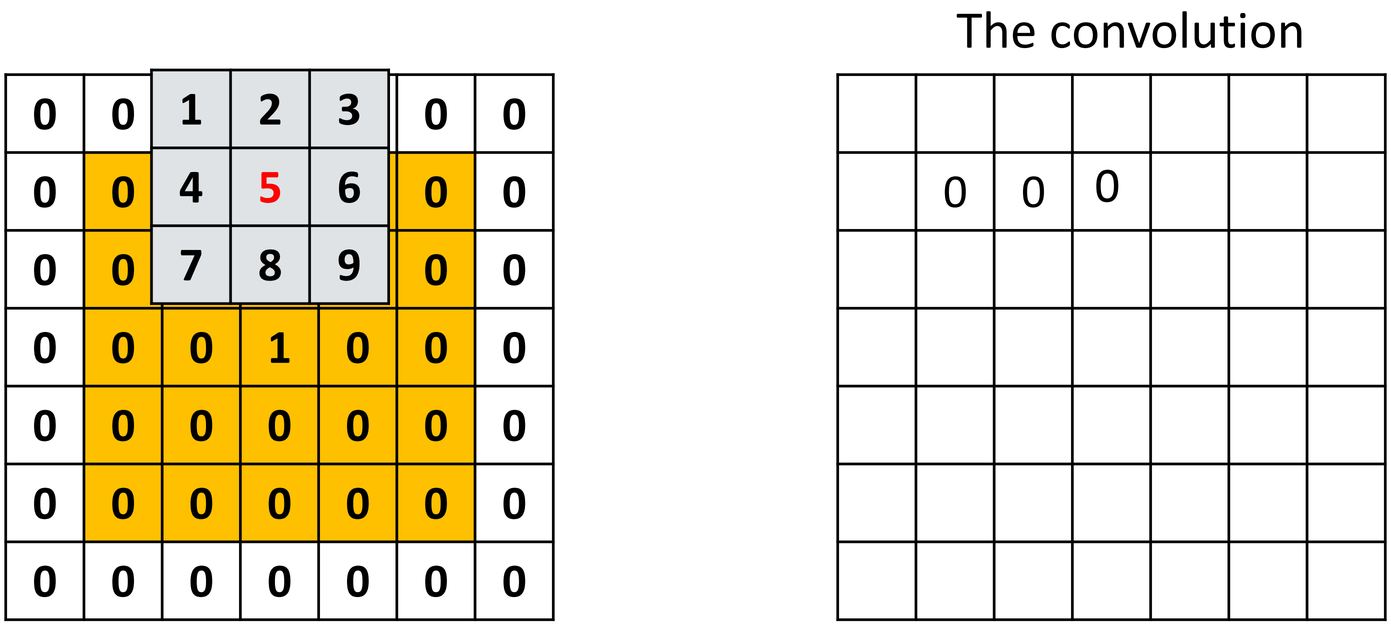
Convolution-2D illustration

The convolution



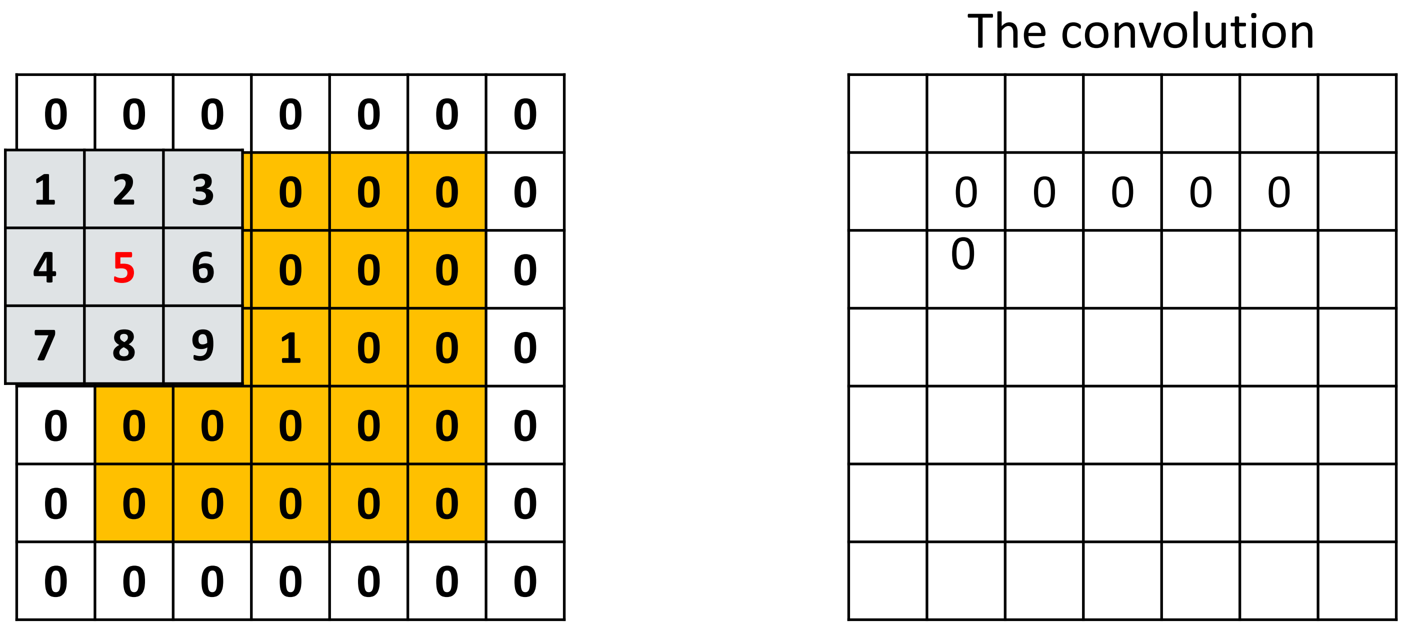
0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

