# Coding Algorithm

- Introduction
- Basic Concepts of Information Theory
- Run-Length Coding
- Variable-Length Coding (VLC)
- Dictionary-based Coding
- Arithmetic Coding
- Lossless Image Compression
- Further Exploration

## Introduction

- **Compression**: the process of coding that will effectively reduce the total number of bits needed to represent certain information.

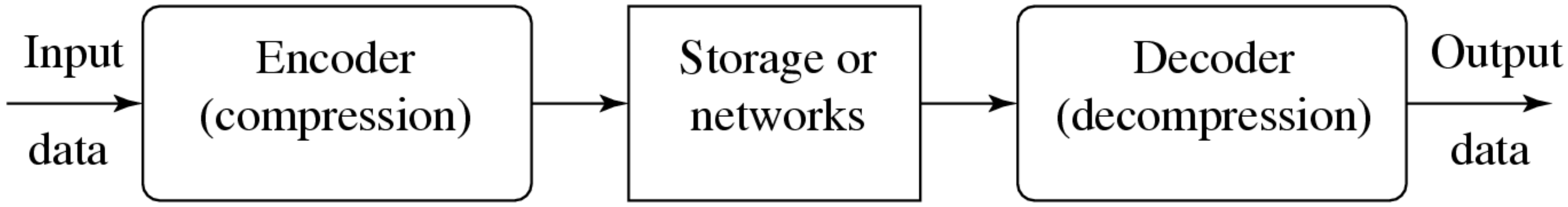


Fig. 7.1: A General Data Compression Scheme.

## Introduction (cont'd)

- If the compression and decompression processes induce no information loss, then the compression scheme is **lossless**; otherwise, it is **lossy**.

**Compression ratio**:

$$compression\,ratio = \frac{B_0}{B_1} \tag{7.1}$$

- $B_0$ – number of bits before compression
- $B_1$ – number of bits after compression

## Basic Concepts of Information Theory

- The *entropy* $\eta$ of an information *source* with alphabet $S = \{s_1, s_2, \ldots, s_n\}$ is:

$$\eta = H(S) = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i} \qquad (7.2)$$

$$= -\sum_{i=1}^{n} p_i \log_2 p_i \qquad (7.3)$$

- $p_i$ – probability that symbol $s_i$ will occur in $S$.
- $\log_2 \frac{1}{p_i}$ – indicates the amount of information (self-information as defined by Shannon) contained in $s_i$, which corresponds to the number of bits needed to encode $s_i$.

- In science, entropy is a measure of the disorder of a system.
- In information theory, entropy is a measure of **uncertainty**.
- The more entropy, the more uncertainty, the more information it carries.

- Example:
- Three types of languages are used in an essay and each letter/Hiragana/character appears averagely.
  - English-26 letters $$\eta = \sum_{i=1}^{26} \frac{1}{26} \log_2 26 = \boxed{4.7}$$
  - Japanese-50 Hiragana $$\eta = \sum_{i=1}^{50} \frac{1}{50} \log_2 50 = \boxed{5.64}$$
  - Chinese-2500 characters (frequently used)

$$\eta = \sum_{i=1}^{2500} \frac{1}{2500} \log_2 2500 = \boxed{11.3}$$

The information carried in a Chinese character is the biggest. Translation of an English essay into Chinese is shorter.

## Distribution of Gray-Level Intensities

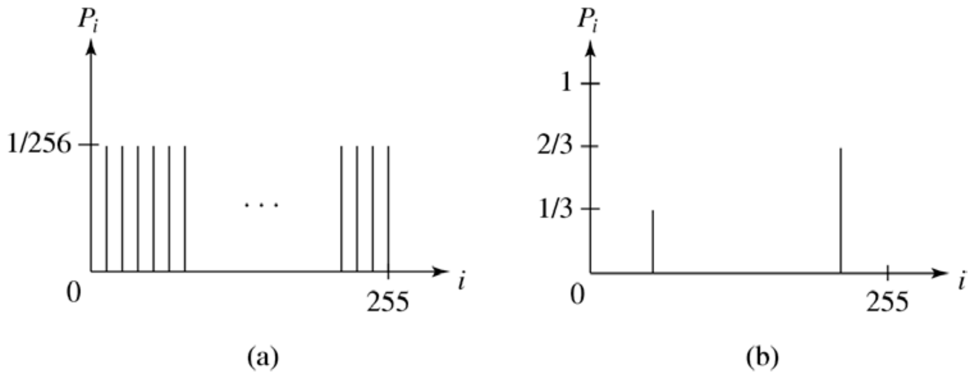


Fig. 7.2 Histograms for two gray-level images.

Fig. 7.2(a) shows the histogram of an image with *uniform* distribution of gray-level intensities. Its entropy is $\eta = 8$.

Fig. 7.2(b) shows the histogram of an image with two possible values. Its entropy is:

$$\eta = \frac{1}{3}\log_2 3 + \frac{2}{3}\log_2 \frac{3}{2} = 0.92 \tag{7.4}$$

## Exercises

1. For a string ‘xxzyyzzz’, calculate its entropy.
2. If a .BMP grayscale image with 800*400 pixels is converted to a .JPG image, the compression ratio is 1.2, calculate the file size of the .JPG image.

## Exercises

1. For a string ‘xxzyyzzzz’, calculate its entropy.

Probability: $P_x$=1/4, $P_z$=1/2, $P_y$=1/4.

$$\eta = -\frac{1}{4}\log_2\frac{1}{4} - \frac{1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} = \frac{3}{2}\log_2 2 = \frac{3}{2}$$

## Exercises

2. If a .BMP grayscale image with 800*400 pixels is converted to a .JPG image, the compression ratio is 1.2, calculate the file size of the .JPG image.

- Original .BMP image size=800*400*1 byte=320kB
- JPB image size=320kB/1.2=266.67kB

## Entropy and Code Length

- As can be seen in Eq. (7.3): the entropy $\eta$ is a weighted-sum of terms $\log_2 \frac{1}{p_i}$ ; hence it represents the *average* amount of information contained per symbol in the source $S$.
- The entropy $\eta$ specifies the lower bound for the average number of bits to code each symbol in $S$, i.e.,

$$\eta \leq \bar{l} \tag{7.5}$$

$\bar{l}$ - the average length (measured in bits) of the codewords produced by the encoder.

## Run-Length Coding

- **Memoryless Source**: an information source that is independently distributed. Namely, the value of the current symbol does not depend on the values of the previously appeared symbols.

- Instead of assuming memoryless source, *Run-Length Coding* (*RLC*) exploits memory present in the information source.

- **Rationale for RLC**: if the information source has the property that symbols tend to form continuous groups, then such symbol and the length of the group can be coded.

## Run-Length Coding

- Example: If a scanline from a binary image is as below,

WWWWWWWWWWWWBWWWWWWWWWWWWBBBWWWWWW
WWWWWWWWWWWWWWWWWWBWWWWWWWWWWWWWW

(67 characters)

- Using RLC, it can be rendered as below,

12W1B12W3B24W1B14W (18 characters)

- Compression ratio=3.72

## Variable-Length Coding (VLC)

## Shannon-Fano Algorithm

A **top-down** approach

1. Sort the symbols according to the **frequency count** of their occurrences.
2. Recursively divide the symbols into two parts, each with approximately the same number of counts, until all parts contain only one symbol.

A natural way of implementing the above procedure is to build a binary tree.

As a convention, assign bit 0 to its left branches and 1 to the right branches.

## Shannon-Fano Algorithm

- Example: coding of "HELLO"

| Symbol | H | E | L | O |
|---|---|---|---|---|
| Count | 1 | 1 | 2 | 1 |

Frequency count of the symbols in "HELLO"

