Optimization

COMP406 - Calculus

Dennis Wong

Optimization problems

Many ***optimization problems*** can be phrased as: For which value of x in the interval $a \leq x \leq b$ is $f(x)$ the largest?

In other words you are given a function f on an interval $[a, b]$ and you must find all ***global maxima*** (or ***global minima***) of f on this interval.

We can use the same technique we learned in graph sketching to find the global maxima or global minima.

The difficulty in optimization problems frequently lies not with the calculus part, but rather with setting up the problem.

Rectangle with largest area

Example: Which rectangle has the largest area, among all those rectangles for which the total length of the sides is 1

Solution: If the sides of the rectangle have lengths x and y, then the total length of the sides is $L = x + x + y + y = 2(x + y)$ and the area is $A = xy$.

Since we are only allowed to consider rectangles with $L = 1$, from this equation we get $L = 1 \Rightarrow y = 1/2 - x$. Therefore

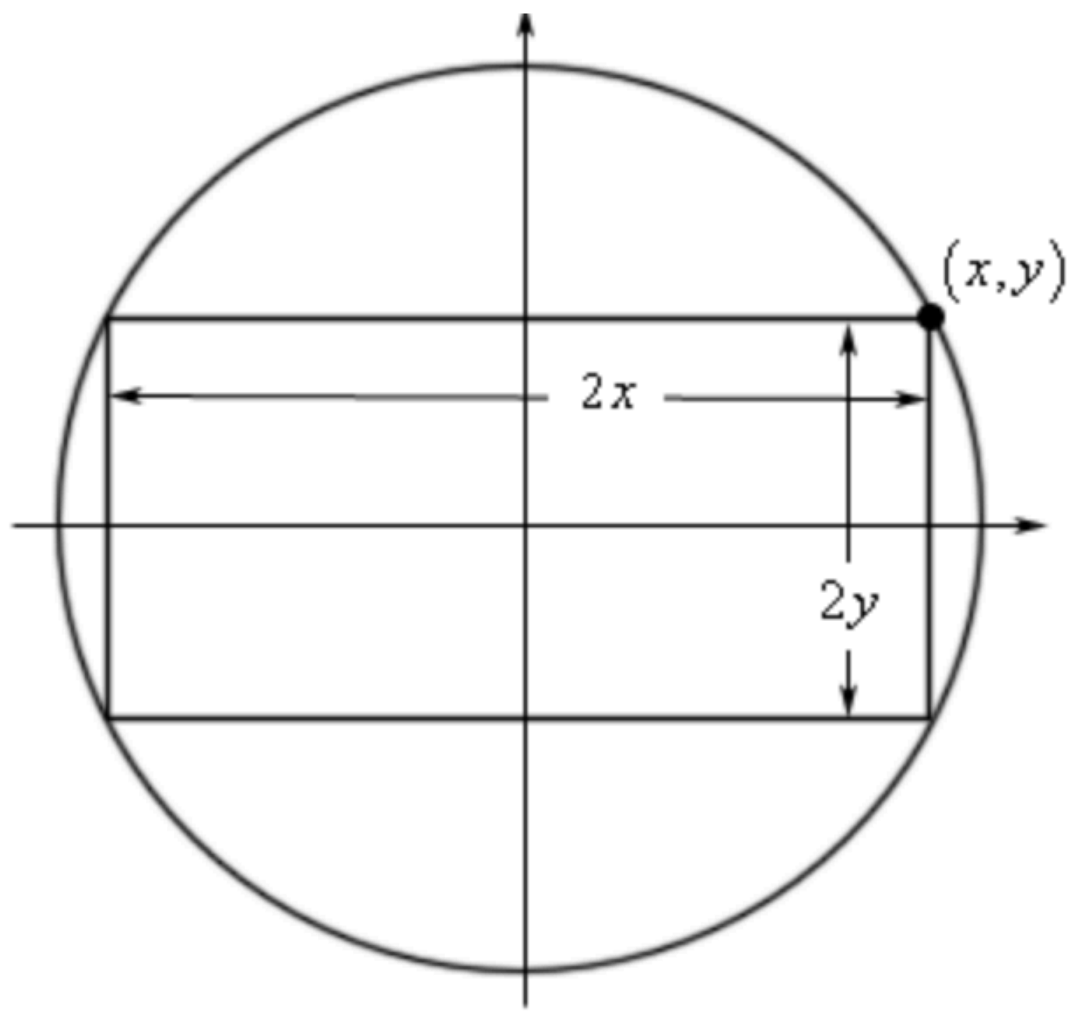
$$A = xy = x(1/2 - x).$$

Therefore, the function we are going to optimize: Find the maximum of the function $f(x) = x(1/2 - x)$ on the interval $0 \leq x \leq 1/2$.

Applying the technique we learned on graph sketching, we conclude that the maxima is attained by $x = 1/4$, and $y = 1/2 - 1/4 = 1/4$ with $A = 1/16$.

Rectangle inscribed in a circle

Example: Determine the area of the largest rectangle that can be inscribed in a circle of radius 4.



Rectangle inscribed in a circle

Solution: The equation of the circle is $x^2 + y^2 = 16$.

The area of the rectangle is thus A = $(2x)(2y) = 4xy$. Therefore we have:

A = $4xy = 4x \sqrt{(16 - x^2)}$ (we remote the negative root)

And the range of x is from 0 to 4.

Applying the technique we learned on graph sketching, we conclude that the maxima is attained by $x = 2\sqrt{2}$, and $y = 2\sqrt{(16 - 2\sqrt{2})} = 2\sqrt{2}$ with A = 32.