Convolution-2D illustration



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

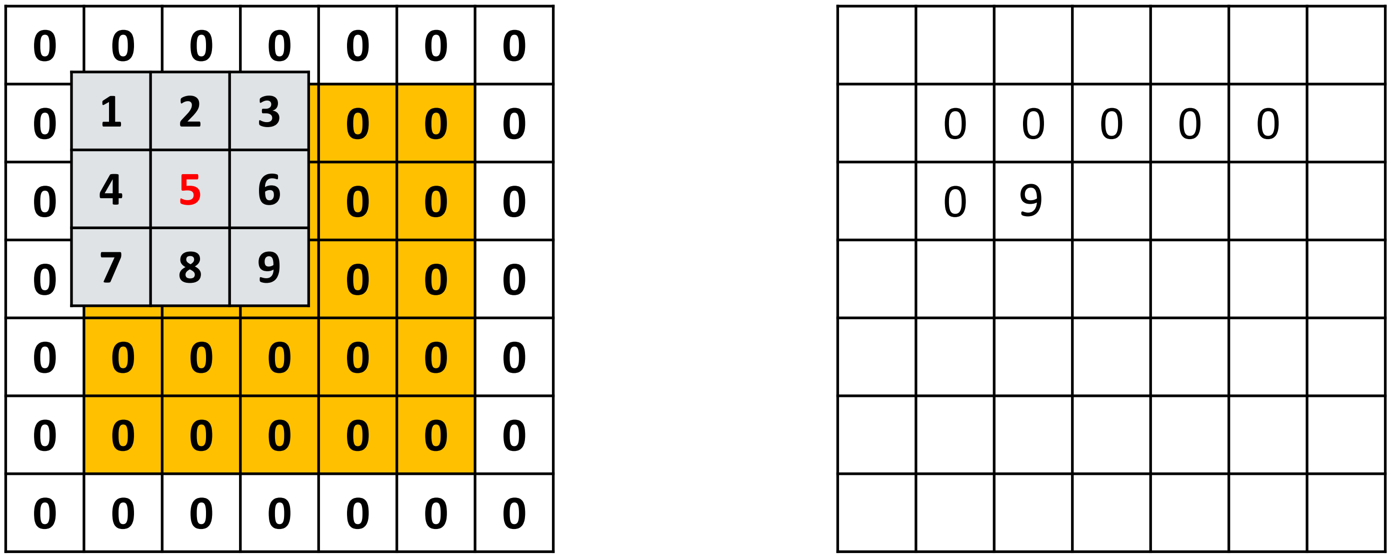
Convolution-2D illustration



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

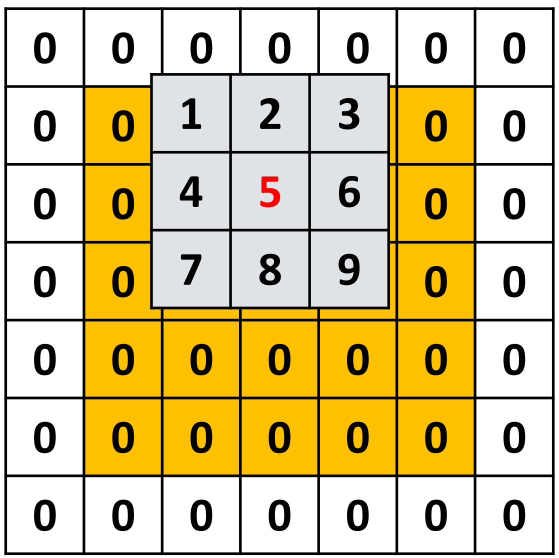
Convolution-2D illustration

The convolution



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+1*9=9

Convolution-2D illustration

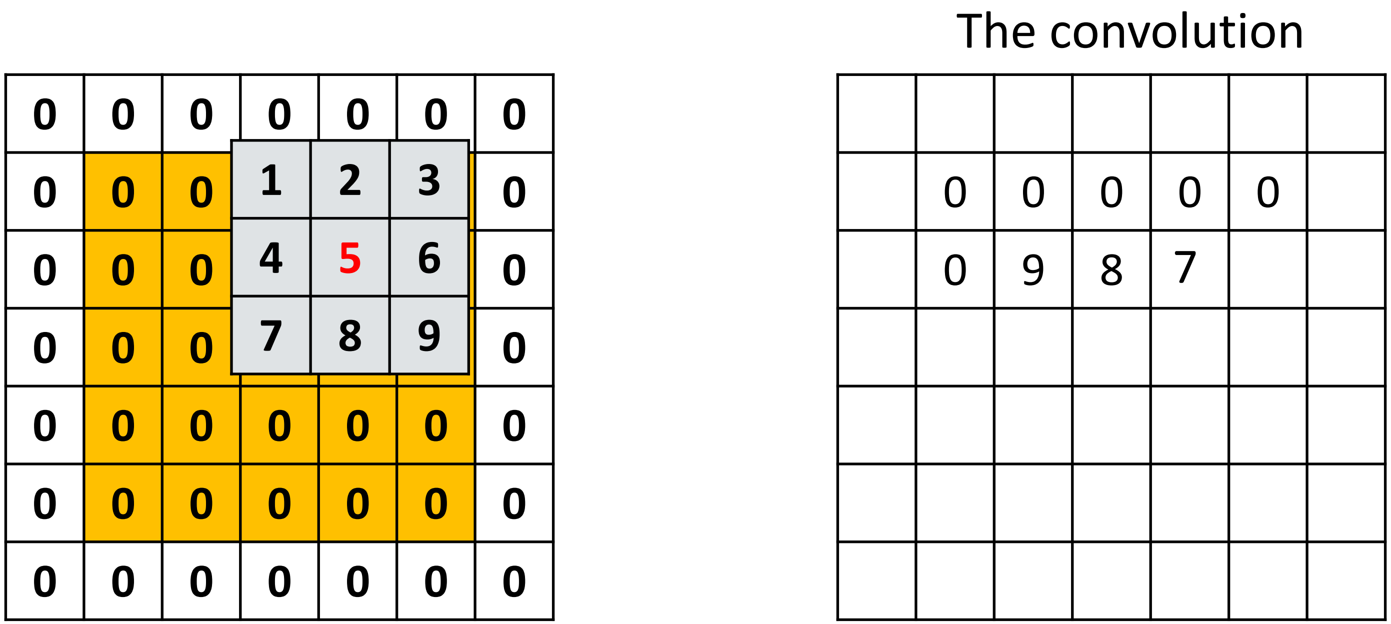


The convolution

	0	0	0	0	0	
	0	9	8			

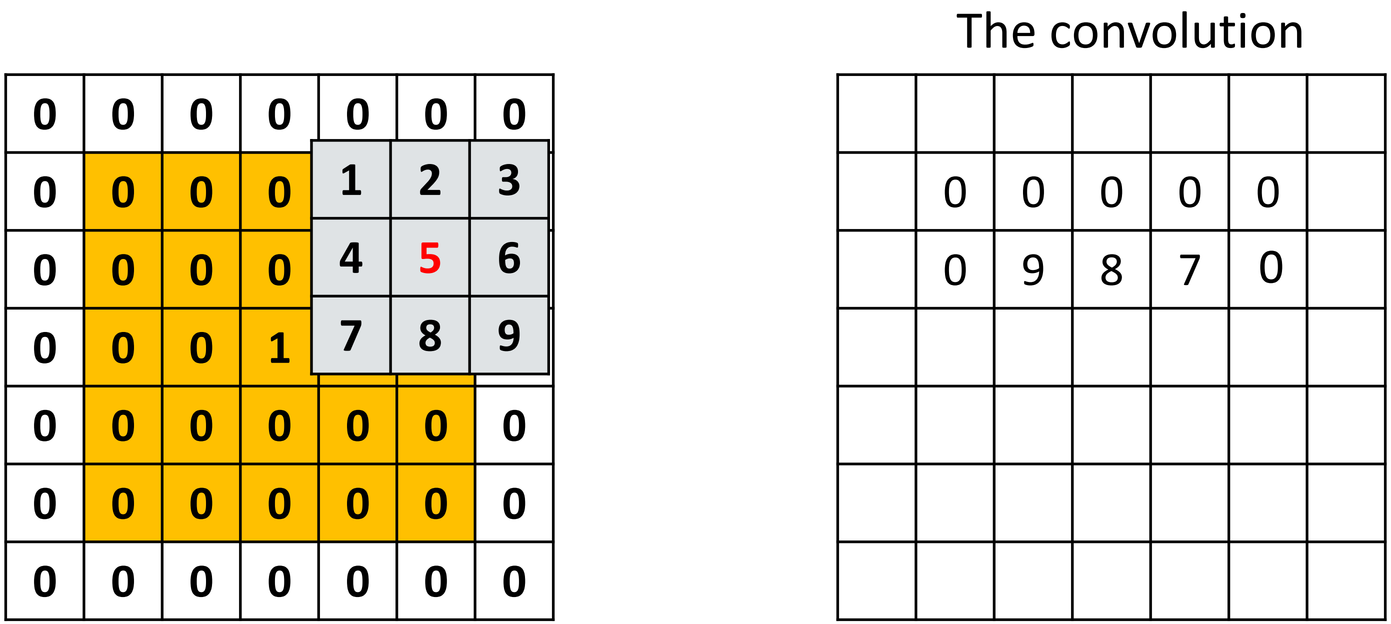
0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+1*8+0*9=8

Convolution-2D illustration



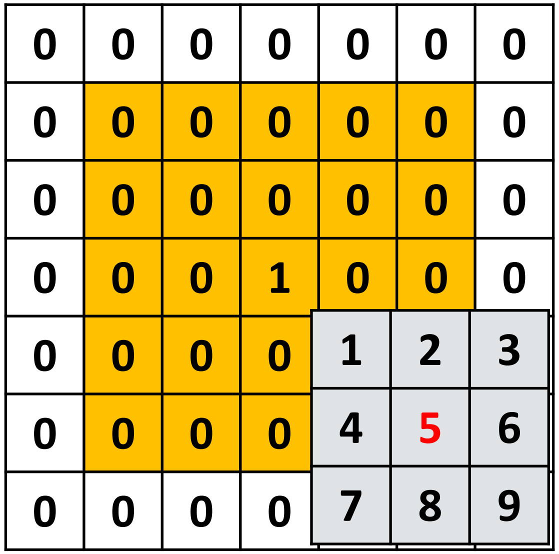
0*1+ 0*2+ 0*3 +0*4+0*5+0*6+1*7+0*8+0*9=7

Convolution-2D illustration



0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+1*9=9

Convolution-2D illustration



The convolution

	0	0	0	0	0	
	0	9	8	7	0	
	0	6	5	4	0	
	0	3	2	1	0	
	0	0	0	0	0	

0*1+ 0*2+ 0*3 +0*4+0*5+0*6+0*7+0*8+0*9=0

Convolution-2D illustration

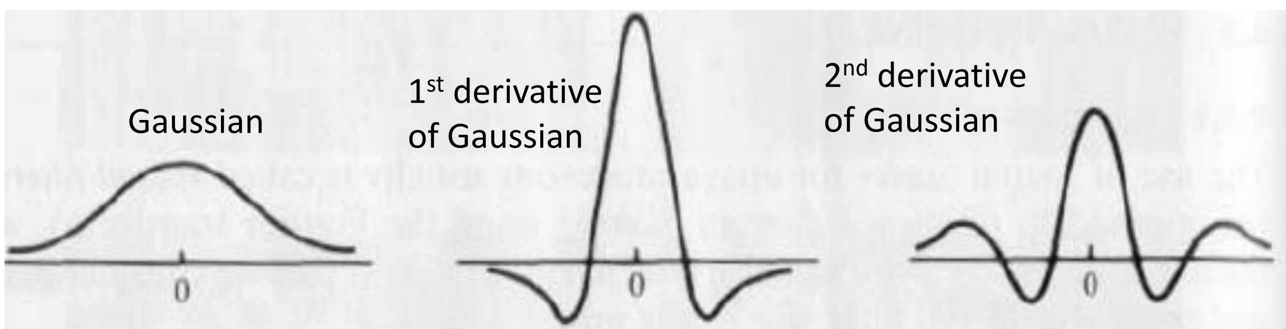
The cropped convolution

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

How to choose the weights of a mask?