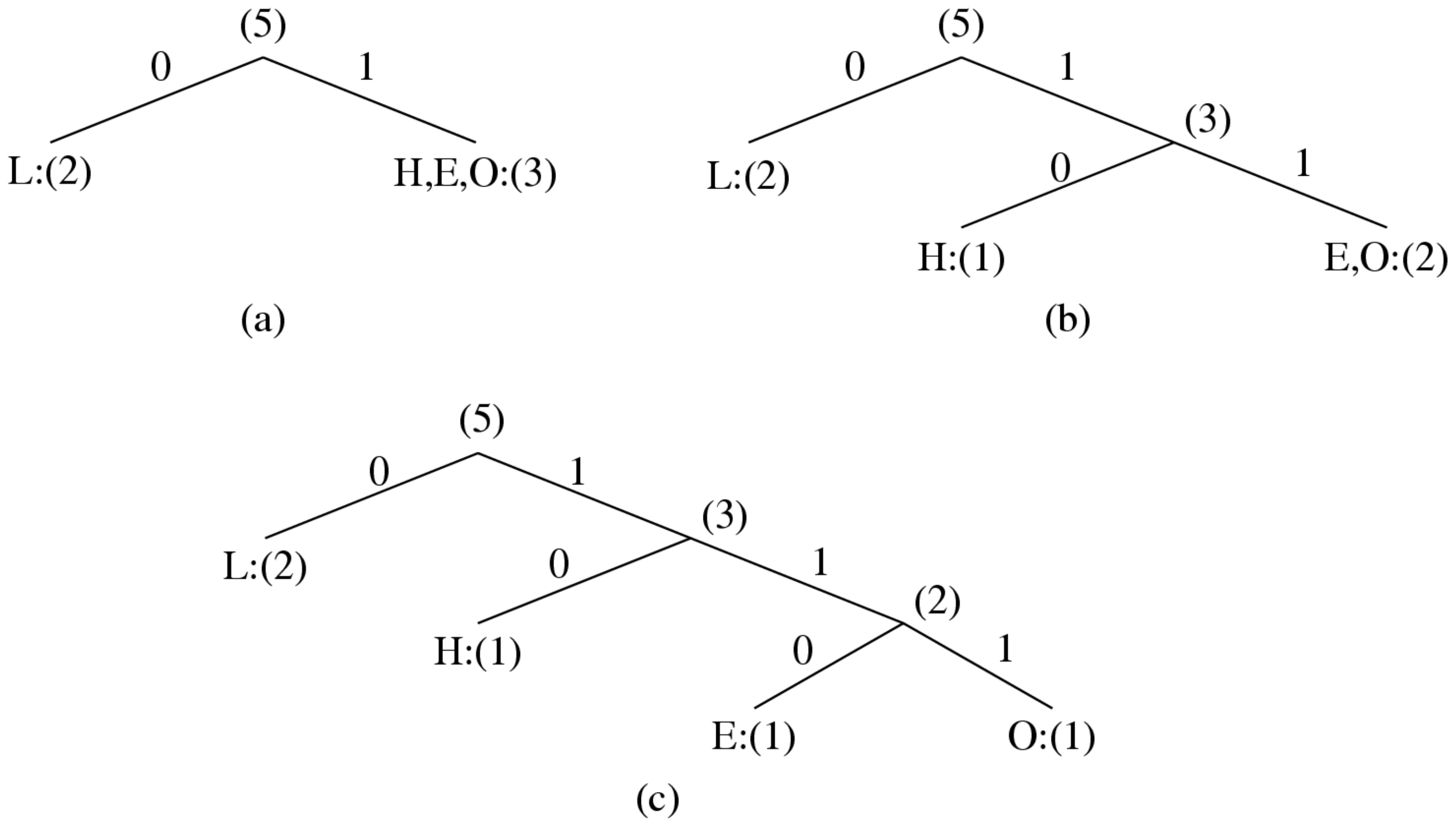


Fig. 7.3 Coding Tree for HELLO by Shannon-Fano.

## Table 7.1: Result of Performing Shannon-Fano on HELLO

| Symbol | Frequency | $\log_2(1/p_i)$ | Code | # of bits used |
|---|---|---|---|---|
| L | 2 | 1.32 | 0 | 2 |
| H | 1 | 2.32 | 10 | 2 |
| E | 1 | 2.32 | 110 | 3 |
| O | 1 | 2.32 | 111 | 3 |
| | | | TOTAL # of bits: | 10 |

$$\eta = 0.4 \times 1.32 + 0.2 \times 2.32 + 0.2 \times 2.32 + 0.2 \times 2.32 = 1.92$$

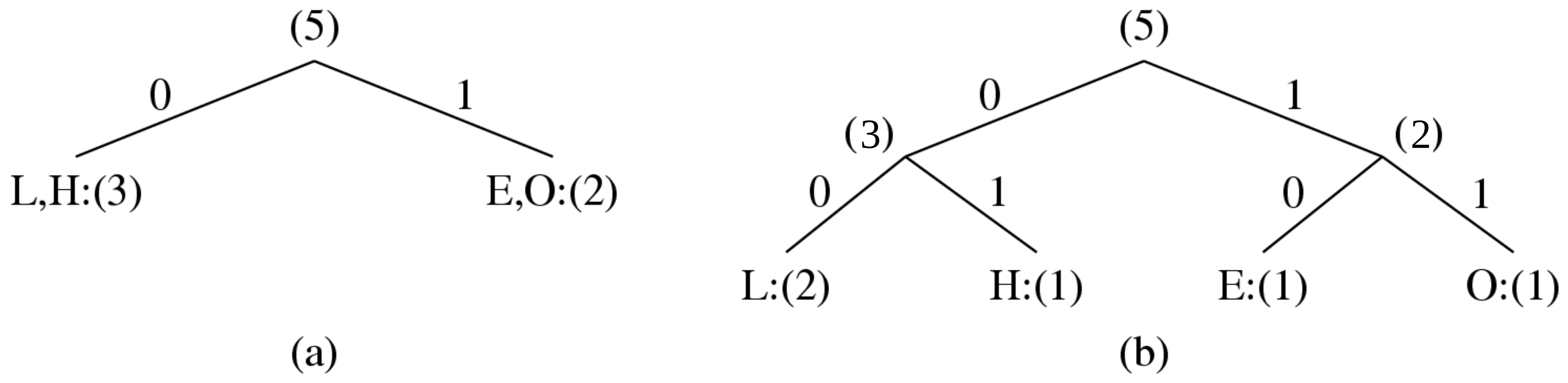


Fig. 7.4 Another coding tree for HELLO by Shannon-Fano.

## Table 7.2: Another Result of Performing Shannon-Fano on HELLO (see Fig. 7.4)

| Symbol | Count | $\log_2 \frac{1}{p_i}$ | Code | # of bits used |
|---|---|---|---|---|
| L | 2 | 1.32 | 00 | 4 |
| H | 1 | 2.32 | 01 | 2 |
| E | 1 | 2.32 | 10 | 2 |
| O | 1 | 2.32 | 11 | 2 |
| | | | TOTAL # of bits: | 10 |

## Exercise

- Using Shannon-Fano Algorithm to encode the following symbols

| Symbol | a | b | c | d | e |
|---|---|---|---|---|---|
| Counts | 1 | 1 | 3 | 4 | 1 |

## Huffman Coding

- A **bottom-up** approach
- **ALGORITHM 7.1-HUFFMAN CODING**

1. Initialization: put all symbols sorted according to their *frequency counts from lowest to highest*.
2. Repeat until the list has only one symbol left:
   a) From the list pick two symbols with the *lowest* frequency counts. Form a Huffman subtree that has these two symbols as child nodes and create a parent node.

   b) Assign the sum of the children's frequency counts to the parent and insert it into the list such that the order is maintained.

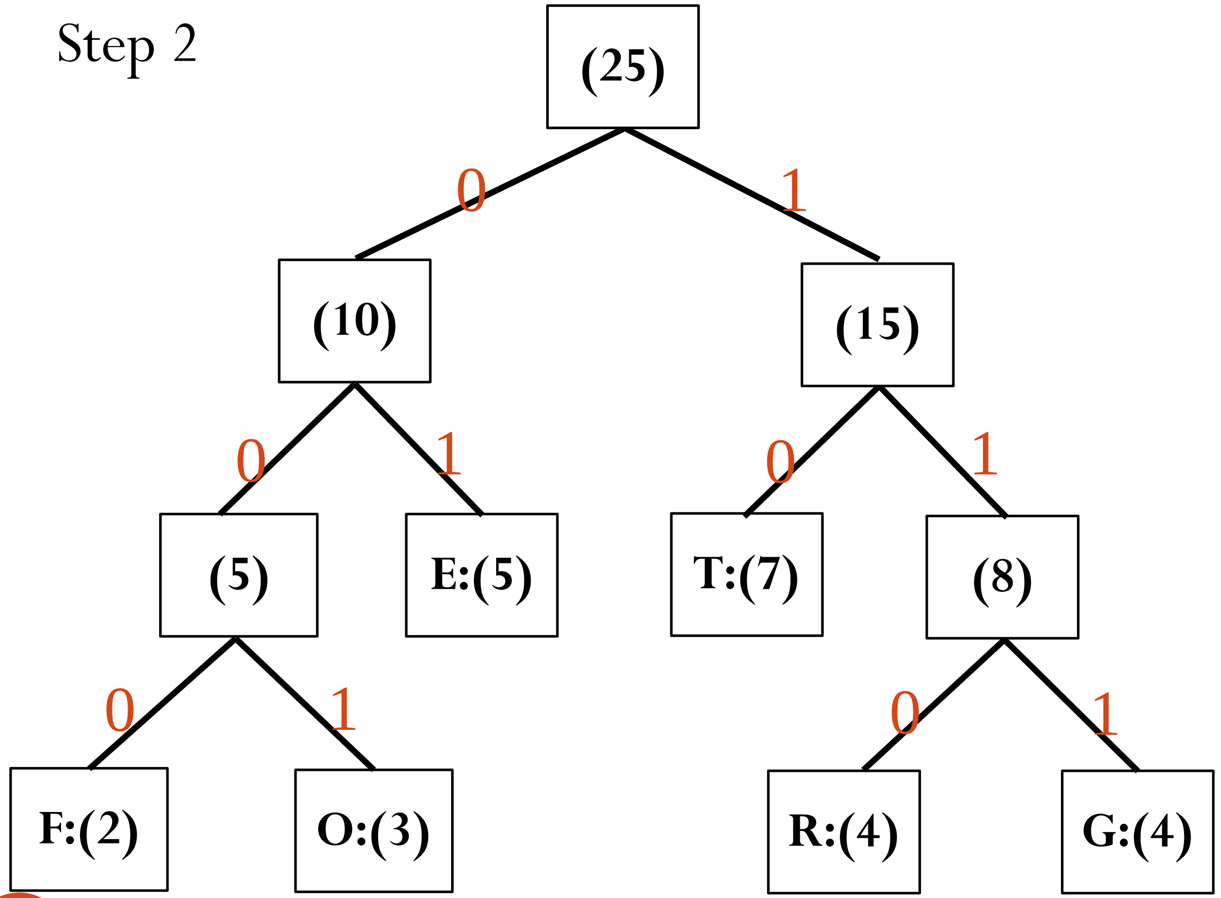
   c) Delete the children from the list.
3. Assign a codeword for each leaf based on the path **from the root**.

