L’Hospital Rule and Newton’s Method

COMP406 - Calculus
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L'Hospital rule

Suppose that we have one of the following cases,

$$\lim_{x \to a} (f(x) / g(x)) = 0/0 \text{ or } \lim_{x \to a} (f(x) / g(x)) = \infty/\infty$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x \to a} (f(x) / g(x)) = 0/0 \text{ or } \lim_{x \to a} (f'(x) / g'(x)) = \pm\infty/\pm\infty.$$

L'Hospital rule

Example: Evaluate the limit of $\lim_{x \to 0} (\sin(x) / x)$.

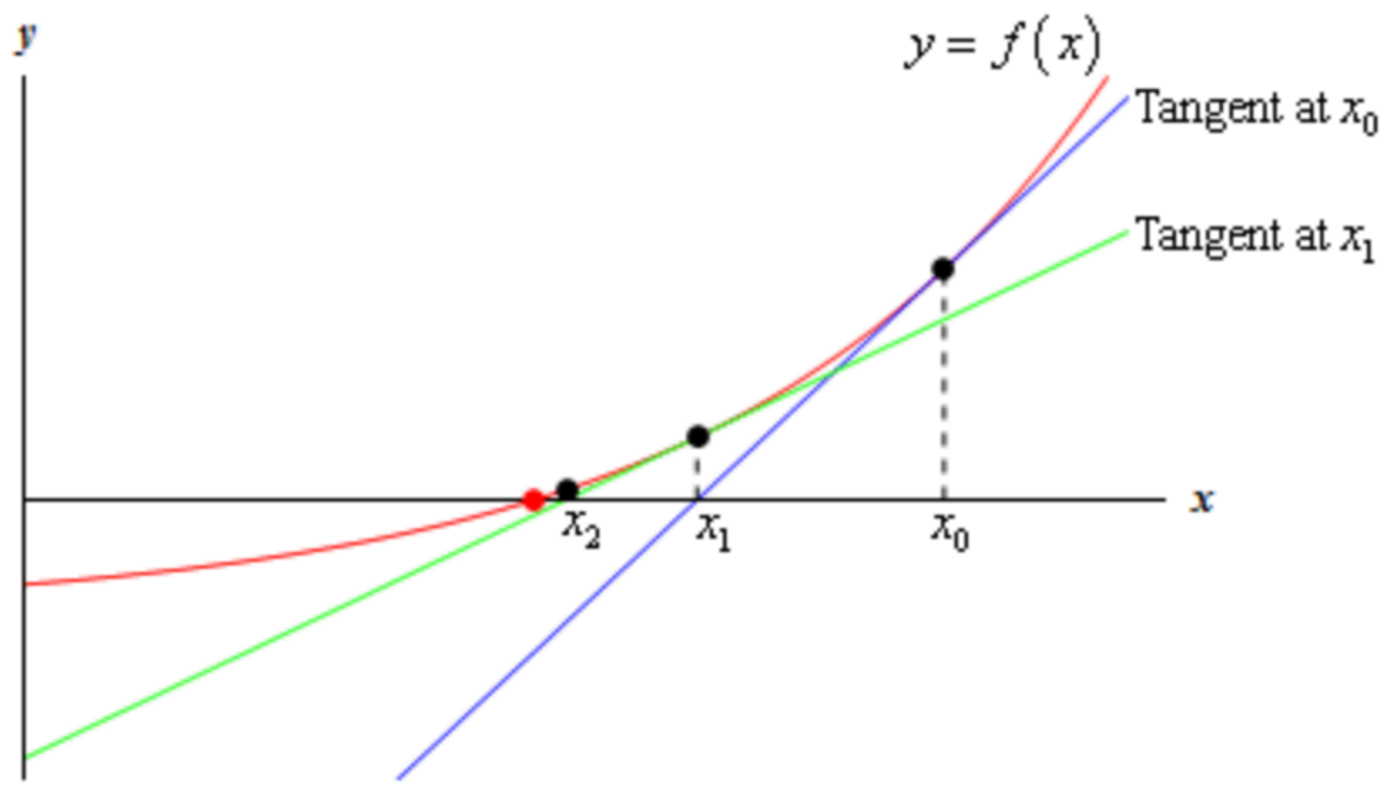
Solution: We have already established that this is a 0/0 indeterminate form, so now let's apply L'Hospital rule.

$$\lim_{x \to 0} (\sin(x) / x) = \lim_{x \to 0} (\cos(x) / 1$$
$$= 1/1$$
$$= 1.$$

Newton’s method

If x_n is an approximation a solution of $f(x) = 0$ and if $f'(x_n) \neq 0$ the next approximation is given by,

$$x_{n+1} = x_n - f(x_n) / f'(x_n).$$



Newton’s method

Example: Use Newton’s Method to determine an approximation to the solution to $\cos(x) = x$ that lies in the interval [0 , 2]. Find the approximation to six decimal places.

Solution: To apply Newton’s method, we must have the function in the form $f(x) = 0$. Therefore, we first rewrite the equation as $f(x) = \cos(x) - x = 0$.

Now we have $x_{n+1} = x_n - f(x_n) / f'(x_n)$. Thus

$$x_{n+1} = x_n - (\cos(x_n) - x_n) / (-\sin(x_n) - 1).$$

Now we use $x_0 = 1$ as our initial guess, thus we have the following:

$x_1 = 1 - (\cos(1) - 1) / (-\sin(1) - 1) = 0.7503638679$
$x_2 = 1 - (\cos(x_1) - x_1) / (-\sin(x_1) - 1) = 0.7391128909$
$x_3 = 1 - (\cos(x_2) - x_2) / (-\sin(x_2) - 1) = 0.7390851334$
$x_4 = 1 - (\cos(x_3) - x_3) / (-\sin(x_3) - 1) = 0.7390851332$

Now we have got two approximations that agree to 9 decimal places and so we can stop. We now assume that the solution is approximately $x_4 = 0.7390851332$.