Typically, by sampling certain functions:

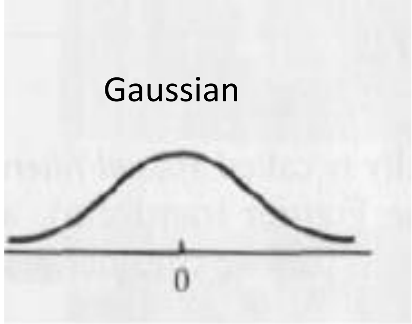
w1	w2	w3
w4	w5	w6
w7	w8	w9



Smoothing Filters (Low-pass)

Smoothing (i.e., low-pass filters)

- Filter-out the high frequencies.
- For blurring and noise reduction. e.g. removing small details prior to object extraction and bridging of small gaps in lines or curves.
- The weights of the mask must be non-negative.
- Sum of weights of the mask is 1.



Smoothing Filters: Average filter

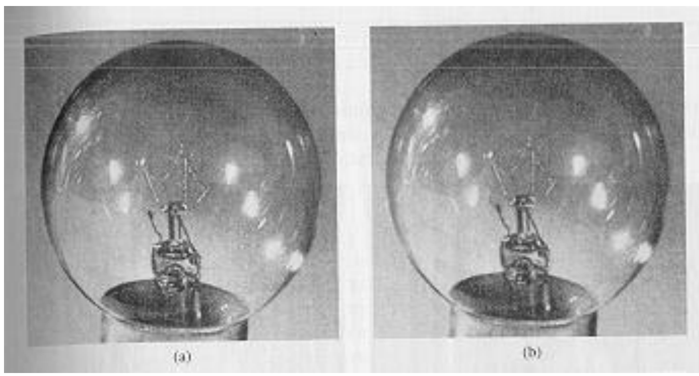
- Average filter is also called box filter.
- All weights are the same, which equals to $\frac{1}{m \times n}$, where m and n are the row number and column number of the filter.
- Normalization is needed to conserve the total energy of the image. (the sum of all intensity levels)

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{25} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{49} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

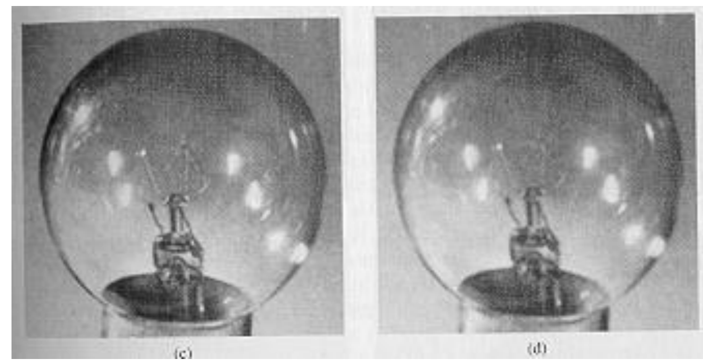
Smoothing Filters: Average filter

Mask size determines the degree of smoothing and loss of details.