Example:

Step 1

| **Symbol** | **F** | **O** | **R** | **G** | **E** | **T** |
|---|---|---|---|---|---|---|
| **Frequency** | **2** | **3** | **4** | **4** | **5** | **7** |

Step 2



Frequency ranking list

Iter1: 2 3 4 4 5 7

Iter2: 4 4 5 5 7

Iter3: 5 5 7 8

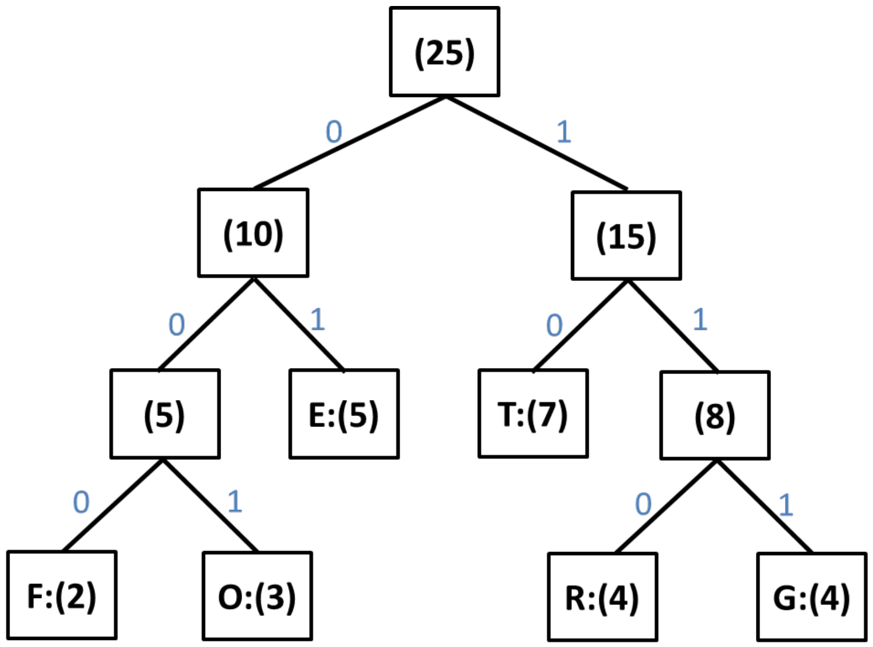
Iter4: 7 8 10

Iter5: 10 15

Iter6: 25

## 3 tips:

1. The frequency count on the same level of nodes should always be : small count on the left and big count on the right.
2. if the count are the same, doesn't matter which is on the right or left.
3. Mark 0 on the left branch and mark 1 on the right branch.



| Symbol | F | O | R | G | E | T |
|---|---|---|---|---|---|---|
| Frequency | 2 | 3 | 4 | 4 | 5 | 7 |
| Code | 000 | 001 | 110 | 111 | 01 | 10 |
| $p_i$ | 0.08 | 0.12 | 0.16 | 0.16 | 0.2 | 0.28 |
| $\log_2(1/p_i)$ | 3.6439 | 3.0589 | 2.6439 | 2.6439 | 2.3219 | 1.8365 |

$$\eta = 2.4832$$

## Properties of Huffman Coding

1. **Unique Prefix Property**: No Huffman code is a prefix of any other Huffman code - precludes any ambiguity in decoding.

2. **Optimality**: *minimum redundancy code* - proved optimal for a given data model (i.e., a given, accurate, probability distribution):

- The two least frequent symbols will have the same length for their Huffman codes, differing only at the last bit.
- Symbols that occur more frequently will have shorter Huffman codes than symbols that occur less frequently.
- The average code length for an information source $S$ is strictly less than $\eta$ + 1. Combined with Eq. (7.5), we have:

$$\eta < \bar{l} < \eta + 1 \tag{7.6}$$

## Exercise

- Using Huffman coding Algorithm to encode the following symbols

| Symbol | a | b | c | d | e |
|---|---|---|---|---|---|
| Counts | 1 | 1 | 3 | 4 | 1 |

# Extended Huffman Coding

- **Motivation**: All codewords in Huffman coding have integer bit lengths. It is wasteful when $p_i$ is very large and hence $\log_2 \frac{1}{p_i}$ is close to 0. (Information carried in this symbol is little.)

- Why not group several symbols together and assign a single codeword to the group as a whole?

- **Extended Alphabet**: For alphabet $S = \{s_1, s_2, \ldots, s_n\}$, if $k$ symbols are grouped together, then the *extended alphabet* is:

$$S^{(k)} = \{\overbrace{s_1 s_1 \ldots s_1}^{k \text{ symbols}}, s_1 s_1 \ldots s_2, \ldots, s_1 s_1 \ldots s_n, s_1 s_1 \ldots s_2 s_1, \ldots, s_n s_n \ldots s_n\}.$$

- — the size of the new alphabet $S^{(k)}$ is $n^k$.

## Extended Huffman Coding (cont'd)

- It can be proven that the average # of bits for each symbol is:

$$\eta \leq \bar{l} < \eta + \frac{1}{k} \qquad (7.7)$$

- An improvement over the original Huffman coding, but not much.

- **Problem**: If $k$ is relatively large (e.g., $k \geq 3$), then for most practical applications where n $\gg$ 1, $n^k$ implies a huge symbol table — impractical.

## Adaptive Huffman Coding

- **Adaptive Huffman Coding**: statistics are gathered and updated dynamically *as the data stream arrives*.

```
ENCODER                       DECODER
--------                     --------
Initial_code();              Initial_code();

while not EOF                while not EOF
 {                           {
       get(c);                     decode(c);
       encode(c);                  output(c);
       update_tree(c);             update_tree(c);
 }                           }
```

## Adaptive Huffman Coding (Cont'd)

- `Initial_code` assigns symbols with some initially agreed upon codes, without any prior knowledge of the frequency counts.

- `update_tree` constructs an Adaptive Huffman tree. It basically does two things:

  a. increments the frequency counts for the symbols (including any new ones).
  b. updates the configuration of the tree.

- The *encoder* and *decoder* must use exactly the same `initial_code` and `update_tree` routines.

## Notes on Adaptive Huffman Tree Updating

- Nodes are numbered in order from left to right, bottom to top. The numbers in parentheses indicates the count.

- The tree must always maintain its ***sibling* property**, i.e., all nodes (internal and leaf) are arranged in the order of increasing counts.

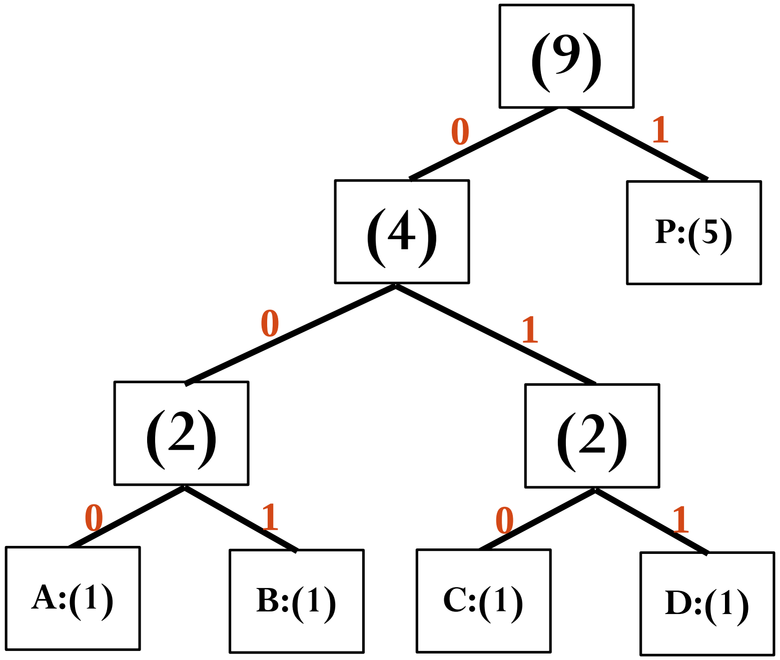
  - If the sibling property is about to be violated, a *swap* procedure is invoked to update the tree by rearranging the nodes.

