COMP408 Assignment 1 P1807524 Jacky

1. $f(x,y) = ax+by$

$f(1,2) = 0 \Rightarrow a+2b=0$ ①

$f(2,3) = 1 \Rightarrow 2a+3b=1$ ②

$2\times① - ② : b=-1 \quad a=-2b=2$

$\therefore f(x,y) = 2x-y$

2. $3x+y-z=1$ ①
$x-y+z=-3$ ②
$2x+y+z=0$ ③

① + ②: $4x=-2$, $x=-\frac{1}{2}$

② + ③: $3x+2z=-3$

$z = \frac{-3-3x}{2} = \frac{-3+\frac{3}{2}}{2} = -\frac{3}{4}$

$y = -2x-z = \frac{7}{4}$

$\therefore x=-\frac{1}{2}, y=\frac{7}{4}, z=-\frac{3}{4}$

3. $P_1 = 1+x+4x^2$

$$\begin{bmatrix}1\\1\\4\end{bmatrix} = \alpha_1\begin{bmatrix}1\\2\\-1\end{bmatrix} + \alpha_2\begin{bmatrix}3\\5\\2\end{bmatrix} = \begin{bmatrix}\alpha_1+3\alpha_2\\2\alpha_1+5\alpha_2\\-\alpha_1+2\alpha_2\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1&3&1\\2&5&1\\-1&2&4\end{array}\right] R_2-2 \sim \left[\begin{array}{cc|c}1&3&1\\0&-1&-1\\-1&2&4\end{array}\right] R_2 \sim \left[\begin{array}{cc|c}1&3&1\\0&-1&-1\\-1&0&2\end{array}\right]$$

$-\alpha_1 = 2$, $\alpha_1 = -2$, $\alpha_2 = 1$

$\therefore [-2,1]$ is the solution of P_1

$\therefore P_1$ is in span

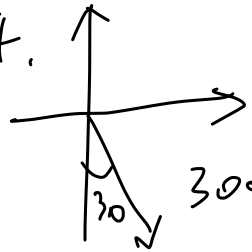
$P_2 = 1+5x+x^2$

$$\begin{bmatrix}1\\5\\1\end{bmatrix} = \alpha_1\begin{bmatrix}1\\2\\-1\end{bmatrix} + \alpha_2\begin{bmatrix}3\\5\\2\end{bmatrix} = \begin{bmatrix}\alpha_1+3\alpha_2\\2\alpha_1+5\alpha_2\\-\alpha_1+2\alpha_2\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1&3&1\\2&5&5\\-1&2&1\end{array}\right] R_2-2 \sim \left[\begin{array}{cc|c}1&3&1\\0&-1&3\\-1&2&1\end{array}\right] R_2 \sim \left[\begin{array}{cc|c}1&3&1\\0&-1&3\\-1&0&7\end{array}\right]$$

no solution, $\therefore P_2$ is not in span.

4.



(1) $300 \times \sin 30° = 150 \text{ km/h}$

$300 \times \cos 30° = 150\sqrt{3} \text{ km/h}$

Plane will fly $150\sqrt{3}$ km/h in the east direction.

(2) $300 \times \sin 30° = 150 \text{ km/h}$

speed on the east: $150 + 150 = 300$ km/h

$300 \times \cos 30° = 150\sqrt{3}$ km/h

speed $= \sqrt{(150\sqrt{3})^2 + (150)^2}$ km/h

5. take two arbitrary points $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$

1. Close under addition:

$3(x_1+x_2) - 2(y_1+y_2) + (z_1+z_2) = 0$

$\Rightarrow (3x_1 - 2y_1 + z_1) + (3x_2 - 2y_1 + z_2) = 0$

$0 + 0 = 0$ $\therefore$ the plane is closed under addition.

2. close under multiplication:

$k \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} kx_1 \\ ky_1 \\ kz_1 \end{pmatrix}$ $\Rightarrow 3kx_1 - 2ky_1 + kz_1 = 0$

$\Rightarrow k(3x_1 - y_1 + z_1) = 0$

the plane is closed under multiplication

$\therefore$ M is a subspace of R^3.

6. $\vec{AB} = (6-4, 4-(-7), 4-9) = (2, 11, -5)$

$\vec{AC} = (7-4, 10-(-7), -6-9) = (3, 17, -15)$

$\vec{BC} = (7-6, 10-4, -6-4) = (1, 6, -10)$

$\vec{AB} \cdot \vec{AC} = 2\times3 + 11\times17 + (-5)\times(-15) = 268 \neq 0$

$\vec{AB} \cdot \vec{BC} = 2\times1 + 11\times6 + (-5)\times(-10) = 118 \neq 0$

$\vec{AC} \times \vec{BC} = (3\times1 + 17\times6 + (-15)\times(-10) = 255 \neq 0$

the dot product among these three vector are not zero, thus it is not a right angle triangle.