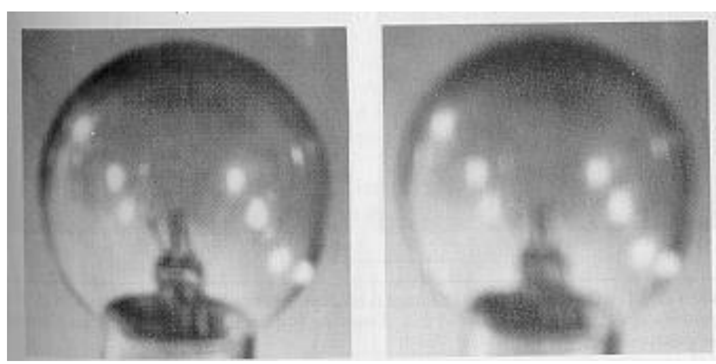
original 3x3



5x5 7x7



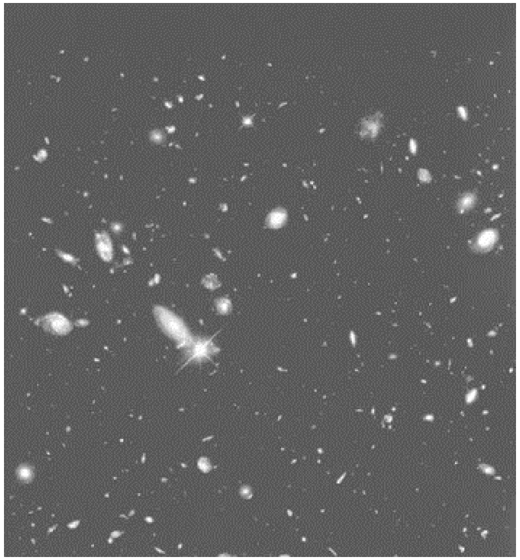
15x15 25x25



Smoothing Filters: Average filter

Example: extract largest and brightest objects.

original



15 x 15 Average

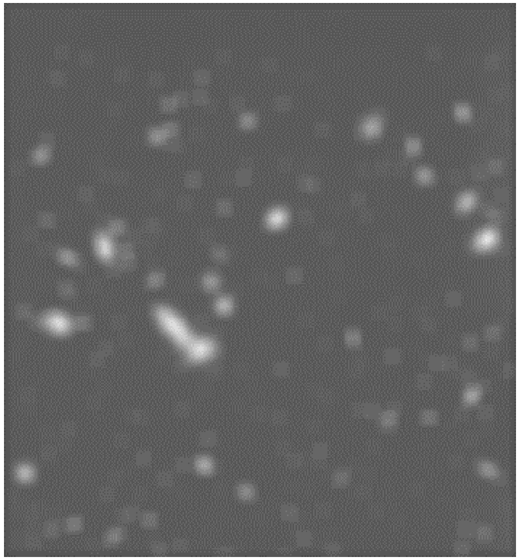
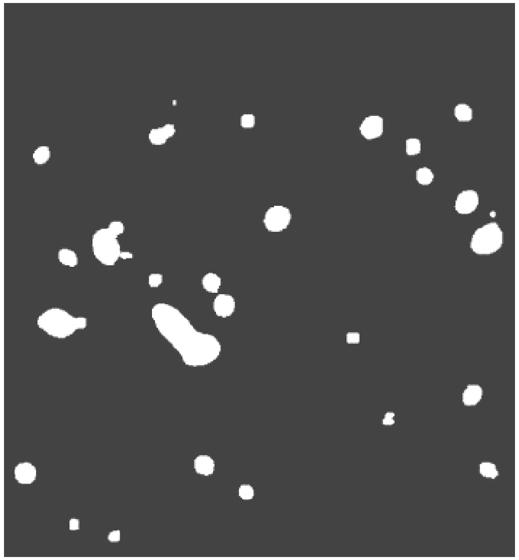


image thresholding



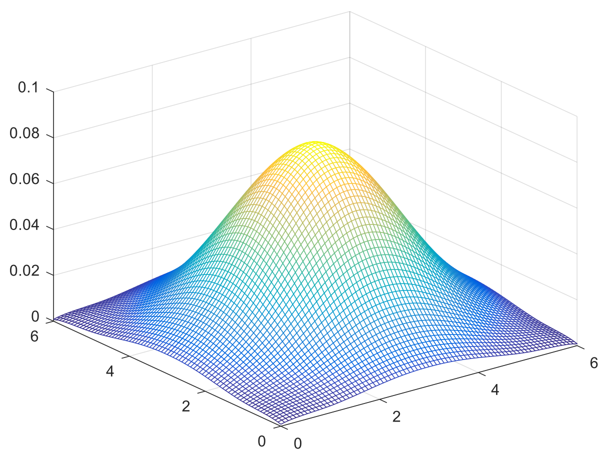
Smoothing filters: Gaussian filter

The weights are samples of the Gaussian function

$$G_\sigma = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

mask size (height or width) $\geq 5\sigma$ (subtends 98.76% of the area.)

e.g. σ = 1.4, mask size=7×7



7×7 Gaussian mask
(before normalization)

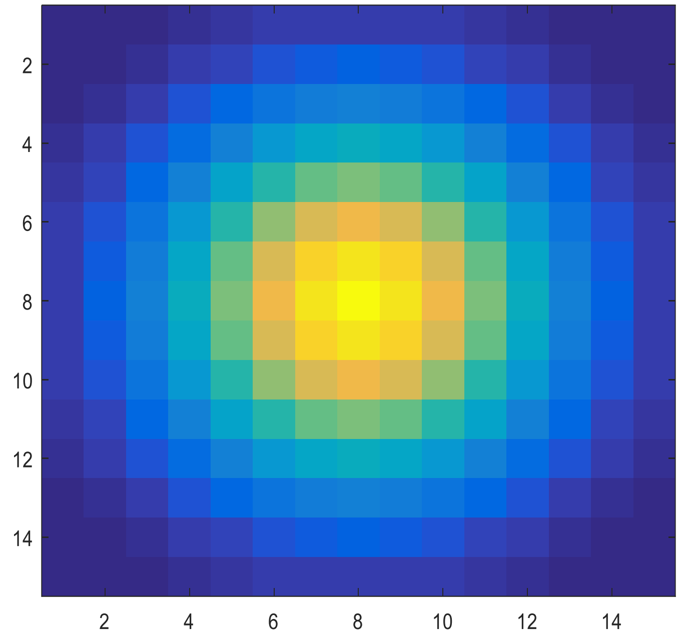
	1	2	3	4	5	6	7
1	1	1	2	2	2	1	1
2	1	2	2	4	2	2	1
3	2	2	4	8	4	2	2
4	2	4	8	16	8	4	2
5	1	2	4	8	4	2	2
6	1	2	2	4	2	2	1
7	1	1	2	2	2	1	1