- When a swap is necessary, the farthest node with count $N$ is swapped with the node whose count has just been increased to $N+1$.

# Adaptive Huffman Tree Updating

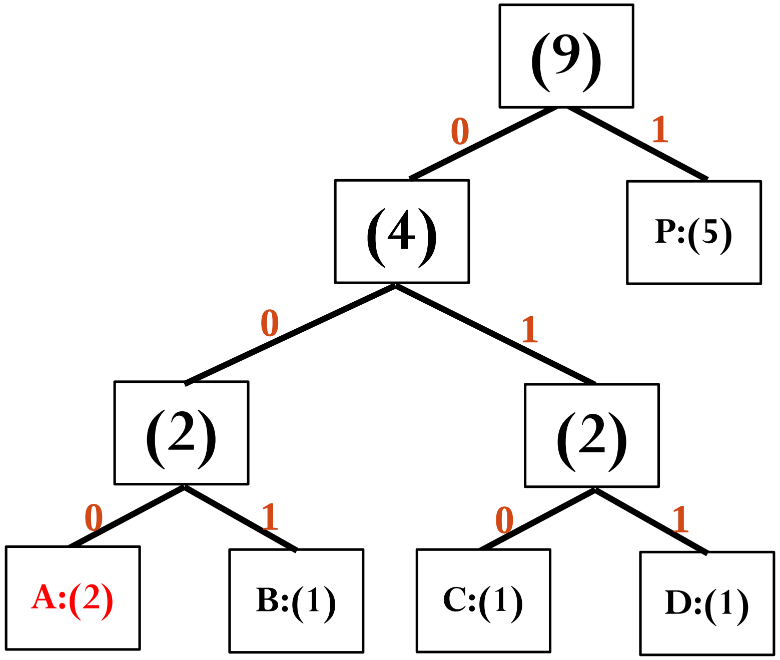
- Example: Symbol input: **PPDCPPBPA…**

| **Symbol** | **A** | **B** | **C** | **D** | **P** |
|---|---|---|---|---|---|
| **Frequency** | 1 | 1 | 1 | 1 | 5 |



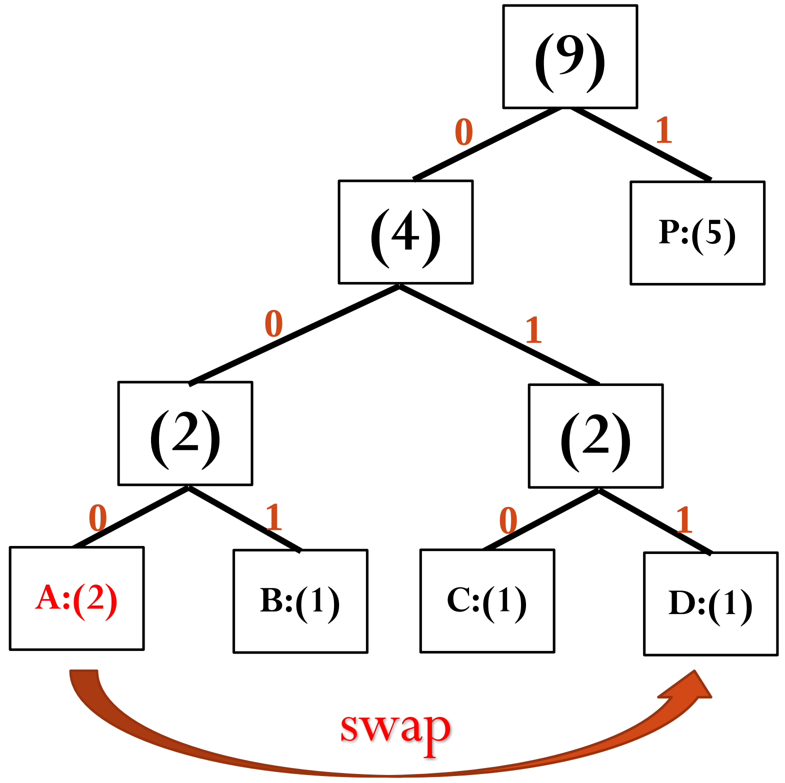
receiving 2nd A

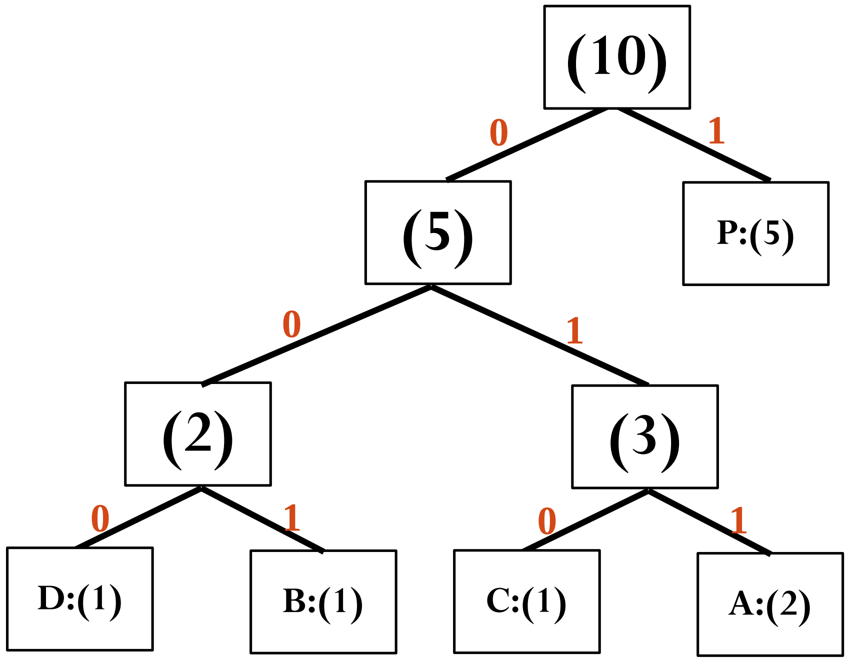
| **Symbol** | **D** | **B** | **C** | **A** | **P** |
|---|---|---|---|---|---|
| **Frequency** | 1 | 1 | 1 | 2 | 5 |



## Receiving 2nd A

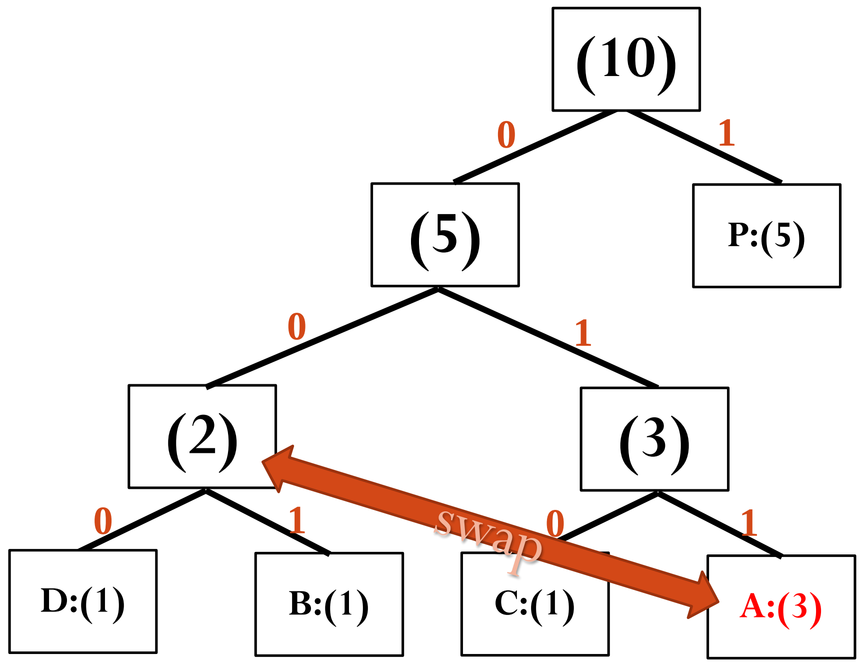
| Symbol | D | B | C | A | P |
|---|---|---|---|---|---|
| Frequency | 1 | 1 | 1 | 2 | 5 |

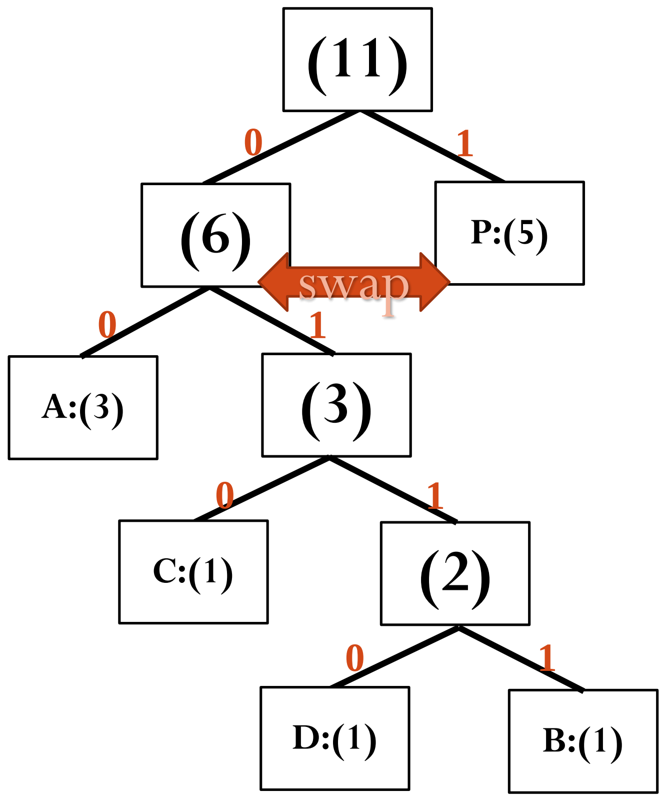


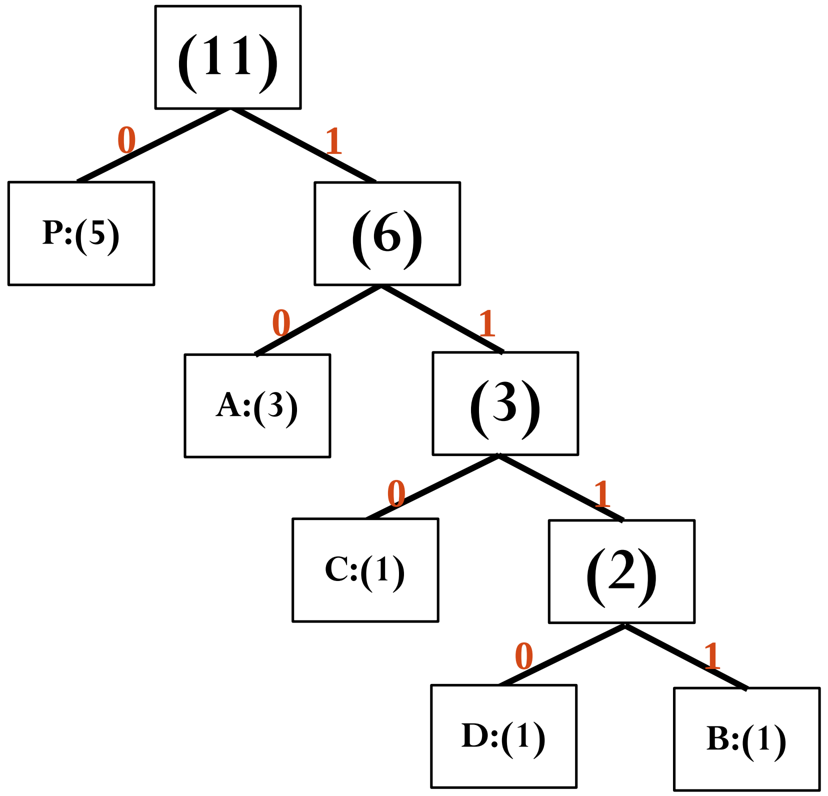


- Receiving 3rd A

| Symbol | D | B | C | A | P |
|---|---|---|---|---|---|
| Frequency | 1 | 1 | 1 | 3 | 5 |






(11)
0
1
P:(5)
(6)
0
1
A:(3)
(3)
0
1
C:(1)
(2)
0
1
D:(1)
B:(1)

## Exercise

- Using Adaptive Huffman coding Algorithm to encode the following string:
- The initial input are ‘xxyzzz…’,work out the Huffman tree.
- When a second ‘y’ comes, how the Huffman tree changes.
- When a third ‘y’ comes, how the Huffman tree changes.
- When a fourth ‘y’ comes, how the Huffman tree changes.

## Another Example: Adaptive Huffman Coding

- This is to clearly illustrate more implementation details. We show exactly what *bits* are sent, as opposed to simply stating how the tree is updated.

- An additional rule: if any character/symbol is to be sent the first time, it must be preceded by a special symbol, NEW. The initial code for NEW is 0. The *count* for NEW is always kept as 0 (the count is never increased); hence it is always denoted as NEW:(0).

Initial Code

| | |
|---|---|
| NEW: | 0 |
| A: | 00001 |
| B: | 00010 |
| C: | 00011 |
| D: | 00100 |
| | . . |
| | . . |
| | . . |

Table 7.3: Initial code assignment for AADCCDD using adaptive Huffman coding. (ASCII)

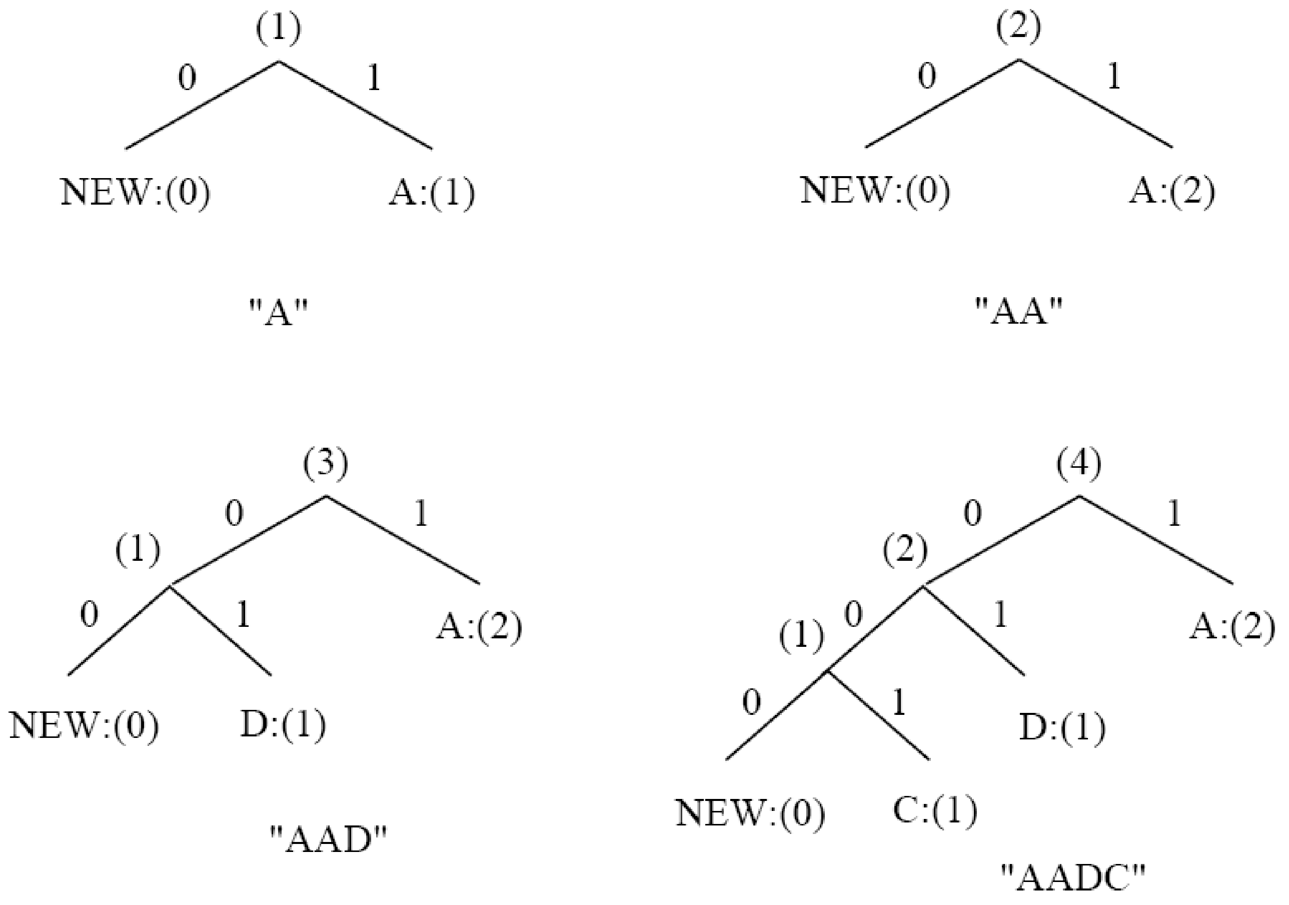


Fig. 7.7 Adaptive Huffman tree for AADCCDD.

| Symbol | NEW | A | A | NEW | D | NEW | C | C | D | D |
|---|---|---|---|---|---|---|---|---|---|---|
| Code | 0 | 00001 | 1 | 0 | 00100 | 00 | 00011 | 001 | 101 | 101 |