Smoothing filters: Gaussian filter

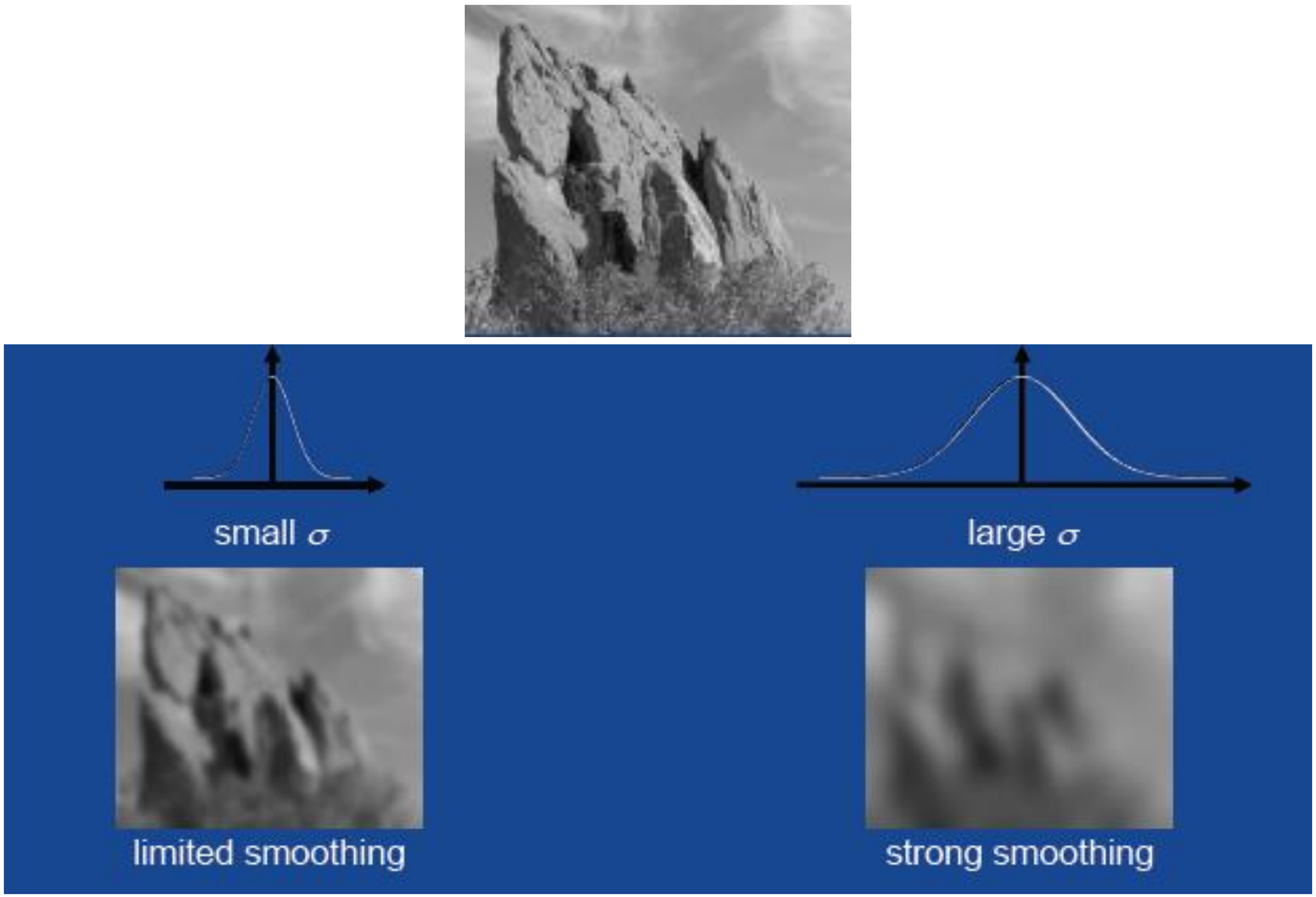
- As σ increases, more samples must be obtained to represent the Gaussian function accurately.
- σ controls the amount of smoothing.

σ = 3
Mask size=15×15

2	2	3	4	5	5	6	6	6	5	5	4	3	2	2
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
6	8	11	13	16	18	19	20	19	18	16	13	11	8	6
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
2	2	3	4	5	5	6	6	6	5	5	4	3	2	2



Smoothing filters: Gaussian filter



Average filter vs Gaussian filter

Average

Gaussian

Smoothing Filters: Median filter

- Non-linear filter
- The response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter.
- Very effective for removing “salt and pepper” noise (i.e., random occurrences of black and white pixels).

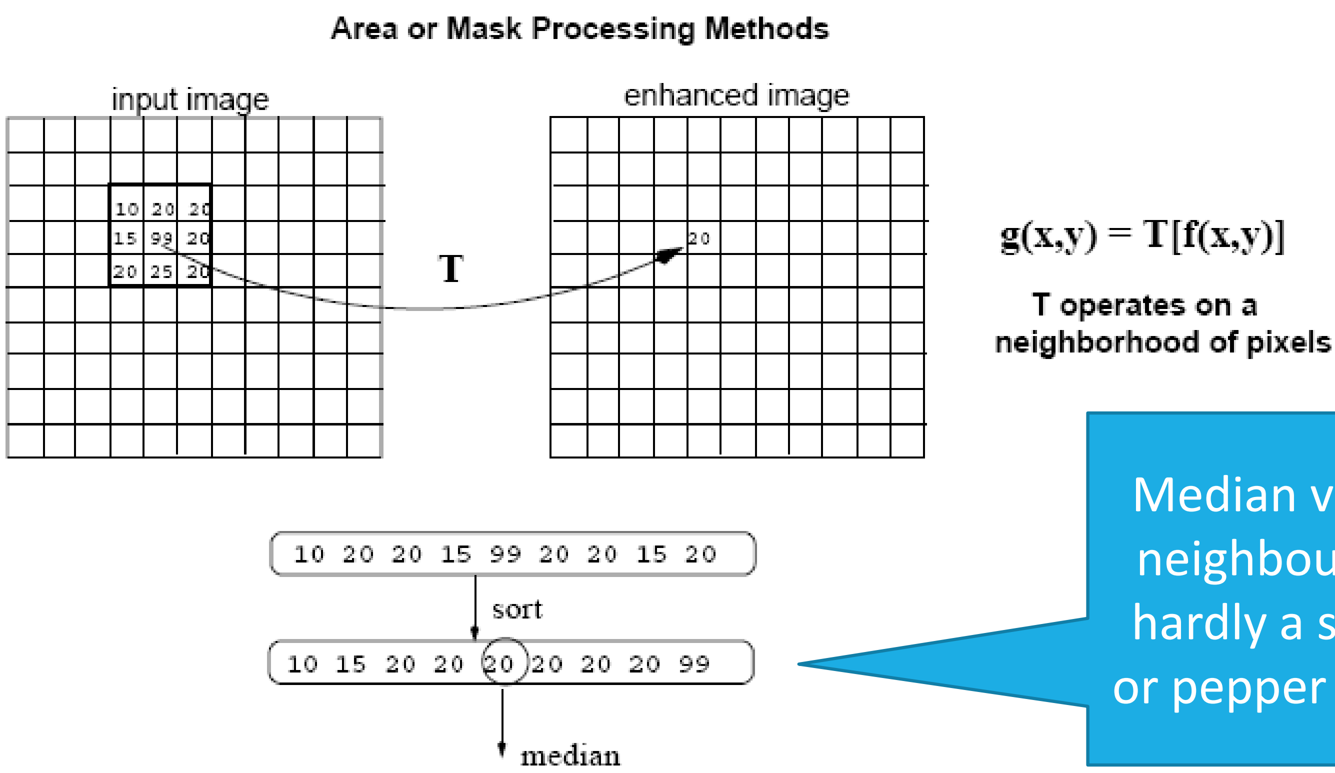
WHY?