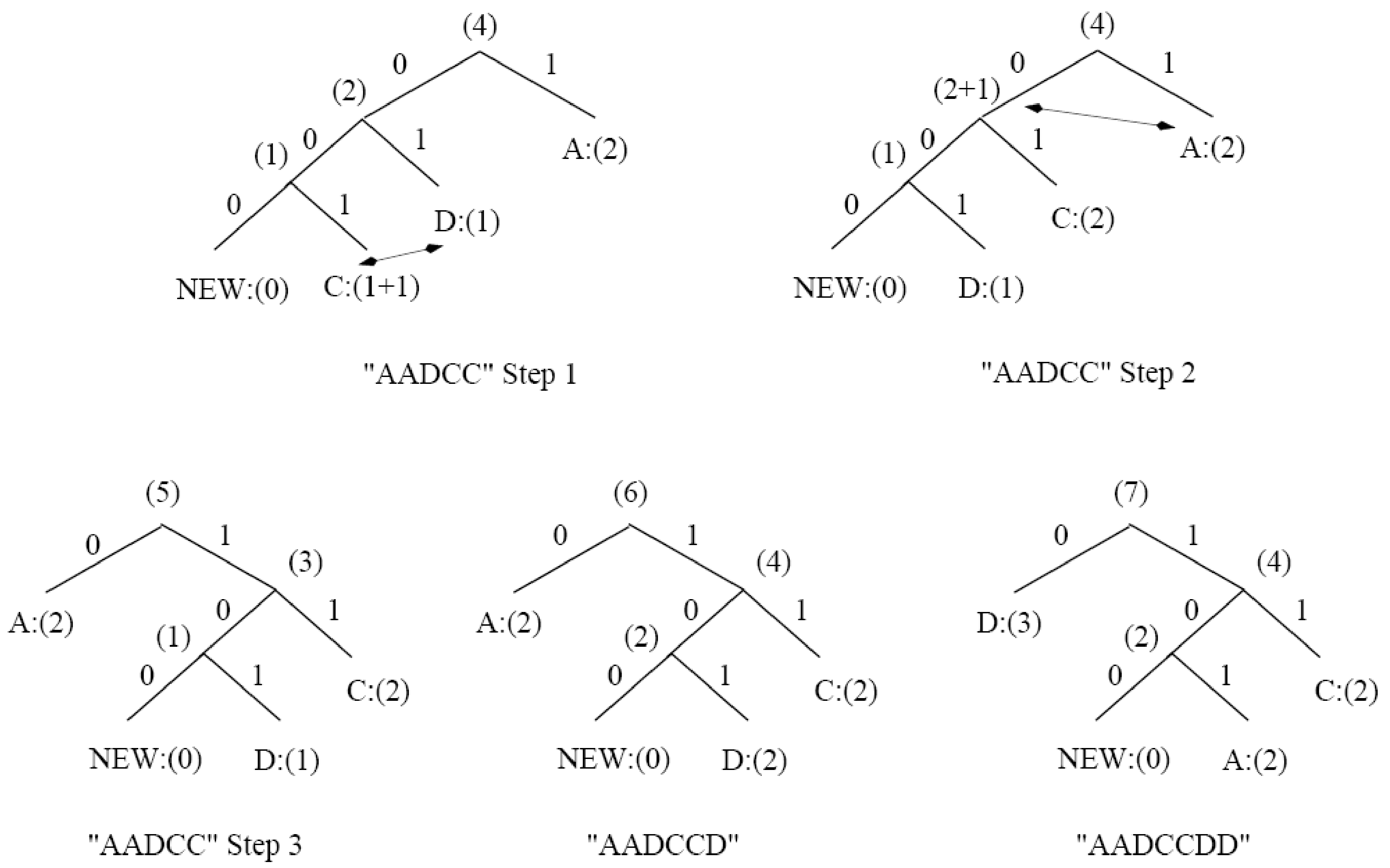


Fig. 7.7 (cont'd) Adaptive Huffman tree for AADCCDD.

Table 7.4 Sequence of symbols and codes sent to the decoder

| Symbol | NEW | A | A | NEW | D | NEW | C | C | D | D |
|---|---|---|---|---|---|---|---|---|---|---|
| Code | 0 | 00001 | 1 | 0 | 00100 | 00 | 00011 | 001 | 101 | 101 |

- It is important to emphasize that the code for a particular symbol changes during the adaptive Huffman coding process.

- For example, after AADCCDD, when the character D overtakes A as the most frequent symbol, its code changes from 101 to 0.

## Exercise

- For ‘xxyzzz’ work out code by using adaptive Huffman coding with NEW symbol.

Initial Code

---------------------

| | |
|---|---|
| NEW: | 0 |
| x: | 11000 |
| y: | 11001 |
| z: | 11010 |

## Dictionary-based Coding

- LZW uses *fixed-length* codewords to represent variable-length strings of symbols/characters that commonly occur together, e.g., words in English text.

- The LZW encoder and decoder build up the *same* dictionary dynamically while receiving the data.

- LZW places longer and longer repeated entries into a dictionary, and then emits the *code* for an element, if the element has already been placed in the dictionary.

## ALGORITHM 7.2 - LZW Compression

```
BEGIN
       s = next input character;
       while not EOF
       {
                c = next input character;
                if s + c exists in the dictionary
                s = s + c;
                else
            {
                output the code for s;
                add string s + c to the dictionary with a new code;
                s = c;
            }
       }
       output the code for s;
END
```

- **Example:**

  **LZW compression for string "ABABBABCABABBA"**

- Let's start with a very simple dictionary (also referred to as a "string table"), initially containing only 3 characters, with codes as follows:

| Code | String |
|---|---|
| 1 | A |
| 2 | B |
| 3 | C |

- Now if the input string is "ABABBABCABABBA", the LZW compression algorithm works as follows:

## Example:

## LZW compression for string “ABABBABCABABBA”

| s | c | s+c | Output | Code | String |
|---|---|---|---|---|---|
| | | | | 1<br>2<br>3 | A<br>B<br>C |
| **A** | B | AB | 1 | 4 | AB |
| **B** | A | BA | 2 | 5 | BA |
| A | B | AB | | | |
| **AB** | B | ABB | 4 | 6 | ABB |
| B | A | BA | | | |
| **BA** | B | BAB | 5 | 7 | BAB |
| **B** | C | BC | 2 | 8 | BC |
| **C** | A | CA | 3 | 9 | CA |
| A | B | AB | | | |
| **AB** | A | ABA | 4 | 10 | ABA |
| A | B | AB | | | |
| AB | B | ABB | | | |
| **ABB** | A | ABBA | 6 | 11 | ABBA |
| **A** | EOF | A | 1 | | |

## Example:

### LZW compression for string "ABABBABCABABBA"

| s | c | s+c | Output | Code | String |
|---|---|---|---|---|---|
| | | | | 1 | A |
| | | | | 2 | B |
| | | | | 3 | C |
| A | B | AB | 1 | 4 | AB |
| B | A | BA | 2 | 5 | BA |
| A | B | AB | | | |
| AB | B | ABB | 4 | 6 | ABB |
| B | A | BA | | | |
| BA | B | BAB | 5 | 7 | BAB |
| B | C | BC | 2 | 8 | BC |
| C | A | CA | 3 | 9 | CA |
| A | B | AB | | | |
| AB | A | ABA | 4 | 10 | ABA |
| A | B | AB | | | |
| AB | B | ABB | | | |
| ABB | A | ABBA | 6 | 11 | ABBA |
| A | EOF | A | 1 | | |

- The output codes are: **1 2 4 5 2 3 4 6 1**. Instead of sending 14 characters, only 9 codes need to be sent (compression ratio = 14/9 = 1.56).

## Input String “ABCBCABCBBCAB” (13 characters)

| **s** | **C** | **s+c** | **Output** | **Code** | **String** |
|---|---|---|---|---|---|
| | | | | 1 | A |
| | | | | 2 | B |
| | | | | 3 | C |
| **A** | B | AB | 1 | 4 | AB |
| **B** | C | BC | 2 | 5 | BC |
| **C** | B | CB | 3 | 6 | CB |
| B | C | BC | | | |
| **BC** | A | BCA | 5 | 7 | BCA |
| A | B | AB | | | |
| **AB** | C | ABC | 4 | 8 | ABC |
| C | B | CB | | | |
| **CB** | B | CBB | 6 | 9 | CBB |
| B | C | BC | | | |
| BC | A | BCA | | | |
| **BCA** | B | BCAB | 7 | 10 | BCAB |
| **B** | EOF | | 2 | | |

Output String “1 2 3 5 4 6 7 2” (8 characters)

## ALGORITHM 7.3 LZW Decompression (simple version)

```
BEGIN
    s = NIL;
    while not EOF
    {
          k = next input code;
          entry = dictionary entry for k;
          output entry;
          if (s != NIL)
                add string s + entry[0] to
                dictionary with a new code;
          s = entry;
    }
END
```

## Example 7.3:

Input codes to the decoder are 1 2 4 5 2 3 4 6 1. The initial string table is identical to what is used by the encoder.