Image Image with noise

Average Median filter

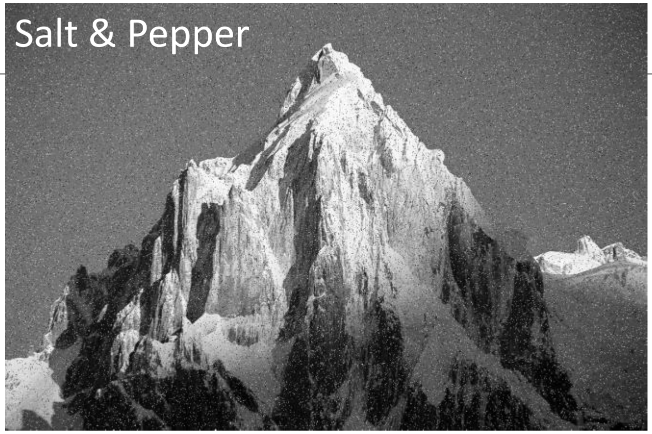
Smoothing Filters: Median filter

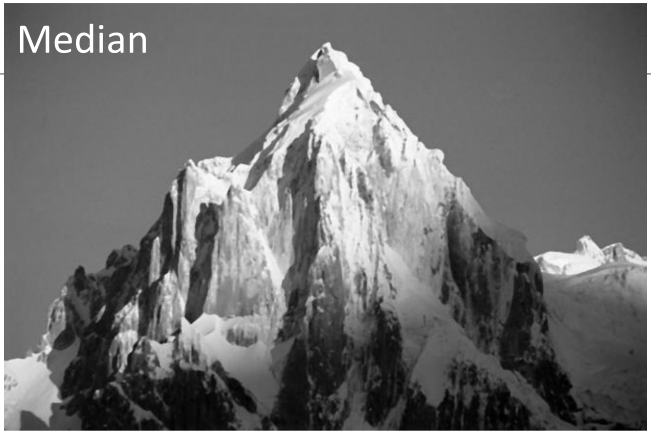
Replace each pixel by the median in a neighborhood around the pixel.

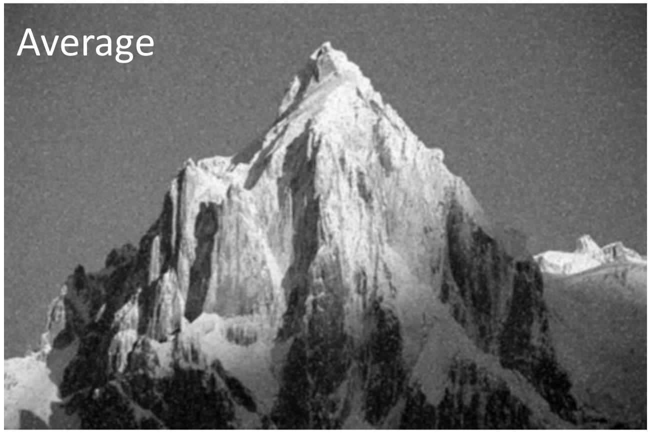


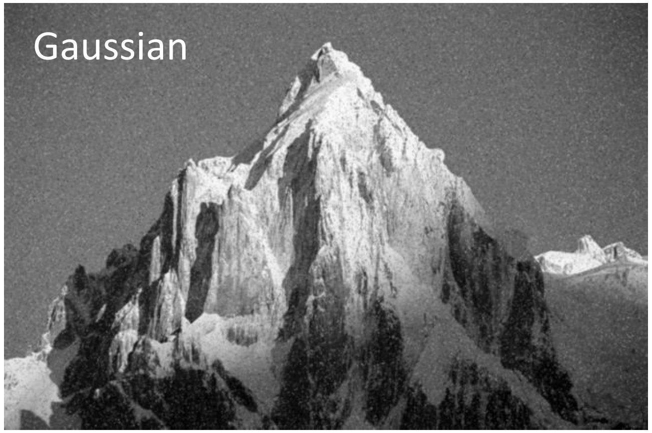
Median value of a neighbourhood is hardly a salt (255) or pepper (0) noise.

Comparison







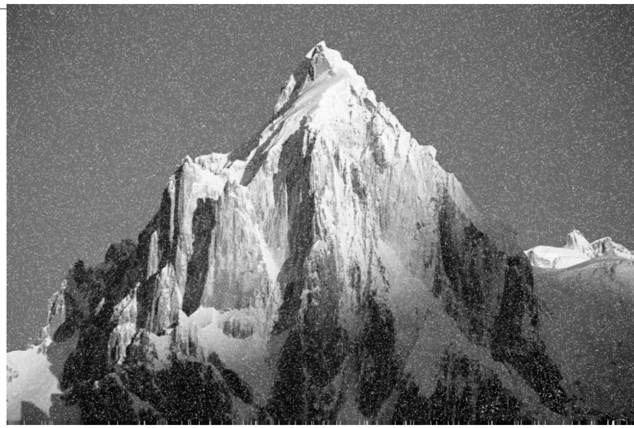


Smoothing Filters: Max filter, Min filter

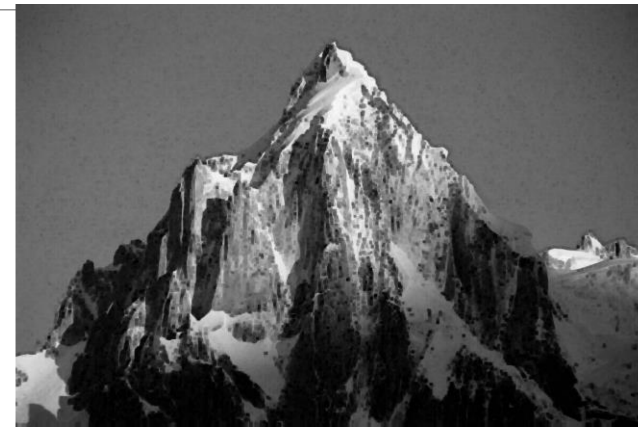
- A max filter or a min filter work similarly as a median filter.
- Non-linear filters
- Max filter
  - takes the max value of a neighbourhood.
  - for images with pepper (0) noise.
- Min filter
  - takes the min value of a neighbourhood.
  - for images with salt (255) noise.

Comparison

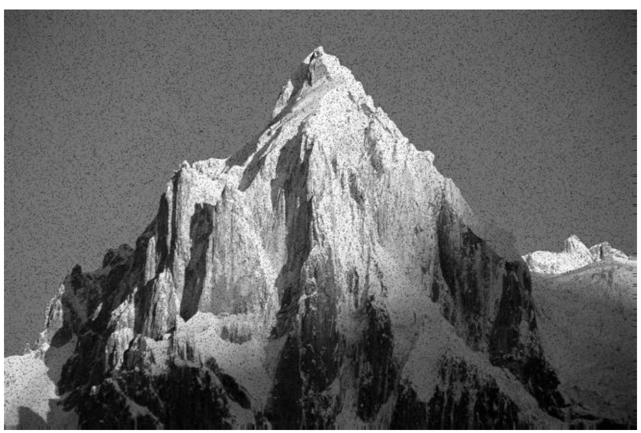
Orignal



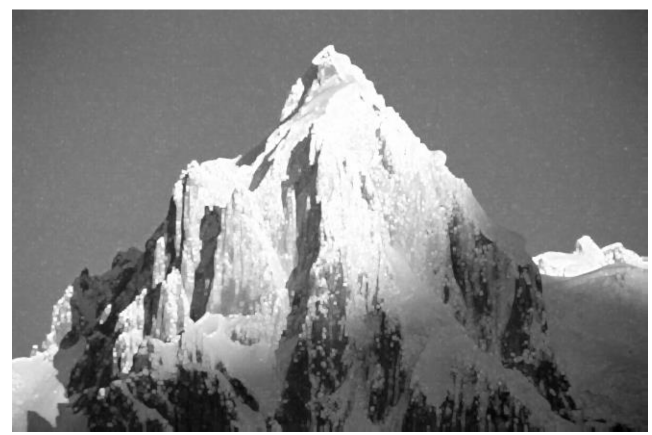
after min filtering



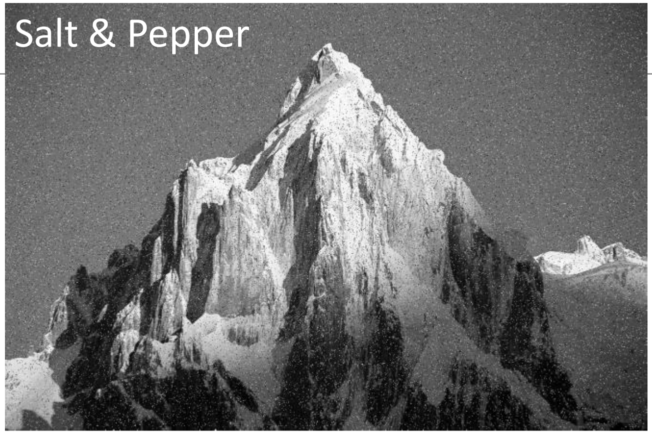
Orignal

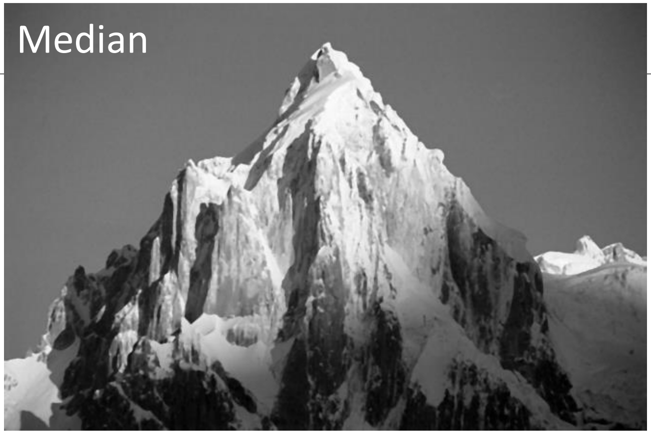


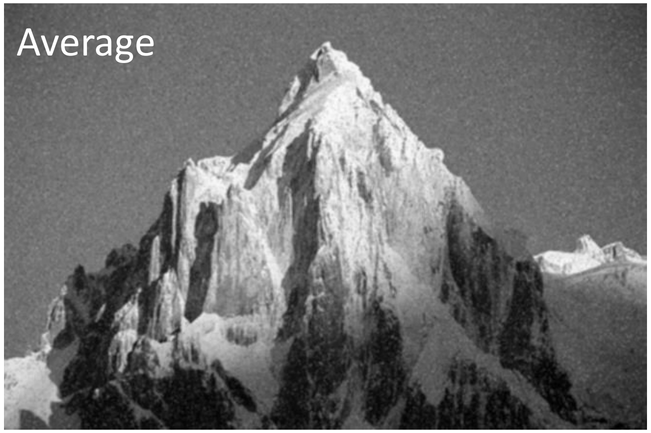
after max filtering

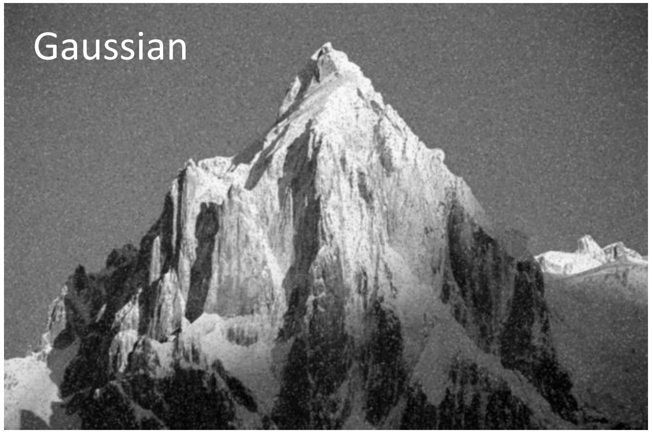


Comparison





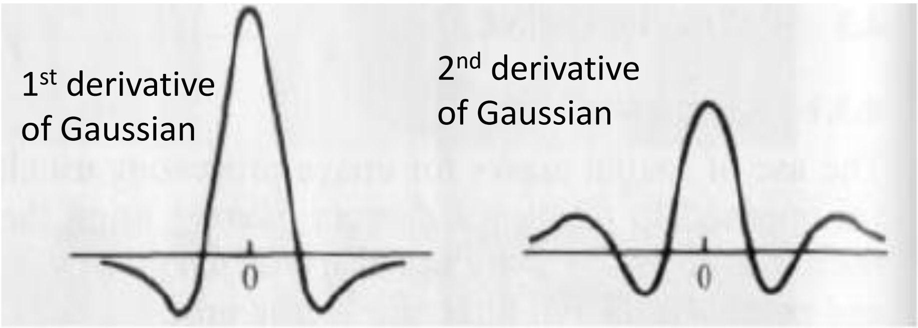




2. Sharpening Filters

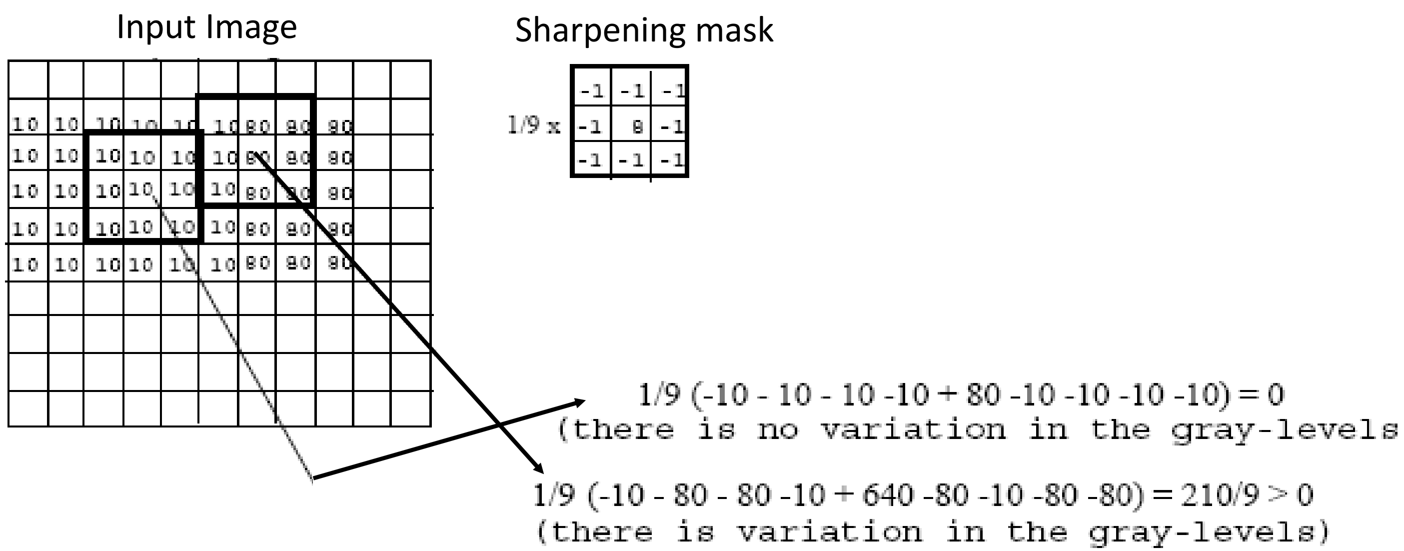
Sharpening

- Highlights the fine details or enhance details that have been blurred.
- The weights of the mask contain both positive and negative values.
- Sum of weights of the mask is 0.



Sharpening Filters

Useful for emphasizing transitions in image intensity (e.g., edges).



Sharpening Filters: Unsharp Masking

Steps:

1. blur the original image by a low-pass filter.

2. subtract the blurred image from the original (the difference is the Mask).

Mask**$_{UM}$* **=** ***Original **-** ***Lowpass***

=

-

3. add the mask to the original.

Enhanced image**$_{UM}$* **=** ***Original **+** ***Mask**$_{UM}$*

=

+

Sharpening Filters: Unsharp Masking

Note that the response of high-pass filtering might be negative.