The LZW decompression algorithm then works as follows:

| S | K | Entry/output | Code | String |
|---|---|---|---|---|
| | | | 1<br>2<br>3 | A<br>B<br>C |
| NIL | **1** | A | | |
| A | **2** | B | 4 | AB |
| B | **4** | AB | 5 | BA |
| AB | **5** | BA | 6 | ABB |
| BA | **2** | B | 7 | BAB |
| B | **3** | C | 8 | BC |
| C | **4** | AB | 9 | CA |
| AB | **6** | ABB | 10 | ABA |
| ABB | **1** | A | 11 | ABBA |
| A | **EOF** | | | |

## The LZW decompression algorithm then works as follows:

| S | K | Entry/output | Code | String |
|---|---|---|---|---|
| | | | 1 | A |
| | | | 2 | B |
| | | | 3 | C |
| NIL | 1 | A | | |
| A | 2 | B | 4 | AB |
| B | 4 | AB | 5 | BA |
| AB | 5 | BA | 6 | ABB |
| BA | 2 | B | 7 | BAB |
| B | 3 | C | 8 | BC |
| C | 4 | AB | 9 | CA |
| AB | 6 | ABB | 10 | ABA |
| ABB | 1 | A | 11 | ABBA |
| A | EOF | | | |

- Apparently, the output string is “ABABBABCABABBA”, a truly lossless result!

## ALGORITHM 7.4 LZW Decompression (modified)

```
BEGIN
   s = NIL;
   while not EOF
   {
         k = next input code;
         entry = dictionary entry for k;

         /* exception handler */
         if (entry == NULL)
                 entry = s + s[0];

         output entry;
         if (s != NIL)
                 add string s + entry[0] to dictionary with a new
   code;
         s = entry;
   }
END
```

## LZW Coding (cont'd)

- In real applications, the code length $l$ is kept in the range of $[l_0, l_{max}]$. The dictionary initially has a size of $2^{l_0}$. When it is filled up, the code length will be increased by 1; this is allowed to repeat until $l = l_{max}$.

- When $l_{max}$ is reached and the dictionary is filled up, it needs to be flushed (as in Unix *compress*, or to have the LRU (least recently used) entries removed).

## Exercise

- Use the LZW method to compress the string "ABCBBAACABAC" with the following initial dictionary.
- Show the compression process step by step and give the code result.

| Code | String |
|---|---|
| 1 | A |
| 2 | B |
| 3 | C |

## Arithmetic Coding

- Arithmetic coding is a more modern coding method that usually **out-performs** Huffman coding.
- Huffman coding assigns each symbol a codeword which has an integral bit length. Arithmetic coding can treat the whole message as one unit.
- A message is represented by a half-open interval $[a, b)$ where $a$ and $b$ are real numbers between 0 and 1. Initially, the interval is [0, 1). When the message becomes longer, the length of the interval shortens and the number of bits needed to represent the interval increases.

- **ALGORITHM 7.5 Arithmetic Coding Encoder**

```
BEGIN
  low = 0.0;     high = 1.0;      range = 1.0;

  while (symbol != terminator)
  {
      get (symbol);
      low = low + range * Range_low(symbol);
      high = low + range * Range_high(symbol);
      range = high - low;
  }

  output a code so that low <= code < high;
END
```

## Example: Encoding in Arithmetic Coding

| Symbol | Probability | Range |
|---|---|---|
| A | 0.2 | [0, 0.2) |
| B | 0.1 | [0.2, 0.3) |
| C | 0.2 | [0.3, 0.5) |
| D | 0.05 | [0.5, 0.55) |
| E | 0.3 | [0.55, 0.85) |
| F | 0.05 | [0.85, 0.9) |
| $ | 0.1 | [0.9, 1.0) |

Probability distribution of symbols.

Fig. 7.8: Arithmetic Coding: Encode Symbols "CAEE$"

## Encode Symbols “CAEE$”

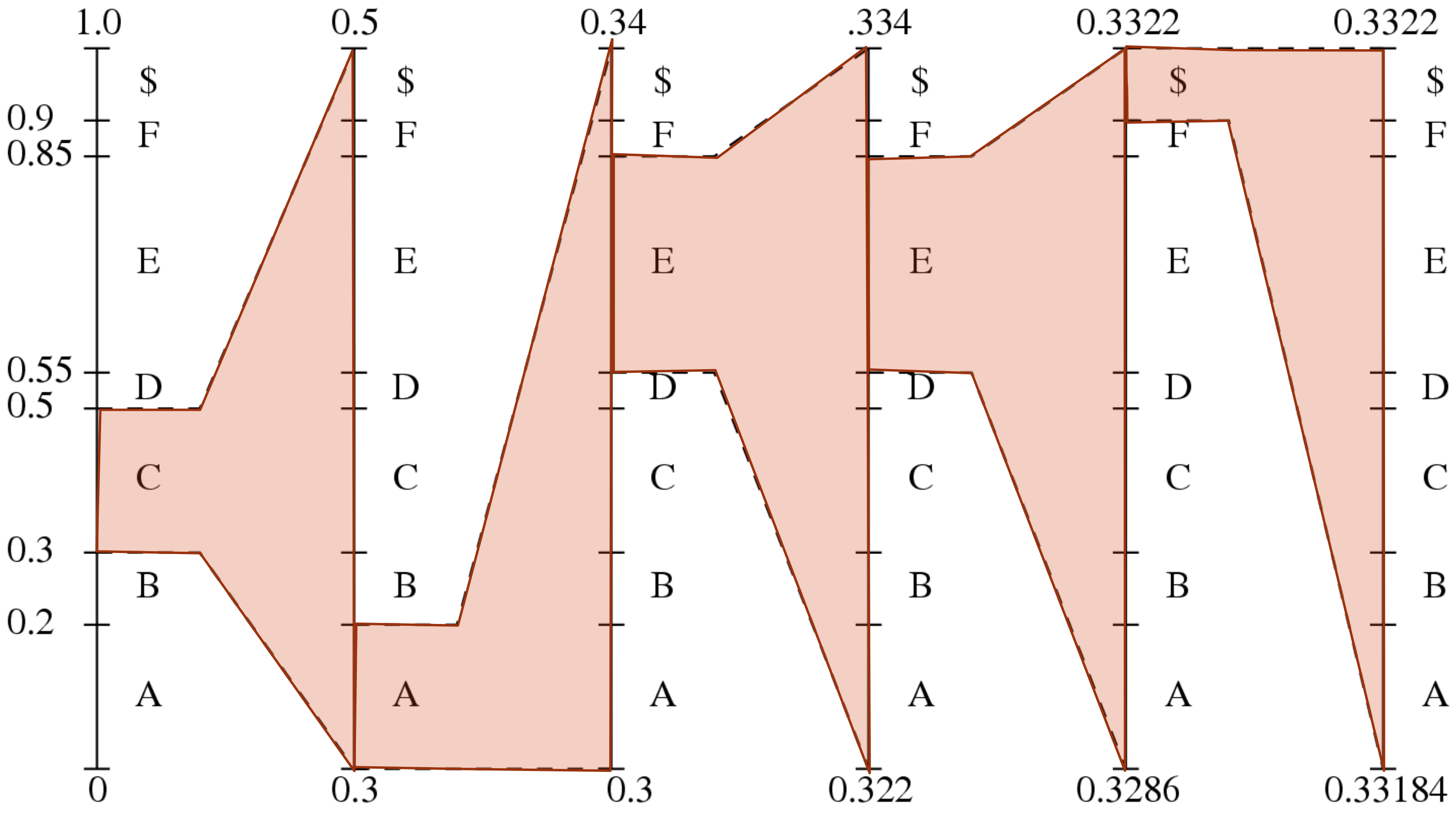


Fig. 7.8(b) Graphical display of shrinking ranges.

## Example: Encoding in Arithmetic Coding

| Symbol | Low | High | Range |
|---|---|---|---|
| | 0 | 1.0 | 1.0 |
| C | 0.3 | 0.5 | 0.2 |
| A | 0.30 | 0.34 | 0.04 |
| E | 0.322 | 0.334 | 0.012 |
| E | 0.3286 | 0.3322 | 0.0036 |
| $ | 0.33184 | 0.33220 | 0.00036 |

New *low*, *high*, and *range* generated.

Fig. 7.8 (cont'd): Arithmetic Coding: Encode Symbols "CAEE$"

$$Range = P_C \times P_A \times P_E \times P_E \times P_S = 0.2 \times 0.2 \times 0.3 \times 0.3 \times 0.1 = 0.00036$$

## PROCEDURE 7.2 Generating Codeword for Encoder

```
BEGIN
   code = 0;
   k = 1;
   while (value(code) < low)
   {
         assign 1 to the kth binary fraction bit
         if (value(code) > high)
         replace the kth bit by 0
         k = k + 1;
   }
END
```

- The final step in Arithmetic encoding calls for the generation of a number that falls within the range [*low*, *high*). The above algorithm will ensure that the shortest binary codeword is found.