Values must be re-mapped to [0, 255]

Original

Mask$_{UM}$

Enhanced image$_{UM}$

Sharpening Filters: High boost

Image sharpening emphasizes edges but details (i.e., low frequency components) might be lost.

High boost filter: amplify input image, then subtract a lowpass image.

$$Mask_{HB} = A*Original - Lowpass$$

$$= (A-1)*Original + (Original - Lowpass)$$

$$= (A-1)* \quad Original \quad + \quad Mask_{UM}$$

$= (A-1)*$

$+$

Sharpening Filters: High boost

If **A=1**, we have unsharp masking.

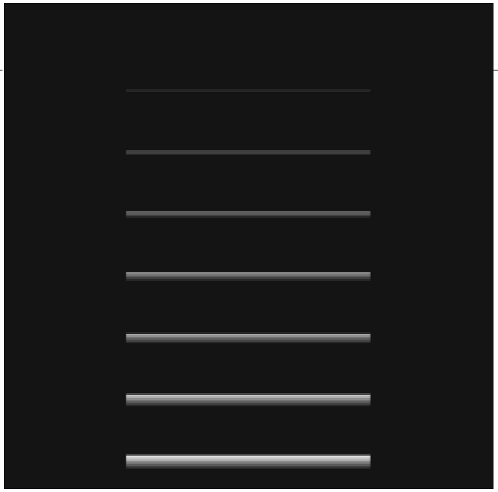
If **A>1**, part of the original image is added back to the high pass filtered image.

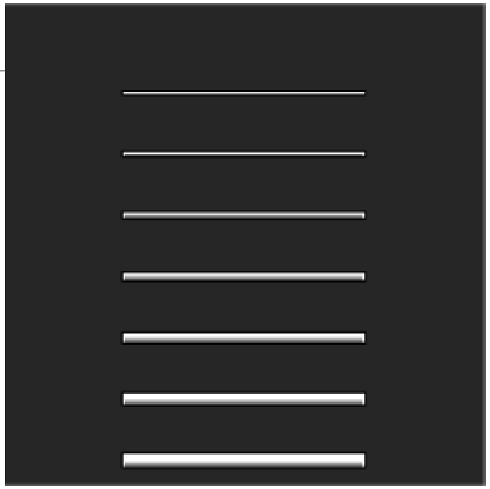
If **A<1**, the contribution of the unsharp mask is de-emphasized.

$$W_{highboost} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9A-1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \times \frac{1}{9}$$

Sharpening Filters: High boost

A=1.9





A=1.4

Summary

- Basics on spatial filtering
  - The filtering process
  - Border problem
  - Convolution and correlation
- Smoothing filter
  - Average filter
  - Gaussian filter
  - Median filter
- Sharpening filter
  - Unsharp masking filter
  - High-boost filter

Q&A