## ALGORITHM 7.6 Arithmetic Coding Decoder

```
BEGIN
   get binary code and convert to
   decimal value = value(code);
   Do
   {
       find a symbol s so that
             Range_low(s) <= value < Range_high(s);
       output s;
       low = Rang_low(s);
       high = Range_high(s);
       range = high - low;
       value = [value - low] / range;
   }
   Until symbol s is a terminator
END
```

## Table 7.5 Arithmetic coding: decode symbols “CAEE$”

| Value | Output Symbol | Low | High | Range |
|---|---|---|---|---|
| 0.33203125 | C | 0.3 | 0.5 | 0.2 |
| 0.16015625 | A | 0.0 | 0.2 | 0.2 |
| 0.80078125 | E | 0.55 | 0.85 | 0.3 |
| 0.8359375 | E | 0.55 | 0.85 | 0.3 |
| 0.953125 | $ | 0.9 | 1.0 | 0.1 |

## Lossless Image Compression

- **Approaches of Differential Coding of Images:**
  - Given an original image $I(x, y)$, using a simple difference operator we can define a difference image $d(x, y)$ as follows:

$$d(x, y) = I(x, y) - I(x - 1, y) \tag{7.9}$$

or use the discrete version of the 2D Laplacian operator to define a difference image $d(x, y)$ as

$$d(x, y) = 4\, I(x, y) - I(x, y - 1) - I(x, y + 1) - I(x+1, y) - I(x - 1, y) \tag{7.10}$$

- Due to *spatial redundancy* existed in normal images $I$, the difference image $d$ will have a narrower histogram and hence a smaller entropy, as shown in Fig. 7.9.

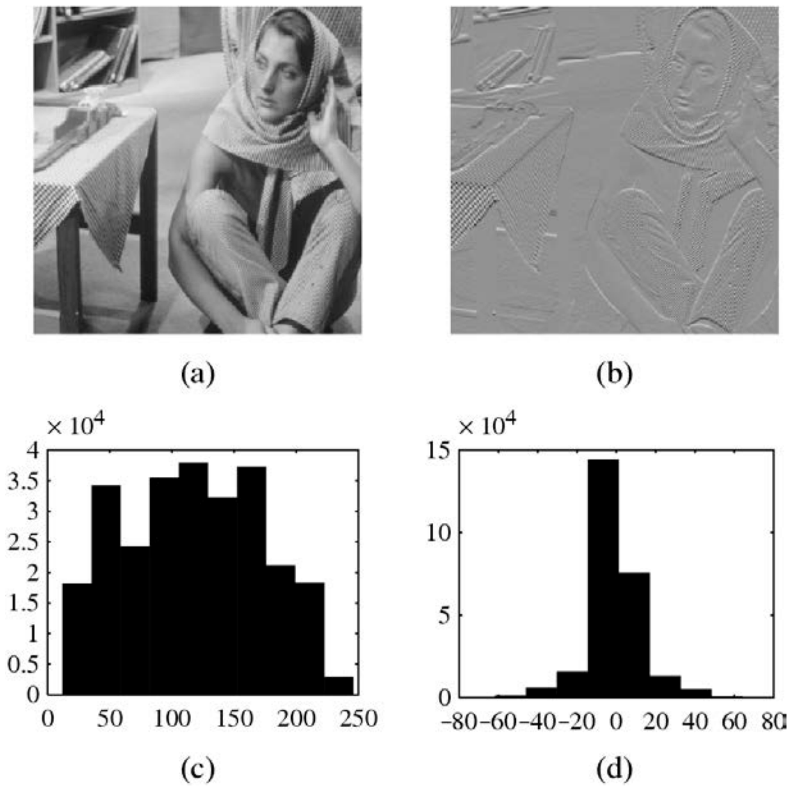


Fig. 7.9: Distributions for Original versus Derivative Images.

(a), (b): Original gray-level image and its partial derivative image

(c), (d): Histograms for original and derivative images.

# Recall: Distribution of Gray-Level Intensities

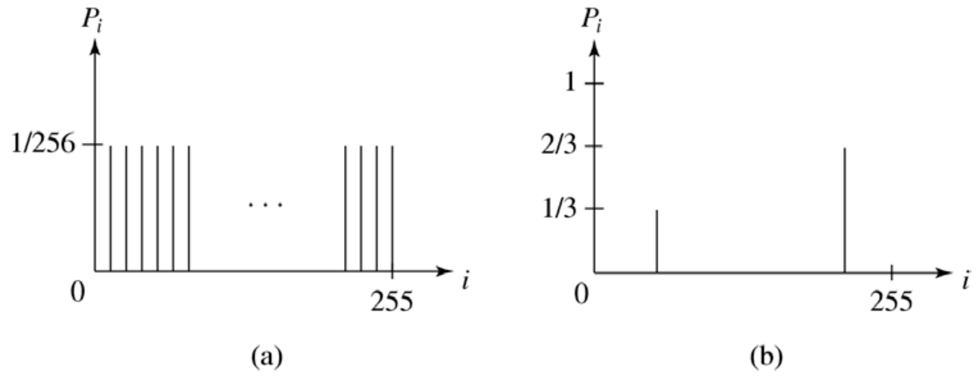


Fig. 7.2 Histograms for two gray-level images.

Fig. 7.2(a) shows the histogram of an image with *uniform* distribution of gray-level intensities. Its entropy is .

Fig. 7.2(b) shows the histogram of an image with two possible values. Its entropy is:

$$\eta = 8 \tag{7.4}$$

$$\eta = \frac{1}{3}\log_2 3 + \frac{2}{3}\log_2 \frac{3}{2} = 0.92$$

## Lossless JPEG

- **Lossless JPEG**: A special case of the JPEG image compression.

**The Predictive method**

1. **Forming a differential prediction:**
   - A predictor combines the values of up to three neighboring pixels as the predicted value for the current pixel, indicated by ‘X’ in Fig. 7.10.
   - The predictor can use any one of the seven schemes listed in Table 7.6.
2. **Encoding:**
   - The encoder compares the prediction with the actual pixel value at the position ‘X’ and encodes the difference using one of the lossless compression techniques we have discussed, e.g., the Huffman coding scheme.

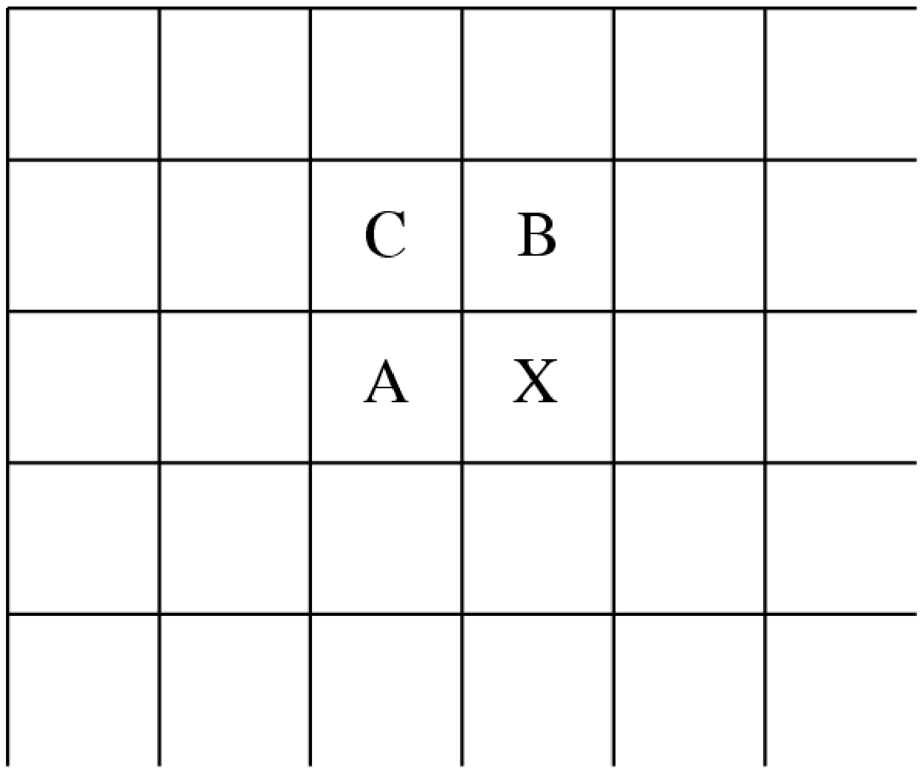


Fig. 7.10: Neighboring Pixels for Predictors in Lossless JPEG.

- **Note**: Any of A, B, or C has already been decoded before it is used in the predictor, on the decoder side of an encode-decode cycle.

## Table 7.6: Predictors for Lossless JPEG

| Predictor | Prediction |
|---|---|
| P1 | A |
| P2 | B |
| P3 | C |
| P4 | A + B – C |
| P5 | A + (B – C) / 2 |
| P6 | B + (A – C) / 2 |
| P7 | (A + B) / 2 |

## Table 7.7: Comparison with other lossless compression programs

| Compression Program | Compression Ratio | | | |
|---|---|---|---|---|
| | Lena | Football | F-18 | Flowers |
| Lossless JPEG | 1.45 | 1.54 | 2.29 | 1.26 |
| Optimal Lossless JPEG | 1.49 | 1.67 | 2.71 | 1.33 |
| Compress (LZW) | 0.86 | 1.24 | 2.21 | 0.87 |
| Gzip (LZ77) | 1.08 | 1.36 | 3.10 | 1.05 |
| Gzip -9 (optimal LZ77) | 1.08 | 1.36 | 3.13 | 1.05 |
| Pack(Huffman coding) | 1.02 | 1.12 | 1.19 | 1.00 |

## Further Exploration

- **Text books:**
  - *The Data Compression Book* by M. Nelson
  - *Introduction to Data Compression* by K. Sayood